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Effects of two-dimensional plasmons on the tunneling density of states

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We demonstrate that gapless plasmons lead to a universal $(\delta\nu(\epsilon)/\nu \propto |\epsilon|/E_F)$ correction to the tunneling density of states of a clean two-dimensional Coulomb interacting electron gas. We also show how this correction affects the conductance of a tunnel bridge in a double-layer system. [S0163-1829(98)50408-0]

The phenomenon of suppression of electron tunneling into interacting conductors, known as ''zero-bias anomaly,'' still remains in the center of current theoretical studies.

This experimentally well-documented phenomenon received its first explanation in the theory of the electron density of states (DOS) in Coulomb interacting disordered metals.[1] This theory, however, was formulated in the diffusive regime and therefore limited to the range of energies ϵ or, correspondingly, bias voltages V small compared to the impurity scattering rate: $\epsilon, V < 1/\tau$.

Recently, an attempt was made to extend the theory of Ref. 1 beyond the diffusive regime.[2] The authors of Ref. 2 found a universal (independent of the strength of Coulomb coupling) correction to the two-dimensional (2D) DOS: $\delta\nu(\epsilon)/\nu \propto -(E_F\tau)^{-1}[\ln(\epsilon/\Delta)]^2$ in the regime $1/\tau \ll \epsilon \ll \Delta$, where the characteristic energy scale $\Delta = v_F\kappa$ is determined by the Debye screening wave vector $\kappa = 2\pi e^2\nu$ proportional to the bare two-spin DOS $\nu = m/\pi$. Also, on the basis of the calculations performed in Ref. 2, a modification of the diffusive correction obtained in Ref. 1 was proposed. Later, the same authors generalized the theory of Refs. 1 and 2 onto the case of nonquantizing magnetic fields.[3]

In this communication we show that the tunneling DOS of a clean 2D Coulomb conductor also contains another universal term $\delta\nu(\epsilon)/\nu \propto |\epsilon|/E_F$, which is completely independent of impurity scattering and may well become dominant in the ballistic regime. With this new term included, the tunneling conductance $G(V)$ acquires a linear cusplike universal contribution $\delta G(V)/G_0 \propto |V|/E_F$.

Unlike Ref. 2 where Matsubara technique was used, we employ the real-time formalism in order to avoid problems with a somewhat intricate procedure of analytical continuation from discrete imaginary frequencies. The two-spin tunneling DOS is defined as

$$\nu(\epsilon,T) = -\frac{2}{\pi}\int \frac{d^2\mathbf{p}}{(2\pi)^2}\,\mathrm{Im}[\mathcal{G}^R(\epsilon,\mathbf{p})]. \quad (1)$$

In the presence of impurities the noninteracting electron Green function has the standard form

$$\mathcal{G}_0^R(\epsilon,\mathbf{p}) = [\mathcal{G}_0^A(\epsilon,\mathbf{p})]^* = \frac{1}{\epsilon - \xi_p + i/2\tau}, \quad \xi_p = \frac{p^2 - p_F^2}{2m}. \quad (2)$$

The interaction correction to DOS is simply related to the electron self-energy

$$\delta\nu(\epsilon,T) = -\frac{2}{\pi}\,\mathrm{Im}\int \frac{d^2p}{(2\pi)^2}[\mathcal{G}_0^R(\epsilon,\mathbf{p})]^2\Sigma^R(\epsilon,\mathbf{p}). \quad (3)$$

