# Screened Coulomb potential for a quantum wire in the Thomas-Fermi approximation

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We derive the dielectric function and screened Coulomb potential of a quantum wire by using a one-dimensional (1D) generalization of the Thomas-Fermi approximation (TFA). To apply the TFA approach to all temperatures we employ a constructive method based on the standard Padé approximant technique to approximate the dependence of the temperature of the 1D TFA parameter $q_s = 2e^2 \partial n / \partial \mu$. [S0163-1829(98)05115-7]

## I. INTRODUCTION

Quasifree carriers in low-dimensional systems can be generated by a variety of processes. For instance, they can be produced by modulation doping or optically by intense illumination of a nominally undoped quantum well. The way these carriers screen the bare Coulomb potential depends on the dimensionality of the system. When the dimensionality of the system is lowered, the screening effects decrease. That is, the influence of screening in two dimensions is considerably weaker than in three dimensions and this trend continues if one passes from two-dimensional (2D) quantum wells to quasi-one-dimensional quantum wires. This occurs because, if we assume that the quasifree carriers are confined in low-dimensional (2D or 1D) systems, screening between any two charges occurs mainly inside the system, corresponding to field lines *inside* the material, whereas the field lines that abandon the system are not directly affected by those charges. Clearly, screening is not decreased in this way in a bulk (3D) material since all field lines are screened by the charge carriers. For example, absorption spectrum calculations for a cylindrical GaAs quantum wire shows that for quasi-1D systems the effect of state filling is a more important source of optical nonlinearities than the effect of screening.[1,2] It might be for this reason, weak screening in one dimension, that the behavior in real space of the screened Coulomb potential in quantum wires has not been investigated in detail. Therefore, the purpose of this paper is to derive expressions for the small-wave-vector dielectric function and for the long-range screened Coulomb potential. We resort to the well-known Thomas-Fermi approximation (TFA), which provides a simple model for screening and has been extensively used in two[3–5] and three dimensions.[6] An advantage of the TFA screening model is that it generally allows one to include in a relatively simple way the dependence on the temperature and on the electron density, unlike more sophisticated models such as the random-phase approximation (RPA).[7] However, it is well known that the TFA has an important disadvantage: It cannot reproduce Friedel oscillations of degenerated systems at low temperatures that arise from the abrupt change in screening at a wave vector of $k = 2k_F$, where $k_F$ is the carrier wave vector at the Fermi surface. These oscillations occur in 3D,[8] 2D,[9] and 1D (Ref. 10) systems, dominate the long-range behavior of the screened potential, and tend to disappear when the carrier occupation probability becomes a smooth function of the wave vector $k$, that is, when the temperature increases sufficiently. In contrast to metals, for which the intrinsic energy scales are usually much larger than the temperature, in low-dimensional semiconductor structures the experimental temperature can be compared to the intrinsic energies. For example, for the quantum wire (QW) to be in the quantum limit, the doping must necessarily be low and hence the Fermi energy is also small. For these reasons we will also study here how the screening parameter changes with temperature. In this way we intend to present a complete and practical TFA model for electrons confined to the lowest subband of a QW. For a semiconductor QW, some of their many-body properties have been discussed, for example, in the extensive work of Hu and Das Sarma[11] and the references quoted therein. These properties can manifest as lattice Peierls distortion, disorder-induced Anderson localization, hole screening effects, plasma effects, and impurity scattering, the last two being the most important ones for actual semiconductor QW's.[11] Because the plasmon dispersion in a QW goes to zero as the momentum $q$ gets small, dynamical effects are expected to be important for small $q$. In fact, Hu and Das Sarma have argued that for 1D systems low-energy virtual plasmon excitations can be crucial in dynamical screening since they cause the Fermi surface to disappear (in the sense that elementary excitations are very different from those of the noninteracting systems), but when impurity scattering is included, the Fermi surface reappears because these plasmons are damped by impurity scattering, which is consistent with Raman scattering and photoluminescence experiments since these experiments are explained successfully on the basis of standard Fermi-liquid theory.[11,12] Therefore, in light of these results, it seems that our static screening theory, which does not include plasma effects, can be safely applied to most semiconductor QW's, but it should not be applied to very clean QW's (where by clean we mean that it lacks impurity scattering).

