Modulation instability of electromagnetic radiation in narrow-gap semiconductors

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The modulation instability of electromagnetic radiation propagating in narrow-gap semiconductor plasma is considered. In a dense n-doped InSb semiconductor plasma the instability can develop in a picosecond time scale for the field intensities $E \approx 1.6\times10^5$ V/cm. The numerical simulations confirm the analytical predictions that a modulated localized long pulse splits into a chain of solitonlike structures. Possible applications of this effect to generate short picosecond pulses and the dynamical light grating of the semiconductor at midinfrared wavelengths are suggested. [S0163-1829(98)01411-8]

The recent progress in femtosecond laser techniques has stimulated investigations on the interaction of ultrashort electromagnetic (EM) pulses with matter. The nonlinear self-interaction of EM waves in different kinds of optical media is a subject of considerable interest. The magnitude and response time of nonlinear refraction that leads to EM wave self-interaction depend on the mechanism of nonlinearity. Different types of optical nonlinearities with fast response time have been studied in semiconductors.[1] In n-doped semiconductors moderately large free-carrier concentration may occur via thermal excitation of impurities. Early research on the free-carrier optical nonlinearities in the semiconductors concentrated on the effects of nonparabolicity of conduction-band electrons. In the pioneering work of Patel, Slusher, and Fleury[2] large optical nonlinearities arising from conduction electrons have been observed in n-doped narrow-gap semiconductors (NGS). This work stimulated the research devoted to the nonlinear modulation interactions of EM radiation propagating through semiconductor plasma. Semiconductor plasma is a reasonably high density conduction-band electron plasma in a fixed neutralizing ionic background. In the two-band approximation of Kane's model the Hamiltonian of the conduction-band electrons is analogous to the relativistic one $H=\gamma m_* c_*^2=(m_*^2 c_*^4+c_*^2 p^2)^{1/2}$.[3] Here $c_*=(E_g/2m_*)^{1/2}$ plays the part of the speed of light, m_* is the effective mass of the electrons at the bottom of the conduction band, E_g is the width of the gap, and p is the electron quasimomentum. The characteristic velocity c_* in the Kane's dispersion law is much less than the speed of light (e.g., $c_* \sim 3\times10^{-3}c$ for InSb). Therefore, the nonlinear effects appear for a considerably lower EM field intensity than that required for the normal gaseous plasma. As a consequence, the methodology of relativistic gaseous plasma has been used to show that due to the velocity-dependent effective mass of the conduction electrons it is possible to have: a self-focusing of laser light in NGS,[4] a parametric amplification of EM waves and different kind of resonant excitations of density waves.[5] Since these investigations have been carried out two decades ago, the dynamical properties of the nonlinear interaction of EM waves in NGS were studied mainly on a nanosecond or even slower time scales. In order to avoid the breakdown of the semiconductor these considerations have been limited to the field intensities much lower than 10^7 W/cm^2 [the surface ionization intensity for InSb is 3×10^7 W/cm^2 (Ref. 4)]. Consequently, ''relativistic'' nonparabolicity factor $p^2/m_*^2 c_*^2 \sim e^2E^2/m_*^2 c_*^2 \omega_o^2$ was much less than unity (E and ω_o are electric field and frequency of laser radiation, respectively).

Current technology, however, has made it possible to produce picosecond intense laser pulses with wavelengths ranging from the ultraviolet to the midinfrared.[6] Since such pulses are much shorter than the breakdown time (which typically develops in 10^{-10} sec) the breakdown does not occur for higher intensities of laser radiation allowing us to consider effects with finite strength of nonparabolicity factor $p^2/m_*^2 c_*^2 (\approx 1)$. Notice however, that the pump fluence should be smaller than 0.5 J/cm^2 otherwise it will melt the semiconductor even on a subpicosecond time scale.[7]

In our recent publication it has been shown that an intense short laser pulse propagating through the semiconductor plasma will generate longitudinal Langmuir waves in its wake.[8] Also, an alternative approach for strong longitudinal wave generation by two near frequency laser beams has been suggested.[9] In these studies we focused basically on the case of transparent plasma with characteristic frequency of the laser radiation ω_0 much larger than the plasma frequency ω_{*e}. Consequently, the laser field distortion due to the modulation instability has been neglected. However, the modulation instability may have fast growth rate for the case of dense plasma. Such an instability leads in general to the spontaneous breakup of the EM beam into a periodic pulse train as a result of an interplay between the nonlinear and dispersive effects.

