

Plasmon excitations in nondegenerate quasi-one-dimensional electron systems

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The many-body dielectric formalism is applied for the laterally confined nondegenerate electron system over the liquid helium surface. The density-density response function dependent on the wave number and frequency for the multisubband quasi-one-dimensional (Q1D) system is evaluated within the random-phase approximation for arbitrary values of wave number and subband index. The spectra of both intrasubband and intersubband plasmons are obtained in the limit of small wave numbers and low temperatures. The intrasubband plasmon dispersion is almost soundlike and similar to that of intrasubband plasmons in degenerate Q1D systems and in the Q1D electron chain. The intersubband mode is optical-like with frequency close to the one-electron frequency related to the lateral parabolic confinement. Intra- and intersubband magnetoplasmon frequencies are also derived. Our results are compared with recent spectroscopic experiment. [S0163-1829(98)50502-4]

A collective excitation spectrum is one of the main features of charge systems which depends crucially on the system dimensionality. As is well known, the dispersion law of two-dimensional (2D) electron systems does not show a gap for a zero wave number in contrast to plasma oscillations in 3D. Hence the great interest in studying plasma oscillations in quasi-one-dimensional (Q1D) electron systems especially in view of the great technological progress in the fabrication of semiconductor structures where motion of carriers is laterally restricted and they behave like a degenerate Q1D system at large densities. Collective oscillations in a degenerate Q1D electron system were investigated rather intensively during the last years both theoretically¹⁻⁹ and experimentally.¹⁰ Das Sarma and co-workers²⁻⁴ and Hu and O'Connell,⁵ by using the many-body dielectric formalism, derived the dielectric function dependent on frequency and wave number for the Q1D degenerate electron system and employed some approaches for the calculation of the spectrum of plasma oscillations in multisubband Q1D systems, which allowed them to consider plasma oscillations not only in isolated quantum wires but also in quantum-wire superlattices.^{3,7} Plasmons have been also investigated within the hydrodynamic model by solving the equation of motion and the continuity equation in the case of a lateral parabolic confinement⁸ and by considering the strip geometry.⁹

Theoretical and experimental studies of Q1D systems in semiconductors stimulated interest for realizing Q1D electrons using the well-known nondegenerate surface electrons (SE's) on the liquid helium surface.¹¹ Such a *classical* Q1D electron system can be used for the understanding of different physical phenomena including collective excitations. Very recently, mobility data of SE's in Q1D arrays over a suspended thick helium film¹⁴ and conductivity measurements,¹⁵ as well the observation of magnetoplasma resonances,¹⁶ in the system of SE's on helium in microfabricated channels have been reported.

In this paper, we investigate from a theoretical point of view the collective excitation spectrum of the electron system confined to a parabolic potential in a Q1D channel on the liquid-helium surface in the framework of the many-body dielectric formalism. We consider a solitary channel filled

with superfluid helium and formed between two polymer sheets meeting at sharp angle.¹² Due to the action of a large holding electric field E_{\perp} along the z direction, electrons are located mainly near the bottom of the channel. The liquid surface profile has approximately a semicircular form which can be described as $z = R(1 - \sqrt{1 - y^2/R^2}) \approx y^2/2R$ for $y \ll R$ where R is the curvature radius for the liquid in the channel. If one moves the electron from the bottom of the channel ($y=0$), it is subjected to the potential $U(y) = eE_{\perp}z(y) \approx m\omega_0^2 y^2/2$ leading to its confinement in the y direction with a characteristic frequency $\omega_0 = \sqrt{eE_{\perp}/mR}$. Hence the electron motion along the y direction has an oscillatorylike behavior and a Q1D multisubband electron system is formed on the helium surface with free electron motion along the x direction. In the presence of a magnetic field B along the z axis,¹³ the one-electron wave function and the energy spectrum can be written as $\psi_{n,l,k_x} = \exp(ik_x x) \varphi_n(y) \chi_l(z)/\sqrt{L_x}$ and $E_{n,l} = \hbar^2 k_x^2/2m^* + (n + 1/2)\hbar\Omega + \Delta_l$, respectively, where k_x is the 1D electron wave number, L_x is the size of the system in the x direction, and $\chi_l(z)$ and Δ_l for $n=0,1,2, \dots, l=1,2,3, \dots$ indicate the electron wave function and eigenenergy in the z direction. The effective electron mass is $m^* = m(\Omega/\omega_0)^2$ and the hybrid frequency is $\Omega^2 = \omega_0^2 + \omega_c^2$, with $\omega_c = eB/mc$. The mean electron distance $\langle z \rangle$ from the surface well satisfies the condition $\langle z \rangle \ll R$ which allows treating the χ_l and Δ_l in the same manner as for SE's over a flat surface.¹⁷ The energy gap between the ground ($l=0$) and the first excited ($l=1$) surface levels is more than 10 K for holding fields above 1000 V/cm. For this reason we can disregard the possibility of electron escape from ground level in the temperature range below 1 K and restrict ourselves by the consideration of the ground surface state with $l=0$. The wave function describing the electron motion along the y direction is

