RAPID COMMUNICATIONS

Modification of the Landau-Lifshitz equation in the presence of a spin-polarized current in colossal- and giant-magnetoresistive materials

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We derive a continuum equation for the magnetization of a conducting ferromagnet in the presence of a spin-polarized current. Current effects enter in the form of a topological term in the Landau-Lifshitz equation. In the stationary situation the problem maps onto the motion of a classical charged particle in the field of a magnetic monopole. The spatial dependence of the magnetization is calculated for a one-dimensional geometry and suggestions for experimental observation are made. We also consider time-dependent solutions and predict a spin-wave instability for large currents. [S0163-1829(98)50406-7]

Phenomena associated with spin-polarized currents in layered materials and in Mn oxides have attracted much interest recently. Efforts are strongly concentrated on theoretical and experimental investigation of large magnetoresistance, which is of great value for future applications. Examples of the effect are giant magnetoresistance (GMR) in layered materials (see Ref. 1), spin valve effect for a particular case of a three-layer sandwich, and colossal magnetoresistance (CMR) in the manganese oxides (see Ref. 2).

The dependence of resistivity on magnetic field is explained conceptually in two steps: first the magnetic field changes the magnetic configuration of the material and that in turn influences the current. Of course, as for any interaction there must be a back action of the current on the magnetic structure. The existence of such back action was explored in Refs. 3 and 4. Several current-controlled microdevices utilizing this principle were proposed.[3] In both papers layered structures with constant magnetization throughout the magnetic layers were considered. In the present paper we derive the equations for a continuously changing magnetization in the presence of a spin-polarized current. This equation takes the form of a Landau-Lifshitz equation with an additional topological term, and admits a useful analogy with a mechanical system. We discuss several solutions in one-dimensional geometries. Our equations also can be viewed as a continuum generalization of Refs. 3, 4, and 6 for layer thickness going to zero.

Consider a current propagating through a conducting ferromagnet. Assume that conducting electrons are free and interact only with local magnetization $\mathbf{M}$. The motion of each individual electron is governed by the Schrödinger equation with a term $J_H \boldsymbol{\sigma}\cdot\mathbf{M}$, where J_H is the value of the Hund's rule coupling or in general of the local exchange. Since spin-up electrons have lower energy a nonzero average spin of conducting electrons $(1/2)\langle\boldsymbol{\sigma}\rangle$ develops. An angular momentum density $(\hbar/2)\langle\boldsymbol{\sigma}\rangle$ is then carried with the electron current so we have a flux of angular momentum. This leads to a nonzero average torque acting on the magnetization which can deflect it from the original direction (see Fig. 1).

Propagation of a current in a ferromagnet should be described by a system of two equations: one for the motion of conducting electrons and another for the magnetization. We derive here the second equation in the limit of small space-time gradients and present several solutions. The case of very large $J_H \to \infty$ is considered, meaning a complete polarization of electron spins in the ferromagnet. This case can be often realized in experiment. In the layered structures magnetic layers can be made of a material with large band splitting, such as Heusler alloys, in which the spin opposite to the magnetization direction cannot propagate. In the CMR materials large Hund's rule coupling is well known[2] and constitutes the basis for a double-exchange mechanism governing their magnetic ordering.

Schrödinger equation: the conducting electrons are considered noninteracting with an $\varepsilon = p^2/2m$ energy spectrum:

$$i\hbar\frac{\partial\psi_\alpha}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi_\alpha + J_H\mathbf{M}(\mathbf{r},t)\cdot\boldsymbol{\sigma}_{\alpha\beta}\psi_\beta . \qquad (1)$$

By writing Eq. (1) we made an assumption of the ballistic electron transport which will be discussed at the end of the paper. We diagonalize the matrix $\mathbf{M}(\mathbf{r},t)\cdot\boldsymbol{\sigma}_{\alpha\beta}$ with a local spin rotation $\phi_\alpha = U_{\alpha\beta}(\mathbf{r},t)\psi_\beta$. The spinor ϕ describes the electron in the coordinate system with the z axis parallel to the local magnetization. Retaining only the first order terms

