Fermi-fluid description of the half-filled Landau level

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We present a many-body approach to calculate the ground-state properties of a system of electrons in a half-filled Landau level. Our starting point is a simplified version of the recently proposed trial wave function where one includes the antisymmetrization operator to the bosonic Laughlin state. Using the classical plasma analogy, we calculate the pair-correlation function, the static structure function, and the ground-state energy in the thermodynamic limit. These results are in good agreement with the expected behavior at $\nu=\frac{1}{2}$. [S0163-1829(98)03315-3]

The fractional quantum Hall effect (FQHE),[1] which is understood to be due to condensation of electrons to unique incompressible states as a result of electron correlations,[2,3] fails to explain the odd behavior of even-denominator filling fractions that lie right in the middle of all the observed FQHE filling factors. It has been experimentally established that at $\nu=\frac{1}{2}$ the system is metallic. (Here $\nu=\phi_0 n_e/B$, where $\phi_0=hc/e$, n_e is the mean electron density, and B is an externally applied magnetic field.) The seemingly metallic behavior observed in transport measurements[4] was confirmed in subsequent surface acoustic wave experiments where it was found that, contrary to the case of odd-denominator filling factors where the conductivity is reduced, at the half-filled Landau level the conductivity is in fact enhanced.[5] Earlier theoretical attempts to understand the nature of the $\nu=\frac{1}{2}$ state, largely pioneered by Halperin[6] and later by others,[7–9] remained mostly inconclusive. While it was known from those theoretical works that the $\frac{1}{2}$ state is compressible, the exact nature of the state remained unclear. For example, working with up to ten electrons in a periodic rectangular geometry and the exact diagonalization of the few-electron Hamiltonian in the lowest Landau level, Haldane[7] found that the excitation spectrum is particle number dependent, the ground-state energy was never at the zero total momentum (contrary to what one expects in a uniform-density liquid), and no clear physical picture could be extracted from those numerical results. The ground-state energy (in fact, the lowest energy) was also dependent on the electron number and extrapolation of the energies to an infinite system led to $E_0\approx -0.465e^2/\epsilon\ell_0$ (Ref. 3) [here $\ell_0=(\hbar c/eB)^{1/2}$ is the magnetic length]. The Laughlin wave function[2]

$$\psi_L=\prod_{i<j}(z_i-z_j)^m\exp\left\{-\sum_k |z_k|^2/4\ell_0^2\right\}, \qquad (1)$$

where $z=x+iy$ is the electron position and $\nu=1/m$ describes a system of particles obeying Bose statistics for $m=2$ and cannot be used for the fermion system without any further modification.

In order to explain the anomalous results at $\nu=\frac{1}{2}$, a very intriguing theory was proposed by Halperin, Lee, and Read (HLR).[10] This theory describes the compressible even-denominator states in terms of a transformation that represents each electron as a Chern-Simons fermion carrying an even number of fictitious magnetic flux quanta pointing in the direction opposite to the external magnetic field. In a mean-field approximation (no interparticle interaction) the average fictitious field cancels the real magnetic field and as a result the transformed fermions experience no net field. They then form a gapless Fermi liquid. Subsequent experimental observation[11] of the geometric resonance of the quasiparticle cyclotron orbits with the acoustic waves, and similar geometrical resonances found in antidot arrays, indicated the existence of a Fermi surface at $\nu=\frac{1}{2}$. These experiments provided strong support for the theoretical picture of HLR. However, fluctuations beyond the mean-field theory, which are essential to explain transport and other long-wavelength results and are expected just to renormalize the Fermi-liquid parameters, instead are found to cause divergences. The situation has not improved much.[12]

Parallel to the above approach, there is an ongoing effort to develop a microscopic approach to $\nu=\frac{1}{2}$ based on the idea of having an improved Laughlin-like wave function as a starting point. One way to do that is to include the antisymmetrization operator to the Laughlin state and have a trial wave function[13,14]

$$\Psi=\mathcal{P}_{LLL}\det M\prod_{i<j}(z_i-z_j)^2\exp\left\{-\sum_k |z_k|^2/4\ell_0^2\right\}. \qquad (2)$$