$$\varphi_n(y) = \frac{1}{\pi^{1/4} y_B^{1/2}} \frac{1}{\sqrt{2^n n!}} \exp\left(-\frac{(y-Y)^2}{2y_B^2}\right) H_n\left(\frac{y-Y}{y_B}\right), \quad (1)$$

where $y_B = \sqrt{\hbar/m\Omega}$, $Y = -\hbar\omega_c k_x/m\Omega^2$ plays the role of the center of the electron orbit, and $H_n(t)$ is the Hermite poly-

nomial. It can be shown that the inequality $\sqrt{\langle y_n^2 \rangle} \ll R$ is fulfilled in a wide range of holding fields giving support to the use of the parabolic potential approximation for confinement in the y direction and typical values of $\sqrt{\langle y_n^2 \rangle}$ are of the order of $10^{-6} - 10^{-5}$ cm even for $n \sim 10^2$.¹¹

First we consider the case of $B=0$. So, $Y=0$, $\Omega = \omega_0$, and y_B coincides with the localization length $y_0 = \sqrt{\hbar/m\omega_0}$.¹⁷ The general expression for the dielectric function in the multisubband Q1D electron system can be written as^{2,5}

$$\epsilon_{ij,nn'}(\omega, q_x) = \delta_{in} \delta_{jn'} - v_{ij,nn'}(q_x) \Pi_{nn'}(\omega, q_x), \quad (2)$$

where $\Pi_{nn'}(\omega, q_x)$ is the density-density response function, δ_{ik} is the Kronecker symbol, and $v_{ij,nn'}$ are the matrix elements of the Coulomb interaction $V = e^{*2}/|\mathbf{r} - \mathbf{r}'|$ acting between electrons, with a renormalized charge $e^* = [2e^2/(1 + \epsilon)]^{1/2}$ where $\epsilon \approx 1.0572$ is the helium dielectric constant, located at coordinates $\mathbf{r} = \{x, y, z\}$ and $\mathbf{r}' = \{x', y', z'\}$. In order to obtain the expression for $v_{ij,nn'}$, we need to evaluate the Fourier transform of the electron-electron interaction. Disregarding small electron displacements along the z direction (of order of y^2/R), we easily obtain the expression²

$$V_0(q_x, y - y') = 2(e^{*2}/L_x) K_0(|q_x||y - y'|), \quad (3)$$

where $K_0(x)$ is the modified Bessel function. In the bare 1D case ($y = y'$), $V_0(q_x, y - y')$ diverges logarithmically for all q_x . However, the finite scale of the electron localization along the y axis, even in the one-electron approximation, allows us to find a finite value of the matrix elements of the Coulomb interaction by simply averaging $V_0(q_x, y - y')$ over $\varphi_n(y)$, which come from the confinement potential:

$$\Pi_{nn'}^{(0)}(\omega, q_x) = \frac{-2N[\exp(-n\hbar\omega_0/T)U(\xi_{nn'}^{(-)}) - \exp(-n'\hbar\omega_0/T)U(\xi_{nn'}^{(+)})]}{\hbar q_x u_T [1 + \coth(\hbar\omega_0/2T)]}, \quad (7)$$

where $u_T = \sqrt{2T/m}$ is the electron thermal velocity and $\xi_{nn'}^{(\pm)}(q_x) = \lambda_{nn'} \pm \hbar q_x / 2m u_T$ with $\lambda_{nn'} = (\omega/q_x u_T) [1 + (\omega_0/\omega)(n - n')]$. The function $U(\xi)$ is given by the integral

$$U(\xi) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-y^2) dy}{y - \xi - i\delta}. \quad (8)$$