In this paper we consider the modulation instability of

EM radiation propagating through NGS plasma. By confining the interacting EM waves in a semiconductor waveguide of small cross-sectional dimensions, typically of the order of the wavelength, high-power densities can be achieved from sources of relatively moderate power. Such a geometry of waveguide allows an one-dimensional description. In what follows we assume that the laser pulse propagates along the z axis. All physical quantities depend only on the space coordinate z and time t. We start from the wave equation for the vector potential $\mathbf{A}$ and the quasihydrodynamic equations for a wave polarized in the xy plane. The validity of the hydrodynamic approach for the semiconductor plasma requires that both the Fermi energy (E_F) and the temperatures are low ($E_g \gg E_F, k_B T$). This ensures that the high-temperature tail of the electron distribution does not contribute significantly. We also assume that the characteristic frequency Ω of wave processes is much higher than the electron collision frequency ν. The system of equations that describes the dynamics of EM waves in NGS is written as (see for details Ref. 8):

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{c^2}{\epsilon_o}\frac{\partial^2 \mathbf{A}}{\partial z^2} + \frac{\omega_{*e}^2}{\gamma}\frac{n}{n_o}\mathbf{A} = 0, \quad (1)$$

$$\epsilon_o \frac{\partial^2 \phi}{\partial z^2} = 4\pi e(n - n_o), \quad (2)$$

$$\frac{\partial p_z}{\partial t} + m_* c_*^2 \frac{\partial \gamma}{\partial z} = e\frac{\partial \phi}{\partial z}, \quad (3)$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z} n \frac{p_z}{m_* \gamma} = 0, \quad (4)$$

where

$$\gamma = \left(1 + \frac{e^2 \mathbf{A}^2}{m_*^2 c_*^2 c^2} + \frac{p_z^2}{m_*^2 c_*^2}\right)^{1/2}. \quad (5)$$

Here ϕ is the scalar potential of exited longitudinal field, $\mathbf{p}$ is the electron fluid momentum, n is the electron density, and ϵ_o is the dielectric constant of the lattice. The effective plasma frequency ω_{*e} is defined as $\omega_{*e} = (4\pi e^2 n_o / m_* \epsilon_o)^{1/2}$, where n_o is the unperturbed density of the electrons and also of the positive charged background. Deriving Eqs. (1)–(5) we used the conservation of transverse canonical momentum: $\mathbf{p}_\perp = (e/c)\mathbf{A}$.

We next carry out a stability analysis for the circularly polarized EM waves. The monochromatic pump EM wave with frequency ω_o and wave number k_o, is described by

$$\mathbf{A_o} = \tfrac{1}{2}(\mathbf{x} + i\mathbf{y})A_o \times \exp(-i\omega_o t + ik_o z) + \text{c.c.}, \quad (6)$$

where $\mathbf{x}$ and $\mathbf{y}$ are the unit vectors, and $A_o = \text{const}$. The dispersion relation that follows from Eqs. (1)–(5) reads

$$\omega_o^2 = \Omega_p^2 + \epsilon_o^{-1} k_o^2 c^2. \quad (7)$$

Here the plasma frequency $\Omega_p = \omega_{*e}/\gamma_o^{1/2}$ is modified by the nonparabolicity factor, $\gamma_o = (1 + |A_o|^2)^{1/2}$. For convenience we introduced the dimensionless vector potential $\mathbf{A} = e\mathbf{A}/(m_* c_* c)$. The unperturbed state of the plasma is characterized by a purely transverse EM mode $\mathbf{A}_o$ ($\phi = 0 = p_z$) and a constant concentration of carriers ($n = n_o$).

