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Phase-space filling and stimulated scattering of composite bosons

A. Imamoḡlu

Department of Electrical and Computer Engineering, University of California, Santa Barbara, California 93106

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We show that the final-state stimulation of a scattering process involving composite bosons is fundamentally limited by the Pauli exclusion of the constituent fermions. Even though we concentrate on the saturation and Pauli blocking of the statistical enhancement factor for exciton-phonon scattering into a many-body electron-hole pair state, our basic result is valid for a large class of composite bosons and for arbitrary dimensionality. The expression that we obtain explicitly shows that the stimulation is a direct result of the many-body coherence in the final state. For a two-dimensional magnetoexciton gas, we analytically demonstrate the transition from stimulated scattering in the low-density bosonic limit to Pauli blocking of scattering in the high-density limit. [S0163-1829(98)51508-1]

Final-state stimulation of a scattering process is the bosonic counterpart of the Pauli exclusion principle. The best known example for *bosonic stimulation* or statistical enhancement of scattering is the stimulated photon emission that provides gain for (photon) laser oscillators. The accumulation of a large fraction of excitons in the ground state during the initial stages of Bose-Einstein condensation is also a direct result of stimulated (exciton-phonon) scattering.[1–3] In the latter case, condensing Bose particles are composed of fermionic constituents, which obey Fermi-Dirac statistics. As the density of composite bosons increases the mean separation between particles becomes comparable to their size (Bohr radius a_B). The stimulated scattering in this limit should be altered due to phase-space filling (PSF) effects of the underlying fermionic particles.

In this paper, we derive an expression for the statistical enhancement factor of composite boson scattering that is valid for all densities. Even though we concentrate on electron-hole-pair phonon interaction in the Born-Markov limit, our basic result can be used for a large class of composite bosons and for arbitrary dimensionality, provided that the Bardeen-Cooper-Schrieffer (BCS) type ground state of the system is predetermined.[4] The expression that we obtain allows us to analyze the saturation of stimulated scattering arising from PSF effects for a two-dimensional (2D) magnetoexciton gas.[5,6] For this system, we analytically demonstrate the transition from stimulated scattering in the low-density bosonic limit to Pauli blocking of scattering in the high-density limit. The scattering rate for 2D magnetoexcitons peaks near the *Mott density* where only 1/2 of the ground-state magnetoexcitons effectively participate in final-state stimulation. Our results indicate that bosonic stimulation is a direct result of the many-body coherence in the final state and in this respect cannot be reduced to a breakdown of bosonic commutation relations.

The starting point of our analysis is the interaction of a specific many-body electron-hole state with the phonon reservoir. The corresponding interaction Hamiltonian is[7]

$$\hat{H}_{\rm int}=\hbar\sum_{k,q}\left[g_{e-{\rm ph}}(q)\hat{e}^{\dagger}_{k+q}\hat{e}_k+g_{h-{\rm ph}}(q)\hat{h}^{\dagger}_{k+q}\hat{h}_k\right](\hat{b}_q+\hat{b}^{\dagger}_{-q}), \tag{1}$$

where $\hat{e}_k$, $\hat{h}_k$, and $\hat{b}_k$ denote the annihilation operators for electron, hole, and phonon modes with momentum k, respectively. $g_{e-{\rm ph}}(q)$ and $g_{e-{\rm ph}}(q)$ are the corresponding electron-phonon and hole-phonon interaction coefficients. The spin index and the vector nature of the momenta are suppressed for simplicity. The same set of variables could also be used to describe an electron-hole-phonon system under large magnetic fields.

Before proceeding, we reiterate that our principal goal is to analyze the Pauli blocking of bosonic enhancement, and not necessarily to evaluate the actual exciton-phonon scattering process at arbitrary densities. To this end, we assume that the initial many-body electron-hole state can be written as[4]

$$|\Psi_{\rm in}\rangle=\hat{C}^{\dagger}_{\nu,K}|\tilde{\Psi}_{\rm BCS}\rangle=\hat{C}^{\dagger}_{\nu,K}\prod_k\,[\tilde{u}(k)+\tilde{v}(k)\hat{e}^{\dagger}_k\hat{h}^{\dagger}_{-k}]|0\rangle \tag{2}$$

where

$$\hat{C}^{\dagger}_{\nu,K}=\sum_p\,\varphi_\nu(p)\hat{e}^{\dagger}_{K/2+p}\hat{h}^{\dagger}_{K/2-p} \tag{3}$$

