Comparison between two models of dephasing in mesoscopic systems

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Two phenomenological models have been proposed in mesoscopic systems to study the role of inelastic scattering on the phase coherent motion of electrons. In the first one, due to Büttiker, one adds a voltage probe into the system (or in the scattering matrix). The second model invokes the complex (or optical) potential in the system Hamiltonian. Studying a simple geometry of a metallic loop in the presence of Aharonov-Bohm magnetic flux, we show that the two probe conductance is symmetric in the reversal of the magnetic field in Büttiker's approach. Whereas the two probe conductance within the complex potential model is asymmetric in the magnetic flux reversal contrary to the expected behavior. [S0163-1829(98)00115-5]

During the last two decades, the study of transport in mesoscopic systems has been actively pursued owing to immense interest from technological as well as fundamental viewpoints.[1–8] Mesoscopic systems are structures made of metallic or semiconducting material on a nanometer scale. The length scale associated with the dimensions in these systems is much smaller than the inelastic mean free path or the phase breaking length (L_ϕ). The phase breaking length (or phase coherence length) L_ϕ is the average diffusion length between the two inelastic collisions. Typically L_ϕ scales with the temperature T in a power-law form, i.e., $L_\phi = T^{-p}$ (p lies in the range 1–2). At low enough temperature when the system size L is much smaller than the phase breaking length L_ϕ, an electron maintains phase coherence across the entire sample. The mesoscopic sample should be treated as a quantum scatterer. Here shape of the sample, quantization of energy levels, and discreteness of charge play a major role. Thus mesoscopic systems have provided an opportunity for exploring truly quantum mechanical effects beyond atomic realm. In the quantum-phase coherent transport regime classical Ohm's law breaks down[6] in the sense that if one adds two resistors having resistance R_1 and R_2 in series then the total resistance R of the system is no longer a sum of the two resistances R_1 and R_2 ($R \neq R_1 + R_2$). Apart from this in lower dimensions resistance is a non-self-averaging quantity in that the resistance fluctuations over the ensemble of macroscopically identical samples dominates the ensemble average.[6,9,10] The quantum resistance of a sample depends on the details of the relative position of scatterers. Thus the mesoscopic system is characterized by the sample specific global resistance. However, as the temperature increases, inelastic scattering effects start dominating, leading to the loss of phase coherence. If the sample size is larger than L_ϕ the sample breaks up dynamically into mutually incoherent domains of size L_ϕ, with transport within each domain remaining phase coherent. Here the self-averaging property of the resistance is automatically realized and classical additivity of resistance is restored, i.e., $R = R_1 + R_2$. In the phase coherent transport regime several, often counterintuitive, new experimental results have been obtained[1–8] and have been successfully explained within a Landauer–Büttiker formalism for dc transport.[11,12]

Although experiments on mesoscopic samples confirm the predictions based on the phase coherent transport theory, a quantitative comparison at finite temperatures requires the loss of phase coherence be included in the theory. There are two widely used phenomenological models that have been proposed for this purpose. In the first method due to Büttiker, one introduces a fictitious voltage probe in the scattering matrix.[6,13,14] The voltage probe breaks the phase coherence by removing electrons from the phase coherent motion in the mesoscopic system and subsequently reinjecting them without any phase relationship. The treatment based on the voltage probe method (which serves as an inelastic scatterer) has been extended to include the realistic physics of inelastic processes occurring uniformly in space.[6] To simulate inelastic scattering another method makes use of complex (or optical) potentials.[15–17] In that case the Hamiltonian becomes non-Hermitian and thus the particle number is not conserved. In these studies the absorption is identified as the spectral weight lost in the inelastic channels. As an example, in the case of one-dimensional double barrier structures the absorbed part is assumed to tunnel through both the left and the right hand sides of the barriers in proportion to the transmission coefficient of each barrier, and this is added to the coherent transmission to get the overall transmission coefficient.[16] It should be noted that in the presence of imaginary potentials the temporal coherence of the wave is preserved in spite of absorption, which causes a particle nonconserving scattering process. The absorption is to be understood as a depletion of the coherent amplitude by the inelastic process. Problems related to the use of complex potentials have been discussed in the earlier literature.[18–20] A recent study identifies the limit in which these two models of dephasing are equivalent and the distribution of conductance in that limit has been calculated.[17]

In our present study we analyze both these models in the presence of magnetic flux, and show that these two models lead to qualitatively different results for the symmetry of the two-probe conductance in the presence of magnetic field. In Büttiker's approach of voltage probe, two probe conductance is symmetric in the reversal of magnetic field as has been observed experimentally.[6] However, the model based on the complex potential makes the two-probe conductance asymmetric in the magnetic field reversal contrary to the expected behavior.[6]