In the quasiballistic regime of large momentum and energy transfers $(1/\tau < v_F q, \omega)$ the important for the correction to the density-of-states part of the self-energy $\Sigma^R(\epsilon,\mathbf{p})$ is given by the expression

$$\Sigma^R(\epsilon,\mathbf{p}) = \int \frac{d\omega\, d^2\mathbf{q}}{(2\pi)^3}(1 + 2\Gamma^A(\omega,q)) \times \mathrm{Im}\,\mathcal{G}_0^A(\epsilon+\omega,\mathbf{p}+\mathbf{q})V^A(\omega,\mathbf{q})\tanh\left(\frac{\epsilon+\omega}{2T}\right), \quad (4)$$

where $\Gamma^A(\omega,q)$ is the impurity vertex correction

$$\Gamma^A(\omega,q) = \frac{1}{\pi\nu\tau}\int \frac{d^2p}{(2\pi)^2}\,\mathcal{G}^R(\epsilon,\mathbf{p})\mathcal{G}^A(\epsilon+\omega,\mathbf{p}+\mathbf{q}) = -\frac{i/\tau}{\sqrt{(\omega - q^2/2m - i/\tau)^2 - v_F^2q^2}}. \quad (5)$$

The dynamically screened 2D Coulomb potential $V^A(\omega,\mathbf{q})$ is

$$V^A(\omega,q) = \frac{V_0(q)}{1 - V_0(q)P^A(\omega,q)}, \quad V_0(q) = 2\pi e^2/q, \quad (6)$$

where $P^A(\omega,q)$ is the polarization operator,

$$P^A(\omega,q) = -\nu\left(1 - \frac{(\omega - q^2/2m)}{\sqrt{(\omega - q^2/2m - i/\tau)^2 - v_F^2q^2}}\right). \quad (7)$$

After the integration in Eq. (3) over the electronic momentum one arrives at the expression

$$\frac{\delta\nu(\epsilon,T)}{\nu} = \int \frac{d\omega}{(2\pi)^2}\tanh\left(\frac{\epsilon+\omega}{2T}\right)\mathrm{Im}\int_0^\infty dq\,q\,V^A(\omega,q) \times \frac{(1+2\Gamma^A(\omega,q))(\omega - q^2/2m - i/\tau)}{[(\omega - q^2/2m - i/\tau)^2 - v_F^2q^2]^{3/2}}. \quad (8)$$

A straightforward analysis of Eq. (8) shows that the range of transferred energies and momenta $1/\tau<\omega<v_F q$, where the Coulomb potential is statically screened ($V(q) = (1/\nu)(\kappa/q+\kappa)$), yields only a small contribution to DOS. The q dependence of $V(q)\approx 1/\nu$ is weak, and, with logarithmic accuracy, this contribution coincides with that of a short-ranged potential $V(q)=V_0$:

$$\frac{\delta\nu(\epsilon,T)}{\nu}=-\frac{V_0\nu}{4\pi\epsilon_F\tau}\ln\frac{\Delta}{\max\{|\epsilon|,T\}}. \tag{9}$$

The authors of Ref. 2 used the coordinate space representation to demonstrate that this term occurs due to the interference between scattering off a single impurity and off Friedel oscillations of the electronic density caused by the same impurity.

However, the overall q integral in Eq. (8) is dominated by the interval of momenta $\omega^2/\Delta<v_F q<\omega$ where the "anti-screened" potential $V(\omega,\mathbf{q})$ develops a plasmon pole at $\omega=v_F(\kappa q/2)^{1/2}$. As we show below, this gapless collective mode plays a role which is somewhat similar to that of a diffusion pole $\omega=iDq^2$ appearing in the disordered regime $\omega,v_F q<1/\tau$.

The contribution resulting from the above range of momenta can be readily found:

$$\begin{aligned}\frac{\delta\nu(\epsilon,T)}{\nu}&=\frac{1}{(2\pi)^2}\frac{\kappa}{\nu}\int d\omega\,\tanh\left(\frac{\epsilon+\omega}{2T}\right)\\&\quad\times\mathrm{Im}\left(\frac{\omega-i/\tau}{\omega}\int_0^{\sim\omega/v_F}\frac{dq}{(\omega-i/\tau)\omega-q\kappa v_F^2/2}\right)\\&=-\frac{2}{(2\pi)^2\nu v_F^2}\int d\omega\,\tanh\left(\frac{\epsilon+\omega}{2T}\right)\\&\quad\times\mathrm{Im}\left[\frac{\omega-i/\tau}{\omega}\left(\ln\frac{\Delta}{|\omega|}+i\pi\frac{\omega}{|\omega|}\right)\right].\end{aligned} \tag{10}$$