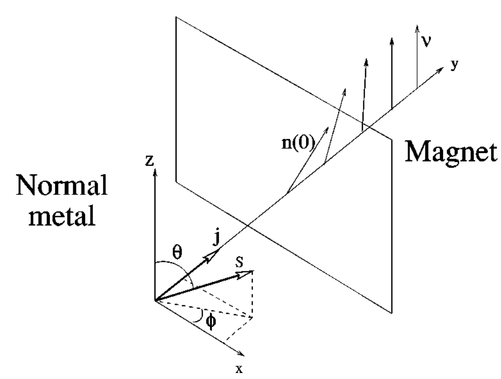

FIG. 1. Experimental setting: spin-polarized current enters a half-infinite magnet from the left. Originally the magnetization is aligned along the easy-axis $\nu \| z$. However if the incoming electrons are spin-polarized in a different direction, their interaction with the magnetization leads to a deflection of the magnetization.

in gradients and using $\phi_{-}\to 0$ for $J_H\to\infty$ we reduce Eq. (1) from a system of two equations to one equation for $\phi_{+}\equiv\phi$ spin amplitude:

$$i\hbar\frac{\partial\phi}{\partial t}=-\frac{\hbar^2}{2m}\Delta\phi+J_H M\phi-\frac{\hbar^2}{2m}(U_{+\beta}\nabla_i U_{\beta+})\nabla_i\phi. \tag{2}$$

The last term in Eq. (2) can be transformed ($\mathbf{M}=M\mathbf{n}$):

$$i\hbar\frac{\partial\phi}{\partial t}=-\frac{\hbar^2}{2m}\Delta\phi+J_H M\phi-\frac{i\hbar^2}{4m}A^k_{\rm mon}(\mathbf{n})\nabla_i n^k\nabla_i\phi, \tag{3}$$

where $\mathbf{A}^{\rm mon}(\mathbf{n})$ is a function satisfying the following equations:

$$\epsilon_{\alpha\beta\gamma}\frac{\partial A^{\rm mon}_{\beta}}{\partial n_{\gamma}}=2n_{\alpha},\ \frac{\partial A^{\rm mon}_{\alpha}}{\partial n_{\beta}}=\epsilon_{\alpha\beta\gamma}n_{\gamma}.$$

If we view $n^2(\mathbf{r},t)=1$ as a sphere, $\mathbf{A}^{\rm mon}$ has the simple interpretation of the vector potential due to a magnetic monopole located at the center of the sphere. The monopole term is known to appear from $U_{+\beta}\nabla_i U_{\beta+}$ in the theory of the Berry phase and is used by other CMR theories in different forms (see Ref. 5). Equation (3) has a form of a Schrödinger equation in a magnetic field expanded up to the linear term in $\mathbf{A}^{\rm eff}$, with vector potential $A^{\rm eff}_i=(i\hbar^2/4m)A^k_{\rm mon}\nabla_i n^k$. It describes the motion of the conducting electrons in the given field $\mathbf{n}(\mathbf{r},t)$. Conversely it gives the interaction between the current and the magnetization. The form of the equation is the same as for an electromagnetic interaction, and hence we can write by analogy

$$E_{\rm int}=\frac{1}{c}j_iA^{\rm eff}_i=-\frac{\hbar}{4}\frac{j_i}{e}A^k_{\rm mon}(\mathbf{n})\nabla_i n^k, \tag{4}$$

where $\mathbf{j}$ is an electric current.