In the linear approximation, for small perturbations of the EM field $\delta\mathbf{A}$ and the plasma density δN we obtain from Eqs. (1)–(5) the following system of equations:

$$\left(\frac{\partial^2}{\partial t^2} - \frac{c^2}{\epsilon_o}\frac{\partial^2}{\partial z^2} + \Omega_p^2\right)\delta\mathbf{A} = -\Omega_p^2 \frac{\delta N}{n_o}\mathbf{A}_o + \frac{\Omega_p^2}{\gamma_o^2}\mathbf{A}_o(\mathbf{A}_o \cdot \delta\mathbf{A}), \quad (8)$$

$$\left(\frac{\partial^2}{\partial t^2} + \Omega_p^2\right)\frac{\delta N}{n_o} = \frac{c_*^2}{\gamma_o^2}\frac{\partial^2}{\partial z^2}(\mathbf{A}_o \cdot \delta\mathbf{A}). \quad (9)$$

Equations (8) and (9) describe field perturbations that are coupled via the pump wave ($\mathbf{A_o}$) and perturbations of the plasma density. To obtain the dispersion equation, we assume that the density perturbations depend on the coordinates and time like $\delta N = \delta N_o \exp(-i\Omega t + ikz)$. Using Eqs. (6)–(9) one can see that the field perturbation has the form

$$\delta\mathbf{A} = \mathbf{A}_+ \exp[-i(\omega_o + \Omega)t + i(k_o + k)z] + \mathbf{A}_- \exp[-i(\omega_o - \Omega)t + i(k_o - k)]. \quad (10)$$

After some algebra Eqs. (8)–(10) lead to the dispersion relation

$$\frac{\Omega_p^2}{2\gamma_o^2}|A_o|^2\left(1 + \frac{k^2 c_*^2}{D_e}\right)\left[\frac{1}{D_+} + \frac{1}{D_-}\right] = 1, \quad (11)$$

where

$$D_\pm = \Omega_p^2 + \epsilon_o^{-1} c^2 (k_o \pm k)^2 - (\Omega \pm \omega_o)^2,$$

$$D_e = \Omega_p^2 - \Omega^2. \quad (12)$$

A dispersion relation similar to Eq. (11) has been derived in Ref. 10 for the interaction of relativistically strong EM radiation with a gaseous electron plasma. However, the term related to the induced variation of density of carriers proportional to $k^2 c_*^2 / D_e$ in Eq. (11) is 10^{-5} ($\sim c_*^2/c^2$) of an analogous term in the gaseous plasma. Straightforward analysis shows that effects related to the induced density variation of the carriers does not contribute significantly in the process of modulation instability and can be safely neglected, considerably simplifying the analysis of Eq. (11).

The modulation instability can be defined as an absolute instability in the group velocity frame of EM wave. Consequently we set $\Omega = kv_g + i\Gamma$, where $v_g = k_o c^2/\omega_o \epsilon_o$ is the group velocity of the pump wave and Γ is the growth rate of instability. We analyze Eq. (11) in two regimes. First we consider the case of short-wave modulations $k \gg k_o$ implying that the frequency of the pump wave is tuned near the plasma frequency ($\omega_o \approx \Omega_p$). One can readily show that for $k < k_l = \epsilon_o^{1/2}(\omega_o/c)(|A_o|/\gamma_o)$, Eq. (11) exhibits the aperiodic instability. The maximum growth rate occurs for

$$k = k_m = \left(\frac{\epsilon_o}{2}\right)^{1/2}\frac{\omega_o}{c}\frac{|A_o|}{\gamma_o}\left(1 - \frac{|A_o|^2}{8\gamma_o^2}\right)^{1/2} \quad (13)$$

and is given by

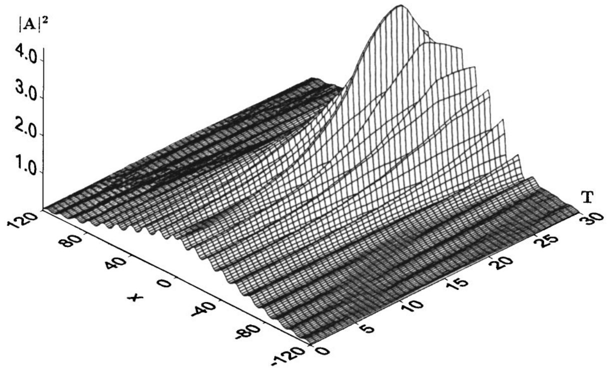

FIG. 1. The field intensity $|A|^2$ is plotted versus X and T for initially Gaussian shaped pulse $|A(X,0)|=A_m\exp(-X^2/2D^2)$ where $A_m=1$, $D=200$.