Note that this function is related to the well-known W function which appears in the plasma theory¹⁸

$$v_{ij,nn'}(q_x) = \int dy \int dy' \varphi_i(y) \varphi_j(y) \times V_0(q_x, y - y') \varphi_n(y') \varphi_{n'}(y'). \quad (4)$$

Hu and O'Connell⁵ have shown that the integrals in Eq. (4) can be calculated analytically for $B=0$ giving a nonzero result when $i + j + n + n'$ is even and zero when the sum is odd. In particular,

$$v_{00,00}(q_x) = \frac{e^{*2}}{L_x} \exp\left(\frac{q_x^2 y_0^2}{4}\right) K_0\left(\frac{q_x^2 y_0^2}{4}\right) \approx \frac{e^{*2}}{L_x} \ln \frac{1}{|q_x y_0|} \quad (5)$$

in the limit of $q_x \rightarrow 0$.

The density-density response function $\Pi_{nn'}(\omega, q_x)$, which appears in Eq. (2), can be taken in the random-phase approximation (RPA) as the noninteracting response function of the multisubband system given as

$$\Pi_{nn'}^{(0)}(\omega, q_x) = \sum_{k_x, \sigma} \frac{f_0(E_{k_x} + \Delta_n) - f_0(E_{k_x + q_x} + \Delta_{n'})}{\hbar \omega + E_{k_x} + \Delta_n - E_{k_x + q_x} - \Delta_{n'} + i\delta}, \quad (6)$$

where $\Delta_n = \hbar \omega_0(n + 1/2)$, $E_{k_x} = \hbar^2 k_x^2 / 2m$, δ is a infinitesimal positive, and σ is the spin index. In degenerate Q1D electron systems, $f_0(E_{k_x} + \Delta_i)$ is simply the Fermi function which is usually approximated by the step function. However, for the nondegenerate electron system f_0 is taken as the Boltzmann function $f_0(E_{k_x} + \Delta_n) = \exp[-(E_{k_x} + \Delta_n)/T]$, normalized by the condition $\sum_{n, k_x, \sigma} f_0(E_{k_x} + \Delta_n) = N$, where N is the total number of particles. Making the substitutions, the general expression of the response function for arbitrary q_x , n , and n' reads as

$$W(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{y \exp(-y^2) dy}{y - \zeta - i\delta}$$

through the following equations: $U(\xi) = (1/\xi) [W(\xi) - 1]$ and $\partial U(\xi)/\partial \xi = -2W(\xi)$, where $\text{Re}[W(\xi)] = 1 - 2\zeta e^{-\zeta^2} \int_0^\zeta \exp(t^2) dt$ and $\text{Im}[W(\xi)] = \sqrt{\pi} \zeta e^{-\zeta^2}$.

In the limit $q_x \ll k_x$ (note that in nondegenerate systems, $k_x \sim k_T = \sqrt{2mT/\hbar}$), the response function can be written as

$$\Pi_{nn'}^{(0)}(\omega, q_x) \approx - \frac{N[\exp(-n\hbar\omega_0/T) + \exp(-n'\hbar\omega_0/T)]}{T[1 + \coth(\hbar\omega_0/2T)]} \times \left[W(\lambda_{nn'}) + \left(\frac{\exp(-n\hbar\omega_0/T) - \exp(-n'\hbar\omega_0/T)}{\exp(-n\hbar\omega_0/T) + \exp(-n'\hbar\omega_0/T)} \right) \frac{m u_T}{\hbar q_x} U(\lambda_{nn'}) \right]. \quad (9)$$

The plasmon spectrum of the multisubband system is determined by the roots of the equation

$$\det|\epsilon_{ij,nn'}|=0. \quad (10)$$

At temperatures $T < 1$ K, the energy gap $\hbar\omega_0$ between subbands with different n is of the order of T (Ref. 11) and the occupation of all subbands have to be taken in account. In such conditions, Eq. (10) can be solved, in the general case, only by numerical methods. Analytical solutions become, however, possible at low temperatures $T \ll \hbar\omega_0$ where only the ground subband $n=0$ is occupied by the electrons.