Here $\mathcal{P}_{LLL}$ is the lowest-Landau-level projection operator. The matrix M has elements that are plane waves, $M_{ij}=e^{i\mathbf{k}_i\cdot\mathbf{r}_j}$, $|\mathbf{k}|<k_F$. A plane-wave state is a reasonable choice for the bound state of zeros and electrons when the effective magnetic field is zero.[13] For $\nu=\frac{1}{2}$, the Fermi wave vector is $k_F=[4\pi n_e/s]^{1/2}=1/\sqrt{s}\ell_0$, where s is the spin degeneracy. For a fully spin-polarized system $s=1$. Because of the projection operator, $\overline{z_i}\rightarrow 2(\partial/\partial z_i)$ and therefore the plane-wave factors act as operators on the Jastrow factor where, as a result, the zeros of ψ_L are displaced.[13,14] The wave function (2) is supposed to have the right statistics and right correlations to describe the Fermi-liquid properties at $\nu=\frac{1}{2}$ and is found to provide a good description of a small-size system at $\nu=\frac{1}{2}$.[14] However, in those numerical studies of the few-electron systems the ''Fermi'' pair-correlation function on a sphere was found to have distinct long-range-type oscillations unlike the dominant short-range order present in a fluid and also not present in the Laughlin (''bo-

son'') state. Further, it is reasonable to question the reliability of a few-electron system result when we are to describe a gapless Fermi liquid. The pair-correlation function and the structure function for the state (2) are the most essential building blocks for any further development in the theory of a compressible fluid. The nature of the correlation functions in the thermodynamic limit, the effective mass, and collective excitations, which are related to those correlation functions, therefore needs careful attention.[13]

In this work we have attempted to fill in for some of those open questions by appealing to the ingenuity of the original Laughlin approach, where one is able to map the electrons onto a classical plasma and make use of the established formalism to calculate various physical quantities. To develop such a many-body scheme to deal with the wave function (2) that includes the projection operator is, however, a nontrivial problem. In our approach, we start with a simplified problem and drop the projection operator from Eq. (2): The wave function is then a simple product of the Slater determinant and the Laughlin function for the $\frac{1}{2}$ state,

$$\Psi_F = \det M \, \psi_L = \Phi \, \psi_L \,. \tag{3}$$

Our justification for that somewhat radical step is that in our choice of the trial wave function Ψ_F the only job of the Slater determinant Φ is to make Ψ_F antisymmetric. When $\Phi = 1$, the wave function describes the correlated boson (Laughlin) fluid and when $\psi_L = 1$ the wave function describes the noninteracting Fermi system. Although we no longer have any explicit projection to the lowest Landau level, the Laughlin wave function, in particular the analytic part of the wave function, already describes the correlations in the lowest Landau level. Also, since we are interested primarily in the correlation functions, structure functions, etc., it should perhaps still be an acceptable step to drop the projection operator, especially since the form of $|\Phi|^2$ is chosen to be of the same form as $|\psi_L|^2$ (see below). We wish to add here that for a Fermi liquid in the *absence* of a magnetic field, a division of labor as for the two functions in Eq. (3) is entirely justified.

Once the choice of the wave function (3) is made the next question is how we deal with Φ. We have already stated that we are mostly interested in the pair-correlation functions where information about $|\Phi|^2$ is all that needs to be known or, more specifically, we need to constrain $|\Phi|^2$ to be positive definite. One available choice in the literature[15] that was quite successful in describing the correlated electron systems in the absence of an external magnetic field is to write

$$\sum_{\sigma} |\Phi(\mathbf{r}_1, \ldots, \mathbf{r}_N)|^2 \approx \prod_{i<j} \phi^2(r_{ij}),$$

$$\phi(r) = e^{-u_I(r)/2}, \tag{4}$$

where the set of spin coordinates is denoted by σ. This means that we expand $|\Phi|^2$ and retain only the two-body term, which is then approximated by a Jastrow-type function. This allows us to write the square of the total wave function as

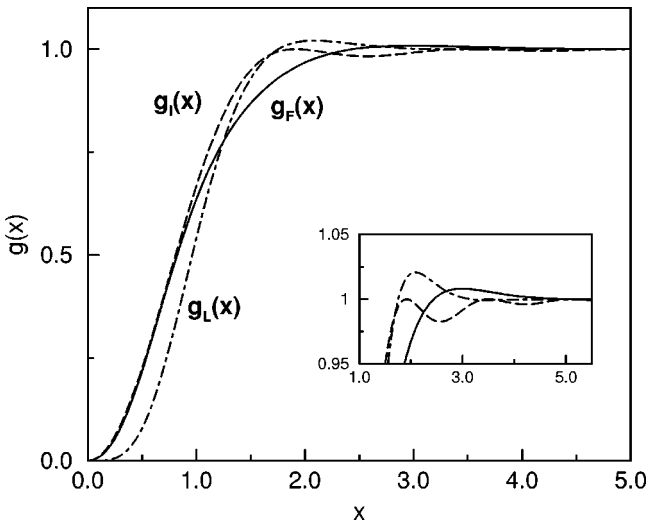

FIG. 1. Pair-correlation function $g(x)$ as a function of $x = r/R$ ($R = \sqrt{2m}\ell_0$) for the noninteracting system $g_I(x)$ [Eq. (7)], the Laughlin state (1) for a Bose system $g_L(x)$, and the Fermi state $g_F(x)$ [Eq. (3)]. A blowup of the region around unity is given in the inset.

$$|\Psi_F(\mathbf{r}_1, \ldots, \mathbf{r}_N)|^2 = \prod_{i<j} e^{-[u_L(r_{ij}) + u_I(r_{ij})]}$$

$$= \prod_{i<j} e^{-u_t(r_{ij})} \tag{5}$$

and the corresponding pair-correlation function

$$g_F(r_{12}) = N(N-1) n_e^2 \int d^2 r_3 \cdots d^2 r_N \times \exp\left(-\sum_{i<j} u_t(r_{ij}) \right) \Big/ \int d^{2N} r \times \exp\left(-\sum_{i<j} u_t(r_{ij}) \right), \tag{6}$$

where N is the particle number.