The first term in Eq. (10) which stems from the real part of the q integral reproduces the correction obtained in Ref. 2:

$$\begin{aligned}\frac{\delta_1\nu(\epsilon,T)}{\nu}&=-\frac{2}{(2\pi)^2}\frac{1}{v_F^2\nu\tau}\int\frac{d\omega}{\omega}\tanh\left(\frac{\epsilon+\omega}{2T}\right)\ln\frac{\Delta}{|\omega|}\\&=-\frac{1}{4\pi E_F\tau}\left(\ln\frac{\Delta}{\max\{|\epsilon|,T\}}\right)^2,\end{aligned} \tag{11}$$

which appears to be greater than Eq. (9) by an extra logarithmic factor.

The second term originating from the imaginary part of the integral over q constitutes our new result:

$$\begin{aligned}\frac{\delta_2\nu(\epsilon,T)}{\nu}&=-\frac{1}{(2\pi)v_F^2\nu}\int d\omega\frac{\omega}{|\omega|}\tanh\left(\frac{\epsilon+\omega}{2T}\right)\\&=-\frac{1}{2\pi v_F^2\nu}\int_0^{\sim\Delta}d\omega\left[\tanh\left(\frac{\epsilon+\omega}{2T}\right)\right.\\&\quad\left.+\tanh\left(\frac{\omega-\epsilon}{2T}\right)-2\right]\\&=\frac{\max\{|\epsilon|,2T\ln 2\}}{2E_F},\end{aligned} \tag{12}$$

where we subtracted a constant term $\delta\nu(0,0)$ to avoid a divergence at the upper limit. The new term (12) exceeds Eq. (11) in the whole range of energies $\tau^{-1}(\ln\Delta\tau)^2<\epsilon<E_F$.

The inequality $\omega>qv_F$ must be satisfied for characteristic momenta $q\sim\omega^2/\Delta v_F$, which means that Eq. (12) is valid under condition $|\epsilon|\ll\min(\Delta,E_F)$.

It is worth mentioning that a universal correction to the tunneling conductance similar to Eq. (12) was obtained in the case of tunneling through a uniform barrier which imposes an additional condition of partial momentum conservation.[4] The physical origin of this effect is, however, completely different from ours: the correction $G(V)/G_0 \propto|V|/E_F$ was obtained in Ref. 4 for the case of a short-range potential and was shown to be due to Friedel oscillations of the electronic density induced by the barrier.

In a double-layer system the above DOS correction gives rise to an applied voltage bias and/or temperature-dependent correction to the differential tunneling conductance.

Below we consider a case of tunneling with no lateral momentum conservation which corresponds to a tunnel bridge between two identical layers.

In the linear-response method the tunneling current

$$I(V)=e\ \mathrm{Im}\,\Pi^R(eV) \tag{13}$$

is simply related to the retarded polarization operator $\Pi^R(\omega)$ with an energy-independent tunneling amplitude t_{12} standing in each of the two momentum-nonconserving vertices, which has to be taken at the external frequency determined by the applied voltage bias V.

For noninteracting electrons the tunneling conductance is given by the expression

$$G_0=\frac{dI}{dV}=\frac{1}{2}e^2\pi|t_{12}|^2\nu_0^2. \tag{14}$$

Corrections to Eq. (14) stem from both intralayer and interlayer Coulomb interactions where the latter is given by the expression

$$U_0(q)=\frac{2\pi e^2}{\epsilon q}\exp(-qd). \tag{15}$$

In Eq. (15), d is the interlayer distance, and, for the sake of simplicity, we put the dielectric constants of the electronic layer and the interlayer media equal to each other.