We here restrict ourselves to a two-subband model and consider $n, n'=0, 1$. In this case Eq. (10) splits into two independent equations

$$1 - v_{00,00}\Pi_{00}^{(0)}(\omega, q_x) = 0, \quad (11a)$$

$$1 - v_{01,01}[\Pi_{01}^{(0)}(\omega, q_x) + \Pi_{10}^{(0)}(\omega, q_x)] = 0 \quad (11b)$$

and the coupling between collective modes appears only if one takes into account higher subbands. Equation (11a) gives the *intrasubband* longitudinal plasma excitations with charge-density oscillations in the x direction (along the channel axis) and Eq. (11b) determines the frequency of *intersubband* excitations in the y direction. Putting $\omega = \omega_q - i\gamma_q$ we obtain, from Eq. (11a) and through Eqs. (5) and (9), the following expression for the dispersion law of intrasubband plasmons in the limit of $\omega_q/|q_x u_T| \gg 1$ and $|q_x y_0| \ll 1$:

$$\omega_l^2 = \frac{2e^{*2}q_x^2}{ma} \ln \frac{1}{|q_x y_0|}, \quad (12)$$

where $a = L_x/N$ is the average distance between electrons along the x direction. Note that the condition $\omega_l/q_x u_T \gg 1$, as can be easily shown, is equivalent to the condition $T \ll e^2/a$. As is seen from Eq. (12), Q1D intrasubband plasmons have approximately a soundlike dispersion with sound velocity $c_p^2 \approx 2e^{*2}/ma$. Furthermore, the y_0 dependence of the dispersion law appears only in the logarithmic factor and hence cannot affect significantly the plasma spectrum. This result enforces the use of the one-electron wave function, given in Eq. (1), for the evaluation of the matrix elements of the Coulomb potential in interacting systems.

We point out that a similar structure of the plasma dispersion has also been obtained for intrasubband plasmons in degenerate Q1D systems, even when another kind of electron confinement for the motion along y direction was employed.² This allows us to state that the dispersion law, given by Eq. (12), is quite general. Also remarkable is the fact that the spectrum of longitudinal plasma oscillations in the Q1D electron chain in the quasicrystalline approximation, obtained from the condition of compatibility of the equations of motion,^{19,20} is given in the limit of $|q_x a| \ll 1$ by the expression

$$\omega_l^2 = \frac{2e^{*2}q_x^2}{ma} \ln \frac{1}{|q_x a|}, \quad (13)$$

which is almost the same as the dispersion law in an itinerant electron system, but now a replaces y_0 in the logarithmic factor. We observe also that the plasma dispersion in the

itinerant phase coincides with the longitudinal branch of the spectrum of the 2D Wigner crystal.

The intersubband plasmon spectrum in the limit of small q_x , which follows from Eq. (11b) and taking $v_{01,01} \approx e^{*2}/L_x$,⁵ can be written as

$$\omega_l = \omega_0 \left\{ 1 - \frac{T}{\hbar\omega_0} \left[1 + \sqrt{1 + \left(1 + \frac{2aT}{e^{*2}} \right) \frac{\hbar^2 q_x^2}{mT}} \right] \right\}. \quad (14)$$

The frequency of this mode starts from the frequency ω_0 of the confinement potential at $q_x=0$ and is slightly shifted, as q_x increases, by the small quantity $2T/\hbar\omega_0$. This is a manifestation of the effect of depolarization shift in Q1D systems.⁵ For a sake of comparison, one could remember that the dispersion of transverse oscillations, calculated in the quasicrystalline approximation, starts from the frequency ω_0 at $q_x=0$ and decreases with increasing q_x .^{11,20} Equation (14) exhibits similar behavior of $\omega_l(q_x)$ in the itinerant phase. Note also that the Landau damping is small, $\gamma_q \ll \omega_q$, for the modes described by Eqs. (12) and (14).

