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Flux-lattice melting and lowest-Landau-level fluctuations

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We discuss the influence of lowest-Landau-level (LLL) fluctuations near $H_{c2}(T)$ on flux-lattice melting in $YBa_2Cu_3O_{7-\delta}$ (YBCO). We show that the specific heat step of the flux-lattice melting transition in YBCO single crystals can be attributed largely to the degrees of freedom associated with LLL fluctuations. These degrees of freedom have already been shown to account for most of the latent heat. We also show that these results are a consequence of the correspondence between flux-lattice melting and the onset of LLL fluctuations. [S0163-1829(98)52014-0]

Recent high-quality specific heat measurements[1–3] on $YBa_2Cu_3O_{7-\delta}$ (YBCO) single crystals have uncovered features that leave little doubt that the flux lattice melts via a first-order phase transition into a new state of matter called a vortex liquid.[4] The sharp spikes observed in these measurements at fields of up to 16 T reinforce the previous indications of flux-lattice melting (FLM) in earlier specific heat measurements[5–7] and resistivity and magnetization experiments.[8–10] There are many theoretical[11–16] treatments and numerical simulations[17–22] which treat the flux-lattice melting in both the low- and high-field regimes. The considerable breadth of the theoretical approaches brought to bear on the question has produced a considerable number of insights, but no clear overall consensus has yet evolved as to the origin of the features that the experiments have revealed.

A prominent feature of the specific heat results is the spikes associated with the heat of melting. Along with these spikes, steps were reported in fields up to 9 T in Ref. 3, with a larger specific heat on the vortex liquid side of the transition. Such steps were also observed in Refs. 2, 5, and 6. Schilling *et al.*[3] found that they were unable to explain the steps in terms of the Abrikosov ratio, the effective Debye temperature, the number of vortices or the vortex degrees of freedom. Indeed, other authors have also shown that degrees of freedom not associated with the vortices contribute a significant amount to the entropy jump (i.e., the specific heat spike) at the FLM transition. For example, Hu and MacDonald[21] found in their Monte Carlo study that 90% of the latent heat at the FLM transition comes primarily from ''the change in entropy content at microscopic length scales associated with a change in the magnitude of the superconducting order parameter and not from changes in the entropy content of vortex configurations.''[21] Alternatively, it was suggested by Volovik[23] that some of the excess entropy could be attributed to ''electronic'' degrees of freedom in the vortex background, that is, to quasiparticle excitations close to the gap nodes of a d-wave superconductor. In any case, it appears that the vortices are not the leading contributors to the latent heat.

In this paper, we address the question of the specific heat *step* which is observed in connection with the spike. We show that, for fields larger than 2–3 T, the entropy from lowest-Landau-level (LLL) fluctuations provides a significant, if not leading, contribution to the steps observed in the specific heat at the FLM transition. Such an explanation for the steps implies a deeper connection between FLM and LLL fluctuations: namely, that flux-lattice melting corresponds with the onset of LLL fluctuations. Evidence for such a correspondence has been presented before,[24] but the arguments for this idea will be reinforced and made more persuasive here. We develop our argument along the following lines: First, we will show that an analysis of the specific data of Ref. 3 in terms of the analytical LLL expressions of Refs. 25 and 26 indicates that a good portion of the ''step'' near the spike can be attributed to the onset of LLL fluctuations. Second, a comparison of the spike positions with LLL predictions[15,27,28] emphasizes the correspondence of flux-lattice melting with the onset of LLL fluctuations.

It is helpful to briefly review fluctuations in superconductors and the terminologies commonly employed to describe them. We are particularly concerned here with LLL fluctuations. In conventional bulk superconductors, there is a phase transition at $H_{c2}(T)$. Whether or not fluctuations are important is determined by the Ginzburg criterion.[29] If fluctuations are negligible, the signature of the transition in specific heat measurements as a function of temperature consists of ramps with the mean-field discontinuity at the transition.[30] If fluctuations are significant, they contribute a ''bump'' on top of this mean-field ramp and there is no sharp discontinuity.[31] These fluctuations might be generally denoted as H_{c2} fluctuations. At higher fields (larger than about 1–2 T as specified below), the system can be treated within a Ginzburg-Landau, lowest-Landau-level formalism. In this case, these fluctuations are called LLL fluctuations. Fluctuations can contribute to the entropy through microscopic order-parameter amplitude fluctuations as well as vortex position fluctuations, represented by zeros of the order parameter. The former are those found responsible in the simulations of Hu and MacDonald[21] for most of the entropy change.