The screened intralayer and interlayer potentials are described by the equations

$$\begin{aligned}V(\mathbf{q},\omega)&=V_0(\mathbf{q})+V_0(\mathbf{q})P(\mathbf{q},\omega)V(\mathbf{q},\omega)\\&\quad+U_0(\mathbf{q})P(\mathbf{q},\omega)U(\mathbf{q},\omega),\\U(\mathbf{q},\omega)&=U_0(\mathbf{q})+V_0(\mathbf{q})P(\mathbf{q},\omega)U(\mathbf{q},\omega)\\&\quad+U_0(\mathbf{q})P(\mathbf{q},\omega)V(\mathbf{q},\omega),\end{aligned} \tag{16}$$

which yield the solution

$$V,U=\frac{1}{2}\frac{V_0+U_0}{1-P(V_0+U_0)}\pm\frac{1}{2}\frac{V_0-U_0}{1-P(V_0-U_0)}. \tag{17}$$

The poles of the two terms in Eq. (17) determine the spectra of the in- (ω_+) and out-of-phase (ω_-) plasmons[5] which at small momenta $q<d^{-1}$ acquire the form

$$\omega_+=v_F(\kappa q)^{1/2}, \quad \omega_-=qv_F\frac{1+\kappa d}{(1+2\kappa d)^{1/2}}. \tag{18}$$

The corrections to the tunneling conductance resulting, respectively, from the intralayer V and the interlayer U potentials are given by the expressions

$$\delta_{11}G(V,T)=e^2\pi|t_{12}|^2 \text{ Im} \int \frac{d\epsilon}{2\pi}\int\frac{d\omega}{2\pi}\frac{\partial S(\epsilon+eV)}{\partial\epsilon} \times S(\epsilon+\omega)\int\frac{d^2\mathbf{q}}{(2\pi)^2}V^R(\mathbf{q},\omega)\int\frac{d^2\mathbf{p}'}{(2\pi)^2}2i \times \text{Im } \mathcal{G}^A(\mathbf{p}',\epsilon)\int\frac{d^2\mathbf{p}}{(2\pi)^2}(\mathcal{G}^A(p,\epsilon))^2 \times\mathcal{G}^R(\mathbf{p}+\mathbf{q},\epsilon+\omega), \tag{19}$$

$$\delta_{12}G(V,T)=\frac{1}{2}e^2\pi|t_{12}|^2 \text{ Im}\int\frac{d\epsilon}{2\pi}\int\frac{d\omega}{2\pi}\frac{\partial S(\epsilon+eV)}{\partial\epsilon} \times[S(\epsilon+\omega)-S(\epsilon-\omega)]\int\frac{d^2\mathbf{q}}{(2\pi)^2}U^R(\mathbf{q},\omega) \times\int\frac{d^2\mathbf{p}'}{(2\pi)^2}\mathcal{G}^A(\mathbf{p}',\epsilon)\mathcal{G}^R(\mathbf{p}'+\mathbf{q},\epsilon+\omega) \times\int\frac{d^2\mathbf{p}}{(2\pi)^2}\mathcal{G}^A(\mathbf{p},\epsilon)\mathcal{G}^R(\mathbf{p}+\mathbf{q},\epsilon+\omega). \tag{20}$$

It is convenient to divide Eqs. (19) and (20) into two contributions $\delta_{11}G=\delta^+_{11}G+\delta^-_{11}G$ and $\delta_{12}G=\delta^+_{12}G-\delta^-_{12}G$ corresponding to the two terms in the right-hand side of Eq. (17).