The advantage of our choice of Eq. (5) is that one can now use established methods such as the celebrated mapping of Laughlin's wave function (1) to a one-component classical plasma,[3,16] which determines the $u_L(r)$ or, equivalently, the pair-correlation function $g_L(r)$. In order to perform similar calculations for $u_t(r) = u_L(r) + u_I(r)$ we first have to determine $u_I(r)$ and then follow the plasma analogy to solve for $u_t(r)$. In a completely degenerate ideal (noninteracting) two-dimensional Fermi system the exact two-body radial distribution function $g_I(r)$ can be calculated to be[17]

$$g_I(r) = 1 - [2J_1(k_F r)/k_F r]^2, \tag{7}$$

where $J_1(k_F r)$ is the Bessel function of the first kind of order one. Since we are considering a fully spin polarized system, $g_I(r)$ vanishes at the origin due to the Pauli exclusion principle (Fig. 1). The corresponding ideal gas static structure function obtained from the two-dimensional Fourier transform of Eq. (7) is[17]

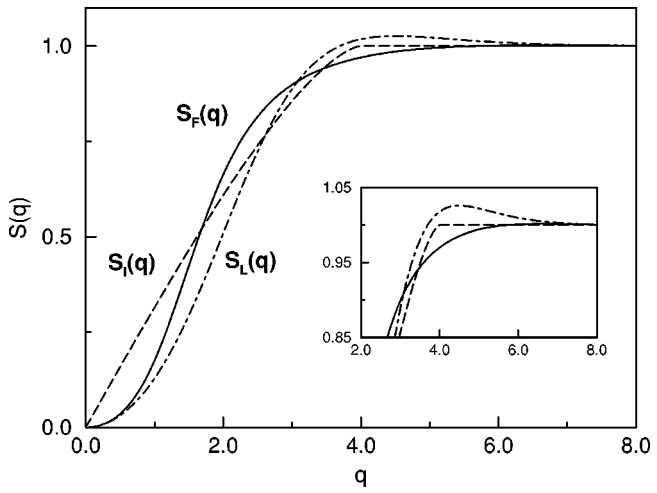

FIG. 2. Static structure factor $S(q)$ as a function of $q=kR$ for the noninteracting system $S_I(q)$ [Eq. (8)], the Laughlin state (1) for a Bose system $S_L(q)$, and the Fermi state $S_F(q)$ [Eq. (3)]. A blowup of the region around unity is given in the inset.

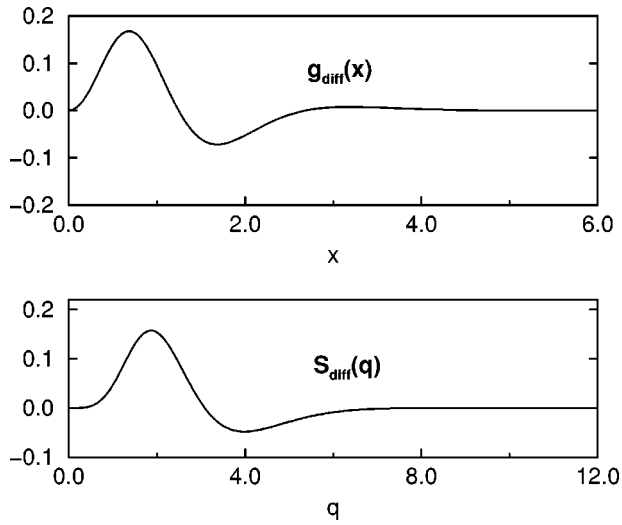

FIG. 3. Difference between the Fermi and Bose functions (a) $g_{diff}(x)$ vs x and (b) $S_{diff}(q)$ vs q.

$$S_I(\kappa)=\begin{cases}\dfrac{2}{\pi}[\sin^{-1}\kappa+\kappa(1-\kappa^2)^{1/2}], & \kappa<1\\ 1, & \kappa>1,\end{cases} \tag{8}$$

where $\kappa=k/2k_F$. For small κ, $S_I(\kappa)$ increases linearly with κ (Fig. 2). Given these two functions, $u_I(r)$ now can be obtained from inverting the hypernetted-chain (HNC) equations[15,3]

$$u_I(r)=-\ln g_I(r)+\frac{1}{n_e}\mathcal{F}\left\{\frac{[S_I(k)-1]^2}{S_I(k)}\right\}, \tag{9}$$

where $\mathcal{F}$ denotes the two-dimensional Fourier transform

$$F(k)=2\pi\int_0^\infty r\,dr\,F(r)J_0(kr),$$

$$F(r)=\frac{1}{2\pi}\int_0^\infty k\,dk\,F(k)J_0(kr),$$

where $J_0(kr)$ is the Bessel function of the first kind of order zero.