Magnetization motion is described by Landau-Lifshitz equations which are obtained from the energy functional. After adding Eq. (4) to the usual energy density of a ferromagnet with uniaxial anisotropy along the axis $\boldsymbol{\nu}$, we obtain

$$E=\int\left(J(\nabla\mathbf{M})^2-K(\boldsymbol{\nu}\cdot\mathbf{n})^2+\frac{1}{c}j_iA^{\rm eff}_i\right)dV, \tag{5}$$

with $K>0$ corresponding to the easy axis and $K<0$ to the easy-plane magnets. The equations of motion then take the form

$$\frac{\partial\mathbf{M}}{\partial t}=\frac{g|e|}{2mc}[\mathbf{f}\times\mathbf{M}], \tag{6}$$

$$\mathbf{f}=-\frac{\delta E}{\delta\mathbf{M}}=J\Delta\mathbf{M}+\frac{2K}{M^2}(\boldsymbol{\nu}\cdot\mathbf{M})\boldsymbol{\nu}+\frac{\hbar}{2M}\frac{j_i}{e}[\nabla_i\mathbf{n}\times\mathbf{n}], \tag{7}$$

where the last term in $\mathbf{f}$ is new and describes the effect of the current. The system of Eq. (3) and Eqs. (6), (7) constitutes a complete set of equations for a magnet with current. Equations (6), (7), generalizing the Landau-Lifshitz equation in the presence of a current, are the central result of this work.

Since magnetization corresponds to angular momentum $\mathbf{L}=\hbar/g\mu_B\mathbf{M}$, an equation of the angular momentum flux continuity follows from Eq. (7):

$$\frac{\partial L_k}{\partial t}+\frac{\partial\Lambda_{ki}}{\partial x_i}=\left[\frac{2g\mu_BK}{\hbar M^2}(\boldsymbol{\nu}\cdot\mathbf{n})\boldsymbol{\nu}\times\mathbf{M}\right]_k, \tag{8}$$

$$\Lambda_{ki}=M^2J[\mathbf{n}\times\nabla_i\mathbf{n}]_k+\frac{\hbar}{2}\left(\frac{j_i}{e}\right)n_k. \tag{9}$$

The flux Λ_{ki} consists of two parts: one due to the spatial derivatives of magnetization and another due to the motion of conducting electrons. In our situation the spins of moving electrons are parallel to $\mathbf{n}$. That is why their contribution is factorized in the form $(\hbar/2)(j_i/e)n_k$.

Consider the stationary case in an experimental setting shown on Fig. 1. For the stationary process the rhs of Eq. (6) vanishes. The current propagates along the $\hat{\mathbf{y}}$ direction. All spatial derivatives reduce to $\boldsymbol{\nabla}\to\nabla_y$. For the reasons immediately following we will denote differentiation with a prime to get a resemblance to a time derivative in notation $\nabla_y\mathbf{n}\equiv\mathbf{n}'$. From Eq. (7) we get an equation on $\mathbf{n}(\mathbf{r},t)$:

$$\widetilde{J}[\mathbf{n}''\times\mathbf{n}]=\left[\left(-\widetilde{K}(\boldsymbol{\nu}\cdot\mathbf{n})\boldsymbol{\nu}-Q\left(\frac{j}{e}\right)[\mathbf{n}'\times\mathbf{n}]\right)\times\mathbf{n}\right], \tag{10}$$

with new parameters

$$\widetilde{J}=\frac{g\mu_BM}{\hbar}J,\quad \widetilde{K}=\frac{2g\mu_B}{\hbar M}K,\quad Q=\frac{g\mu_B}{2M}.$$

Since $\mathbf{n}$ in the stationary case depends on y only, we can interpret y as a fictitious time; together with $\mathbf{n}^2=1$, Eq. (10) can then be interpreted as the equation of motion for a particle of a mass $\widetilde{J}$ confined to the surface of a unit sphere and experiencing two forces: (a) a force of magnitude $-\widetilde{K}(\boldsymbol{\nu}\cdot\mathbf{n})\boldsymbol{\nu}$ parallel to the anisotropy axis, and (b) a Lorentz force, due to a field $\mathbf{H}_{\rm mon}=-Q(j/e)\mathbf{n}$ of a magnetic monopole positioned in the center of the sphere.

The vector product ensures that only tangential components of the total force act on the particle. The normal component is compensated by the reaction forces. Such an analogy enables one to visualize the solutions of the original equation (10) as trajectories of a massive particle on the sphere.