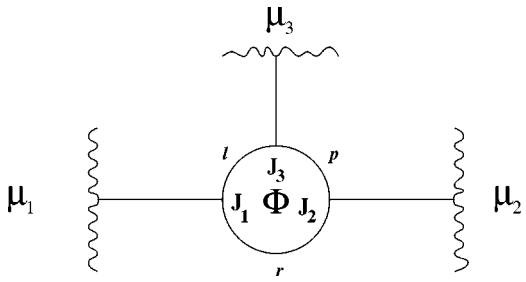

FIG. 1. A metallic loop connected to three reservoirs in the presence of magnetic flux Φ.

To this end we consider a simple geometry of a one-dimensional metallic ring in the presence of Aharonov-Bohm (AB) flux as shown in Figs. 1 and 2. In Fig. 1 we have attached an additional lead at point x on the upper arm of the loop that acts as a voltage probe. In Fig. 2 we have introduced a δ-function optical potential of strength iV at the same point x in the upper arm that acts as an absorber. The length of the upper arm is $l+p$ and that of the lower arm is r. The total circumference of the loop is $L=l+p+r$.

The two-probe phase coherent conductance of a mesoscopic sample at zero temperature is given by the Landauer formula[11]

$$G=\left(\frac{e^2}{h}\right)T, \tag{1}$$

where T is the transmission probability for carriers to traverse the sample. Here the transmission probability is taken at the Fermi energy. From Eq. (1) we introduce a dimensionless conductance $g=(h/e^2)G=T$.

To study the effect of dephasing in the presence of AB flux via Büttiker's approach we consider a mesoscopic open ring connected to three electron reservoirs at chemical potential μ_1, μ_2, and μ_3 as shown in Fig. 1. An AB flux ϕ is present at the center. We focus on the situation when the third lead is used as a voltage probe to measure the chemical potential μ_3. The net current in the third lead is zero. If we denote transmission probabilities of carriers incident in lead j to reach lead i by T_{ij} $(i,j=1,2,3)$, then the two-probe conductance (in the dimensionless units) of the AB ring is given by[6]

$$g_B=T_{21}+\frac{T_{31}T_{23}}{T_{31}+T_{32}}. \tag{2}$$

We see that the two port conductance is a sum of two parts. The first part T_{21} arises due to those electrons that traverse the ring without ever entering into the third reservoir; this corresponds to the elastic transmission probability. The second part, i.e., $T_{31}T_{23}/(T_{31}+T_{32})$, describes electrons that

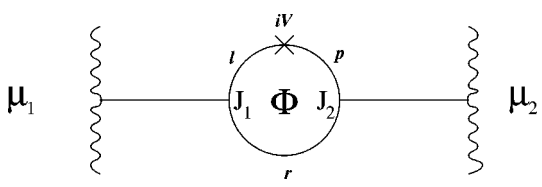

FIG. 2. A metallic loop connected to two reservoirs in the presence of a magnetic flux Φ and a δ-function imaginary potential iV.

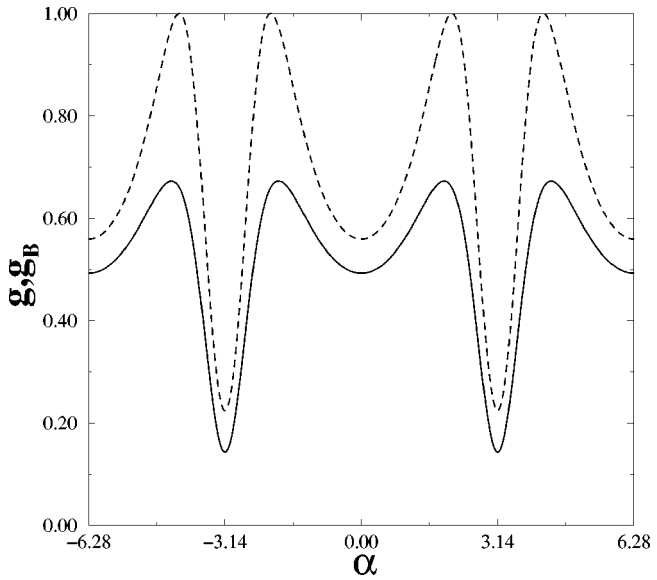

FIG. 3. The plots of g (dotted line) and g_B (solid line) vs $\alpha=2\pi\phi/\phi_0$.

emanate from port 1, reach reservoir 3 where their energy and phases are randomized, and from reservoir 3 in an additional step reach reservoir 2. In this sense the third lead connected to the reservoir acts like an inelastic scatterer.

