

Moving vortex line: Electronic structure, Andreev scattering, and Magnus force

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The wave functions of quasiparticles in a vortex line, moving with velocity \vec{v}_L relative to the lattice when a transport current with drift velocity \vec{v}_T is applied, are calculated by solving the time-dependent Bogoliubov–de Gennes equations for a high- κ superconductor in contact with a reservoir of chemical potential μ . Far away from the vortex core the pair potential has the constant modulus Δ_∞ . Comparison with the wave functions of a vortex at rest shows that vortex motion modifies the amplitudes, the radial wave numbers of the states with energy $E > \Delta_\infty$, and the penetration lengths of states with energy $E < \Delta_\infty$ by a term $\pm \varepsilon_v \cos \Theta$. Here Θ is the azimuthal angle of cylinder coordinates with the z direction parallel to the vortex axis, and $\varepsilon_v = \hbar k_\rho v$; $v = |\vec{v}_T - \vec{v}_L|$ and $k_\rho = \sqrt{(2m/\hbar^2)\mu - k_z^2}$, with k_z being the wave number of propagation in the z direction. If one neglects terms of the order of ε_v^2 in the spectrum of the bound states, one obtains the same eigenvalues as for the vortex at rest. The supercurrent force on the corresponding quasiparticles, caused by Andreev scattering at the core boundary, is calculated with the v -modified wave functions. It transfers half of the Magnus force from the moving condensate to the unpaired quasiparticles in the vortex core. [S0163-1829(98)05113-3]

I. INTRODUCTION

The motion of vortex lines in type-II superconductors dissipates energy, because the unpaired electrons in the vortex core are subject to frictional forces from the lattice. This limits the technological applications of conventional and especially high-temperature superconductors (HTSC's). Discovery of the HTSC's has stimulated anew intensive research in vortices and their motion, documented in recent reviews.^{1–3} While being disadvantageous from a technological point of view, vortex motion has challenged experimental and theoretical physicists with a wealth of interesting problems. One of the oldest is the question, which force acting on a moving vortex line is in balance with the frictional force \vec{F} from the lattice? Different answers have been given by the theories of Bardeen and Stephen (BS),⁴ on the one hand, and Nozières, Vinen, and Warren (NVW),^{5,6} on the other hand. While NVW show that this force should be the Magnus force,

$$-\vec{F} = \vec{F}_{\text{Magnus}} = n_s e L (\vec{v}_T - \vec{v}_L) \times \vec{\Phi}, \quad (1)$$

which has been confirmed recently by Berry-phase considerations,^{7–9} BS have instead

$$-\vec{F} = -n_s e L \vec{v}_L \times \vec{\Phi}; \quad (2)$$

here n_s is the density of the superconducting electrons, $e = -|e|$ is the charge of an electron, L is the length of the vortex line which is assumed to be parallel to the z axis, \vec{v}_T is the drift velocity of the applied transport supercurrent, \vec{v}_L is the velocity of the vortex line, and $\vec{\Phi} = \vec{e}_z \Phi_0$, with $\Phi_0 = h/2|e|$ being the flux quantum.

The basic difference between the two theories consists in an ‘‘interface force,’’ introduced phenomenologically in the NVW theory by momentum-balance arguments and absent in the BS theory. It is required for the complete transfer of the Magnus force from the superconducting condensate outside

the core to the unpaired electrons inside the core which are in equilibrium with the lattice. Nozières and Vinen state, ‘‘It is essential to know, how the Magnus force is shared between the bulk of the core and the ‘interface’ with the superfluid.’’⁵ The bulk share turned out to be half of the Magnus force.^{5,6} It originates from the electrostatic field in the core of radius r_c :⁶

$$\vec{E}_c = \frac{1}{2\pi r_c^2} (\vec{v}_L - 2\vec{v}_T + \vec{v}_{nc}) \times \vec{\Phi} = \frac{1}{2\pi r_c^2} (\vec{v}_L - \vec{v}_T) \times \vec{\Phi}. \quad (3)$$

The second equality holds if no pinning centers inhibit the motion of the vortex so that the drift velocity inside the core \vec{v}_{nc} is equal to \vec{v}_T . (\vec{v}_T is \vec{v}_{s1} in Ref. 6.) The interface force, however, which has to provide the other half of the Magnus force,⁶ has remained unclear in its physical nature until recently. In a previous Letter¹⁰ we have shown by a semiclassical calculation that this interface force is the result of momentum transfer from the Cooper pairs in the condensate to the core electrons by Andreev scattering at the core boundary. It is a special and important manifestation of a second force, called supercurrent force $\vec{f}_{\Delta 2}$,^{11,12} involved in Andreev scattering.¹³ This force is the fourth member of the full set of forces acting on quasiparticles in an inhomogeneous, current-carrying superconductor with pair potential $\Delta(\vec{r}) = |\Delta(\vec{r})|e^{i\varphi(\vec{r})}$, vector potential \vec{A} , and scalar potential V . These forces are obtained from the time-dependent Bogoliubov–de Gennes equations (TdBdGE's) (Refs. 14 and 15) and the resulting generalized Ehrenfest theorem for quasiparticles with electron component $u(\vec{r}, t)$ and hole component $v(\vec{r}, t)$.¹¹ Accordingly, the total force on a quasiparticle, defined as

$$\vec{f} \equiv \frac{d}{dt} \int d^3r \psi^\dagger(\vec{r}, t) \hat{p} \psi(\vec{r}, t), \quad (4)$$

with

$$\psi(\vec{r}, t) = \begin{pmatrix} u(\vec{r}, t) \\ v(\vec{r}, t) \end{pmatrix} \quad (5)$$

being a solution of the TdBdGE's, and

$$\hat{p} \equiv \begin{pmatrix} \vec{p}_e & 0 \\ 0 & \vec{p}_h \end{pmatrix} \equiv \begin{pmatrix} \left[\frac{\hbar}{i} \vec{\nabla} - e\vec{A}(\vec{r}, t) \right] & 0 \\ 0 & \left[\frac{\hbar}{i} \vec{\nabla} + e\vec{A}(\vec{r}, t) \right] \end{pmatrix} \quad (6)$$

being the quasiparticle momentum operator, is the sum of the four forces:

$$\vec{f} = \vec{f}_e + \vec{f}_h + \vec{f}_{\Delta 1} + \vec{f}_{\Delta 2}. \quad (7)$$

It consists of the usual diagonal electromagnetic forces on electrons (e) and holes (h),

$$\vec{f}_e = \int d^3 r u^* \left[\frac{e}{2m} (\vec{p}_e \times \vec{B} - \vec{B} \times \vec{p}_e) - e \frac{\partial}{\partial t} \vec{A} - \vec{\nabla} V \right] u, \quad (8)$$

$$\vec{f}_h = \int d^3 r v^* \left[\frac{e}{2m} (\vec{p}_h \times \vec{B} - \vec{B} \times \vec{p}_h) + e \frac{\partial}{\partial t} \vec{A} + \vec{\nabla} V \right] v, \quad (9)$$

and two off-diagonal pair potential forces

$$\vec{f}_{\Delta 1} = -2 \text{Re} \int d^3 r u^* v (\vec{\nabla} |\Delta|) e^{i\varphi}, \quad (10)$$

$$\vec{f}_{\Delta 2} = 4 \frac{m}{\hbar} \int d^3 r v_s \text{Im}(u^* v \Delta). \quad (11)$$

Here $\vec{B} = \vec{\nabla} \times \vec{A}$, and

$$\vec{v}_s = \frac{\hbar}{2m} \left[\vec{\nabla} \varphi - \frac{2e}{\hbar} \vec{A} \right] \quad (12)$$

is the gauge-invariant Cooper pair velocity. The off-diagonal force $\vec{f}_{\Delta 1}$ is responsible for the change of quasiparticle momentum from a value above to a value below the Fermi surface (or vice versa) in usual Andreev scattering. The second off-diagonal force $\vec{f}_{\Delta 2}$, which is present if there is a supercurrent flowing with Cooper pair velocity \vec{v}_s , is the new supercurrent force. In this paper we present a complete quantum mechanical calculation of this supercurrent force by solving the time-dependent Bogoliubov–de Gennes equations for an isolated moving vortex line and show that the screening current around the vortex line and the modification of the quasiparticle wave functions by the motion of the vortex relative to the superconducting condensate result indeed in half of the Magnus force.

Šimánek¹⁶ has calculated the force on the center of a moving vortex line in the adiabatic limit with the help of the instantaneous eigenstates of the superfluid and the Bogoliubov transformation of the many-body Hamiltonian. The wave functions in this transformation are the solutions of the stationary Bogoliubov–de Gennes equations. The way they enter Šimánek's equations (5), (10), and (15) shows that his

force “obtained in a form that involves virtual transitions between the core levels around the Fermi level” is due to the same physical mechanisms, i.e., Andreev scattering and supercurrent force, which give rise to

$$\vec{f}_{\Delta} \equiv \vec{f}_{\Delta 1} + \vec{f}_{\Delta 2} = \frac{2}{\hbar} \text{Im} \int d^3 r u^* v \left(\frac{\hbar}{i} \vec{\nabla} - 2e\vec{A} \right) \Delta. \quad (13)$$

The sum of \vec{f}_{Δ} over all occupied quasiparticle states yields the total off-diagonal force on the vortex. Because of symmetry reasons, only $\vec{f}_{\Delta 2}$ has a component perpendicular to $\vec{v}_T - \vec{v}_L$ and may therefore be involved in the Magnus-force transfer across the core boundary. Therefore, it is sufficient to calculate $\vec{f}_{\Delta 2}$.