In both Eqs. (19) and (20) the typical values of the transferred frequency ω are determined by the largest between eV and T, and in what follows we will concentrate on deviations of the tunneling I-V characteristic from the linear (ohmic) dependence at small temperatures ($T\ll eV$). The corresponding values of the transferred momentum q can be found from the relations $\omega_\pm(q)\sim eV$. For the condition $q<d^{-1}$ to be satisfied the applied voltage has to be smaller than $V_1=(v_F/e)(\kappa/d)^{1/2}$.

For large voltages, $V_1<V<\Delta/e$, the effects of the interlayer coupling U are small, and therefore the following relations hold:

$$\delta^+_{11}G\approx\delta^-_{11}G, \quad \delta^+_{12}G\approx\delta^-_{12}G. \tag{21}$$

Hence the result for $\delta G(V)$ amounts to the above DOS correction:

$$\frac{\delta G(V)}{G}=\int d\epsilon\frac{\partial S(\epsilon+eV)}{\partial\epsilon}\frac{\delta\nu(\epsilon)}{\nu}=\frac{|V|}{E_F}, \tag{22}$$

where the zero-bias value of the renormalized conductivity $G=G(0)$ includes the (negative) V-independent term which had been subtracted in Eq. (12).

At biases smaller than V_1 the interlayer coupling becomes important, and the correction to the tunneling conductance can no longer be expressed solely in terms of DOS. We separate the frequency and the momentum integrations in $\delta^\pm_{11,12}G$ as follows:

$$\delta^\pm_{11,12}G/G=\int\frac{d\omega}{4\pi}f(\omega/T)I^\pm_{11,12}(\omega), \tag{23}$$

where

$$f(\omega/T)=\int\frac{d\epsilon}{2\pi}\frac{\partial S(\epsilon+eV)}{\partial\epsilon}S(\epsilon+\omega),$$

and the momentum integrals read as

$$I^+_{11}=\text{Im}\int_0^{\sim 1/d}qdq\,\frac{\omega}{\Omega^2}\frac{1}{(1+q/2\kappa)\Omega-\omega},$$

$$I^+_{12}=-\text{Im}\int_0^{\sim 1/d}qdq\,\frac{1}{\Omega}\frac{1}{(1+q/2\kappa)\Omega-\omega},$$

$$I^-_{11}=\text{Im}\int_0^{\sim 1/d}qdq\frac{\omega}{\Omega^2}\frac{1}{(1+(\kappa d)^{-1})\Omega-\omega},$$

$$I^-_{12}=-\text{Im}\int_0^{\sim 1/d}qdq\frac{1}{\Omega}\frac{1}{(1+(\kappa d)^{-1})\Omega-\omega}, \tag{24}$$

where $\Omega=\sqrt{(\omega+i/\tau)^2-v_F^2q^2}$.

To compute these integrals, we first change from real to the imaginary values of ω, do the momentum integrations, and then perform analytic continuation back to the real frequencies: $\omega\to -i\omega+0$.

A straightforward analysis shows that at all frequencies in the range $1/\tau<\omega<eV_1$ the integrals $I^+_{11,12}(\omega)$ are governed by the momenta $v_Fq\sim\omega^2/\Delta\ll\omega$ corresponding to the in-phase plasmon mode $\omega_+(q)$. Therefore, when calculating these integrals, one can put $\Omega\approx\omega$ and obtain

$$I^+_{11}\approx -I^+_{12}\approx\frac{2\pi}{\nu_0v_F^2}\frac{\omega}{|\omega|}. \tag{25}$$

Hence the in-phase terms $\delta^+_{11}G$ and $\delta^+_{12}G$ nearly cancel out in the whole interval of biases $1/e\tau<V<V_1$.