The implication of Eqs. (7)–(9) is that in the two-body level, $\phi(r)$ in Eq. (4) can be chosen such that we reproduce the exact two-body radial distribution function and the static structure factor corresponding to the *full determinant*. We note that $u_I(r)$ is a numerically strictly decreasing function, but it is a long-ranged function [because for small q, $\tilde{u}_I(q)\equiv\mathcal{F}(u_I(r))\sim q^{-1}$]. Therefore, one needs to pay particular attention to the long- and short-range behaviors of $u_t(r)$ (Ref. 15) while solving the HNC equations for the one-component plasma. In our numerical calculations we have used the dimensionless variables $x=r/R$ and $q=kR$, where $R=\sqrt{2m}\ell_0$ is the ion-disk radius.[3] The method of deriving the HNC equations for $u_t(x)$ is similar to that for $u_L(x)$ and with proper choice of the long- and short-range functions[3,15] a numerically rapidly convergent set of equations is obtained that leads to $g_F(x)$ and its Fourier transform $S_F(q)$. In the case of $u_I(x)=0$, the pair-correlation function $g_L(x)$ for the Laughlin state is plotted in Fig. 1. The ground-state energy corresponding to that state is $E_L=-0.480e^2/\epsilon\ell_0$. The pair-correlation function corresponding to state (3) is denoted by $g_F(x)$ in Fig. 1. Clearly, the ''Fermi hole'' is not much affected by the introduction of the Laughlin correlation function, but marked deviations of $g_F(x)$ from $g_I(x)$ occur near the maximum of $g_L(x)$.

The static structure functions $S(q)$ vs q for the various cases are shown in Fig. 2. As with the pair-correlation functions, $S_I(q)$ corresponds to the ideal system result, $S_F(q)$ is the present result for the Fermi-fluid state (3) at $\nu=\frac{1}{2}$, and $S_L(q)$ is the structure function for the Laughlin state (1). Interestingly, for small q we find that $S_F(q)$ is different from $\frac{1}{2}q^2$ and therefore the intra–Landau-level excitation spectrum in this case should not have a gap.[18] The difference between the present results and the Laughlin results, $g_{diff}=g_F(x)-g_L(x)$ and $S_{diff}=S_F(q)-S_L(q)$, is plotted in Fig. 3. These are oscillatory functions with a rapidly decreasing amplitude. In finite-size system calculations,[14] a sinusoidal oscillation in $g_{diff}(x)$ was taken as an indication of a Fermi-fluid behavior. We note that the correlation functions for the nonideal system show many fewer oscillations around unity in accordance with the properties of a uniform density fluid (inset of Fig. 1) and therefore the difference in correlation functions is also rapidly damped. Interestingly, $S_{diff}(k)$ develops a positive peak slightly below $k\sim 2k_F$ and a negative peak beyond that k. Finally, we find the ground-state energy for the state (3) and for the Coulomb potential to be $E_F=-0.448e^2/\epsilon\ell_0$, which is very different from the energy of the Laughlin state E_L but very close to the energy value E_0, extrapolated for an infinite system from the finite-size system results mentioned in the introduction. This agreement between the energy of the state (3) and the estimate E_0 is a strong indication that our Fermi-liquid description has the right correlations and correct statistics needed to describe a Fermi-liquid behavior at $\nu=\frac{1}{2}$.

In summary, our simplified choice (3) of the ground-state wave function for the Fermi-fluid state at $\nu=\frac{1}{2}$ has led to a microscopic approach where we can calculate the physical quantities in the thermodynamic limit. The pair-correlation

function, the structure function, and the ground-state energy are in good agreement with the expected behavior at $\nu=\frac{1}{2}$. This approach can also be suitably modified to calculate the one-body density matrix and the nature of the off-diagonal long-range order,[19] which will provide more information about the Fermi nature of the proposed state. In defense of our choice of Eq. (3), we should mention that the wave function that would result from the operation of $\mathcal{P}_{LLL}$ will be a wave function in the lowest Landau level and therefore be similar to the Laughlin-like wave function (but with correct statistics). Hence our choice of Eq. (5), which is formally similar to the Laughlin approach, should be a suitable approximation for the full wave function (2). This is supported by our numerical results presented here.

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