The paper is structured as follows. In Sec. II we derive the TFA dielectric function, which agrees, as expected, in the corresponding limit, with those of Lee and Spector[7] calculated in the self-consistent approximation or RPA. In Sec. III

 

we find for all temperatures an approximated analytical expression of the TFA screening parameter $q_s$. In Sec. IV we derive the asymptotic behavior of the 1D screened potential in terms of $q_s$. Finally, Sec. V is devoted to some concluding remarks.

## II. TFA DIELECTRIC FUNCTION

To develop a 1D TFA, we use the standard 3D procedure.[6] We will consider only the presence of a periodic background potential through an effective mass $m^*$. If we have a charge particle (for convenience, let us assume that it is positive) placed at a given position in an electron gas and rigidly held there, it will attract electrons, creating a surplus of negative charge in its neighborhood, which reduces or screens its field. Let us introduce two electrostatic potentials. The first $\phi^{\text{ext}}$ arises only from the positive-charged particles and satisfies the Poisson equation given by

$$-\nabla^2\phi^{\text{ext}}=4\frac{\pi}{\kappa}\rho^{\text{ext}}(\vec{r}), \tag{2.1}$$

where $\rho^{\text{ext}}$ is the particle charge density and $\kappa$ is the dielectric constant of the medium. The second $\phi$ is the full physical potential produced by both the positive-charged particle and the cloud of screening electrons; it satisfies

$$-\nabla^2\phi=4\frac{\pi}{\kappa}\rho(\vec{r}), \tag{2.2}$$

where $\rho$ is the full density,

$$\rho(\vec{r})=\rho^{\text{ext}}(\vec{r})+\rho^{\text{ind}}(\vec{r}), \tag{2.3}$$

and $\rho^{\text{ind}}$ is the charge density induced in the electron gas by the presence of the external particle. In a static model, since the external charge has an electrostatic influence over a finite vicinity that surrounds it, $\phi$ has to have a nonlocal relation with $\phi^{\text{ext}}$ given by

$$\phi^{\text{ext}}=\int d\vec{r}'\,\frac{\epsilon(\vec{r}-\vec{r}')}{\kappa}\phi(\vec{r}'), \tag{2.4}$$

which implies that the corresponding Fourier transform satisfies

$$\phi(q)=\frac{\kappa\phi^{\text{ext}}(q)}{\epsilon(q)}, \tag{2.5}$$

where $\epsilon(q)$ is the wave-vector-dependent dielectric constant of the metal.

The most natural quantity to be calculated is not the dielectric constant but the charge density $\rho^{\text{ind}}$ induced in the electron gas by the total potential $\phi$. When $\rho^{\text{ind}}$ and $\phi$ are linearly related (for sufficiently weak $\phi$), then their Fourier transform satisfies

$$\rho^{\text{ind}}(q)=\chi(q)\phi(q). \tag{2.6}$$

We can relate $\epsilon$ to $\chi$ by taking the Fourier transforms of the Poisson equations (2.1) and (2.2). Since these transforms depend on the dimensionality of the system involved, we perform for $z$-oriented 1D systems the $z$ Fourier transform to yield

$$\left(q_z^2-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\phi(x,y,q_z)=4\frac{\pi}{\kappa}\rho(x,y,q_z). \tag{2.7}$$