In the case of the integrals $I^-_{11,12}(\omega)$, they are both dominated by the momenta $q\sim\omega/(1+\kappa d)^{1/2}<\omega$ corresponding to the out-of-phase plasmon mode $\omega_-(q)$ only if the frequency is large enough: $\omega>eV_2=eV_1(1+\kappa d)^{-1/2}$ (obviously, the new scale V_2 different from V_1 can only appear for $\kappa d\gg 1$). If this is the case, the two integrals appear to be simply related:

$$I^-_{11}\approx -I^-_{12}\frac{1+\kappa d}{\kappa d}\approx\frac{\pi}{\nu_0v_F^2}\frac{\omega}{|\omega|}, \tag{26}$$

and, consequently, in the range of biases $V_2<V<V_1$ the conductance correction gets reduced by a factor of 2 compared to Eq. (22):

$$\delta G/G=\frac{|V|}{4E_F}\frac{1+2\kappa d}{1+\kappa d}\approx\frac{|V|}{2E_F}. \tag{27}$$

On the other hand, at smaller frequencies $1/\tau<\omega<eV_2$ the integrals $I^-_{11,12}(\omega)$ receive equally important contributions of

opposite signs from both the momenta $v_F q \sim \omega/(1+\kappa d)^{1/2} < \omega$ and $v_F q \sim \omega$, since the latter region of momenta now lies under the upper bound $\sim 1/d$. The competition between these terms further reduces the imaginary part of I_{12}^- while in the case of I_{11}^- a nearly perfect compensation occurs:

$$I_{12}^- \approx -\frac{\kappa d}{1+\kappa}\frac{\pi}{2\nu_0 v_F^2}\frac{\omega}{|\omega|}, \quad I_{11}^- \approx 0 \tag{28}$$

Thus at low biases $1/e\tau < V < V_2$, the correction to the conductance becomes

$$\delta G/G = \frac{|V|}{8E_F}\frac{\kappa d}{1+\kappa d}. \tag{29}$$

A systematic estimate of higher-order Coulomb corrections to $G(V)$ can be made by means of the method of the tunneling action developed in Ref. 6. Employing this formalism, we obtain the tunneling conductivity

$$\delta G(V) \propto \mathrm{Im} \int_0^\infty \frac{dt}{t} \exp(iS(t) - iVt), \tag{30}$$

in terms of the time-dependent action of the electrostatic potential excited in the process of tunneling: $S(t) = \int d\omega |J(\omega,t)|^2 S(\omega)$. Here, $J(\omega,t) = (1-e^{i\omega t})/\omega$ is a spectral function of the tunneling electron's density, and the kernel

$$S(\omega) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{V_0^A(q) - U^A(q)}{1 - P^A(\omega,q)(V_0^A(q) - U_0^A(q))} \tag{31}$$

is given solely in terms of the out-of-phase combination $V^A - U^A$.

Equation (31) can be readily interpreted as a sum of the partial actions associated with the Coulomb energies due to the (repulsive) self-interaction of the excessive electronic density and that of the hole left behind immediately after the tunneling event, which is reduced by the amount of energy of their mutual attraction.

At weak coupling the exponent in Eq. (31) can be expanded in powers of the action $S(t)$ which approaches a constant value as the charge-spreading time $t \sim V^{-1}$ tends to infinity at vanishingly small biases $(S(t) = \mathrm{const} + O(1/t))$. Then the first-order correction to the conductivity can be cast in the form

$$\frac{\delta G(V)}{G} = \mathrm{Im} \int_0^V \frac{d\omega}{\omega^2} S(\omega). \tag{32}$$

By using Eq. (32) one can reproduce the above results (22) and (27).

As derived, Eq. (32) neglects the tunneling electron's recoil, which is only permissible as long as the relation $\omega \gg v_F q$ holds for all relevant transferred momenta and frequencies. Therefore, at biases smaller than V_2 the tunneling action has to be modified in order to include the recoil and to recover the low-bias asymptotic of Eq. (29).

To conclude, in the present paper we demonstrated that gapless 2D plasmons affect the tunneling DOS even in the ballistic regime. We found a new, impurity-independent, nonanalytic correction which also leads to a linear cusplike contribution to the conductance of a tunnel bridge in a double-layer system.

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