A fringe benefit of our going beyond the adiabatic limit in solving the TdBdGE's is the discovery of a new sort of quasiparticle states. They may be called “angular bound states,” because in two angular ranges their wave functions decay exponentially outside the core, and in the two complementary angular ranges they spread as undamped scattering states.

The paper is organized as follows. In Sec. II, by a Galilei transformation, the TdBdGE's in the lattice frame of reference (LFR) are changed into stationary BdGE's in the vortex frame of reference (VFR). In Sec. III we study the asymptotic behavior of the quasiparticle wave functions of the vortex system and discuss the three types of quasiparticle states: bound, scattering, and angular bound states. Section IV describes how matching of the solutions outside the core to the ones inside the core is achieved with the help of a Fourier transformation of the angle-dependent matching conditions. The complete, velocity-dependent wave functions for the case of a moving vortex with a square-well pair potential are presented. With these wave functions we calculate the supercurrent-force contribution to the Magnus force in Sec. V and find that it is equal to one-half of the Magnus force, if the normal vortex core has a diameter of about one coherence length. A summary and outlook concludes the paper.

II. GALILEI TRANSFORMATION OF THE TDBDGE'S

We consider a low- T_c , high- κ type-II superconductor. In the absence of external fields its pair potential is homogeneous, isotropic, and small compared to the Fermi energy. In this superconductor vortices are induced by an applied magnetic field B (parallel to the z direction) which satisfies $B_{c1} < B \ll B_{c2}$, so that vortex-vortex interactions can be ignored, and it is sufficient to consider an isolated vortex. This vortex is driven by an applied supercurrent with constant drift velocity \vec{v}_T . It moves with constant velocity \vec{v}_L . Both velocities lie in the x - y plane:

$$\vec{v}_T = v_{Tx} \vec{e}_x + v_{Ty} \vec{e}_y, \quad (14)$$

$$\vec{v}_L = v_{Lx} \vec{e}_x + v_{Ly} \vec{e}_y. \quad (15)$$

The system is described by the mean-field TdBdGE's,^{14,15} with a pair potential which in the lattice frame of reference and the corresponding coordinates \vec{r}' and t' is given by

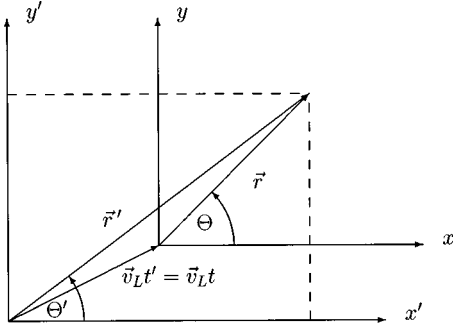


FIG. 1. Frames of reference: the coordinates (\vec{r}', t') with the Cartesian components (x', y') and the angle Θ' correspond to the lattice frame of reference (LFR) while the coordinates (\vec{r}, t) with the Cartesian components (x, y) and the angle Θ correspond to the vortex frame of reference (VFR) which moves with velocity \vec{v}_L relative to the LFR.

$$\tilde{\Delta}(\vec{r}', t') = \Delta(\vec{r}' - \vec{v}_L t'). \quad (16)$$

Here

$$\Delta(\vec{r}) = \Delta(r, \Theta) = \Delta_0(r) e^{-i\Theta} \quad (17)$$

is the pair potential of a vortex at rest; i.e., the coordinates (\vec{r}, t) are those of a frame of reference fixed to the vortex. The radial part $\Delta_0(r)$ of the pair potential vanishes at the center of the vortex and assumes the constant value Δ_∞ far from the core. The phase of the pair potential furnishes the superfluid velocity

$$\vec{v}_{s,0} = -\frac{\hbar}{2m\vec{r}} \vec{e}_\Theta \quad (18)$$

of the screening current.

With Eqs. (16) and (17) and Fig. 1 the pair potential in the lattice frame of reference has the form

$$\begin{aligned} \tilde{\Delta}(\vec{r}', t') = \Delta_0 & \left[\sqrt{(r' \cos \Theta' - v_{Lx} t')^2 + (r' \sin \Theta' - v_{Ly} t')^2} \right] \\ & \times \exp \left(-i \arctan \frac{r' \sin \Theta' - v_{Ly} t'}{r' \cos \Theta' - v_{Lx} t'} \right). \end{aligned} \quad (19)$$

The magnetic field of the vortex may be neglected^{5,17,18} as well as the electric field induced by the motion of the vortex.^{5,6,19} The chemical potential μ in the TdBdGE's is that of the energy and particle reservoir to which the superconductor is coupled. We neglect all influences of entropy production associated with vortex motion on the chemical potential, because the number of degrees of freedom of the reservoir is assumed to be very much larger than that of the superconductor.¹⁵ Then μ is the same as in the case of no vortex motion and equal to the Fermi energy ε_F of the superconductor. The vector potential in the LFR is determined by considering a region far away from the vortex core. There $\vec{v}_{s,0}$ of Eq. (18) vanishes, and so does $\vec{\nabla} \varphi$ in Eq. (12), and $\vec{v}_s = \vec{v}_T$. With that Eq. (12) turns into

$$\vec{A} = -\frac{m}{e} \vec{v}_T \equiv \vec{A}. \quad (20)$$

Thus, in the LFR the TdBdGE's of an isolated moving vortex line are

$$i\hbar \frac{\partial}{\partial t'} \tilde{\Psi}(\vec{r}', t') = \tilde{H}(\vec{r}', t') \tilde{\Psi}(\vec{r}', t'). \quad (21)$$

$\tilde{\Psi}(\vec{r}', t')$ is the spinor quasiparticle wave function (with electron component \tilde{u} and hole component \tilde{v}) in the LFR, and the matrix Hamiltonian

$$\tilde{H}(\vec{r}', t') = \begin{pmatrix} \tilde{H}_e(\vec{r}') & \tilde{\Delta}(\vec{r}', t') \\ \tilde{\Delta}^*(\vec{r}', t') & -\tilde{H}_e^*(\vec{r}') \end{pmatrix}, \quad (22)$$

contains the pair potential of Eq. (19) and the single-electron Hamiltonian

$$\begin{aligned} \tilde{H}_e(\vec{r}') &= \frac{1}{2m} (-i\hbar \vec{\nabla}' - e\vec{A})^2 - \varepsilon_F \\ &= \frac{1}{2m} (-i\hbar \vec{\nabla}' + m\vec{v}_T)^2 - \varepsilon_F. \end{aligned} \quad (23)$$

In order to avoid the rather complicated structure (19) of the pair potential in the LFR we now switch over to the VFR by means of the Galilei transformation

$$\vec{r}' = \vec{r} + \vec{v}_L t, \quad t' = t. \quad (24)$$

This transformation shifts the pair potential $\tilde{\Delta}(\vec{r}', t')$ back to the pair potential $\Delta(\vec{r})$ of the vortex at rest. The wave functions $\Psi(\vec{r}, t)$ in the VFR follow from the LFR wave functions $\tilde{\Psi}(\vec{r}', t')$ by the same transformation:

$$\Psi(\vec{r}, t) \equiv \tilde{\Psi}(\vec{r} + \vec{v}_L t, t). \quad (25)$$

Applying the Galilei transformation to the differential operators,

$$\vec{\nabla}' = \vec{\nabla}, \quad (26)$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} - v_{Lx} \frac{\partial}{\partial x} - v_{Ly} \frac{\partial}{\partial y}, \quad (27)$$

we transform the TdBdGE (21) to the VFR:

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t) = \begin{pmatrix} H_e^0(\vec{r}) + H_1(\vec{r}) & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & -H_e^0(\vec{r}) + H_1(\vec{r}) \end{pmatrix} \Psi(\vec{r}, t). \quad (28)$$

Here H_e^0 and H_1 are, appropriate to the symmetry of our problem, written in cylinder coordinates (r, Θ, z) ,

$$\begin{aligned} H_e^0(r, \Theta, z) &= -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \Theta^2} + \frac{\partial^2}{\partial z^2} \right) \\ &\quad - \left(\varepsilon_F - \frac{1}{2} m v_T^2 \right) \end{aligned} \quad (29)$$

and

$$H_1(r, \Theta, z) = -i\hbar \left[(-v_x \sin \Theta + v_y \cos \Theta) \frac{1}{r} \frac{\partial}{\partial \Theta} + (v_x \cos \Theta + v_y \sin \Theta) \frac{\partial}{\partial r} \right], \quad (30)$$

where the definitions

$$\vec{v} \equiv \vec{v}_T - \vec{v}_L, \quad v \equiv |\vec{v}|, \quad (31a)$$

$$v_x \equiv v_{Tx} - v_{Lx}, \quad v_y \equiv v_{Ty} - v_{Ly} \quad (31b)$$

have been introduced. From now on we assume, without loss of generality, $v_y = 0$ and therefore $v = v_x$.