Analytical expressions for the specific heat of two-dimensional (2D), layered, and three-dimensional (3D) superconductors have been derived[25,26] through the use of a

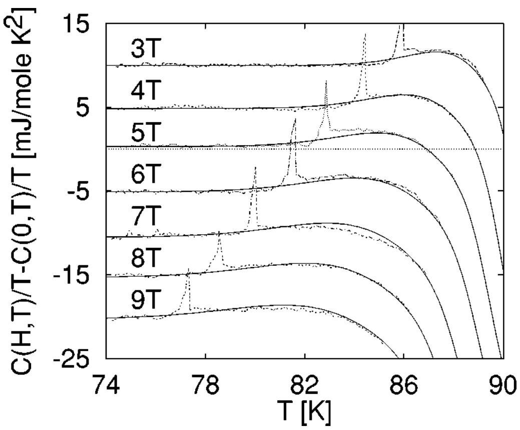

FIG. 1. The $H\|c$ YBCO specific heat data of Ref. 3 along with fits, discussed in the text, to the 3D specific heat function of Ref. 26.

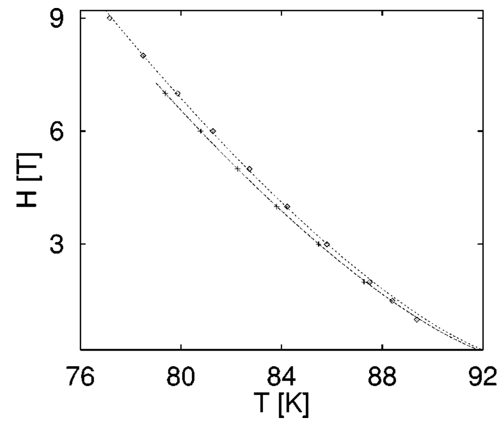

FIG. 2. The positions of the FLM features of Ref. 3 (triangles) and Ref. 6 (plus signs) plotted in H-T space along with their respective fits (see text) to Eq. (1).

Ginzburg-Landau LLL approach. We will use these theoretical results to fit the YBCO, $H\|c$ data (for field values 3–9 T) of Ref. 3. We will use the 3D expressions,[26] since YBCO is relatively isotropic when compared to the bismuth-, mercury-, or thallium-based copper oxides. This 3D function is written as Eq. (27) of Ref. 26 [and in more detail as Eq. (2) of Ref. 24] and it provides a good fit to the data for fields $H \gtrsim 2$ T for YBCO-type materials.[26,32,33] In the data we consider from Ref. 3, the zero-field data was subtracted off. Since 3D LLL expressions do not apply to zero field, we will approximate the theoretical zero-field contribution by a mean-field expression. This is valid for the temperature ranges that we are investigating since the fluctuations in zero field are negligible here.

In Fig. 1, the best fit to the Schilling *et al.* data[3] is shown. The fits are quite satisfactory, except of course in the regions of the spikes, and the curves provide a smooth crossover from the vortex solid phase to the vortex liquid phase. Thus, we can see that a major portion of the step near the spike can be attributed to the onset of of H_{c2} or LLL fluctuations. The remaining part of the steps might be explained, at least to some extent, in terms of the thermodynamic-equilibrium properties of the first-order vortex-lattice phase transition discussed in Ref. 3 and perhaps also in part by the quasiparticle excitations.[23] The fitting parameters used in Fig. 1 are the c-axis coherence length $\xi_c = 7.218$ Å, the ratio of the slope of $H_{c2}(T)$ to the Ginzburg-Landau parameter κ: $H'_{c2}/\kappa = 3.40\times 10^{-2}$ T/K, the mean-field transition temperatures $T_c(H) = 91.33$, 90.86, 90.50, 90.25, 89.8, 89.51, 89.2 K for $H = 3-9$ T, and the parameters $Q = 8.29$, $K = -1.44$, and $M = 5.21$ of Ref. 26. The values of all parameters are reasonable. For the YBCO materials typical values are $H'_{c2} = 1.8$ T/K, $\kappa = 52$, and $\xi_c \simeq 3$ Å. The values of $T_c(H)$ do produce an H'_{c2} which is large. The most likely reason for this, in our opinion, is the fact that the function does not account for 3D/2D crossover. An extensive discussion of this point is in Ref. 24.