$$\Gamma_m=\frac{\omega_o}{4}\frac{|A_o|^2}{\gamma_o^2}. \tag{14}$$

In the case of long-wave modulations $k\ll k_o$, Eq. (11) leads to the modulation instability provided that $k<k_l$. The maximum growth rate occurs for $k_m=k_l/2^{1/2}$ and reads

$$\Gamma_m=\frac{\omega_o}{4}\left(\frac{\Omega_p}{\omega_o}\right)^2\frac{|A_o|^2}{\gamma_o^2}. \tag{15}$$

As follows from Eqs. (14) and (15) the growth rate of the modulation instability can reach a considerable value in the dense semiconductor plasma embedded in the field of strong EM radiation. Indeed, for $|A_o|=1$ the growth rate of instability is as high as $\omega_o/8$ in the near critical case, i.e., $\omega_o \approx\Omega_p$ and it is $(\omega_o/8)(\Omega_p/\omega_o)^2$ in the case of underdense plasma $\omega_o\gg\Omega_p$. The wave number of fastest growing modes can be estimated to be $k\approx\epsilon_o^{1/2}\omega_o/2c$, i.e., the wavelength of the modulation is of the same order as a vacuum wavelength of the pump.

The theory is now applied to the case of InSb plasma for which the relevant parameters are $T=77$ K, $m_*=m_e/74$, $\epsilon_o=16$, $c_*=c/253$, and $E_g=0.234$ eV. To avoid one and two-photon absorption one must use the CO_2 laser with the wavelength $\lambda_v\approx10.8\mu$ (i.e., $\omega_o=1.74\times10^{14}$ rad/sec). For the ''relativistic'' case when $|A_o|\approx1$, the corresponding intensity of the EM field is $E\approx1.6\times10^5$ V/cm. This field is one order of magnitude smaller than what is required to bring the electrons to the boundary of the Brillouin zone. Consequently we can assume that deviation of the electron Hamiltonian from Kane's law as well as possible complications related to Zener breakdown should not be important.

In a dense plasma of InSb the estimated characteristic spatial length of exponentially growing modulations is $\lambda=2\pi k^{-1}\approx10\mu$ and growth time $\tau=\Gamma_m^{-1}=45$ fs for the near critical case (i.e., $n_o\approx n_c=2\times10^{18}$ cm^{-3}). For a moderately doped sample with carrier density $n_o=10^{17}$ cm^{-3} the growth time is $\tau=0.9$ ps. However, in a weakly doped InSb with the carrier density $n_o\approx10^{14}$ cm^{-3}, as suggested in Refs. 8 and 9, the growth time of modulations is 0.9 ns and would not harm the process of wake field generation, which takes place on the picosecond time scale. Our model of the modulation instability is valid provided the dissipation of the fields is small and its influence can be neglected. One of the principal differences between semiconductor plasmas and the usual gaseous ones concerns the collisionality. The comparatively short collision times in semiconductors hinder the excitation of collective plasma modes. The relaxation time of free carriers $\tau_o\sim\nu^{-1}$ depends on the mechanism of scattering, which is related to the quality of the sample, the level of doping, and the temperature of the semiconductors. In InSb the electron mobility turns out to be extremely high. Nevertheless, due to the relatively small effective masses of conduction electrons the experimentally measured value of relaxation time for InSb appears to be quite short and lies in the range of $\tau_o=10^{-11}-10^{-13}$ sec. However, the strong laser field itself contributes much towards increasing the relaxation time and it helps in driving the plasma to a collisionless state. Indeed, it is generally assumed that there exists, for the relaxation time, a power law of the type $\tau_r=\tau_o(\overline{e_E}/\overline{e_0})^{s/2}$,[1] where $\overline{e_E}$ is the total average energy in the presence of the field (consisting of the thermal and drift part) and $\overline{e_0}$ is the average energy in absence of the field. Notice that $s=3$ if the momentum losses occur predominantly through the scattering on ionized impurities (which is likely to be case for doped InSb at temperature about 77 K). Considering numbers presented above one gets $\tau_r\approx10^2\tau_o$. Thus, for the case of modulation instability that essentially develops in a subpicosecond time scale the collisionless assumption remains valid.