Since the Hamiltonian in Eq. (28) does not depend explicitly on time t and the z coordinate, the ansatz

$$\Psi(\vec{r}, t) = \begin{pmatrix} u(r, \Theta) \\ v(r, \Theta) \end{pmatrix} e^{-iEt/\hbar} e^{ik_z z} \quad (32)$$

leads to the stationary Bogoliubov–de Gennes equations for the electron and hole components $u(r, \Theta)$ and $v(r, \Theta)$ of the radial and angular wave functions:

$$E \begin{pmatrix} u(r, \Theta) \\ v(r, \Theta) \end{pmatrix} = \begin{pmatrix} H_e^0(r, \Theta) + H_1(r, \Theta) & \Delta(r, \Theta) \\ \Delta^*(r, \Theta) & -H_e^0(r, \Theta) + H_1(r, \Theta) \end{pmatrix} \begin{pmatrix} u(r, \Theta) \\ v(r, \Theta) \end{pmatrix}. \quad (33)$$

Here the pair potential is given by Eq. (17), and

$$H_e^0(r, \Theta) = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \Theta^2} \right) - \varepsilon_\rho, \quad (34)$$

$$\varepsilon_\rho \equiv \varepsilon_F - \frac{1}{2} m v_T^2 - \frac{\hbar^2 k_z^2}{2m}, \quad (35)$$

$$H_1(r, \Theta) = -i\hbar v \left[-\sin \Theta \frac{1}{r} \frac{\partial}{\partial \Theta} + \cos \Theta \frac{\partial}{\partial r} \right]. \quad (36)$$

Apart from the small term $\frac{1}{2} m v_T^2$, which can be disregarded, $H_e^0(r, \Theta)$ is the same as for a vortex at rest. The effects of the applied supercurrent as well as of the motion of the vortex are contained in the term $H_1(r, \Theta)$. Note that this term depends only on the relative velocity $\vec{v} = \vec{v}_T - \vec{v}_L$ between vortex and superconducting condensate, exactly as the Magnus force, Eq. (1), does. For a vortex at rest and not immersed in an applied supercurrent the term $H_1(r, \Theta)$ vanishes as well as the Magnus force. The same is true for a vortex drifting along with the condensate ($\vec{v}_L = \vec{v}_T$).

If the vortex is immersed in an applied supercurrent but does not drift along with it ($\vec{v}_L \neq \vec{v}_T$), $H_1(r, \Theta)$ must be taken into account. In this case, the wave functions will not be the same as those of the vortex at rest. Moreover, the term $H_1(r, \Theta)$ breaks the symmetry of the problem: The wave functions can no longer be separated into radial and angular functions. This drastically complicates the problem of solving the BdGE's. Fortunately, the solutions in the asymptotic limit $r \rightarrow \infty$ open up a way which will lead us in a sufficiently good approximation to the full solutions to be used in Eq. (11). The two steps on this way, taken in Secs. III and IV, are guided by the following considerations.

The dynamic, symmetry-breaking quantities which generate the supercurrent force are the superfluid velocity \vec{v}_{s0} of the screening current, Eq. (18), and the vortex velocity \vec{v} relative to the condensate, Eq. (31). In the asymptotic limit \vec{v}_{s0} becomes vanishingly small so that \vec{v} dominates in the asymptotic wave functions. These are calculated in Sec. III.

Then, in Sec. IV, the influence of \vec{v}_{s0} is taken into account by functions which are independent of \vec{v} and multiply the \vec{v} -dependent asymptotic wave functions. Each such product represents a solution of the BdGE's outside the vortex core. Appropriate matching to the solutions inside the core will finally provide the energy eigenvalues and wave functions of the bound states needed for the calculation of the supercurrent force.

III. ASYMPTOTIC WAVE FUNCTIONS

A. Nonlinear differential equations and proper solutions

The electron and hole components $u^A(r, \Theta)$ and $v^A(r, \Theta)$ of the asymptotic wave functions are solutions of the asymptotic BdGE's which result from Eq. (33) for $r \rightarrow \infty$:

$$(H_e^0 + H_1 - E)u^A(r, \Theta) + \Delta_\infty e^{-i\Theta} v^A(r, \Theta) \xrightarrow{r \rightarrow \infty} 0, \quad (37)$$

$$(-H_e^0 + H_1 - E)v^A(r, \Theta) + \Delta_\infty e^{i\Theta} u^A(r, \Theta) \xrightarrow{r \rightarrow \infty} 0, \quad (38)$$

where $\Delta_\infty \equiv \Delta_0(r \rightarrow \infty)$. In Eqs. (37) and (38) only those terms are kept which do not vanish for $r \rightarrow \infty$.

Inspired by the asymptotic solutions of the BdGE's for a vortex at rest,^{17,18} we start with the ansatz

$$u_{\nu,n}^A(r, \Theta) = e^{in\Theta} e^{ik_\nu(\Theta)r}, \quad (39)$$

$$v_{\nu,n}^A(r, \Theta) = c_\nu(\Theta) e^{i(n+1)\Theta} e^{ik_\nu(\Theta)r}. \quad (40)$$

Here n is an integer and ν will be explained later. The functions $u_{\nu,n}^A(r, \Theta)$ and $v_{\nu,n}^A(r, \Theta)$ differ from the solutions for a vortex at rest in so far, as they include the angular-dependent functions $c_\nu(\Theta)$ and $k_\nu(\Theta)$ in places, where the latter ones have constants. Finding the functions $c_\nu(\Theta)$ and $k_\nu(\Theta)$ is equivalent to finding the asymptotic wave functions $u_{\nu,n}^A(r, \Theta)$ and $v_{\nu,n}^A(r, \Theta)$. We are looking for those solutions which in the limit $v \rightarrow 0$ turn into the known asymptotic solutions of the vortex at rest.

Inserting the ansatz of Eqs. (39) and (40) into the asymptotic BdGE's (37) and (38) we find after some algebra the coupled nonlinear differential equations for $k_\nu(\Theta)$ and $c_\nu(\Theta)$,

$$0 = \frac{\hbar^2}{2m} \{k_\nu^2(\Theta) + [k'_\nu(\Theta)]^2\} - \varepsilon_\rho - \hbar v [k'_\nu(\Theta) \sin \Theta - k_\nu(\Theta) \cos \Theta] - E + \Delta_\infty c_\nu(\Theta), \quad (41)$$

$$0 = -\frac{\hbar^2}{2m} \{k_\nu^2(\Theta) + [k'_\nu(\Theta)]^2\} + \varepsilon_\rho - \hbar v [k'_\nu(\Theta) \sin \Theta - k_\nu(\Theta) \cos \Theta] - E + \Delta_\infty c_\nu^{-1}(\Theta), \quad (42)$$

where the slash denotes differentiation with respect to Θ :

$$k'_\nu(\Theta) \equiv \frac{d}{d\Theta} k_\nu(\Theta). \quad (43)$$

In order to decouple the system of differential equations (41) and (42) we rewrite Eq. (41),

$$\Delta_\infty c_\nu(\Theta) = -\frac{\hbar^2}{2m} \{k_\nu^2(\Theta) + [k'_\nu(\Theta)]^2\} + \varepsilon_\rho + \hbar v [k'_\nu(\Theta) \sin \Theta - k_\nu(\Theta) \cos \Theta] + E, \quad (44)$$

multiply Eq. (42) by $\Delta_\infty c_\nu(\Theta)$, and insert Eq. (44):

$$0 = \left[\frac{\hbar^2}{2m} \{k_\nu^2(\Theta) + [k'_\nu(\Theta)]^2\} - \varepsilon_\rho \right]^2 - \{E + \hbar v [k'_\nu(\Theta) \sin \Theta - k_\nu(\Theta) \cos \Theta]\}^2 + \Delta_\infty^2. \quad (45)$$

We have to solve Eq. (45) for $k_\nu(\Theta)$, and then insert $k_\nu(\Theta)$ into Eq. (44) in order to obtain $c_\nu(\Theta)$.

Before doing so, let us see how $k_\nu(\Theta)$ and $c_\nu(\Theta)$ are related to the corresponding quantities in the asymptotic wave functions of the vortex at rest. This will help us to select the physically appropriate solutions. In this case $v=0$, and Eq. (45) reduces to

$$\frac{\hbar^2}{2m} \{k_\nu^2(\Theta) + [k'_\nu(\Theta)]^2\} - \varepsilon_\rho = \pm \sqrt{E^2 - \Delta_\infty^2}. \quad (46)$$

There are two types of solutions.

In the first place, $k_\nu(\Theta)$ might be k_ν , independent of Θ . Then Eq. (46) is easily solved for k_ν , resulting in

$$k_\nu = \pm \sqrt{\frac{2m}{\hbar^2} (\varepsilon_\rho \pm \sqrt{E^2 - \Delta_\infty^2})}, \quad (47)$$

where $\nu \in (1,2,3,4)$ counts the four solutions due to the four possible combinations of the + and - signs. From Eq. (44) with $v=0$ and Eq. (47) one finds

$$c_\nu = \frac{E}{\Delta_\infty} \mp \sqrt{\frac{E^2}{\Delta_\infty^2} - 1}. \quad (48)$$

Inserting these k_ν and c_ν into Eqs. (39) and (40) yields the four well-known^{17,18} asymptotic solutions for the vortex at rest.