Now, we use the same approach to study the influence of a magnetic field on the dielectric response of the nondegenerate Q1D electron system. In the presence of a magnetic field applied in the z direction, the matrix elements of the Coulomb interaction will also depend on k_x because $\varphi_n(y)$ is dependent on k_x through Y . Hence the calculation of the spectrum of collective modes becomes intractable. To overcome this difficulty, Li and Das Sarma⁴ proposed a perturbation scheme which allows expanding $\varphi_n(y)$ in a series of Y , and a general method of expansion was developed later by Wendler and Grigoryan.⁶ Here we use the perturbation approach which, according to the results of Ref. 4, allows describing the influence of the magnetic field on the dielectric function of Q1D electron systems for intermediate B satisfying the condition $\omega_c < \omega_0$.

The wave-function expansion is valid if $|Y(k_x)| < y_B$, and one considers Q1D degenerate electrons with the Fermi momentum playing the role of the effective k_x . For nondegenerate electrons, the thermal wave number k_T , which depends, in the presence of a magnetic field, on the effective electron mass m^* , is the characteristic value of k_x . So we obtain, from the condition $|Y(k_T)| < y_B$, that the inequality

$$T < \hbar\Omega\omega_0^2/2\omega_c^2 \quad (15)$$

must hold in order that the expansion method be valid in the nondegenerate regime. As was shown by Sokolov, Hai, and Studart,²¹ this criterion arises from the use of standard normalization of both the electron wave function and the Boltzmann distribution function in the presence of B . Observe that for $\omega_c < \omega_0$, the inequality provides a lower bound for the temperature instead, $T \ll \hbar\Omega$, which gives the limit of full occupation of the lowest subband. Assuming that the latter condition is fulfilled, we conclude that the intrasubband plasmons, in the lowest-order approximation and in the limit of small q_x , are described by the relation

$$\omega = \omega_l \omega_0 / \Omega, \quad (16)$$

where ω_l is given by Eq. (12) with the localization length y_B instead of y_0 . As can be seen from Eq. (16), the increase of

the magnetic field leads to a decrease of the plasma frequency in comparison with the case of $B=0$. This behavior of the frequency is qualitatively similar to the magnetic field dependence of the mode observed in the edge-magnetoplasmon spectrum of 2D systems. This result is also in agreement with a recent experiment of Valkering and van der Heijden,¹⁶ who observed that the resonance frequency has a linear dependence on $1/B$. However, the dependence of the resonance frequency on the holding electric field (decreasing of the resonance magnetic field for an increasing electric field) is different from our result, since increasing the holding field could reduce the effective localization length and hence, as shown in Eq. (16), the resonance will be shifted to higher magnetic fields, contrary to what was observed. This unexpected result was attributed to profile effects which obviously depend on the holding field. We must emphasize that the electron profile in the experiment is much more complicated than the one considered here and the population of higher subbands must be taken into account for reliable comparisons since the experiments were performed at 0.6 K.²²

The intersubband mode, when the same condition is satisfied, is described by an expression similar to Eq. (14) but with Ω replacing ω_0 . It is also interesting to observe that the dispersion law in the presence of magnetic field resembles that one in the quasicrystalline chain. In this approximation,²⁰ one of the branches of the plasma spectrum

starts from the hybrid frequency Ω in the limit of $q_x \rightarrow 0$ and the other one is described by Eq. (16) but with ω_l given by Eq. (13).

In conclusion, we have used the dielectric formalism for describing many-body properties of the multisubband Q1D classical electron system localized in a single parabolic channel on the liquid-helium surface. A general expression was obtained for the density-density response function in the RPA for arbitrary wave numbers. The dispersion law for the nondegenerate Q1D electron system was derived, when the lowest subband is occupied by the electrons, from the determinantal equation given by the zeroes of the dielectric function at low temperatures. The intrasubband (longitudinal) plasmon has a soundlike dispersion apart from a logarithmic factor and is similar for both intrasubband plasmons in degenerate Q1D systems and in the quasicrystalline phase. By contrast, the intersubband (transverse) plasmon has an optical branch and starts from a threshold frequency which is slightly shifted from the confinement frequency. The influence of magnetic field on the dispersion law was analyzed for both intra- and intersubband plasmons.

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