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Theory of friction: Coulomb drag between two closely spaced solids

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We calculate the friction force between two metallic bodies with flat surfaces separated by a vacuum slab of thickness d, and moving with a relative velocity v. The separation d is assumed to be so large that the only interaction between the bodies is via the Coulomb field. The friction force depends linearly on the velocity v (for small v) and decays rapidly with increasing d. In most practical cases, the Coulomb drag makes a negligible contribution to the friction force, but it may make an important contribution in some special cases. [S0163-1829(98)06312-7]

I. INTRODUCTION

The interaction between two stationary bodies is often dominated by the van der Waals interaction. The van der Waals interaction is long ranged, with the interaction energy falling off as $1/d^2$, where d is the separation between two flat surfaces (we neglect retardation, which is a good approximation for $d<100$ Å). The van der Waals interaction energy (per unit area) between two metallic bodies with flat surfaces separated by $d\sim 10$ Å is typically of the order of 1 meV/Å^2, which is a factor of ~ 100 smaller than the interaction energy that occurs when the (clean) surfaces are in direct atomic contact. For an elastically soft solid, such as rubber, the van der Waals interaction can lead to a large enhancement of the area of real contact when such a solid is in contact with a substrate.

Because of surface roughness, the area of real (atomic) contact between two bodies is usually a very small fraction of the apparent area of contact. For example, if a cubic steel block with a side of length 10 cm is placed on a steel substrate, the area of real contact is of the order ~ 0.001 cm^2, i.e., only a fraction $\sim 10^{-5}$ of the apparent area. [The area of real contact ΔA can usually be estimated by assuming the plastic yielding has occurred in all contact ''points,'' so that $\sigma_c \Delta A = L$, where σ_c is the yield stress (indentation hardness) and L the total load.] Thus, if the amplitude of the surface roughness is small, the van der Waals interaction between the noncontacting surface areas makes an important contribution to the overall interaction energy. It is interesting to ask if the long-range interaction between the noncontacting surface areas also makes an important contribution to the sliding friction force.

In this paper we will address the fundamental problem of the contribution to the sliding friction from the long-range Coulomb interaction between two uncharged bodies.[1] We show that in contrast to the important contribution from the (conservative) van der Waals interaction to the adhesion energy between two solids, in most cases the dissipative part of the long-ranged interaction makes a negligible contribution to the friction force. That is, the friction force is determined mainly by the area of real (or atomic) contact; the surfaces area separated by more than ~ 10 Å makes an extremely small contribution to the friction force, even if this noncontacting surface area is many orders of magnitude larger than the area of atomic contact.

In two earlier papers Schaich and Harris[2] and Levitov[3] have estimated the contribution from the long-range Coulomb interaction to the sliding friction when two systems move slowly relative to each other. For the case of two metallic bodies with flat surfaces Schaich and Harris found that the friction force for parallel motion is independent of any metal property. However, our study shows that this result needs to be improved, as we find that the metal properties enter via the surface response function $g(q,\omega)$. Nevertheless, the dependence of the friction force on the temperature T and on the separation d between the surfaces is correctly described by the theory of Schaich and Harris. The friction force obtained by Levitov vanishes when retardation is neglected (i.e., when the light velocity $c\rightarrow\infty$). This result is incorrect. [It is not easy to check where the calculation of Levitov is incorrect because he does not present the details of the calculation.]

In an interesting preprint, Pendry considers the same problem as discussed below.[4] However, he focusses only on the zero-temperature limit where the friction force is a nonlinear function of the sliding velocity v. This nonlinear contribution is derived from diagram (c) in Fig. 3. The formalism used by Pendry is very similar to ours, and we agree with the results he derives.

II. THEORY

We consider two semi-infinite metallic bodies with flat surfaces separated (with vacuum) by a distance d. The upper body moves parallel to the lower body with the velocity v. We show below that the friction force $F=\gamma A v$, where A is the surface area and γ the friction factor. Note that the friction force is proportional to the velocity v (for small veloci-

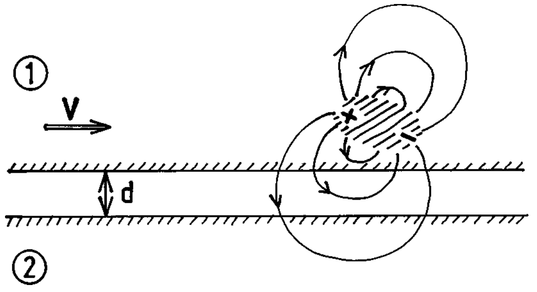

FIG. 1. Two semi-infinite metals, 1 and 2, separated by a vacuum ''slab'' of thickness d. Solid 1 moves with the velocity v relative to solid 2. The figure illustrates a thermal or a quantum fluctuation, which gives rise to a temporal charge imbalance and an electric field. The electric field penetrates into solid 2 where it creates electron-hole pair excitations.

ties). The origin of this result is that in the present case no rapid motion occurs at the interface at any stage of the sliding (as would be the case if, e.g., elastic instabilities would occur) and we therefore expect $F \sim v$ as $v \rightarrow 0$. We consider only metallic (or conducting) bodies as the electronic friction vanishes for insulating bodies (no low-energy electronic excitations are possible for insulators).