In the second place, there are solutions of the type $k_\nu(\Theta) = k_\nu \cos(\Theta - \Theta_0)$, where the k_ν are those of Eq. (47) and the Θ_0 are arbitrary. Inserting these solutions into Eqs. (39) and (40) delivers functions containing plane waves propagating in the x and y directions:

$$u_\nu^A(r, \Theta) = e^{in\Theta} e^{ik_\nu r \cos(\Theta - \Theta_0)} = e^{in\Theta} e^{i(k_x x + k_y y)}, \quad (49)$$

$$v_\nu^A(r, \Theta) = c_\nu e^{i(n+1)\Theta} e^{ik_\nu r \cos(\Theta - \Theta_0)} = c_\nu e^{i(n+1)\Theta} e^{i(k_x x + k_y y)}, \quad (50)$$

where $k_x = k_\nu \cos \Theta_0$, $k_y = k_\nu \sin \Theta_0$, and c_ν is given again by Eq. (48). These asymptotic solutions are not the cylindrical plane waves required for a vortex at rest. Thus, they are dismissed. The corresponding solutions of Eq. (45) with $v \neq 0$ will be dismissed, too.

Equation (45) for $k_\nu(\Theta)$ is rewritten as

$$0 = \frac{\hbar^2}{2m} \{k_\nu^2(\Theta) + [k'_\nu(\Theta)]^2\} - \varepsilon_\rho + q_1 \sqrt{\{E + \hbar v [k'_\nu(\Theta) \sin \Theta - k_\nu(\Theta) \cos \Theta]\}^2 - \Delta_\infty^2}, \quad (51)$$

where $q_1 \in (-1, +1)$ denotes the two sheets of the square root. In order to solve this nonlinear differential equation we introduce the substitution

$$\varepsilon_\nu(\Theta) \equiv \sqrt{\{E + \hbar v [k'_\nu(\Theta) \sin \Theta - k_\nu(\Theta) \cos \Theta]\}^2 - \Delta_\infty^2}. \quad (52)$$

The function $\varepsilon_\nu(\Theta)$ has no immediate physical meaning, but for the vortex at rest it becomes

$$\varepsilon_\nu(\Theta) \xrightarrow{v \rightarrow 0} \sqrt{E^2 - \Delta_\infty^2}, \quad (53)$$

which is just the term giving the dependence of the radial wave number k_ν , Eq. (47), on the quasiparticle energy E .

Subsequently we will express k_ν and k'_ν by $\varepsilon_\nu(\Theta)$. This will lead us to an algebraic equation for $\varepsilon_\nu(\Theta)$ which can be solved approximately.

Inserting Eq. (52) into (51) results in

$$0 = \frac{\hbar^2}{2m} \{k_\nu^2(\Theta) + [k'_\nu(\Theta)]^2\} - \varepsilon_\rho + q_1 \varepsilon_\nu(\Theta). \quad (54)$$

In order to eliminate $k'_\nu(\Theta)$ we multiply Eq. (54) by $(2m/\hbar^2) \sin^2 \Theta$,

$$0 = k_\nu^2(\Theta) \sin^2 \Theta + [k'_\nu(\Theta)]^2 \sin^2 \Theta - \left(k_\rho^2 - q_1 \frac{2m}{\hbar^2} \varepsilon_\nu(\Theta) \right) \sin^2 \Theta, \quad (55)$$

define $k_\rho \equiv \sqrt{2m\varepsilon_\rho}/\hbar$, and insert $k'_\nu(\Theta) \sin \Theta$, as obtained from Eq. (52),

$$k'_\nu(\Theta)\sin\Theta = \frac{q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E}{\hbar v} + k_\nu(\Theta)\cos\Theta, \quad (56)$$

where $q_2 \in (-1, +1)$, and arrive at the quadratic equation for $k_\nu(\Theta)$:

$$0 = k_\nu^2(\Theta) + 2k_\nu(\Theta) \frac{q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E}{\hbar v} \cos\Theta + \left(\frac{q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E}{\hbar v} \right)^2 - \left(k_\rho^2 - q_1 \frac{2m}{\hbar^2} \varepsilon_\nu(\Theta) \right) \sin^2\Theta. \quad (57)$$

The eight solutions of Eq. (57) are

$$k_\nu(\Theta) = -\frac{q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E}{\hbar v} \cos\Theta + q_3 \left[k_\rho^2 - q_1 \frac{2m}{\hbar^2} \varepsilon_\nu(\Theta) - \left(\frac{q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E}{\hbar v} \right)^2 \right]^{1/2} \sin\Theta, \quad (58)$$

where $q_3 \in (-1, +1)$, like q_1 and q_2 , denotes the sheets of the square root.

Finally, we eliminate $k_\nu(\Theta)$ and $k'_\nu(\Theta)$ from Eq. (51) by inserting Eq. (58) and its derivative. This results in the equation

$$0 = \frac{d\varepsilon_\nu(\Theta)}{d\Theta} \left[\frac{q_2\varepsilon_\nu(\Theta)}{\hbar v \sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2}} \cos\Theta + q_3 \left(q_1 \frac{m}{\hbar^2} + q_2 \frac{\varepsilon_\nu(\Theta)[q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E]}{(\hbar v)^2 \sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2}} \right) \times \left[k_\rho^2 - q_1 \frac{2m}{\hbar^2} \varepsilon_\nu(\Theta) - \left(\frac{q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E}{\hbar v} \right)^2 \right]^{-1/2} \sin\Theta \right] \quad (59)$$

for $\varepsilon_\nu(\Theta)$. This equation is simpler than Eq. (51). Its solutions determine $k_\nu(\Theta)$ according to Eq. (58). Furthermore, by inserting Eqs. (54) and (56) into Eq. (44) we can express $c_\nu(\Theta)$ by $\varepsilon_\nu(\Theta)$:

$$c_\nu(\Theta) = \frac{q_1\varepsilon_\nu(\Theta) + q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2}}{\Delta_\infty}. \quad (60)$$

Therefore, the problem of solving the asymptotic BdGE's (37) and (38) has now been reduced to solving Eq. (59).

Equation (59) can be satisfied in two ways: first, by $\varepsilon_\nu(\Theta) = \text{const}$ so that

$$\frac{d\varepsilon_\nu(\Theta)}{d\Theta} = 0. \quad (61)$$

This gives solutions $u_\nu^A(r, \Theta)$ and $v_\nu^A(r, \Theta)$ which for $v \rightarrow 0$ become functions of the kind given in Eqs. (49) and (50).¹⁹ As we already discussed, these are not the appropriate solutions for the vortex at rest and therefore are to be dismissed. Consequently, here we dismiss the solutions with $\varepsilon_\nu(\Theta) = \text{const}$ as well. Second, Eq. (59) is satisfied if the expression in the exterior square brackets vanishes. Thus, writing the two terms in this expression on one common denominator, one obtains

$$0 = q_1\varepsilon_\nu(\Theta)q_3\{\varepsilon_\nu^2 - 2\varepsilon_m q_1\varepsilon_\nu(\Theta) - [q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E]^2\}^{1/2} \cos\Theta + \{q_1\varepsilon_\nu(\Theta)[q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E] + \varepsilon_m q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2}\} \sin\Theta. \quad (62)$$

Here we have introduced the definitions

$$\varepsilon_m \equiv m v^2 \quad (63)$$

and

$$\varepsilon_\nu \equiv \hbar k_\rho v. \quad (64)$$

ε_m is the kinetic energy of an electron pair moving with velocity v . The energy ε_ν can formally be interpreted as a Doppler term which shifts the energy of a quasiparticle moving with momentum $\hbar k_\rho$ parallel to the superconducting condensate drifting with velocity v .²⁰

By squaring out the roots Eq. (62) could be transformed into an algebraic equation of eighth degree which might easily be solved numerically. This would be a convenient method of solving exactly the system of coupled nonlinear differential equations (41) and (42). However, we are interested in analytical approximations for $k_\nu(\Theta)$. These are obtained by the following reasoning.

The energies ε_m and ε_ν are connected by the relation

$$\varepsilon_m = \frac{\varepsilon_\nu^2}{2\varepsilon_\rho}. \quad (65)$$

With the exception of a negligible number of quasiparticles traveling nearly parallel to the vortex axis, ε_ρ is of the order of the Fermi energy. Thus it follows from Eq. (65) that ε_m is much smaller than ε_ν or Δ_∞ , and we can expand the right-hand side (rhs) of Eq. (62) with respect to ε_m . Equation (58), written in terms of ε_m and ε_ν ,

$$k_\nu(\Theta) = -\frac{q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E}{\hbar v} \cos\Theta + q_3 \frac{\{\varepsilon_\nu^2 - 2\varepsilon_m q_1\varepsilon_\nu(\Theta) - [q_2\sqrt{\varepsilon_\nu^2(\Theta) + \Delta_\infty^2} - E]^2\}^{1/2}}{\hbar v} \sin\Theta, \quad (66)$$

can be expanded in terms of ε_m as well. Carefully executing these expansions and comparing with the results for the vortex at rest, we find that—in order to get a $k_\nu(\Theta)$ different from k_ν of Eq. (47) and thus have an effect of $H_1(r, \Theta)$, i.e., vortex motion, on wave propagation—we have to expand up to first order in ε_m . In $c_\nu(\Theta)$, on the other hand, one may neglect ε_m altogether and yet get a dependence on ε_v . After a straightforward but rather lengthy calculation, presented elsewhere,¹⁹ we find four solutions for $k_\nu(\Theta)$:

$$k_\nu(\Theta) = \alpha k_\rho - \alpha \beta \frac{k_\rho}{2\varepsilon_\rho} \sqrt{(E - \alpha \varepsilon_v \cos \Theta)^2 - \Delta_\infty^2}, \quad (67)$$

where $k_\rho/2\varepsilon_\rho = \varepsilon_m/\hbar v \varepsilon_v$, and $c_\nu(\Theta)$ becomes

$$c_\nu(\Theta) = \frac{E - \alpha \varepsilon_v \cos \Theta}{\Delta_\infty} + \beta \left(\frac{(E - \alpha \varepsilon_v \cos \Theta)^2}{\Delta_\infty^2} - 1 \right)^{1/2}. \quad (68)$$

The coefficients $\alpha = \pm 1$ and $\beta = \pm 1$ will be associated with the index ν in the way given in Table I.