In the presence of absorbing potential (Fig. 2) the sum of transmission (T) and reflection coefficient (R) is not unity. The absorption coefficient is given by $A=1-T-R$. In this model of dephasing the absorbed part is assumed to be re-emitted to the right and left in proportion to the transmission coefficient at the right and left hand side of the absorber. In our case absorbed flux of particles is re-emitted equally on both sides of the absorber and consequently the dimensionless conductance in this model is given by

$$g_i=T+A/2. \tag{3}$$

To calculate conductances g, g_B, and g_i we need to know transmission and reflection coefficients. To calculate them we follow our earlier method of quantum waveguide transport on networks.[21–24] Our calculation is for a noninteracting system of electrons. We set units of h, e, and m to be unity. We do not assume any particular form for the scattering matrix for the junctions J_1, J_2, and J_3, but scattering at junctions follows from first principles using quantum mechanics. We have imposed Griffith's boundary conditions (conservation of current) and single valuedness of the wave functions at the junctions. After calculating different transmission and reflection coefficients we substitute them back into Eqs. (2) and (3) to get the analytical expressions for the conductances g_B and g_i. However, the analytical expressions are too long to be reproduced here. In the following we present our results graphically.

In Fig. 3 we plot the dimensionless conductance g (dotted line) and g_B (solid line) as a function of the dimensionless flux $\alpha=2\pi\phi/\phi_0$, where $\phi_0=hc/e$ is the elementary flux quantum. We choose $kL=5$, $l/L=0.15$, $p/L=0.3$, and $r/L=0.55$. Both g and g_B oscillate with a period ϕ_0 and are symmetric with respect to the field reversal as expected for the two-probe conductance. As g_B includes the effect of

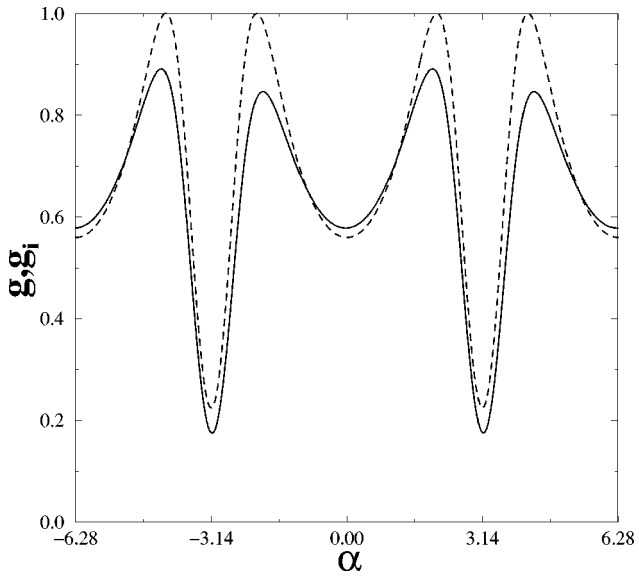

FIG. 4. The plots of g (dotted line) and g_i (solid line) vs $\alpha = 2\pi\phi/\phi_0$.

dephasing due to an additional voltage probe, the amplitude of oscillations in g_B is smaller than that observed for g, the phase coherent conductance. This is expected from the fact that inelastic effects reduce the amplitude of conductance oscillations (or interference effects).[7]

In Fig. 4 we plot g (dotted line) and g_i (solid line) as a function of α. The length parameter values are the same as used for Fig. 3. The strength of the imaginary potential in evaluating g_i is taken to be $VL = 3$. Both g and g_i are periodic in flux with a period ϕ_0. The amplitude of oscillations in g_i is smaller than that observed for g. However, g_i is asymmetric in field reversal in contrast to the expected behavior. This also follows from our analysis of the symmetries of the transmission and reflection coefficients under the field reversal in the presence of complex potential (iV), namely, $T(V,\phi) \neq T(V,-\phi)$ and $R(V,\phi) = R(V,-\phi)$.

In conclusion we have compared two phenomenological models for dephasing in mesoscopic systems in the presence of Aharonov-Bohm flux. The model due to Büttiker based on addition of voltage probe to simulate inelastic scattering leads to the two-probe conductance that is symmetric in magnetic field. On the other hand the model based on the use of the complex potential leads to two-probe conductance, which is asymmetric in the magnetic field, contrary to the expectation based on experimental as well as theoretical predictions. We would also like to emphasize that the use of imaginary potentials is justified in the case of optical wave propagation in an absorbing or a lasing medium (random dielectric media). In the electromagnetic wave propagation the bosonic nature of light quanta (photons) brings in both features, namely, amplification as well as attenuation as the photon number is not conserved.[25–27]

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