Let us first present a qualitative discussion about the origin of the friction force. At first it might seem a paradox that there should be friction forces between perfectly flat metal surfaces separated by vacuum, since on the *average* there is no electric field outside such surfaces, which could couple the surfaces together. However, due to *thermal* or *quantum fluctuations*, local charge imbalance will occur *temporarily* in the metals, which gives rise to electric field patches extending from one solid into the other solid; see Fig. 1. The fluctuating electric field will induce electric currents in the solids, which are damped due to, e.g., normal ''Ohmic'' processes such as scattering of conduction electrons against imperfections. For stationary surfaces (no sliding) such energy transfer takes place in both directions and there is no net energy transfer from one solid to the other (thermal equilibrium). However, during sliding a net energy and momentum transfer will occur from the sliding body to the stationary body, leading to a friction force. A detailed study shows that there is a fundamental difference between quantum and thermal fluctuations: Thermal fluctuations will make a contribution to the friction force already to the lowest order in the (electric field) coupling between the solids, while quantum fluctuations only contribute to the linear (in v) sliding friction in the second (or higher) order of perturbation theory. Furthermore, within the jellium model there is no contribution to the linear sliding friction from quantum fluctuations so that the linear sliding friction vanishes at zero temperature in this model. (Note that in the jellium model the ion cores of the solid are replaced by a uniform positive background.) For real (crystalline) solids the contribution from quantum fluctuations is nonzero, but decays extremely rapidly with increasing separation d between the surfaces, $\gamma \sim \exp(-2Gd)$, where $G = 2\pi/a$ is the (smallest) reciprocal lattice vector of the surface unit cell (see Appendix B). The contributions from thermal fluctuations to the friction force are more long ranged, asymptotically falling off as $1/d^6$ (or as $1/d^4$, see below). On the other hand, the thermal contribution depends on temperature as $\sim T^2$ so that at low enough temperature the contribution from quantum fluctuations will dominate for any fixed separation d. We note that the thermal contribution to the friction force requires that *both* solids are metallic, while the contribution from quantum fluctuations only requires that at least *one* of the solids is metallic (see Appendix B).

Let us define the linear response function $g(q,\omega)$, which is of central importance for a large class of surface processes.[5] Assume that a semi-infinite metal occupies the half space $z \leqslant 0$. A charge distribution in the half space $z > d$ gives rise to an (external) potential that must satisfy the Laplace equation for $z < d$ and that therefore can be written as a sum of evanescent plane waves of the form

$$\phi_{\text{ext}} = \phi_0 e^{qz} e^{i\mathbf{q}\cdot\mathbf{x} - i\omega t},$$

where $\mathbf{q} = (q_x, q_y)$ is a two-dimensional (2D) wave vector. This potential will induce a charge distribution in the solid (occupying $z < 0$), which in turn gives rise to an electric potential that must satisfy the Laplace equation for $z > 0$, and that therefore can be expanded into evanescent plane waves that decay with increasing $z > 0$. Thus the total potential for $0 < z < d$ can be expanded in functions of the form[6,7]

$$\phi = \phi_0 (e^{qz} - g e^{-qz}) e^{i\mathbf{q}\cdot\mathbf{x} - i\omega t},$$

where the reflection factor $g = g(q,\omega)$.

The g function introduced above has the same central role for dynamical processes at surfaces as the dielectric function $\epsilon(\omega,k)$ has for processes in the bulk. During the last ~ 20 years much effort has been devoted to calculating $g(q,\omega)$ for simple metals. There are several contributions to Im g that can be distinguished by the source of the momentum involved in the excitation process. For a semi-infinite jellium for small frequencies ($\omega \ll \omega_F$) only the surface can supply with momentum and[6,7]

$$(\text{Im} g)_{\text{surf}} = 2\xi(q) \frac{\omega}{\omega_p} \frac{q}{k_F}, \tag{1}$$

where ω_p, ω_F, and k_F are the plasma frequency, the Fermi frequency, and the Fermi wave vector, respectively, and where $\xi(q)$ depends on the electron density parameter r_s but typically $\xi(0) \sim 1$. For real metals, in addition to this *surface* contribution there will be a *bulk* contribution to Img derived from ''normal'' Ohmic processes (e.g., scattering of the conduction electrons from imperfections), as characterized by a bulk mean free path l, and given by[6,7]

$$(\text{Im} g)_{\text{bulk}} = 4 \frac{\omega_F}{\omega_p} \frac{1}{k_F l} \frac{\omega}{\omega_p}. \tag{2}$$

Let g_1 and g_2 be the g functions of the two solids in Fig. 1. Using first-order perturbation theory, and taking into account screening, one can show that the thermal fluctuations make the following contribution to the friction parameter (see Appendix A):

$$\gamma = 0.167 \frac{(k_B T)^2}{\hbar} \int_0^\infty dq\, q^3 \frac{e^{-2qd}}{|1 - g_1(q,0) g_2(q,0) e^{-2qd}|^2} \times \lim_{\omega \to 0} \left(\frac{\mathrm{Im} g_1(q,\omega) \mathrm{Im} g_2(q,\omega)}{\omega^2} \right), \tag{3}$$

which is symmetric in g_1 and g_2. If $\mathrm{Im} g_1$ and $\mathrm{Im} g_2$ are of the form (1) and if we approximate $g_1(q,0) = g_2(q,0) = 1$, which are good approximations for separations d beyond a few angstroms, then the surface contribution to γ is obtained from Eqs. (1) and (3);

$$\gamma_{\mathrm{surf}} = 1.298 \left(\frac{k_B T}{\hbar \omega_p} \right)^2 \frac{\hbar \xi^2(0)}{k_F^2 d^6}. \tag{4}$$

Similarly, using Eqs. (2) and (3) gives the bulk contribution

$$\gamma_{\mathrm{bulk}} = 1.204 \left(\frac{k_B T}{\hbar \omega_p} \right)^2 \left(\frac{\omega_F}{\omega_p} \right)^2 \frac{\hbar}{(k_F l)^2 d^4}. \tag{5}$$

Note that for large enough d, the bulk contribution dominates over the surface contribution, but for $d < l$ the surface contribution dominates.