Inserting Eqs. (67) and (68) into Eqs. (39) and (40) yields the four asymptotic wave functions of the quasiparticles in a superconductor with a moving vortex line. As for $v \rightarrow 0$, the $k_\nu(\Theta)$ and $c_\nu(\Theta)$ approach the expressions (47) and (48), and the wave functions for the moving vortex smoothly change over to the wave functions for the vortex at rest.

The deviation of the function $k_\nu(\Theta)$, Eq. (67), from the wave number k_ν , Eq. (47), as well as the deviation of $c_\nu(\Theta)$, Eq. (68), from c_ν , Eq. (48), is caused by ε_v . If one wants to neglect the ε_v term and thereby replace the wave functions for the moving vortex by those for the vortex at rest, one has to demand that ε_v be small compared to the lowest-energy eigenvalues of the bound quasiparticles in a vortex at rest. These are of the order $\Delta_\infty^2/\varepsilon_F$,¹⁷ so that $\varepsilon_v < \Delta_\infty^2/\varepsilon_F$ is required. Replacing k_ρ in Eq. (64) by its maximum value k_F and using $\xi \approx \hbar^2 k_F / (\pi m \Delta_\infty)$ for the coherence length ξ , the last equation can be rewritten as $v < v_c \equiv (\Delta_\infty / \hbar k_F)(1/k_F \xi)$, where typically $k_F \xi \approx 10^3$. This corresponds to Simánek's condition for the validity of the adiabatic approximation.^{21,16}

Subsequently, we will not make the adiabatic approximation but rather assume that v is only about an order of magnitude smaller than $\Delta_\infty / \hbar k_F$.

B. Angular bound states

Vortex motion creates a new type of quasiparticle states which are hybrids between bound and scattering states. This can be seen from the energy and angle dependence of the $k_\nu(\Theta)$ of Eq. (67) in the asymptotic solutions (39) and (40).

TABLE I. Labeling of the different combinations of α and β values in Eqs. (67) and (68), and thereafter, by the index ν .

ν	α	β
1	+1	-1
2	-1	+1
3	+1	+1
4	-1	-1

As is well known, in the case of a vortex at rest one has bound states for $|E| < \Delta_\infty$ and scattering states for $|E| \geq \Delta_\infty$.^{17,18} This is because the wave numbers k_ν in Eq. (47) are complex for bound states and real for scattering states. In the case of a moving vortex the functions $k_\nu(\Theta)$ depend not only upon E and Δ_∞ but also on ε_v . Furthermore, they are functions of the angular coordinate Θ and thus not quantum numbers. According to Eq. (67) there are scattering states with real $k_\nu(\Theta)$ for all Θ only if $|E| \geq \Delta_\infty + \varepsilon_v$. Likewise, the $k_\nu(\Theta)$ will be complex for all Θ only if $|E| < \Delta_\infty - \varepsilon_v$. Then the wave functions with $\nu=1$ and $\nu=2$ vanish for $r \rightarrow \infty$ and are those of bound quasiparticles; the wave functions with $\nu=3$ and $\nu=4$ are to be dismissed, because they diverge for $r \rightarrow \infty$.

Wave functions with energies $\Delta_\infty - \varepsilon_v \leq |E| < \Delta_\infty + \varepsilon_v$ cannot be classified as belonging to bound or scattering states. For energies in this range it depends on the angular coordinate Θ whether $k_\nu(\Theta)$ is real or complex. For complex $k_\nu(\Theta)$ the wave functions with $\nu=3$ and $\nu=4$ increase exponentially with r and have to be dismissed as in the case of the bound quasiparticles, whereas the wave functions with $\nu=1$ and $\nu=2$ decrease exponentially. Let us look into the latter ones in some more detail for $E > 0$ and $\varepsilon_v < \Delta$.

First, we consider the wave functions with $\nu=1$. From Eq. (67) it follows, that $k_1(\Theta)$ will be real for

$$\cos \Theta \leq \frac{E - \Delta_\infty}{\varepsilon_v}. \quad (69)$$

With the definition

$$\Theta_v \equiv \arccos \frac{E - \Delta_\infty}{\varepsilon_v}, \quad (70)$$

where $0 \leq \Theta_v \leq \pi$, it follows from Eq. (69) that $k_1(\Theta)$ is real in the interval

$$\Theta \in [\Theta_v, 2\pi - \Theta_v] \quad (71)$$

and complex in the interval

$$\Theta \in] - \Theta_v, \Theta_v [. \quad (72)$$

Consequently, the wave functions behave like those of scattering states in the interval (71) and like those of bound states in the interval (72).

Second, we make the same considerations for the wave functions with $\nu=2$ and find that $k_2(\Theta)$ is real for

$$\cos \Theta \geq - \frac{E - \Delta_\infty}{\varepsilon_v}, \quad (73)$$

i.e., for

$$\Theta \in [\Theta_v - \pi, \pi - \Theta_v], \quad (74)$$

and complex for

$$\Theta \in] \pi - \Theta_v, \pi + \Theta_v [. \quad (75)$$

Therefore, the wave functions with $\nu=2$ behave like those of scattering states in the interval (74) and like those of bound states in the interval (75).

The real part of $k_\nu(\Theta)$ is always approximately given by αk_ρ . Thus, the wave functions with $\nu=1$ have positive ra-

dial momentum and describe electrons moving away from and holes moving towards the vortex center in the angular range defined by Eq. (71), while the wave functions with $\nu = 2$, having negative radial momentum, describe electrons moving towards and holes moving away from the vortex center in the angular range defined by Eq. (74).

In the angular range defined by Eq. (72) the wave functions with $\nu = 1$ are exponentially damped. The same is true for the wave functions with $\nu = 2$ in the angular range defined by Eq. (75). Therefore, we call these states tentatively ‘‘angular bound states.’’ In these two angular ranges the current contributions from the outgoing and the incoming waves do not cancel. Their sum yields a net current flow in the $-x$ direction, opposite to the flow of the condensate. This quasiparticle countercurrent, stimulated by the condensate flow, corresponds to the quasiparticle countercurrent in superconducting-normal-superconducting (SNS) junctions which is responsible for the oscillations of the Josephson current.²²

The width of the energy interval $[\Delta_\infty - \varepsilon_\nu, \Delta_\infty + \varepsilon_\nu]$, for which there is only a limited range of directions in which a quasiparticle can move freely, is $2\varepsilon_\nu$. As $v \rightarrow 0$ this interval vanishes and there are no angular bound states in a vortex at rest. For finite vortex velocities $v > v_c$ the angular bound states may be neglected if $\varepsilon_\nu \ll \Delta_\infty$. In the following we will assume that this condition holds so that there are essentially only bound and scattering states. Of these only the bound states are important for Cooper pair destruction and creation in Andreev scattering and the resulting supercurrent force.

IV. VORTEX CORE AND ITS VICINITY

We use the model of Nozières, Vinen, and Warren,^{5,6} in which the vortex has a normal core of radius r_c with a superfluid of uniform density outside the core. Thus, in the pair potential of Eq. (17), $\Delta_0(r) = \Delta_\infty$ for $r > r_c$ and $\Delta_0(r) = 0$ for $r < r_c$, with $r_c \approx \xi$. In Ref. 18 it has been shown that the bound states calculated with this model do not deviate significantly from those obtained with a spatial variation of the pair potential one finds from the Ginzburg-Landau equations. Furthermore, in Ref. 23 (and for the example of superconducting multilayers) it has been shown that, and how, one may replace self-consistent pair potentials by equivalent square-well pair potentials of appropriate heights and widths. By this method one could determine Δ_∞ and r_c with the help of a self-consistent pair potential, if one were interested in, e.g., very accurate energy spectra. For our purpose, however, it is sufficient to consider the two quantities as free parameters and see if for physically reasonable values of these parameters the supercurrent force may be equal to one-half of the Magnus force. As it will turn out in Sec. V, only the ratio r_c/ξ (i.e., the product $r_c\Delta_\infty$) matters and is reasonable, indeed.

Inside the normal core, for $r < r_c$, the wave functions

$$u_n^N(r, \Theta) = e^{-i(\varepsilon_\nu/2\varepsilon_\rho)k_\rho r \cos \Theta} e^{in\Theta} J_n(k_e r), \quad (76)$$

$$v_n^N(r, \Theta) = e^{i(\varepsilon_\nu/2\varepsilon_\rho)k_\rho r \cos \Theta} e^{in\Theta} J_n(k_h r), \quad (77)$$

with

$$k_e = \left(\frac{2m}{\hbar^2} \right)^{1/2} \sqrt{\varepsilon_\rho + E_n + \frac{\varepsilon_m}{2}} \approx k_\rho + \frac{k_\rho}{2\varepsilon_\rho} \left(E_n + \frac{\varepsilon_m}{2} \right), \quad (78)$$

$$k_h = \left(\frac{2m}{\hbar^2} \right)^{1/2} \sqrt{\varepsilon_\rho - E_n + \frac{\varepsilon_m}{2}} \approx k_\rho - \frac{k_\rho}{2\varepsilon_\rho} \left(E_n - \frac{\varepsilon_m}{2} \right), \quad (79)$$

$$k_\rho \equiv \frac{\sqrt{2m}}{\hbar} \sqrt{\varepsilon_F - \frac{\hbar^2 k_z^2}{2m}}, \quad (80)$$

are exact solutions of the BdGE's (33) with $\Delta_0(r) = 0$ and $E \equiv E_n$; here $J_n(\zeta)$ is the Bessel function of the first kind and order n .