The equation of particle motion in the field of a magnetic monopole (10) has two first integrals.[7]

$$W=\frac{\widetilde{J}\mathbf{n}'^2}{2}+\frac{\widetilde{K}(\boldsymbol{\nu}\cdot\mathbf{n})^2}{2}=\text{const.} \tag{11}$$

$$D_\nu=\widetilde{J}([\mathbf{n}\times\mathbf{n}']\nu)+Q\left(\frac{j}{e}\right)(\mathbf{n}\cdot\boldsymbol{\nu})=\text{const.} \tag{12}$$

Together they provide a way to solve Eq. (10) for arbitrary initial conditions. Expressing everything through the Euler angles $\{\phi(y),\theta(y)\}$ (defined in Fig. 1) of the vector $\mathbf{n}$, we obtain

TABLE I. Characteristic lengths and currents for magnetic materials.

Material	L_m [Å]	j_0 [A/cm^2]	Ref.
CMR: $La_{0.66}Ca_{0.33}MnO_3$	>130	$<4\times10^7$	8
Fe	40	1.1×10^8	9
Heusler Alloy: PtMnSb	50–100	$\sim5\times10^7$	10

$$\phi' = \frac{D_\nu - Q(j/e)\cos\,\theta}{\tilde{J}\,\sin^2\,\theta},$$

$$\theta' = \sqrt{\frac{2W-\tilde{K}\,\cos^2\,\theta}{\tilde{J}} - \frac{[D_\nu - Q(j/e)\cos\,\theta]^2}{\tilde{J}^2\,\sin^2\,\theta}}$$

$$\equiv \mathcal{F}(\theta). \tag{13}$$

The problem for θ is solved by the implicit function

$$y = y(0) + \int_{\theta(0)}^{\theta} \frac{d\theta}{\mathcal{F}(\theta)}; \tag{14}$$

afterwards ϕ can be found from the first equation in Eqs. (13).

Assume that deep inside the magnet ($y\to\infty$) the magnetization resumes its original direction along the anisotropy axis $\mathbf{n}\to\boldsymbol{\nu}$, $\mathbf{n}'\to 0$. From this the values of the first integrals can be found and substituted into Eq. (13). Natural length and current scales appear in the calculation:

$$L_m = \sqrt{\frac{JM^2}{8K}}, \quad j_0 = \frac{e}{\hbar}\sqrt{KJM^2}, \tag{15}$$

through which the material parameters J,K,M enter the problem. Their values for different materials are given in Table I.
The integral (14) can be then expressed in elementary functions but the formula is long and will be detailed in a later paper. Instead, the results are presented in Fig. 2. It is seen that magnetization relaxes in a distance $\approx 10L_m$, which is about the width of the domain wall in the material.

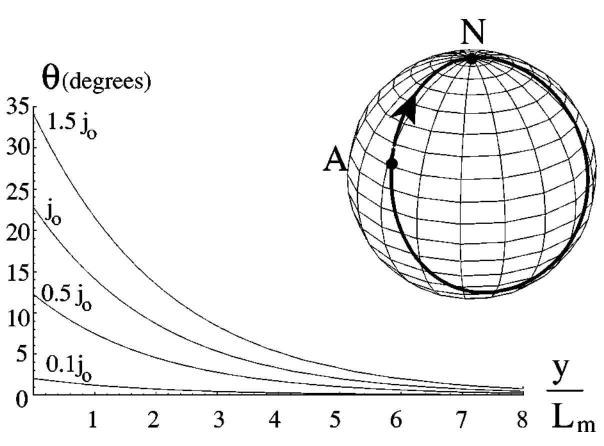

FIG. 2. Magnetization deflection angle $\theta(y)$. Curves correspond to different values of current, $\theta_0=\pi/2$, $p=0.5$. Inset: trajectory of particle on the sphere for $j=j_0$. Starting point A corresponds to the conditions of the main graph.

In the ''particle picture'' the motion starts at some point A on the trajectory, yet to be determined from the boundary condition on the normal metal-magnet interface, and ends on the north pole. The particle has just enough energy to climb the potential hill and come to rest on the top. The particle trajectory is bent by the monopole field. In the absence of the monopole the particle would go along the meridian.