III. APPLICATIONS

Consider two silver bodies, treated as semi-infinite jellium bodies. In this case for $d = 10$ Å Eq. (4) gives $\gamma \sim 10^{-7}$ Ns/m^3, which at the sliding velocity $v = 1$ m/s corresponds to a frictional stress $F/A \sim 10^{-7}$ N/m^2. This stress is extremely small compared with the frictional stress $\sim 10^8$ N/m^2 occurring in the areas of atomic contact even for (boundary) lubricated surfaces. Furthermore, while the Coulomb drag force is proportional to the sliding velocity, the friction force when a block is slid on a substrate is nearly velocity independent at low velocities as indeed expected if the main part of the friction force is generated in the area of real contact (where, e.g., elastic instabilities may occur).

In spite of its small magnitude, the friction force associated with thermal fluctuations has been observed in several elegant experiments.[8] The samples used for these experiments are modulation-doped semiconductor heterostructures grown by molecular beam epitaxy. The experiments consist of two thin slabs of electron (or hole) gases separated by a barrier that is high and wide (typically $d \sim 100$ Å) enough to prevent tunneling of electrons, while thin enough to allow for strong Coulomb interaction between carriers on the opposite sides of the barrier. In the experiments the frictional drag of one electron-gas layer (layer 1) on another (layer 2) was probed by studying how an electric current in one layer induces a current in the other layer. If no current is allowed to flow in layer 2 (open circuit) an electric field E_2 develops whose influence cancels the frictional force between the layers. If the current in layer 1 is denoted by $J_1 = n_1 e v$, where v is the drift velocity and n_1 the carrier concentration (per unit area), then the friction force $F = \gamma A v$ will act on layer 2. This must equal the force $F = A n_2 e E_2$, where E_2 is the electric field in layer 2 induced by layer 1. Thus $\gamma = n_2 e E_2 / v = n_1 n_2 e^2 E_2 / J_1$. The experiments showed that E_2 / J_1 is independent of J_1, i.e., the friction force depends linearly on the sliding velocity v. Furthermore, γ was found to be nearly proportional to T^2 as expected based on Eq. (3). In Appendix C we show how the present theory reproduces the results of Gramila *et al.*[8] when the g functions of 2D electron gas layers are substituted in Eq. (3).

The friction derived from quantum fluctuations (van der Waals friction) gives a very important contribution to the sliding friction force of physisorbed adsorbate layers on metal surfaces. As an example,[9] the friction force observed when an incommensurate Xe monolayer (or bilayer) film is sliding on a flat Ag(111) surface was found to be proportional to the sliding velocity, with the friction factor $\gamma \approx 1000$ Ns/m^3. This is of the same order of magnitude as the friction factor calculated by assuming that quantum fluctuations make the dominating contribution to the friction force [the contribution from quantum fluctuations decay very rapidly with the distance d, but for the short d relevant for a physisorbed monolayer film, the contribution from quantum fluctuations is very important; note that thermal fluctuations cannot contribute to γ in this case because one ''solid'' (the Xe film) is insulating].

In the discussions above we have assumed perfect (single crystal) solids. Most real solids consist of grains that may expose different facets with different work functions. This gives rise to a (static) inhomogeneous electric field distribution in the vicinity of the surfaces. Similarly, adsorbed atoms (e.g., alkali atoms) may give rise to strong local electric fields at the surfaces. It is clear that when two macroscopic bodies (with static inhomogeneous electric field distributions at their surfaces) are slid relative to each other without direct contact, a finite contribution to the friction force will arise from surface imperfections. However, even in these cases the contribution to the friction force from the noncontacting surface area is negligible compared to the contribution from the area of atomic contact.

IV. SUMMARY AND CONCLUSION

We have calculated the friction force between two metallic bodies with flat surfaces separated by a vacuum slab of thickness d, and moving with a relative velocity v. The separation d is assumed to be so large that the only interaction between the bodies is via the Coulomb field, but small enough to allow negligence of retardation effects. The coupling between the solids occurs via the electric field from *thermal* or *quantum fluctuations* in the solids. The friction force is proportional to the velocity v (for small v) and decays rapidly with increasing d. Thus, quantum fluctuations make a contribution $\sim \exp(-2Gd)$ (where $G = 2\pi/a$ is the smallest reciprocal lattice vector) to the friction force, which is negligibly small already for $d = 10$ Å [note that with $a = 3$ Å and $d = 10$ Å one gets $\exp(-2Gd) \sim 10^{-10}$]. The contribution from thermal excitations is proportional to T^2, and decays with increasing d as $1/d^4$ (for the bulk contribution) or $1/d^6$ (for the surface contribution). In most practical cases, involving sliding of a block on a substrate, the Coulomb drag makes a negligible contribution to the friction force (the main part of the friction arises from the regions of real contact between the solids). However, in some special cases the Coulomb drag force is very important. For example, the friction force from vacuum fluctuations contributes in an important manner to the sliding friction acting on thin physisorbed

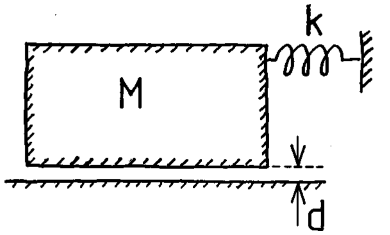

FIG. 2. A block (mass M) separated by a distance d to a substrate. The block is connected by a spring (spring constant k) to a rigid fixed stage. The block-spring system forms a quantum oscillator with the resonance frequency $\omega=(k/M)^{1/2}$.

layers of atoms sliding on metallic surfaces, and the contribution from thermal fluctuations gives the dominating drag force in some experiments involving parallel 2D electron systems.