The quasiparticle wave functions outside the vortex core, $r > r_c$, which solve the BdGE's (33) with the pair potential, Eq. (17) have been calculated in Ref. 19. We do not reproduce the rather lengthy calculations here, but indicate only the principal steps and approximations. We restrict the analysis to bound states with $|E_n| < \Delta_\infty - \varepsilon_\nu$ and $n \ll k_\rho r_c$, and to low velocities, so that $\varepsilon_\nu / \pi \Delta \ll 1$.

For the solutions in $r > r_c$, we make the ansatz

$$u_{\nu,n}^S(r, \Theta) = f_{\nu,n}(r, \Theta) u_{\nu,n}^A(r, \Theta), \quad (81)$$

$$v_{\nu,n}^S(r, \Theta) = g_{\nu,n}(r, \Theta) v_{\nu,n}^A(r, \Theta), \quad (82)$$

where the deviations of the wave functions from the asymptotic wave functions of Eqs. (39) and (40) are given by the functions $f_{\nu,n}(r, \Theta)$ and $g_{\nu,n}(r, \Theta)$ and are due to the screening current around the vortex core.

Since the deviations from the asymptotic wave functions are small (in the sense that they vary slowly in space, as one knows from the vortex at rest¹⁸), one may neglect the small v in $f_{\nu,n}(r, \Theta)$ and $g_{\nu,n}(r, \Theta)$. As a consequence these functions become independent of Θ and are found to be

$$f_{\nu,n}(r, \Theta) = \hat{f}_{\nu,n}(r) = \xi_{\nu,n}(r) e^{\hat{F}_{\nu,n}(r)}, \quad (83)$$

$$g_{\nu,n}(r, \Theta) = \hat{g}_{\nu,n}(r) = \xi_{\nu,n}(r) e^{\hat{G}_{\nu,n}(r)}, \quad (84)$$

with

$$\xi_{\nu,n}(r) = \frac{1}{\sqrt{r}} \exp\left(\frac{in^2}{2K_\nu r} \right), \quad (85)$$

$$\hat{F}_{\nu,n}(r) = -\frac{i}{8K_\nu r} - \frac{\Delta_\infty(n + \frac{1}{2})}{2\varepsilon_\rho} \hat{c}_\nu e^{\hat{q}r} E_1(\hat{q}r), \quad (86)$$

$$\hat{G}_{\nu,n}(r) = \frac{i(n + \frac{1}{2})}{K_\nu r} - \frac{i}{8K_\nu r} - \frac{\Delta_\infty(n + \frac{1}{2})}{2\varepsilon_\rho} \hat{c}_\nu^{-1} e^{\hat{q}r} E_1(\hat{q}r), \quad (87)$$

where

$$\hat{c}_\nu = \exp\left((-1)^\nu i \arccos \frac{E_n}{\Delta_\infty} \right), \quad (88)$$

$$\hat{q} = \frac{k_\rho}{\varepsilon_\rho} \sqrt{\Delta_\infty^2 - E_n^2}, \quad (89)$$

$$K_\nu = -(-1)^\nu k_\rho + i \frac{k_\rho}{2\varepsilon_\rho} \sqrt{\Delta_\infty^2 - E_n^2}, \quad (90)$$

$$E_1(\hat{q}r) = \int_r^\infty \frac{e^{-\hat{q}t}}{t} dt. \quad (91)$$

In r_c the quasiparticle wave functions $u^N(r, \Theta)$, $v^N(r, \Theta)$ of the bound states for $r < r_c$ must smoothly join the wave functions $u^S(r, \Theta)$, $v^S(r, \Theta)$ for $r > r_c$ which have the correct asymptotic properties discussed in the preceding section. In view of the angle-dependent matching conditions

$$u^N(r_c, \Theta) = u^S(r_c, \Theta), \quad (92)$$

$$v^N(r_c, \Theta) = v^S(r_c, \Theta), \quad (93)$$

$$\left. \frac{\partial}{\partial r} u^N(r, \Theta) \right|_{r=r_c} = \left. \frac{\partial}{\partial r} u^S(r, \Theta) \right|_{r=r_c}, \quad (94)$$

$$\left. \frac{\partial}{\partial r} v^N(r, \Theta) \right|_{r=r_c} = \left. \frac{\partial}{\partial r} v^S(r, \Theta) \right|_{r=r_c}, \quad (95)$$

the superpositions

$$u(r, \Theta) = \Theta(r_c - r) \sum_{n=-\infty}^{\infty} A_n u_n^N(r, \Theta) + \Theta(r - r_c) \times \sum_{\nu=1}^2 \sum_{n=-\infty}^{\infty} D_{\nu,n} u_{\nu,n}^S(r, \Theta), \quad (96)$$

$$v(r, \Theta) = \Theta(r_c - r) \sum_{n=-\infty}^{\infty} B_n v_n^N(r, \Theta) + \Theta(r - r_c) \times \sum_{\nu=1}^2 \sum_{n=-\infty}^{\infty} D_{\nu,n} v_{\nu,n}^S(r, \Theta) \quad (97)$$

are formed, and the coefficients A_n and B_n , which appear in the $u^N(r, \Theta)$ and $v^N(r, \Theta)$, and the $D_{1,n}$ and $D_{2,n}$, which appear in the $u^S(r, \Theta)$, $v^S(r, \Theta)$, have to be determined in such a way that the matching conditions are satisfied and the wave functions are normalized.

After a Fourier transformation with respect to Θ the matching conditions turn into a system of four equations, in which products of the coefficients A_n , B_n , and $D_{\nu,n}$ with the Fourier transforms of the $u_{(\nu),n}$ and $v_{(\nu),n}$ at r_c are summed over all n from $-\infty$ to $+\infty$. Each of these products contains a Bessel function $J_{n-n'}(\zeta)$. Since ζ is of the order of $\varepsilon_\nu / \pi \Delta \ll 1$, these Bessel functions may be replaced by $J_0(\zeta) \delta_{n,n'}$. Thus, only one term is significant in each of the sums over n , and one can write down explicitly the energy-eigenvalue equation of the bound states for $n \ll k_\rho r_c$. If one neglects terms of second and higher order in v , this eigenvalue equation becomes

$$\arccos \frac{E_n}{\Delta_\infty} = \frac{k_\rho r_c E_n}{\varepsilon_\rho} + \frac{\pi}{2} + N\pi - \frac{\sqrt{\Delta_\infty^2 - E_n^2}}{\varepsilon_\rho} \times \left(n + \frac{1}{2} \right) e^{\hat{q}r_c} E_1(\hat{q}r_c). \quad (98)$$

This is the same eigenvalue equation as that for a vortex at rest. For $r_c \approx \xi$ and not too small k_ρ solutions with $E_n < \Delta_\infty$ exist only for $N=0$. Thus, the energy eigenvalues of the bound states depend on the angular momentum quantum number n and the wave number k_z of propagation parallel to the direction of the magnetic field. (For the sake of brevity we designate the eigenvalues E_{n,k_z} just by E_n .)

The vortex velocity v does not influence the energy eigenvalues within our approximation, because in the Fourier-transformed matching conditions only the arguments ζ of the Bessel functions

$$J_{n-n'}(\zeta) \approx J_0(\zeta) \delta_{n,n'} \quad (99)$$

depend upon v . Expansion of these Bessel functions with respect to v provides only quadratic or higher-order contributions of v to the eigenvalue equation. These have to be neglected within our linear approximation.

Because of Eq. (99), only the coefficients A_n , B_n , and $D_{\nu,n}$ which belong to a given n and E_n are nonzero in Eqs. (96) and (97). Thus, the eigenfunctions of the bound states with energy E_n are

$$\Psi_n(\vec{r}, t) = \begin{pmatrix} u_n(r, \Theta) \\ v_n(r, \Theta) \end{pmatrix} e^{-iE_n t / \hbar} e^{ik_z z}, \quad (100)$$

with

$$u_n(r, \Theta) = \Theta(r_c - r) A_n u_n^N(r, \Theta) + \Theta(r - r_c) [D_{1,n} u_{1,n}^S(r, \Theta) + D_{2,n} u_{2,n}^S(r, \Theta)], \quad (101)$$

and

$$v_n(r, \Theta) = \Theta(r_c - r) B_{n+1} v_{n+1}^N(r, \Theta) + \Theta(r - r_c) \times [D_{1,n} v_{1,n}^S(r, \Theta) + D_{2,n} v_{2,n}^S(r, \Theta)]. \quad (102)$$

For the absolute squares of the coefficients we find within our approximations

$$|A_n|^2 = |B_{n+1}|^2 = |D_{1,n}|^2 2\pi k_\rho e^{-r_c / \lambda_0}, \quad (103)$$

$$|D_{2,n}|^2 = |D_{1,n}|^2, \quad (104)$$

where from normalization follows

$$|D_{1,n}|^2 = (8\pi r_c L e^{-r_c / \lambda_0} + D_0)^{-1}, \quad (105)$$

with

$$D_0 = 8L \int_0^\pi d\Theta \lambda_2(\Theta) \exp\left(-\frac{r_c}{\lambda_2(\Theta)}\right) \quad (106)$$

and

$$\lambda_0 = \frac{\hbar^2 k_\rho}{2m} \text{Re}(\Delta_\infty^2 - E_n^2)^{-1/2}. \quad (107)$$

The velocity- and angle-dependent quasiparticle decay length $\lambda_2(\Theta)$ is defined in Eq. (113).