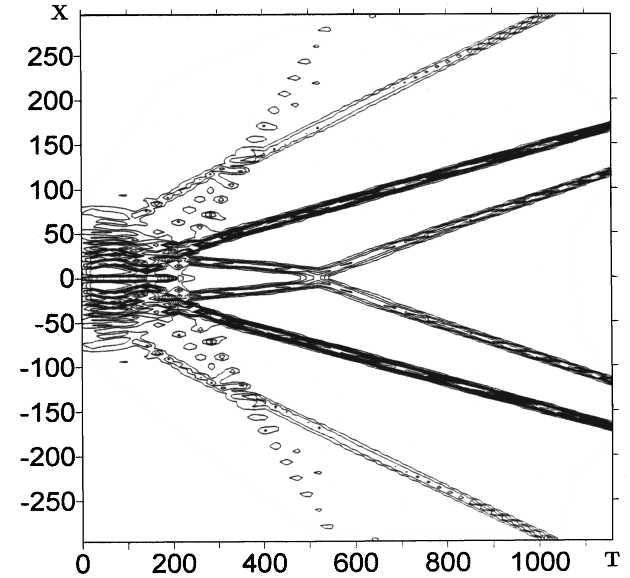

FIG. 2. Intensity contour plot for a long time evolution. Solitary waves are generated and they resist the mutual collisions.

In the nonlinear stage of the modulation instability the initial, wide laser pulse will break down into narrower solitonlike pulses with characteristic length corresponding to the optimum scale of instability ($\sim\lambda$). In the near critical case, the group velocity of the EM waves is negligibly small and the instability, in fact, leads to dynamical light grating of the semiconductor. However, for the underdense case it may lead to generation of a train of subpicosecond solitary pulses.

The nonlinear evolution of the modulated localized EM pulse in the underdense plasma is studied by numerical simulations of Eq. (1), neglecting the density variation of carriers. The behavior of an initially Gaussian-shaped pulse envelope

$|A(X,0)|=A_m \exp(-X^2/2D^2)$ in the dimensionless moving coordinates $X=\epsilon_o^{1/2}(\omega_o/c)(z-v_g t)$ and $T=(\omega_{*e}^2/2\omega_o)t$ is presented in Fig. 1. The pulse amplitude and its width are, respectively, $A_m=1$ and $D=200$. The amplitude of the initially imposed modulation is 0.05 and the wave vector $k=0.5$ is chosen in the range $k<k_l$ according to previous estimations. In agreement with the analytical predictions, the modulation instability occurs resulting in the pulse splitting into a set of narrow solitary pulses. The characteristic time for this process ranges in the real units from 0.23 to 2.3 ps, corresponding to the plasma densities from 10^{18} to 10^{17} cm^{-3}. The solitary wave generation becomes more evident when the long time evolution of the modulation instability is carried out (see Fig. 2).

In conclusion we considered dynamics of modulation instability of EM waves propagating in the NGS plasma. It is shown that in a dense n-doped InSb semiconductor plasma the instability can develop on picosecond time scale. The numerical simulations confirm the analytical predictions that a modulated localized long pulse splits into a chain of solitary pulses. The production of subpicosecond pulses at wavelength $\lambda_o \sim 10\mu$ is based on challenging nonlinear optical parametric down-conversion processes and pushes the femtosecond technology to its current limits. We hope that using the modulation instability, it will be easier and less expensive to generate short subpicosecond pulses at midinfrared wavelengths.

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