denotes the creation operator of an electron-hole pair with a center-of-mass momentum K and an internal motion desribed by the (reciprocal space) wave function $\varphi_\nu(p)$. The coefficients $\tilde{u}(k)$ and $\tilde{v}(k)$ satisfy the normalization condition $\tilde{u}^2(k)+\tilde{v}^2(k)=1$ and $\Sigma_p|\tilde{v}(p)|^2=N$. The final (electronic) state of the scattering process that we choose to consider is

$$|\Psi_{\rm fin}\rangle=|\Psi_{\rm BCS}\rangle=\prod_k\,[u(k)+v(k)\hat{e}^{\dagger}_k\hat{h}^{\dagger}_{-k}]|0\rangle \tag{4}$$

where $\Sigma_p|v(p)|^2=N+1$ is the mean number of electron-hole pairs in the final state. In the following discussion, we will assume that $N\gg1$, so that the difference between $v(p)$ ($u(p)$) and $\tilde{v}(p)$ ($\tilde{u}(p)$) is negligible $\forall p$.

Let $m_{\rm exc}$ and $\tilde{V}_K$ denote the electron-hole pair mass and the effective pair-pair interaction energy, respectively. Both the initial state of Eq. (2) and the final state of Eq. (4) can be considered as (approximate) eigenstates of the many-body electron-hole system, provided that $K\gg\sqrt{4N\tilde{V}_K m_{\rm exc}}$. As we shall see shortly, for such high center-of-mass momentum (particlelike) excited states, the effective exciton-phonon interaction coefficients are small, justifying the Born-Markov approximation that we use.

The next step is to obtain the scattering rate in the Born-Markov approximation using the Fermi golden rule. For simplicity, we will assume that the lattice is at zero temperature and only consider the spontaneous phonon processes. We then obtain the scattering rate

$$W_s=2\pi\sum_q\,|g_{e-\rm ph}(q)M(q)+g_{h-\rm ph}(q)M(-q)|^2 \times\delta(\omega_{\rm in}-\omega_{\rm fin}-\omega_q), \tag{5}$$

where

$$M(q)=\sum_k\,\varphi^*_\nu\left(k+\frac{q}{2}\right)u^*(k)v(k)[1-|v(k+q)|^2]. \tag{6}$$

Here, $\hbar\omega_{\rm in}$ and $\hbar\omega_{\rm fin}$ denote the energies of the initial and final many-body eigenstates, respectively. ω_q is the frequency of the emitted phonon. We reiterate that Eq. (5) is derived using the interaction Hamiltonian of Eq. (1) and no assumption regarding the *bosonic character* of electron-hole pairs was made. The product $u^*(k)v(k)$ is proportional to the electron-hole pair wave function. In the Hartree-Fock approximation, $u^*(k)v(k)$ can be determined from the semiconductor Bloch equation for the polarization term[4]

$$i\hbar\frac{d\sigma(k)}{dt}=[\epsilon_e(k)+\epsilon_h(k)]\sigma(k) -\sum_q\,V(q)[(1-\sqrt{1-4|\sigma(k-q)|^2})\sigma(k) -\sigma(k-q)\sqrt{1-4|\sigma(k)|^2}], \tag{7}$$

where $\sigma(k)=\langle\hat{h}_{-k}\hat{e}_k\rangle=\langle\Psi_{\rm BCS}|\hat{h}_{-k}\hat{e}_k|\Psi_{\rm BCS}\rangle=u^*(k)v(k)$. $V(q)$ is the Fourier transform of the Coulomb interaction energy; $\epsilon_e(k)$ and $\epsilon_h(k)$ denote the bare electron and hole energies, respectively. For low-density excitons (with $Na_B^d/L^d\ll1$, where d is the dimensionality and L is the lateral size), this equation reduces to the Wannier equation in real space and yields the $1s$ exciton wave function $\varphi_{1s}(p)$. In the high-density limit ($Na_B^d/L^d\gg1$), it is the counterpart of the superconducting gap equation.[8] In many cases, however, Hartree-Fock approximation is not valid and the calculation of the pair wave function would require the inclusion of screening and scattering terms.

The factor $[1-|v(k+q)|^2]$ in Eq. (6) gives the correction to the electron-hole-phonon scattering arising from the fact that the presence of a BCS ground state with a large number of composite bosons modify the commutation relation of high-momentum electron-hole pairs as well. To the extent that $K\gg k_F$ where k_F is the Fermi wave vector, the contribution of this term is negligible. If we in addition assume $K\sim q\gg\pi/a_B$, we can also neglect the k dependence of $\varphi^*_\nu(k+q/2)$, provided that φ_ν is a hydrogenic wave function. In this limit, we obtain