We solve this equation for $\phi(x,y,q_z)$ by using the corresponding Green function in cylindrical coordinates $r,\theta,z$ of this inhomogeneous Hemholtz equation;[13] we find

$$\phi(r,\theta,q_z)=\frac{2}{\kappa}\int K_0(q_z s(r,r',\theta))\rho(r',\theta',q_z)r'dr'd\theta', \tag{2.8}$$

where $K_0(q_z s(r,r',\theta))$ is the modified Bessel function of second class and $s(r,r',\theta)=\sqrt{r^2+(r')^2-2rr'\cos\theta}$. Finally, since the 1D potential can be calculated as $\phi_{1\text{D}}(q_z)=\lim_{q_z a\to 0}\phi(r,\theta,q_z)$, where $a$ is the radius of the cylindrical section of the QW (or yields the order of magnitude of the cross section of the QW for other noncircular sections) and we have that[17] $K_0(q_z s(r,r',\theta))\approx-\ln(q_z a/2)-\ln[s(r,r',\theta)/a]-\gamma\approx-\ln(q_z a/2)$ for very small values of $q_z a$, we thus arrive at

$$\phi_{1\text{D}}(q_z)=-\frac{2}{\kappa}\ln\left(\frac{q_z a}{2}\right)\rho_{1\text{D}}, \tag{2.9}$$

where $\rho_{1\text{D}}=\int\rho r'dr'd\theta'$ is the lineal charge density. By performing the same treatment for $\phi^{\text{ext}}$ we obtain

$$\phi_{1\text{D}}^{\text{ext}}(q_z)=-\frac{2}{\kappa}\ln\left(\frac{q_z a}{2}\right)\rho_{1\text{D}}^{\text{ext}}. \tag{2.10}$$

Together with Eq. (2.3) for 1D densities and Eq. (2.6), these equations give

$$\phi_{1\text{D}}=\frac{\phi_{1\text{D}}^{\text{ext}}}{1+2\ln\left(\frac{q_z a}{2}\right)\frac{\chi(q)}{\kappa}}. \tag{2.11}$$

Comparing this equation with Eq. (2.5) leads to the relation

$$\frac{\epsilon(q_z)}{\kappa}=1+2\ln\left(\frac{q_z a}{2}\right)\frac{\chi(q)}{\kappa}. \tag{2.12}$$

Except for the assumption that the externally applied charge is weak enough to produce only a linear response in the electron gas, the foregoing analysis has been exact. The following step is to calculate $\chi$ using the Thomas-Fermi method, which has the advantage that it is applicable even when a linear relation between $\rho^{\text{ind}}$ and $\phi$ does not hold, but has the limitation that is reliable only for a very slowly varying external potential.

Let us consider $\phi$ as a slowly varying function of $z$ in the sense that the energy of the charge carrier that is under its influence is given by

$$E(k_z)=\frac{\hbar^2k_z^2}{2m^*}-e\phi \tag{2.13}$$

and thus the energy is modified from its free-electron value by the total local potential. This only makes sense in terms of wave packets since we must require that $\phi(z)$ varies slowly on the scale of the Fermi wavelength.

To calculate the charge density produced by these electrons we substitute Eq. (2.13) into the 1D electronic number density $n=N/L$, where $L$ is the length of the system, to find

$$n[\mu+e\phi(z)]=2\int \frac{dk_z}{\pi}\frac{1}{\exp[\beta(\hbar^2k_z^2/2m^*-e\phi-\mu)]+1}, \tag{2.14}$$

where $\beta=1/\kappa_B T$ with a similar expression for $n_0$, but with $\mu=0$. Thus the induced charge density

$$\rho^{\text{ind}}=-e\{n[\mu+e\phi(z)]-n_0(\mu)\}, \tag{2.15}$$

which is the basic equation of the nonlinear Thomas-Fermi theory. If we expand Eq. (2.15) we obtain to leading order

$$\chi(q_z)=-e^2\frac{\partial n_0}{\partial\mu}. \tag{2.16}$$

Comparing Eq. (2.16) with Eq. (2.6) we find that

$$\chi(q_z)=-e^2\frac{\partial n_0}{\partial\mu}, \tag{2.17}$$

where $\partial n_0/\partial\mu$ is independent of $q$. Substitution into Eq. (2.12) gives the Thomas-Fermi dielectric response function