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APPENDIX A

In this Appendix we derive Eq. (3). Instead of considering two bodies with the relative velocity v, it is easier to assume that one body oscillates with the frequency ω relative to the second body. The friction force calculated for this case will reduce to that for the former case as $\omega\rightarrow 0$. Thus, we consider the sliding configuration shown in Fig. 2. To the block (mass M) is connected a spring (spring constant k) attached to a rigid fixed stage. The block-spring system forms a harmonic oscillator with the resonance frequency $\omega=(k/M)^{1/2}$. Assume that this (quantum) oscillator is in its first vibrational exited state $n=1$. The vibrational coordinate can be written as

$$u=u_0(be^{-i\omega t}+b^+e^{i\omega t}).$$

We show below that the friction force F depends linearly on the sliding velocity of the block so that we can write $F=-A\gamma\dot{u}$ and the power absorption $P=-\langle F\dot{u}\rangle=A\gamma\langle\dot{u}^2\rangle$, where $\langle\cdots\rangle$ stands for time averaging. Since we treat the oscillator quantum mechanically we must interpret

$$\langle\dot{u}^2\rangle=\langle 1|\dot{u}^2|1\rangle-\langle 0|\dot{u}^2|0\rangle=2\omega^2u_0^2.$$

If $w_\downarrow$ and $w_\uparrow$ denote the (quantum mechanical) rates for the transition $n=1\rightarrow 0$ and $n=0\rightarrow 1$, respectively, then we have

$$P=(w_\downarrow-w_\uparrow)\hbar\omega.$$

Thus we get

$$\gamma=\frac{\hbar(w_\downarrow-w_\uparrow)}{2\omega Au_0^2}. \tag{A1}$$

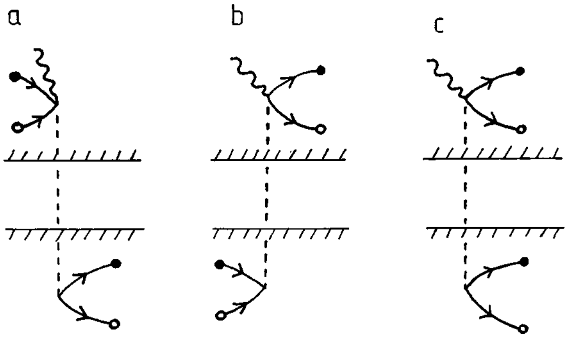

FIG. 3. First-order processes that contribute to the sliding friction. Black dots denote electrons above the Fermi surface and circles denote holes below the Fermi surface.

We will now calculate the rates $w_\downarrow$ and $w_\uparrow$. Let us first focus on $w_\downarrow$. The coupling between the two bodies occurs via the electric field associated with the quantized excitations (e.g., surface plasmons and electron-hole pairs). To lowest order in the coupling between two solids, the three processes indicated in Fig. 3 may occur. In (a) a thermally excited electron-hole pair recombines in solid 1 while simultaneously the oscillator goes from $n=1\rightarrow 0$ and an electron-hole pair is excited in solid 2. This process gives rise to energy and momentum transfer from solid 1 to solid 2 and will contribute to the sliding friction. For the present purposes, we can treat the electron-hole pair excitations as bosons. As shown in Ref. 10, the Hamiltonian for the total system can be written as

$$H=\sum_{\mathbf{q}\alpha_1}\hbar\omega_{\mathbf{q}\alpha_1}b^\dagger_{\mathbf{q}\alpha_1}b_{\mathbf{q}\alpha_1}+\sum_{\mathbf{q}\alpha_2}\hbar\omega_{\mathbf{q}\alpha_2}b^\dagger_{\mathbf{q}\alpha_2}b_{\mathbf{q}\alpha_2}+\hbar\omega b^\dagger b$$

$$+\sum_{\mathbf{q}\alpha_1 n}C_{q\alpha_1}e^{-qz_n}(b_{\mathbf{q}\alpha_1}e^{i\mathbf{q}\cdot(\mathbf{x}_n+\mathbf{u})}+\mathrm{H.c.}). \tag{A2}$$