The boundary condition on the metal-magnet interface, $y=0$ is derived from the continuity of the angular momentum flux. Such a condition ensures that there is no torque concentrated on the boundary consistent with the assumption of slow spatial changes of the magnetization.

The reflection of the down-spin electron component occurs on the length scale of the electron wavelength. On this distance magnetization is almost constant and solving the one-particle reflection problem we find the jump of the electron flux component in the $\hat{y}$ direction $\Sigma_i\equiv\Sigma_{iy}$ to be

$$\boldsymbol{\Sigma}^{\text{electron}}_{\text{metal}} - \boldsymbol{\Sigma}^{\text{electron}}_{\text{magnet}} = \boldsymbol{\Sigma} - \mathbf{n}(\boldsymbol{\Sigma}\cdot\mathbf{n}), \tag{16}$$

where $\boldsymbol{\Sigma}$ is the average injected flux. From Eq. (9) the flux inside the magnet is

$$\Lambda_{iy} = \boldsymbol{\Sigma}^{\text{electron}}_{\text{magnet}} + M^2 J[\mathbf{n}_0\times\mathbf{n}_0']_i\,. \tag{17}$$

In the metal, only the electron part of the flux is present. Then continuity gives the boundary condition

$$\boldsymbol{\Sigma} - (\boldsymbol{\Sigma}\cdot\mathbf{n}_0)\mathbf{n}_0 = M^2 J[\mathbf{n}_0\times\mathbf{n}_0']. \tag{18}$$

Note that it involves both the vector $\mathbf{n}$ and its derivative on the boundary.

Condition (18) can be transformed into a system of two algebraic equations and an inequality:

$$\frac{1}{2}\left(\frac{pj}{j_0}\right)^2(1-y^2) = 1-x^2,$$

$$p(\cos\,\theta_0 - xy) = 1-x,$$

$$y^2+x^2-2yx\,\cos\,\theta_0 \leqslant \sin^2\,\theta_0\,, \tag{19}$$

where $\Sigma=\boldsymbol{\Sigma}\mathbf{e}$, $\cos\,\theta_0=\mathbf{e}\cdot\boldsymbol{\nu}$, $x=\boldsymbol{\nu}\cdot\mathbf{n_0}$, and $y=\mathbf{e}\cdot\mathbf{n_0}$. The parameter $p=e\Sigma/\hbar j$, $p\in[0,1/2]$, describes the ''degree of polarization'' of the incident electrons. The inequality in Eq. (19) is a geometrical constraint on x and y arising from their definition.

The trajectory is determined by three parameters: $(j/j_0,p,\theta_0)$. We can plot a domain of existence of a solution to (19) in the 3D space of these parameters. A typical 2D section of this diagram for constant θ_0 is shown in Fig. 3. A solution is absent in regions B and C which means that for larger currents and spin polarizations no smooth stationary solution approaching the easy-axis direction at infinity is available. Either a nonstationary solution or a solution which never approaches $\boldsymbol{\nu}$ will be realized in that region.

Time-dependent solutions of Eq. (7) can be found in some cases. We again assume the current $\mathbf{j}$ to be uniform. We rewrite Eq. (7) through $\mathbf{n}(\mathbf{r},t)$:

$$\frac{\partial\mathbf{n}}{\partial t} = [\mathbf{g}\times\mathbf{n}] - Q\left(\frac{j_i}{e}\right)\nabla_i\mathbf{n},$$

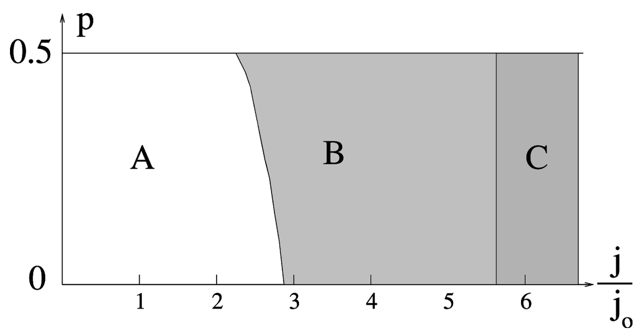

FIG. 3. A typical 2-D section of the phase diagram plotted for $\theta_0 = \pi/3$. A: domain of existence of a solution to (19); B: no solution, b.c. at $y=0$ can not be satisfied; C: no solution, b.c. at $y \to \infty$ can not be satisfied and spin wave instability occurs.