$$M(q)\simeq\varphi^*_\nu\left(\frac{q}{2}\right)\sum_k\,u^*(k)v(k), \tag{8}$$

and

$$W_s\simeq2\pi\sum_q\,\left|g_{e-\rm ph}(q)\varphi^*_\nu\left(\frac{q}{2}\right)+g_{h-\rm ph}(q)\varphi^*_\nu\left(\frac{-q}{2}\right)\right|^2 I \times\delta(\omega_{\rm in}-\omega_{\rm fin}-\omega_q), \tag{9}$$

where

$$I=\left|\sum_k\,u^*(k)v(k)\right|^2. \tag{10}$$

The expression for I given in Eq. (10) is the principal result of this paper as it contains the statistical enhancement factor for the scattering of a phonon by a many-body composite boson (electron-hole pair) system. In the low-density limit where $v(k)\simeq\sqrt{N}\varphi_{1s}(k)$, we obtain $I\propto N$ for all composite bosons, as expected. In the high-density limit, the qualitative nature of saturation and Pauli blocking of statistical enhancement factor strongly depends on the particular BCS state [i.e., the coherence factors $u(k),v(k)$]. The differences between the low- and high-density limits become more apparent if we recall Eq. (10): In the low-density limit [$u(k)\sim1,\forall k$], bosonic enhancement arises from a constructive interference of the contributions from all partially occupied pair states. In the opposite high-density limit, only the states around Fermi level for which $u^*(k)v(k)\neq 0$ contribute to I. Equivalently for this latter case, the electron-hole pairs with $k \ll k_F$ have exhausted the phase space available for them $[v(k)\simeq 1]$ and can no longer participate in stimulated scattering. Physically, this is due to the fact that the mean separation of the electron-hole pairs is less than their size, which makes the Pauli exclusion dominant.

Equation (10) shows that the stimulated scattering explicitly depends on the overlap $u^*(k)v(k)$. Therefore it is the coherence between the electron-hole pair states that results in bosonic enhancement. Conversely, if the ground state of the many-body system is an electron-hole plasma state where $u^*(k)v(k)=0, \forall k$, there is no final-state stimulation at any electron-hole pair density. We remark that even though we assume a BCS state in our analysis, the assumption of a well-defined condensate phase should not be relevant for the bosonic enhancement factor.

Next, we consider the special case of two-dimensional (2D) magnetoexcitons. It has been shown by several authors that in the strong magnetic-field limit where the magnetic length $a_0=\sqrt{\hbar/eB}$ is much smaller than a_B, the magnetoexcitons become ideal noninteracting bosons. More specifically, Paquet *et al.*[6] have shown that the single-particle wave function remains unchanged for all occupancies of the lowest exciton band. For this system we have

$$v(k)=v=\sqrt{2\pi a_o^2 N/L^2}, \tag{11}$$

where L is the transverse size of the 2D structure. The evaluation of the stimulated scattering contribution is then straightforward:

$$I=N\left(1-\frac{2\pi a_0^2 N}{L^2}\right)\frac{L^2}{2\pi a_0^2}. \tag{12}$$

The analytical expression given in Eq. (12) is valid for $1 \leqslant N \leqslant L^2/2\pi a_0^2$. In the low-density limit $(N \ll L^2/2\pi a_0^2)$, magnetoexcitons behave as ideal bosons $(I \propto N)$. The total scattering rate peaks at $N_{\max}=N_M=L^2/4\pi a_0^2$ where only half of the magnetoexcitons contribute to stimulation. For $N>N_M$, stimulated scattering rate into the ground state starts to decrease and goes to zero as $N\rightarrow 2N_M$; at this occupancy, all the underlying electron and hole fermionic phase space is exhausted and it is not possible to create another ground-state magnetoexciton. Here, N_M/L^2 is the counterpart of the *Mott density* for this system.

We believe that the results presented in this paper are important for the analysis of recently proposed semiconductor matter lasers or bosers.[9,10] Stimulated composite-boson scattering provides a gain mechanism for these *devices*. The saturation and eventual Pauli blocking of the stimulated scattering due to PSF effects that we predict would therefore result in unique gain saturation mechanism. This is especially important for large Bohr-radius exciton lasers in III-V and II-VI semiconductors.

In summary, we have derived an expression for bosonic stimulation of exciton-phonon scattering, in the limit where Born-Markov approximation is applicable. Pauli exclusion of the fermionic constituents that form the composite bosons (excitons) results in a saturation of the stimulated scattering at high densities where the separation between the bosons is less than or equal to their Bohr radius. In the special case of two-dimensional magnetoexcitons, we obtain analytical expressions for the stimulated scattering term: as the electron-hole pair density increases, the stimulated scattering rate first saturates and then experiences complete Pauli blocking for $N\rightarrow 2N_M$.

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