Here $\omega_{\mathbf{q}\alpha_1}$, $b^\dagger_{\mathbf{q}\alpha_1}$, and $b_{\mathbf{q}\alpha_1}$ are the angular frequency and creation and annihilation operators for the bosons (of solid 1) with the quantum numbers $(\mathbf{q},\alpha_1)$, and $C_{\mathbf{q}\alpha_1}$ parameters determining the coupling between the boson excitations in solid 1 with the electrons in solid 2. Similarly, $b^\dagger_{\mathbf{q}\alpha_2}$ and $b_{\mathbf{q}\alpha_2}$ are creation and annihilation operators for bosons in solid 2, and $(\mathbf{x}_n,z_n)$ is the position operator of electron n in solid 2, which in principle could be expressed in terms of the operators $b^\dagger_{\mathbf{q}\alpha_2}$ and $b_{\mathbf{q}\alpha_2}$, but for the present purpose this is not necessary. As shown in Ref. 9, $C_{\mathbf{q}\alpha_1}$ can be related to $\mathrm{Im}g_1(q,\omega)$ via

$$\sum_{\alpha_1}|C_{\mathbf{q}\alpha_1}|^2\delta(\omega-\omega_{q\alpha_1})=\frac{2e^2\hbar}{Aq}\,\mathrm{Im}g_1(q,\omega). \tag{A3}$$

We expand

$$e^{i\mathbf{q}\cdot\mathbf{u}}\approx 1+i\mathbf{q}\cdot\mathbf{u}=1+iq_xu_0(b+b^\dagger),$$

where we have taken the direction of $\mathbf{u}$ as the x axis. Substituting this in Eq. (A2) gives

$$H=H_0+V_0+V_1, \tag{A4}$$

where

$$V_0=\sum_{\mathbf{q}\alpha_1 n} C_{q\alpha_1}e^{-qz_n}(b_{\mathbf{q}\alpha_1}e^{i\mathbf{q}\cdot\mathbf{x}_n}+\text{H.c.}), \tag{A5}$$

$$V_1=\sum_{\mathbf{q}\alpha_1 n} C_{q\alpha_1}e^{-qz_n}i\mathbf{q}\cdot\mathbf{u}_0(b+b^\dagger)(b_{\mathbf{q}\alpha_1}e^{i\mathbf{q}\cdot\mathbf{x}_n}-\text{H.c.}). \tag{A6}$$

We can now calculate the decay rate $n=1\rightarrow 0$ using lowest-order perturbation theory in V_1, with H_0+V_0 as the zero-order Hamiltonian. For the moment we will neglect the V_0 term, which describes screening, and return to this term later. Now, there are two possible decay channels: $n=1\rightarrow 0$ while $(n_{\mathbf{q}\alpha_1},n_{\mathbf{q}\alpha_2})\rightarrow(n_{\mathbf{q}\alpha_1}+1,n_{\mathbf{q}\alpha_2}-1)$ or $\rightarrow(n_{\mathbf{q}\alpha_1}-1,n_{\mathbf{q}\alpha_2}+1)$, corresponding to Figs. 3(b) and 3(a), respectively. Let us first consider the first process. Using the ''golden rule'' formula for the transition rate gives

$$w_\downarrow=\frac{2\pi}{\hbar^2}\sum_{\mathbf{q}\alpha_1\alpha_2}\sum_{n_{\mathbf{q}\alpha_1}}\sum_{n_{\mathbf{q}\alpha_2}} P_1(n_{\mathbf{q}\alpha_1})P_2(n_{\mathbf{q}\alpha_2})\delta(\omega+\omega_{\mathbf{q}\alpha_2}-\omega_{\mathbf{q}\alpha_1})(\mathbf{q}\cdot\mathbf{u}_0)^2|C_{q\alpha_1}|^2e^{-2qd}|\langle n_{\mathbf{q}\alpha_1}+1,\ n_{\mathbf{q}\alpha_2}-1|\sum_n e^{-q(z_n-d)}$$
$$\times e^{-i\mathbf{q}\cdot\mathbf{x}_n}b^\dagger_{\mathbf{q}\alpha_1}|n_{\mathbf{q}\alpha_1},n_{\mathbf{q}\alpha_2}\rangle|^2$$

or

$$w_\downarrow=\frac{2\pi}{\hbar^2}\sum_{\mathbf{q}\alpha_1\alpha_2}\sum_{n_{\mathbf{q}\alpha_1}}\sum_{n_{\mathbf{q}\alpha_2}} P_1(n_{\mathbf{q}\alpha_1})P_2(n_{\mathbf{q}\alpha_2})\delta(\omega+\omega_{\mathbf{q}\alpha_2}-\omega_{\mathbf{q}\alpha_1})(\mathbf{q}\cdot\mathbf{u}_0)^2|C_{q\alpha_1}|^2e^{-2qd}(n_{\mathbf{q}\alpha_1}+1)|\langle n_{\mathbf{q}\alpha_2}-1|\sum_n$$
$$\times e^{-q(z_n-d)}e^{-i\mathbf{q}\cdot\mathbf{x}_n}|n_{\mathbf{q}\alpha_2}\rangle|^2, \tag{A7}$$

where $P_1(n_{\mathbf{q}\alpha_1})$ is the probability that solid 1 has $n_{\mathbf{q}\alpha_1}$ thermally excited quanta in boson mode $\mathbf{q}\alpha_1$, i.e.,

$$P_1(n_{\mathbf{q}\alpha_1})=\frac{e^{-\beta\hbar\omega_{\mathbf{q}\alpha_1}n_{\mathbf{q}\alpha_1}}}{Z_1},$$

where

$$Z_1=\sum_{n_{\mathbf{q}\alpha_1}} e^{-\beta\hbar\omega_{\mathbf{q}\alpha_1}n_{\mathbf{q}\alpha_1}}.$$