$$\epsilon(\vec{q}_z)=\kappa-q_s\ln(q_z a), \tag{2.18}$$

where we have introduced the dimensionless Thomas-Fermi parameter $q_s=2e^2(\partial n_0/\partial\mu)$. It is straightforward to calculate $\partial n_0/\partial\mu$ at 0 K by using the definition $n_0=2\int_0^\infty dk/\pi=(2/\pi)(\sqrt{2m^*E_F}/\hbar)$, which leads directly to $\partial n_0/\partial\mu=2m^*/\pi k_F\hbar^2$, where $k_F$ is the Fermi wave number. Notice that from the definition of the parameter $q_s$, its units depend on the dimensionality since $n_0$ depends on the dimension of the system.

We point out that Eq. (2.18) is consistent with the dielectric response function performed by Lee and Spector[7] in the RPA because the latter reduces to Eq. (2.18) when $q_z a\to 0$. To show this, we take $\epsilon^{\text{Q1D}}$ from Eq. (13) of Ref. 7, which is given by

$$\epsilon^{\text{Q1D}}=\kappa+\frac{8e^2}{Lq_z^2a^2}[K_1(q_za)I_1(q_za)-1/2]F(q_z,\omega), \tag{2.19}$$

where $I_1, K_1$ are the modified Bessel functions of first and second classes and $F(q_z,\omega)$ is the temperature-dependent Lindhard function, which in the long-wavelength limit (for which the RPA and TFA coincide; $q\ll 2k_F$) turns out to be $F(q,0)=-2m^*L/\pi\hbar^2k_F$. In Eq. (2.19) we wrote Q1D as a superscript of $\epsilon$ to emphasize that Lee and Spector[7] calculated $\epsilon$ for a quasi-1D system for which, unlike our case, more than the lowest subband is taking into account. Hence, by substituting the dominant terms of the Bessel function for small arguments, Eq. (2.19) reduces to Eq. (2.18).

## III. TFA PARAMETER

The TFA can be considered a useful and simple model of screening when the system does not exhibit Friedel oscillations since these oscillations cannot be reproduced by the TFA. As mentioned in Sec. I, it is known[10] that in low-temperature 1D systems these oscillations also dominate, as in 3D and 2D systems, the long-range behavior of the screened Coulomb potential. Since Friedel oscillations tend to disappear when the carrier occupation probability (the Fermi distribution) becomes a smooth function of the wave vector, then a necessary condition for the TFA to be able to yield a better description of the system including screening effects, is to increase the temperature. Therefore, in order to present a complete one-parameter TFA theory it is important to be able to find the dependence of $q_s$ on the temperature.

We now proceed in the same way as in Refs. 14 and 15 to calculate $q_s=2e^2(\partial n_0/\partial\mu)$ approximately as a function of the temperature. Following the standard procedure, we perform a series representation for $n(\mu)$ given by Eq. (2.14), which involves the expansion of the kernel $[z^{-1}\exp(\beta\hbar^2k_z^2/2m^*)+1]^{-1}$, valid for $\text{Re}\{z\}>-1$,[16] which leads to

$$\nu=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}z^n}{\sqrt{n}}, \tag{3.1}$$

where $\nu=(n/2)\sqrt{\pi\beta\hbar^2/2m^*}$ and $z=e^{\beta\mu}$. We reverse this series by using the well-known rules of series reversion,[17] which gives

$$e^{\beta\mu}=\nu+\sum_{n=2}^{\infty}\alpha_n\nu^n. \tag{3.2}$$

Up to third order in $\nu$ this expression can be approximated as[14]

$$\beta\mu\approx\ln\nu+\sum_{n=1}^{3}A_n\nu^n, \tag{3.3}$$

where $A_1=-1/\sqrt{2}\approx-0.7071$, $A_2=3/4-1/\sqrt{3}\approx 0.1726$, and $A_3=4/\sqrt{6}-1/2-8/3\sqrt{2}\approx-0.7526$. We performed, in the same way as Aguilera-Navarro *et al.*[15] did, a Padé summation in order to extend or accelerate the convergence of the partial summation given by Eq. (3.3). To do this, we first transform Eq. (3.3) into a power series by using the related function