With the help of these functions we will show in the next section that the supercurrent force on the quasiparticles localized in the vortex core by Andreev scattering transfers half of the Magnus force from the Cooper pair condensate to the unpaired core electrons.

V. SUPERCURRENT FORCE AND MAGNUS FORCE

We insert the wave functions of Eqs. (101) and (102) into Eq. (11). This yields the supercurrent force on a bound quasiparticle in the state characterized by the quantum numbers n and k_z for angular momentum around and momentum along the z axis; the radial quantum number N is 0. The superfluid velocity $\vec{v}_s = \vec{v}_s(r)$ is the sum

$$\vec{v}_s(r) = \vec{v}_{s0}(r) + \vec{v} \quad (108)$$

of the screening current velocity of Eq. (18) and the relative velocity of Eq. (31). Since $v \ll \Delta_\infty / (\hbar k_F)$, the relative velocity \vec{v} is small compared to the screening current velocity $\vec{v}_{s0}(\xi)$ in the vicinity of the core and will be neglected in $\vec{v}_s(r)$. The pair potential $\Delta = \Delta(r, \Theta)$ is given by Eq. (17). As in Sec. IV we adopt the local model for the considered high- κ superconductor: The pair potential depression near the vortex center is replaced by a normal core of radius $r_c \approx \xi$, i.e., $\Delta(r < r_c, \Theta) \equiv 0$, while outside of this core the modulus of the pair potential is replaced by the value Δ_∞ . We stay within the approximations introduced in Sec. IV.

According to Eq. (11) contributions to the supercurrent force $\vec{f}_{\Delta 2}$ come from the regions of finite Δ , i.e., $r > r_c$. Since the supercurrent force acts only on quasiparticles which create or destroy Cooper pairs in Andreev scattering,¹⁰⁻¹² the u and v functions in Eq. (11) are the exponentially decaying wave functions of the bound states, given by Eqs. (101) and (102) for $r > r_c$.

Within our approximations we may replace the functions $\hat{f}_{v,n}(r)$ and $\hat{g}_{v,n}(r)$ by $\xi_{v,n}(r)$ [the small functions $\hat{F}_{v,n}(r)$ and $\hat{G}_{v,n}(r)$ are only important for the derivation of the eigenvalue equation (98)], and write the wave functions of the bound states in the superconducting region outside the core as

$$u_{1,n}^S(r, \Theta) = \xi_{1,n}(r) e^{in\Theta} e^{ik_\rho r} e^{-r/2\lambda_1(\Theta)}, \quad (109)$$

$$u_{2,n}^S(r, \Theta) = \xi_{2,n}(r) e^{in\Theta} e^{-ik_\rho r} e^{-r/2\lambda_2(\Theta)}, \quad (110)$$

$$v_{1,n}^S(r, \Theta) = \xi_{1,n}(r) c_1(\Theta) e^{i(n+1)\Theta} e^{ik_\rho r} e^{-r/2\lambda_1(\Theta)}, \quad (111)$$

$$v_{2,n}^S(r, \Theta) = \xi_{2,n}(r) c_2(\Theta) e^{i(n+1)\Theta} e^{-ik_\rho r} e^{-r/2\lambda_2(\Theta)}, \quad (112)$$

where the $c_\nu(\Theta)$ are given by Eq. (68) and the $\lambda_\nu(\Theta)$ are quasiparticle decay lengths derived from Eq. (67),

$$\begin{aligned} \lambda_\nu(\Theta) &= [2 \operatorname{Im} k_\nu(\Theta)]^{-1} \\ &= \frac{\hbar^2 k_\rho}{2m} \operatorname{Re} \{ \Delta_\infty^2 - [E_n + (-1)^\nu \varepsilon_v \cos \Theta]^2 \}^{-1/2}. \end{aligned} \quad (113)$$

Inserting the superfluid velocity (18), the wave functions

$$u_n^S(r, \Theta) = D_{1,n} u_{1,n}^S(r, \Theta) + D_{2,n} u_{2,n}^S(r, \Theta), \quad (114)$$

$$v_n^S(r, \Theta) = D_{1,n} v_{1,n}^S(r, \Theta) + D_{2,n} v_{2,n}^S(r, \Theta), \quad (115)$$

and the pair potential $\Delta(r, \Theta) \equiv 0$ for $r < r_c$ and $\Delta(r, \Theta) = \Delta_\infty e^{-i\Theta}$ for $r \geq r_c$ into Eq. (11) we find

$$\begin{aligned} \vec{f}_{\Delta 2}(k_z, n) &= \frac{\hbar \sqrt{k_F^2 - k_z^2} e \Phi_0 L}{\pi m} |D_{1,n}|^2 \\ &\times \int_0^{2\pi} d\Theta \vec{e}_\Theta \int_{r_c}^\infty \frac{dr}{r} \left(\frac{e^{-r/\lambda_2(\Theta)}}{\lambda_2(\Theta)} - \frac{e^{-r/\lambda_1(\Theta)}}{\lambda_1(\Theta)} \right), \end{aligned} \quad (116)$$

where we neglected the integrals over the rapidly oscillating products $[u_{1,n}^S(r, \Theta)]^* v_{2,n}^S(r, \Theta)$ and $[u_{2,n}^S(r, \Theta)]^* v_{1,n}^S(r, \Theta)$ and made use of the relation

$$\Delta_\infty \operatorname{Im}[c_\nu(\Theta)] = (-1)^\nu \frac{\hbar^2 \sqrt{k_F^2 - k_z^2}}{2m} \lambda_\nu^{-1}(\Theta), \quad (117)$$

which follows from Eq. (68) with Eq. (113) for $|E_n| \leq \Delta - \varepsilon_v$.

In order to evaluate further Eq. (116) we make a crude approximation: We replace r^{-1} in the integral by its maximum value r_c^{-1} . Thus, all the following expressions for the supercurrent force would have to be multiplied by a numerical factor smaller than 1 in order to get quantities which would correspond to the exact integral. This way we can carry out the integration over r and obtain an approximate analytical expression for $\vec{f}_{\Delta 2}$. In order to perform the integration over Θ we write the angular unit vector \vec{e}_Θ in terms of Cartesian unit vectors $\vec{e}_\Theta = -\vec{e}_x \sin \Theta + \vec{e}_y \cos \Theta$. Using furthermore the relation $\lambda_1(\Theta) = \lambda_2(\pi - \Theta)$, which follows directly from Eq. (113), and observing that both $\lambda_\nu(\Theta)$ depend on the angular coordinate Θ only via $\cos \Theta$, we find

$$\begin{aligned} \vec{f}_{\Delta 2}(k_z, n) &= \vec{e}_y \frac{4\hbar \sqrt{k_F^2 - k_z^2} e \Phi_0 L}{\pi m r_c} |D_{1,n}|^2 \\ &\times \int_0^\pi d\Theta \cos \Theta \exp\left(-\frac{r_c}{\lambda_2(\Theta)}\right). \end{aligned} \quad (118)$$

The supercurrent force on a quasiparticle has only a component in the y direction, i.e., perpendicular to the relative velocity $\vec{v} = \vec{v}_T - \vec{v}_L$, just like the Magnus force of Eq. (1).

Numerical integration of Eq. (118) yields in the supercurrent force as a function of $E \equiv E_{n,k_z}$. This is shown in Fig. 2 for $k_z = 0$ (solid line).

The supercurrent force depends nearly linearly on the quasiparticle energy for $|E| < \Delta_\infty - \varepsilon_v$ and vanishes for $|E| > \Delta_\infty - \varepsilon_v$. Note that, in Fig. 2, $\varepsilon_v = 0.1\Delta_\infty$, which is rather large. For smaller values of ε_v the supercurrent force may well be approximated by a linear function in E for $|E| < \Delta_\infty$ and by zero for $|E| > \Delta_\infty$, as shown by the dashed line in Fig. 2.

Analytically a linearized expression for the supercurrent force can be obtained by a first-order Taylor expansion around $E_n = 0$. With the definition

$$k_z \equiv k_F \cos \omega, \quad (119)$$

this yields

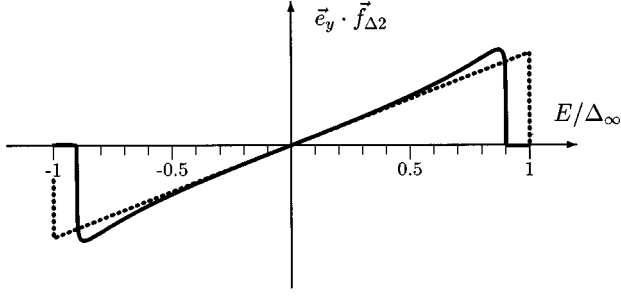


FIG. 2. The magnitude $\vec{e}_y \cdot \vec{f}_{\Delta 2}$ of the supercurrent force $\vec{f}_{\Delta 2}$ on quasiparticles in a moving vortex line as a function of the quasiparticle energy E , in arbitrary units. Here quasiparticles with wave number $k_z=0$ parallel to the vortex axis are considered. Solid line, $\vec{f}_{\Delta 2}$ from numerical integration of Eq. (118); dashed line, $\vec{f}_{\Delta 2}$ linearized according to Eq. (120).

$$\vec{f}_{\Delta 2}(\omega, n) = \vec{e}_y \frac{ev\Phi_0 k_F}{2\pi\Delta_\infty} \frac{E_n \sin \omega}{[r_c + \hat{\lambda}_0(\omega)]}, \quad (120)$$

where

$$\hat{\lambda}_0(\omega) = \frac{\hbar^2 k_F \sin \omega}{2m\Delta_\infty}. \quad (121)$$

In deriving Eq. (120) we have kept only terms linear in v .