To simplify Eq. (A7), let us write

$$\delta(\omega+\omega_{\mathbf{q}\alpha_2}-\omega_{\mathbf{q}\alpha_1})=\int d\omega'\,\delta(\omega'-\omega_{\mathbf{q}\alpha_1})\delta(\omega-\omega'+\omega_{\mathbf{q}\alpha_2}). \tag{A8}$$

Substituting Eq. (A8) in Eq. (A7) and using Eq. (A3) gives

$$w_\downarrow=\frac{u_0^2e^2}{\pi\hbar}\int d^2q\,\frac{q_x^2}{q}\,e^{-2qd}\int d\omega'[n(\omega')+1]\text{Im}g_1(q,\omega')M_q(\omega'-\omega), \tag{A9}$$

where

$$M_q(\omega)=\sum_{\alpha_2}\sum_{n_{\mathbf{q}\alpha_2}} P_2(n_{\mathbf{q}\alpha_2})\delta(\omega-\omega_{\mathbf{q}\alpha_2})|\langle n_{\mathbf{q}\alpha_2}-1|\sum_n e^{-q(z_n-d)}e^{-i\mathbf{q}\cdot\mathbf{x}_n}|n_{\mathbf{q}\alpha_2}\rangle|^2=\sum_{\alpha_2} n(\omega)\delta(\omega-\omega_{\mathbf{q}\alpha_2})$$
$$\times|\langle 0|\sum_n e^{-q(z_n-d)}e^{-i\mathbf{q}\cdot\mathbf{x}_n}|1\rangle|^2. \tag{A10}$$

But it has been shown elsewhere that[6,7]

$$\frac{\hbar Aq}{2\pi^2e^2}\,\text{Im}g_2(q,\omega)=\sum_{\alpha_2}\ \delta(\omega-\omega_{\mathbf{q}\alpha_2})|\langle 0|\sum_n e^{-q(z_n-d)}e^{-i\mathbf{q}\cdot\mathbf{x}_n}|1\rangle|^2$$

so that

$$M_q(\omega)=\frac{Aqn(\omega)}{2\pi^2e^2}\,\text{Im}g_2(q,\omega). \tag{A11}$$

Substituting this result in Eq. (A9) gives

$$w_{\downarrow}=\frac{Au_0^2}{2\pi^2}\int_0^{\infty}dq\,q^3e^{-2dq}\int d\omega'[n(\omega')+1]n(\omega'-\omega)\mathrm{Im}g_1(q,\omega')\mathrm{Im}g_2(q,\omega'-\omega). \tag{A12}$$

It is clear by symmetry that process (a) in Fig. 3 will give an identical contribution to the decay rate so that we must multiply Eq. (A12) with an extra factor of 2.

Let us now consider the effect of V_0. This term gives rise to screening. It can be taken into account as follows: First note that Eq. (A10) contains the electrostatic interaction energy $\sim\Sigma_n\phi(\mathbf{x}_n)$ where

$$\phi=e^{-qz}e^{-i\mathbf{q}\cdot\mathbf{x}}$$

is (proportional to) the electric potential (for $z>0$) from the boson excitation $|\alpha\mathbf{q}\rangle$ in solid 1. This electric potential will induce (via the V_0 term) a (screening) charge density in solid 2 that will give rise to an electric potential that in turn will polarize solid 1. The electric potential from this induced charge density in solid 1 will give an additional electrostatic potential that for $z>0$ is of the form

$$[g_1(q,0)g_2(q,0)e^{-2qd}]\phi,$$

where we have assumed that the screening is essentially statistical, as the real excitations involved have very low energy (see below). The argument above can now be repeated so that after infinitely many reflections the effective (or mean) field takes the form

$$\phi+[g_1(q,0)g_2(q,0)e^{-2qd}]\phi+[g_1(q,0)g_2(q,0)e^{-2qd}]^2\phi+\cdots=\frac{\phi}{1-g_1(q,0)g_2(q,0)e^{-2qd}}.$$

Substituting this result in Eq. (A10) gives the final result

$$w_{\downarrow}=\frac{Au_0^2}{\pi^2}\int_0^{\infty}dq\,q^3e^{-2dq}\int d\omega'[n(\omega')+1]n(\omega'-\omega)\,\frac{\mathrm{Im}g_1(q,\omega')\mathrm{Im}g_2(q,\omega'-\omega)}{|1-g_1(q,0)g_2(q,0)e^{-2qd}|^2}. \tag{A13}$$

Let us now consider the rate for the oscillator to become thermally excited from $n=0\rightarrow1$. This rate is given by the same formula as above with $\omega\rightarrow-\omega$, i.e.,

$$w_{\uparrow}=\frac{Au_0^2}{\pi^2}\int_0^{\infty}dq\,q^3e^{-2dq}\int d\omega'[n(\omega')+1]n(\omega'+\omega)\,\frac{\mathrm{Im}g_1(q,\omega')\mathrm{Im}g_2(q,\omega'+\omega)}{|1-g_1(q,0)g_2(q,0)e^{-2qd}|^2}. \tag{A14}$$

Let us change $\omega'\rightarrow\omega'+\omega$ in the integral (A13). Next, if we note that for small ω, $\mathrm{Im}g_1(q,\omega)\sim\omega$ and similarly for $\mathrm{Im}g_2$, and if we use that

$$[n(\omega'+\omega)+1]n(\omega')-[n(\omega')+1]n(\omega'+\omega)=n(\omega')-n(\omega'+\omega)\rightarrow-\frac{dn(\omega')}{d\omega'}\,\omega$$

as $\omega\rightarrow0$, we get

$$w_{\downarrow}-w_{\uparrow}=\frac{Au_0^2\omega}{\pi^2}\int_0^{\infty}dq\,q^3\,\frac{e^{-2dq}}{|1-g_1(q,0)g_2(q,0)e^{-2qd}|^2}\int_0^{\infty}d\omega'\left(-\frac{dn(\omega')}{d\omega'}\right)\mathrm{Im}g_1(q,\omega')\mathrm{Im}g_2(q,\omega'). \tag{A15}$$