$$\nu\frac{d\beta\mu}{d\nu}\approx 1+A_1\nu+2A_1\nu^2+3A_2\nu^3. \tag{3.4}$$

Since the right-hand side of this equation is a simple power series in $\nu$, it is now possible to represent it by a Padé approximant.[18] The Padé approximant technique consists of systematically expressing the power series of Eq. (3.4) as the ratio of two polynomials

$$A_0+A_1x+A_2x^2+\cdots\doteq\left[\frac{L}{M}\right](x)\equiv\frac{p_0+p_1x+\cdots+p_Lx^L}{1+q_1x+\cdots+q_Mx^M}. \tag{3.5}$$

The symbol $\doteq$ means ''represented by.'' The coefficients $p_i$ and $q_i$ are determined from the A's by $L+M$ linear equations, which follows from the basic definition

$$\left[\frac{L}{M}\right](x)-\sum_{k=0}^{L+M} A_k x^k \equiv O(x^{L+M+1}). \tag{3.6}$$

From Eq. (3.6) it is readily seen that the Padé approximant matches the power-series expansion up to order $L+M$. An interesting feature of the Padé successions is that when the poles of the function are beyond the interval to be represented, the Padé approximant converges faster than the partial summations of the series. Since $\nu d\mu/d\nu$ is not expected to have poles for $\nu \geqslant 0$, the Padé approximant should be a better representation than the power series. We thus write

$$\nu\frac{d\beta\mu}{d\nu} \doteq \left[\frac{L}{M}\right](\nu), \quad L+M \leqslant 4, \tag{3.7}$$

where $[L/M]$ denotes the ratio of two polynomials of order $L$ and $M$, respectively. Four Padé forms, namely, $[L/0]$ ($l=0,\ldots,3$), are excluded from the present study since they are simply partial sums to order $L$ of the right-hand side of Eq. (3.4) and are thus trivial approximations. On the other hand, since $\beta\mu$ is a monotonically increasing function of $\nu$, only the $[L/M]$ Padé approximation that would yield a monotonically increasing behavior of $\beta\mu$ with respect to $\nu$ is acceptable. In particular, since the approximations $[0/M]$ ($M=1,2,3$) are decreasing functions of $\nu$, these forms will not be considered in the present analysis. The same criterion applies to $[1/M]$ ($M=2,3$). In general, the form $[L/L]$ is known to be useful for approximating functions that approach a constant value for large values of $\nu$. This feature is enough reason for not considering further forms $[2/2]$ and $[1/1]$. The surviving term $[2/1]$ does not have singularities in the $\nu$ range $0<\nu<\infty$, as should be expected, and this term can be written as

$$\left[\frac{2}{1}\right](\nu)=1+K_1\nu+\frac{K_2\nu}{1+K_3\nu}, \tag{3.8}$$

where we have already imposed the condition $\beta\mu \to \ln \nu$ as $\nu$ tends to zero, which is required physically at low densities (classical limit). Finally, in order to have an approximation for $\nu(d\beta\mu/d\nu)$ valid for every value of $\mu$, even for very low temperatures or high densities, we shall add to our Padé representation given by Eq. (3.8) the asymptotic limit of $\beta\mu \to 2\pi\nu^2$ (Ref. 19) valid for large values of $\nu$:

$$\nu\frac{d\beta\mu}{d\nu}=1+K_1\nu+\frac{K_2\nu}{1+K_3\nu}+2\pi\nu^2, \tag{3.9}$$