$$\mathbf{g} = \tilde{J}\Delta\mathbf{n} + \tilde{K}(\boldsymbol{\nu}\cdot\mathbf{n})\boldsymbol{\nu}, \tag{20}$$

and suppose $n_0(\mathbf{r})$ solves $[\mathbf{g}\times\mathbf{n_0}]=0$, i.e., represents a static solution in the absence of the current. Then

$$n(\mathbf{r},t) = n_0\left[\mathbf{r} + Q\left(\frac{\mathbf{j}}{e}\right)t\right] = n_0\left(\mathbf{r} + \frac{\omega_0 L_m}{\sqrt{2}}\frac{j}{j_0}t\right), \tag{21}$$

where $\omega_0 = \tilde{K}$ is a solution of Eq. (20) for a nonzero current. For instance, a moving Bloch wall will be a solution when current is flowing perpendicular to it (provided pinning is absent).

Another particular solution is a spin wave in the presence of a current. We search for a solution (7) in the form of a spin wave: $\{\theta = \text{const}, \phi = \mathbf{kr} - \omega t\}$. This gives the spectrum

$$\omega = \tilde{J}k^2 - Q\frac{\mathbf{j}\cdot\mathbf{k}}{e} + \tilde{K}. \tag{22}$$

As we see, the current changes the energy gap of spin waves and shifts the position of the minimum:

$$\omega_{\min} = \tilde{K} - \left(Q\frac{j}{e}\right)^2 \frac{\cos^2\alpha}{4\tilde{J}} = \omega_0\left(1 - \frac{j^2}{32 j_0^2}\cos^2\alpha\right),$$

where α is the angle between j and k and $\omega_0 = \tilde{K}$ is the gap of spin wave in an anisotropic ferromagnet. For large enough current $j > 4\sqrt{2}j_0$ an instability occurs. That is also the condition which leads in region C in Fig. 3 to the loss of any trajectory approaching $\boldsymbol{\nu}$ at infinity as the integral (14) becomes undetermined. A spin-wave instability is also predicted in other models of spin-polarized transport.[6]

Impurity scattering is not taken into account in our derivation of Eqs. (6) and (18), but we argue that those equations will not be changed. Collisions without spin flip do not transfer angular momentum and cannot enter in Eq. (6). Also if the boundary roughness is smaller than elastic mean free path l_t the change of Eq. (18) must be negligible. Spin-flip collisions in the ferromagnet do contribute to random angular momentum exchange with a rate measured by the spin-diffusion length l_s^{FM}. They can be neglected if the rate of ordered transfer of angular momentum is much greater, i.e., if $L_m \ll l_s^{FM}$. Note that for $J_H \to \infty$ spin flip cannot happen because there is no phase space for outgoing electrons with the wrong spin direction; for finite spin polarization spin flipping is partially suppressed and $l_s^{FM} > l_s^{NM}$, where l_s^{NM} is the normal metal value. Experiments[11] show $l_t \sim L_m \ll l_s^{NM}$, $l_s^{NM} \sim 0.3$ mm in Al, which validates our approach.

Discussing possible experiments we note that the characteristic current is large, but such densities are in fact common for layered metallic structures and $j \sim j_0$ is experimentally possible. Then the calculated magnetization deviates by $\sim 20°$ on the boundary (Fig. 2). Spin polarized current can be created by another magnetic electrode which should be placed within a distance $d < l_s^{NM}$ from the first one, and by changing d the degree of polarization of injected current can be controlled. Detection of the effect is difficult, but element-specific x-ray magnetic circular dichroism[12] (MXCD) could be used for a quantitative measurement of the deviation on the boundary. A single layer of a different magnetic element grown on the boundary will give a separate MXCD signal from which $\theta(0)$ can be extracted. Optical detection methods could also be possible.

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