We turn now to the second point. Our evidence that the LLL fluctuations contribute more to the specific heat on the vortex liquid side of the transition than the vortex-lattice side implies that FLM coincides with the onset of LLL fluctuations. To develop this correspondence, we turn to the 3D LLL calculation prediction that the melting temperature $T_M(H)$ should occur at a fixed value of the reduced temperature $y \equiv [T_M - T_c(H)]/(T_M H)^{2/3} = \text{constant}$. Herbut and Tešanović[15,27] have calculated the value of the scaling constant using density-functional theory finding,

$$[T_M - T_c(H)]/(T_M H)^{2/3} = [32\pi^2\sqrt{10.5T_{c0}}\kappa^2\xi_{ab}^2 k_B/(\phi_0^2 H'_{c2}\xi_c)]^{2/3}. \quad (1)$$

(A similar value for the constant was calculated by Hikami *et al.*[28] using perturbative series expansions.) Here we compare the experimental features in the specific heat data, which mark the melting, to Eq. (1). We have done this for two sets of data, as shown in Fig. 2. The spikes of Ref. 3 are denoted by the triangles. The dashed line through them is a two-parameter fit of this theory to the positions of the spikes, using $H'_{c2} = 1.8$ T/K, and a linear $T_c(H)$. We find a mean-field transition temperature $T_{c0} = 93.07$ K and that the constant in the above equation equals to 0.1379 K$^{1/3}$/T$^{2/3}$. The standard deviation is 0.05. The value of the constant can be calculated from the right-hand side of the equation using $H'_{c2} = 1.8$ T/K, $\kappa = 52$, $\xi_c = 3$ Å, and $\xi_{ab} = 17.8$ Å. These are all within reasonable range. We have done a similar analysis for the features observed in Ref. 6 (plus signs in the figure) associated with FLM. This fit is also shown in Fig. 2. We find $T_{c0} = 92.92$ K and const$=0.1427$ K$^{1/3}$/T$^{2/3}$ which would correspond to $\xi_{ab} = 18.26$ Å. As one can see in Fig. 2, the fits to both sets of data are very good. There is somewhat more deviation at the lower fields ($H \sim 1-2$ T), which is reasonable since that is where the LLL approximation is expected to break down. That the values of the fitting parameters to data from two separate YBCO samples are reasonable and nearly the same reinforces the idea that FLM corresponds with the onset of LLL fluctuations.

Evidence for the correspondence of FLM with the onset of LLL fluctuations has been previously found[24] using the approach of Roulin *et al.*[6] These authors identified the peaks in the differential $C(H+\delta H,T) - C(H,T)$ with the flux-lattice melting temperature. In Ref. 24, it was shown that the peaks in the differential could be partially accounted for by the onset of LLL fluctuations. In particular, peaks in the differential of ''theoretical'' data, generated using the functions of Ref. 26, were used to identify the temperatures of the onset of LLL fluctuations which were then shown to correspond with flux-lattice melting temperatures found in experiments.[6,9]

Associating FLM with the onset of LLL fluctuations may have escaped previous researchers because one does not expect such fluctuations to extend to temperatures so much lower than $T_{c2}(H)$. Yet, simple estimates using the Ginzburg number do reveal that in zero field fluctuations can become significant at temperatures on the order of five K (Ref. 15) (even more in the presence of a magnetic field) below this temperature. Furthermore such a correspondence is not inconsistent with the behavior of conventional superconductors where FLM and $H_{c2}(T)$ are indistinguishable since the Ginzburg criterion is several orders of magnitude smaller than in high-temperature superconducting materials.

The statement that FLM corresponds with the onset of LLL fluctuations could be recast in terms of a field-dependent Ginzburg number $\mathrm{Gi}(H)$. One can simply say that FLM is determined by $\mathrm{Gi}(H)$. The usual Ginzburg number is only defined in zero field. The field-dependent Ginzburg criterion says that fluctuations become important when $[T - T_c(H)]/T_c(H) \simeq \mathrm{Gi}(H)$. Since we have found evidence that the fluctuations become important at the FLM temperature, $\mathrm{Gi}(H)$ could then be introduced from Eq. (1). One finds,

$$\mathrm{Gi}(H) \simeq H^{2/3}[32\pi^2\sqrt{10.5}\kappa^2\xi_{ab}^2 k_B/(\phi_0^2 H_{c2}'\xi_c)]^{2/3}. \quad (2)$$

This value is seven times larger than the estimate given by Blatter *et al.*[34]

In summary, we have shown that the specific heat steps observed at the FLM transition in the high-quality specific heat measurements of Schilling *et al.*[3] originate mainly in the entropy associated with lowest-Landau-level fluctuations. Thus, the step appears to be amenable to the same explanations as those for the large entropy jumps given in Ref. 21. We have further shown that the FLM features correspond with the onset of LLL fluctuations and have derived an expression for the field-dependent Ginzburg number that applies to fields where the LLL approximation is valid. We speculate that at lower fields, FLM corresponds with the onset of what we have called H_{c2} fluctuations.

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