where the coefficients $K_1$, $K_2$, and $K_3$ are determined by expanding this expression in a Taylor series around zero up to order 3 and equating each power coefficient with the ones of Eq. (3.4) leading to $K_1=A_1-4(\pi-A_2)^2/3A_3 \approx 14.91$, $K_2=4(\pi-A_2)^2/3A_3 \approx -15.61$, and $K_3=(3/2)A_3/(\pi-A_2) \approx -0.3802$. It is interesting to remark that, in contrast to the approximation performed by Aguilera-Navarro *et al.*[15] for the 3D case, that the asymptotic behavior of $\mu$ is given by an integer power of $\nu$ in the 1D case allows us to match a *unique* expression for $\mu$ on its whole domain. Thus, by integrating Eq. (3.9) we find that $\mu$ is given by

$$\mu=\frac{\pi^2\hbar^2n^2}{8m^*}+K_1\frac{n\hbar}{2}\sqrt{\frac{\pi}{2m^*\beta}}+\frac{1}{\beta}\ln\left[\frac{n\hbar}{2}\sqrt{\frac{\pi\beta}{2m^*}}\right] + \frac{K_2}{\beta K_3}\ln\left[1+K_3\frac{n\hbar}{2}\sqrt{\frac{\pi\beta}{2m^*}}\right]+C, \tag{3.10}$$

where $C$ is a numerical constant that can be found by integrating numerically $n(\mu)$. Since $q_s=2e^2(\partial n_0/\partial\mu)$, we have

$$\frac{1}{q_s}=\frac{\pi^2\hbar^2n}{8m^*e^2}+K_1\frac{\hbar}{4e^2}\sqrt{\frac{\pi}{2m^*\beta}}+\frac{1}{2e^2n\beta} + K_2\frac{\dfrac{\hbar}{4e^2}\sqrt{\dfrac{\pi}{m^*\beta}}}{1+K_3\dfrac{n\hbar}{2}\sqrt{\dfrac{\pi\beta}{2m^*}}}. \tag{3.11}$$

## IV. SCREENING POTENTIAL

Now let us suppose that the external potential $\phi^{\text{ext}}$ is that of a point charge that is $\phi=e/r$. If this charge is confined in a QW of radius $a$ its Fourier transform potential is given by

$$\phi^{\text{ext}}(q_z)=-2\frac{e}{\kappa}\ln(q_z a). \tag{4.1}$$

The total potential in the semiconductor will then be

$$\phi(q_z)=\frac{\phi^{\text{ext}}(q_z)}{\epsilon(q_z)}=\frac{-2e\,\ln(q_z a)}{1-q_s\,\ln(q_z a)}. \tag{4.2}$$

The Fourier transform can be inverted to give

$$\phi(z)=-\frac{2e}{\kappa}\int_0^\infty \frac{dq_z}{2\pi}e^{iq_z z}\frac{\ln(q_z a)}{1-q_s\,\ln(q_z a)}. \tag{4.3}$$

This expression does not converge formally for a real value of $z$, but we shall perform an analytical extrapolation by extending $z$ to the complex domain and keeping just the real part of the resulting expression. For both small and large values of $q_s$, we can expand Eq. (4.3) in a geometrical series and identify each of its terms[20] leading, respectively, to the expressions

$$\phi(z)=-\frac{e}{\kappa z}+\frac{e}{\kappa}\frac{q_s}{\pi}\sum_{n=0}^{\infty} q_s^n \times \mathrm{Re}\left[\frac{\partial^{n+2}}{\partial\nu^{n+2}}\left(\frac{\Gamma(\nu)}{\mu^\nu}\right)\right]\Bigg|_{\nu=1,\mu=-iz}, \quad q_s<1 \tag{4.4}$$

$$\phi(z)=-\frac{e}{\kappa q_s}\delta(z)+O\left(\frac{1}{q_s^2}\right), \quad q_s \gg 1, \tag{4.5}$$

where $\Gamma$ denotes the Gamma function,[13] $\delta$ is the Dirac delta function, and Re indicates the real part of the quantity in

square brackets. By using Eq. (4.359(1-5)) of Ref. 20 it is easy to show that Eq. (4.4) can be written explicitly up to order 4 in $q_s$ as