Now,

$$\int_0^{\infty}d\omega'\left(-\frac{dn(\omega')}{d\omega'}\right)\omega'^2=\frac{(k_BT)^2}{\hbar^2}\int_0^{\infty}dx\,\frac{e^x}{(e^x-1)^2}\,x^2 \tag{A16}$$

and combining Eqs. (A1), (A15), and (A16) gives

$$\gamma=I\,\frac{(k_BT)^2}{\hbar}\int_0^{\infty}dq\,q^3\,\frac{e^{-2qd}}{|1-g_1(q,0)g_2(q,0)e^{-2qd}|^2}\lim_{\omega\rightarrow0}\left(\frac{\mathrm{Im}g_1(q,\omega)\mathrm{Im}g_2(q,\omega)}{\omega^2}\right), \tag{A17}$$

where

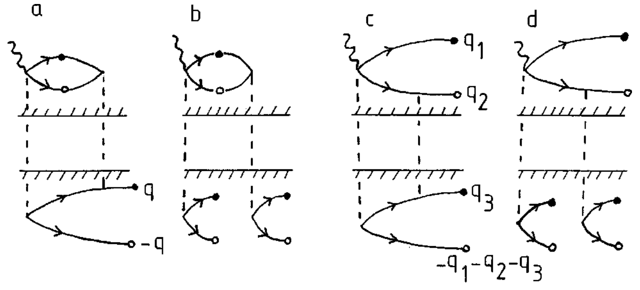

FIG. 4. Second-order processes that contribute to the sliding friction.

$$I=\frac{1}{2\pi^2}\int_0^\infty dx\,\frac{x^2e^x}{(e^x-1)^2}\approx 0.167. \tag{A18}$$

Finally, it is easy to show that process (c) in Fig. 3 gives $w_\downarrow - w_\uparrow \sim \omega^2$ so that, as $\omega\rightarrow 0$, this process gives a vanishing contribution to the friction coefficient γ. This result is physically plausible, since in this case no thermally excited electron-hole pair is involved before the decay $n=1\rightarrow 0$. But only if a thermally excited electron-hole pair is involved before the decay would one expect a frictional coupling between the solids to occur to linear order in v.

APPENDIX B

We present a brief discussion about the contribution from quantum fluctuations (zero temperature) to the friction force. Schaich and Harris[2] have argued that, within the jellium model description of the metals, $\gamma=0$ at $T=0$, i.e., the linear (in the sliding velocity) friction vanishes at zero temperature. The arguments given by Schaich and Harris are not fully transparent to us, so we have considered the problem in some detail. Figure 4 shows four different second-order processes, which we now discuss. [The first-order process (c) in Fig. 3 gives a decay rate proportional to ω^2 and will therefore not contribute to the linear (in v) friction force as $\omega\rightarrow 0$. This result is, in fact, true not only within the jellium model but also when the lattice structure of real solids is taken into account. Thus, quantum fluctuations can only contribute to second order.] Now, it is immediately clear that the second-order processes (a) and (b) cannot, within the jellium model, give a contribution to the sliding friction, since no momentum transfer between the two solids are involved in either of these processes. For a real solid the lattice ions break the parallel translational invariance, and processes (a) and (b) could in principle contribute to the friction force by transferring momentum in units of the reciprocal lattice vector $G=2\pi/a$. However, it is easy to show that even for real solids process (b) does not contribute to the friction force since this process gives $w_\downarrow \sim \omega^2$. As shown in Ref. 11 (see also Ref. 12 and below), for real solids process (a) contributes. Finally, let us consider processes (c) and (d). We have not evaluated these diagrams but we suspect that, because of the small phase space for the final states, they gives decay rates $w_\downarrow$ that vanish faster than linear with ω as $\omega\rightarrow 0$, so that this diagram too will not contribute within the jellium model.

When the lattice structure of the solids is taken into account, the linear sliding friction is nonzero even at $T=0$ K, and can be estimated as follows. In a simple description we treat the metal conduction band as free-electron-like and associate with the ion cores the atomic polarizability $\alpha(\omega)$. The polarizability α is assumed to be due to atomiclike transitions between the ground state A and an exited state B. A quantum fluctuation $A\rightarrow B\rightarrow A$ associated with an atom in, say, solid 1, will give rise to a fluctuating electric field that couples to solid 2. This process is described by the diagram shown in Fig. 4(a). If we neglect the interaction between the transition dipoles on the different atoms, it is easy to extend the study in Ref. 10 to a periodic lattice of atoms. For a simple cubic lattice (lattice constant a) we obtain the friction coefficient

$$\gamma=\frac{e^2n^2}{2\pi\omega}\sum_{\mathbf{G}}|V_{\mathbf{G}}|^2 G\ \mathrm{Im}g_2(G,\omega), \tag{B1}$$