The total force $\vec{F}_{\Delta 2}$ acting on the core because of Andreev scattering is the sum of all supercurrent forces acting on the bound quasiparticles. In the sum of $\vec{f}_{\Delta 2}$, approximated by Eq. (120), over all occupied quasiparticle states

$$\vec{F}_{\Delta 2} = 2 \sum_{k_z} \sum_n \vec{f}_{\Delta 2}(\omega, n) f_0(E_n), \quad (122)$$

the spin degeneracy is taken into account by a factor of 2 and $f_0(E_n)$ is the Fermi distribution function. For the sake of simplicity we limit the calculation to $T=0$ K so that only the quasiparticle states with negative energy—which make up the ground state of the normal core—are occupied: $f_0(E_n) = 1$ for $E_n < 0$ and $f_0(E_n) = 0$ for $E_n > 0$.

The sum over k_z is transformed in an integral over ω :

$$\sum_{k_z} \rightarrow \frac{L}{2\pi} \int_{-k_F}^{k_F} dk_z = \frac{Lk_F}{2\pi} \int_0^\pi d\omega \sin \omega. \quad (123)$$

For fixed ω the energetic separation between two energy eigenvalues differing in n by 1 is very small.¹⁸ Therefore, we can transform the sum over n in an integral over $E_n \equiv E$:

$$\sum_n \rightarrow \int_{-\Delta_\infty}^0 dE \left(\frac{\partial E}{\partial n} \right)_\omega^{-1}; \quad (124)$$

the lower integration limit has been approximated by $-\Delta_\infty$, in the spirit of the linear approximation of Fig. 2 and Eq. (120). Thus the total supercurrent force is

$$\vec{F}_{\Delta 2} = \frac{Lk_F}{\pi} \int_0^\pi d\omega \sin \omega \int_{-\Delta_\infty}^0 dE \left(\frac{\partial E}{\partial n} \right)_\omega^{-1} \vec{f}_{\Delta 2}(\omega, E). \quad (125)$$

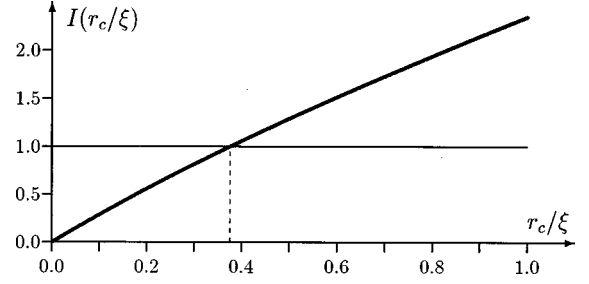


FIG. 3. The function $I(r_c/\xi)$ as defined by Eq. (130).

The term $(\partial E/\partial n)_\omega^{-1}$ can be obtained from the eigenvalue equation (98). Approximating in it $e^x E_1(x) \approx 1/x$ turns it into

$$\arccos \frac{E}{\Delta_\infty} - \frac{k_F r_c E \sin \omega}{\varepsilon_\rho} = \frac{\pi}{2} + N\pi - \frac{n + \frac{1}{2}}{k_\rho r_c}, \quad (126)$$

with $N=0$ for not too small ω . From Eq. (126) one finds

$$\left(\frac{\partial E}{\partial n} \right)_\omega^{-1} = \frac{k_F r_c \sin \omega}{\sqrt{\Delta_\infty^2 - E^2}} + \frac{2mr_c^2}{\hbar^2}. \quad (127)$$

We insert Eqs. (120) and (127) into Eq. (125), assume that the spectrum and the density of all bound states can be approximately described by Eqs. (126) and (127), and carry out the integration over E :

$$\begin{aligned} \vec{F}_{\Delta 2} = \vec{e}_y \frac{ev\Phi_0 L k_F^2}{2\pi^2 \Delta_\infty} \int_0^\pi d\omega \frac{\sin^2 \omega}{r_c + \hat{\lambda}_0(\omega)} \\ \times \left[-\Delta_\infty k_F r_c \sin \omega - \frac{\Delta_\infty^2 m r_c^2}{\hbar^2} \right]. \end{aligned} \quad (128)$$

With $\hat{\lambda}_0(\omega)$ according to Eq. (121), the coherence length $\xi = \hbar^2 k_F / (\pi m \Delta_\infty)$, and the electron density $n_s = k_F^3 / (3\pi^2)$, Eq. (128) results in

$$\vec{F}_{\Delta 2} = \frac{1}{2} \vec{F}_{\text{Magnus}} I\left(\frac{r_c}{\xi}\right), \quad (129)$$

with \vec{F}_{Magnus} given by Eq. (1) and

$$I\left(\frac{r_c}{\xi}\right) = \frac{3r_c}{\pi\xi} \int_0^\pi d\omega \sin^2 \omega \frac{r_c/\xi + \pi \sin \omega}{r_c/\xi + (\pi/2) \sin \omega}. \quad (130)$$

The function $I(r_c/\xi)$ is plotted in Fig. 3. It assumes the value 1 for $r_c \approx 0.4\xi$. Considering the approximations made on the way to Eq. (129) this is reasonable. Without the approximation of replacing $1/r$ by $1/r_c$ in the integral (116),

$$\vec{F}_{\Delta 2} = \frac{1}{2} \vec{F}_{\text{Magnus}}, \quad (131)$$

would result for a somewhat larger value of r_c .

VI. SUMMARY AND OUTLOOK

The motion of a vortex line relative to an applied supercurrent causes an angular asymmetry in the wave functions of the unpaired quasiparticles bound in the vortex core: Outside the core, in a given direction characterized by the azi-

muthal angle Θ , the damping of the wave functions which, at the core boundary, match to radially outgoing electron—and radially ingoing hole—wave functions is different from the damping of the wave functions which match to radially outgoing hole—and radially ingoing electron—wave functions. The different penetration lengths $\lambda_1(\Theta)$ and $\lambda_2(\Theta)$ in Eq. (116) are responsible for the supercurrent force and the resulting half of the Magnus force. This angle-dependent different damping of outgoing and ingoing waves corresponds to the semiclassically computed¹⁰ different rates of Cooper pair formation and destruction—and the associated different momentum transfers from the circulating condensate to the core electrons—in electron→hole and hole→electron scattering processes at the core boundary. The detailed quantum mechanical calculations presented in this paper confirm our earlier semiclassical considerations: The sum of all supercurrent forces, which originate from Cooper pair momentum transfers to the core electrons by Andreev scattering, is equal to one-half of the Magnus force. It explains microscopically the “interface force” in the Nozières-Vinen-Warren theory of vortex motion.^{5,6}

Recently Stone²⁴ showed by a quasiclassical geometric optics model that in a moving vortex spectral flow is thwarted by an analog of Bloch oscillations originating from the discrete nature of the core state spectrum. This is a consequence of the fact that due to the supercurrent force the Andreev reflection fails to be perfectly retroreflective and causes the core bound states to precess in a sense opposite to that of the superflow. Therefore, momentum transferred to the vortex core can only escape via relaxation processes. We have implicitly taken into account such processes by assuming a constant drift velocity \vec{v} of the core quasiparticles rela-

tive to the condensate. That means that all momentum gains from the supercurrent force and the electrostatic field in the core are assumed to be dissipated right away to the lattice. [This is also the condition for the validity of Eq. (1).] This way the nonequilibrium effects relevant in our context have been incorporated. The constant drift velocity causes the asymmetry in the wave functions, discussed above, which gives rise to $\vec{F}_{\Delta 2}$.

Since we have worked with the mean-field TdBdGE's, our analysis is, strictly speaking, only valid for conventional type-II superconductors. In order to extend it to the strongly correlated high-temperature superconductors one must use the time-dependent density-functional Bogoliubov–de Gennes equations (TdDFBdGE's).²⁵ The integral equations defining their vector and pair potentials in terms of exchange correlation functionals of the gauge-invariant current density \vec{J} and the anomalous density Δ_{IP} (which measures off-diagonal long-range order) are difficult to solve, if one takes into account all electromagnetic fields. If, however, one neglects the corresponding scalar and vector potentials—as one does when calculating the electronic structure of vortices in conventional superconductors—things become simpler. Then, at $T=0$ K the TdDFBdGE's are essentially given by Eqs. (21)–(23) with a pair potential which is the sum of the mean-field pair potential and the (negative) variational derivative of the exchange-correlation functional $Q_{xc}[\vec{J}, \Delta_{\text{IP}}]$ with respect to Δ_{IP}^* .^{15,26} Thus, the calculation of an appropriate exchange-correlation functional is crucial for the analysis of the influence of Andreev scattering on vortex motion in high-temperature superconductors. This is a task for further research.

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