$$\phi(z)=-\frac{e}{\kappa z}+\frac{e}{\kappa}q_s\frac{C+\ln z}{z}+\frac{eq_s^2}{\kappa z}\left\{\frac{\pi^2}{8}-\frac{3}{2}[C^2+\zeta(2,1)]+3C\ln z-\frac{3}{2}\ln^2 z\right\}-\frac{eq_s^3}{\kappa z}\left\{2C^3+\frac{\pi^2}{2}C-6C\zeta(2,1)+4\zeta(3,1)-\ln z\left(6C+C\frac{\pi^2}{2}+6\zeta(2,1)\right)+2C^2\ln^2 z-2\ln^3 z\right\}+O(q_s^4). \tag{4.6}$$

Here $C\approx 0.577$ is the Euler constant and $\zeta(j,k)=\Sigma_{l=0}^{\infty}1/(l+k)^j$ is the Rieman zeta function. Note that both Eqs. (4.5) and (4.6) do not depend on $a$, the radius of the QW, which means that these results are the same for any small $a$.

Equations (4.4) and (4.5) illustrate the two limiting behaviors of $\phi$. The former shows that $\phi$ reduces to the Coulomb potential when $q_s\to 0$, as it should be expected. This limit is the appropriate limit to be taken in the TFA since we recall that the TFA is valid for a small perturbation of the Fermi distribution. The other limit of small $q_s$ shown in Eq. (4.5) exhibits a very confined behavior, which, strictly speaking, cannot be represented by the TFA. However, it is interesting to note that even within the TFA that confined behavior can never be reached because the value of $q_s$ is bounded, as can be seen from Eq. (3.11). For instance, since the larger terms in Eq. (3.11) are positive, this equation shows that $q_s$ has its larger values for small temperatures ($\beta\to\infty$) and in fact $q_s$ cannot be larger than $\pi^2\hbar^2 n/8m^*e^2$.

## V. CONCLUDING REMARKS

In summary, our main results within the TFA valid for very thin QW's are given by asymptotic expressions of the screened potential [Eqs. (4.5) and (4.6)] and a Padé approximant expression for $q_s$ appropriate for all temperatures [Eq. (3.11)]. These expressions are relatively simple and can be useful for 1D systems that, as discussed in the Introduction, satisfy the following two conditions. First, the temperature of the system must be such that the carrier distributions are sufficiently smooth and thus they do not exhibit Friedel oscillations. Second, the system has enough impurity scattering in order to damp plasmon screening effects. This latter condition seems to be satisfied in most of the actual semiconductor QW's.[11,12]

Let us compare the asymptotic behaviors for large $q_s$ of the 3D,[6] 2D,[5] and 1D screened Coulomb systems, which are proportional to $e^{-q_s r}$, $1/q_s^2$, and $1/q_s$ [see Eq. (4.5)], respectively. From these expressions it can be seen that the potentials decrease more slowly as a function of $q_s$ when the dimensionality decreases. That is to say, by lowering the dimensionality the screening effect gets diminished. This result is reinforced by noting that the correcting terms to the Coulomb potential in Eq. (4.6) contain terms proportional to $1/z$ so that this potential still has a long scope. In contrast, the 2D and 3D screened potentials have as first correcting terms those proportional to larger inverse powers of their respective variables, that is, $1/\rho^2$ and $1/r^2$.

Another point to emphasize is a common characteristic of the TFA potential for all of the three dimensionalities (3D, 2D, and 1D) that consists in the existence of at least one bounded state for a large value $q_s$. This follows for the 3D case from the well-known general theorem for 3D central force systems,[21] while for the 2D case it was shown numerically by Spector *et al.*[5] and for the 1D case the existence, in the extreme case of an attractive Dirac $\delta$ potential, of a bounded state is well known.

We hope that this work may stimulate further theoretical and experimental work in the study of screening in 1D systems and other nanostructured systems.

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