where $n=1/a^2$ is the number of atoms per unit surface area, and where

$$V_{\mathbf{G}}=2i\alpha(0)G\int d^2q'\ \frac{q''_x(\mathbf{q}'\cdot\mathbf{q}''-q'q'')}{q'q''(G+q'+q'')}\,\frac{e^{-(q'+q'')d}}{1-e^{-(q'+q'')a}}, \tag{B2}$$

with $\mathbf{q}''=\mathbf{G}-\mathbf{q}'$. Substituting Eqs. (1) and (B2) in Eq. (B1) gives

$$\gamma=\frac{4e^2\alpha^2(0)n^2}{\pi k_F\omega_p}\sum_{\mathbf{G}} G^4\xi(G)\times\left|\int d^2q'\ \frac{q''_x(\mathbf{q}'\cdot\mathbf{q}''-q'q'')}{q'q''(G+q'+q'')}\,\frac{e^{-(q'+q'')d}}{1-e^{-(q'+q'')a}}\right|^2.$$

In most cases it is enough to include $\mathbf{G}=\pm G\hat{x}$, where $G=2\pi/a$ in the sum over reciprocal lattice vectors. After some simplifications we get

$$\gamma=J(Gd)\,\frac{e^2\alpha^2(0)n^2}{k_F\omega_p}\,G^4\xi(G)\,\frac{e^{-2Gd}}{d^4}, \tag{B3}$$

where $J(Gd)$ is a relatively slowly varying function of Gd.

APPENDIX C

In this appendix we show how Eq. (3) reduces to the result of Sivan *et al.*[13] and Gramila *et al.*[9] for parallel 2D electronic systems. Electron gas layers can be treated as 2D electronic systems for which[14]

$$g=1-\frac{1}{1-\chi v_q}, \tag{C1}$$

where $\chi=\chi(q,\omega)$ is the density-density correlation function for a 2D electron gas and where $v_q=2\pi e^2/\epsilon q$ (ϵ is the dielectric function of the surrounding material). Substituting Eq. (C1) in Eq. (3) gives

$$\gamma=0.167\frac{(k_BT)^2}{\hbar}\int_0^\infty dq\,q^3\,\frac{v_q^2e^{-2qd}}{|[1-v_q\chi_1(q,0)][1-v_q\chi_2(q,0)]-v_q^2\chi_1(q,0)\chi_2(q,0)e^{-2qd}|^2}\lim_{\omega\to 0}\left(\frac{\mathrm{Im}\chi_1(q,\omega)\mathrm{Im}\chi_2(q,\omega)}{\omega^2}\right). \tag{C2}$$

This result agrees with Sivan *et al.*[13] Next, note that the density-density correlation function χ for the 2D electron gas has been calculated (within the random phase approximation) by Stern. For the case $q \ll k_F$, where k_F is the Fermi wave vector of the degenerate electron gas system, one has

$$\chi(q,\omega)=\frac{n}{\epsilon_F}[\eta(\eta^2-1)^{-1/2}-1], \tag{C3}$$

where $\eta=(k_F/q)(\omega/2\omega_F)$. At low enough temperatures, we can take the $\omega\to 0$ limit in Eq. (C3) to get

$$\chi(q,\omega)=\frac{n}{\epsilon_F}(-i\eta-1).$$

Substituting this in Eq. (C1) and assuming $2\pi e^2n/(\epsilon_F q) \gg 1$ gives

$$g\approx 1+i\frac{\epsilon\hbar k_F}{4\pi ne^2}\omega.$$

Substituting this in Eq. (3) gives

$$\gamma=0.167\frac{(k_BT)^2}{\hbar d^4}\left(\frac{\epsilon\hbar k_F}{4\pi ne^2}\right)^2, \tag{C4}$$

which agrees with the result of Gramila *et al.*[8]

Equation (C4) is strictly valid only for 3D electronic systems. For 2D systems the electron-electron scattering cross section[15] and the friction factor γ have logarithmic corrections, e.g., $\gamma\sim T^2[a+b\ \ln(T/T_F)]$. However, the logarithmic correction arises from backscattering contributions ($q \approx 2k_F$), i.e., from processes involving a large momentum transfer, and because of the factor $\exp(-2qd)$ in the Coulomb coupling between the layers, the backscattering contribution is strongly suppressed $[b\sim\exp(-2k_Fd)]$, leading to a nearly $\sim T^2$ dependence of the friction parameter γ. Finally, let us note that since the two electron-gas slabs are separated by a *solid* layer rather than vacuum, it is also possible for the conduction electrons in the two layers to ''communicate'' by emission and absorption of phonons. It has been shown[16] that exchange of *virtual phonons* gives a small contribution to the coupling of the two conducting layers, which can explain the (small) deviation of the observed temperature dependence from the expected $\sim T^2$ dependence. For large separation d between the conducting layers, the phononic contribution will dominate, since it falls off much slower with increasing d as compared with the electronic contribution (which is proportional to $1/d^4$ for 2D systems).

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[1] For general information about sliding friction, see B. N. J. Persson, *Sliding Friction, Physical Principles and Applications* (Springer, Berlin, 1998); *Micro/Nanotribology and its Applications*, edited by B. Bhushan (Kluwer, Dordrecht, 1997); *Physics of Sliding Friction*, edited by B. N. J. Persson and T. Tosatti (Kluwer, Dordrecht, 1996); F. P. Bowden and D. Tabor, *Friction and Lubrication* (Methuen, 1967); E. Rabinowicz, *Friction and Wear of Materials* (John Wiley and Sons, New York, 1965).

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