Anelastic contributions and transformed volume fraction during thermoelastic martensitic transformations

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The internal-friction spectra of a thermoelastic martensitic transformation are usually obtained as a function of temperature with $\dot{T} \neq 0$ and show three—phase transition, transitory, and intrinsic—contributions. This paper proposes a systematic method to analyze these internal-friction spectra. Indeed, two different procedures are derived. The first one, with the support of an isothermal spectrum ($\dot{T}=0$), allows one to decompose the internal-friction spectrum into its three contributions and in addition to calculate the volume fraction of transformed material. The second one is based on an iterative process, and allows one to separate the intrinsic term from the other two contributions directly related to the martensitic transformation, and to calculate the volume fraction of transformed material, without the aid of an isothermal measurement. This method is successfully applied to a thermoelastic Cu-Al-Ni shape-memory alloy. Then, the dependence of the phase transition term on the volume fraction of transformed material is found. This allows us to reject the models proposed to explain this term that do not take into account this dependence. [S0163-1829(98)00809-1]

I. INTRODUCTION

In recent years there has been an increasing attention to the physics of martensitic phase transformations in metals and alloys, and particularly in shape-memory alloys [see (Ref. 1) for a general overview]. The principal features of interest are related to the technological application of shape memory alloys,^{2,3} that behave as smart materials,^{4–6} and to the fact that this kind of alloy makes a good testing bench to study the physical behavior near the critical temperature of a first-order structural transition like the martensitic transformation.

One of the main characteristics used to define a martensitic transformation is the displaciveness of the latticedistortive type involving a shear-dominant shape change.¹ This means that during martensitic transformation an atomic shearing is responsible for the structural transition between the high-temperature phase (austenite) and the lowtemperature phase (martensite). In fact, due to the elastic instability of their crystalline lattice, the high-temperature phase undergoes a spontaneous homogeneous strain⁷ and a combination of elastic modulii vanishes at the critical temperature, giving rise to a soft mode. This behavior has been widely observed and reviewed.⁸⁻¹⁰ Nevertheless, it should be stressed that the modes never become completely softened¹¹ because the nucleation of martensite takes place through a localized soft mode around the defects^{12–15} in such a way that the incompleteness of the lattice instability leads to a first-order transition.¹⁶ This way, standard soft-mode theory is not applicable because there is a finite discontinuity in the microscopic order parameter at the transition due to the firstorder nature of the martensitic transformation. Fortunately, the relatively small jump observed in the order parameter gives a weakly discontinuous character to the martensitic transformation so that it can be still regarded as a nearly continuous phase transition.^{11,17} Accordingly, the martensitic transformation has been treated in the framework of the Landau-Ginzburg theory using a variety of approaches.^{17–22}

From an experimental point of view, it is not very easy to check the ability of these theories to explain the observed experimental behavior, and for that reason they have always been controversial.¹ Some aspects of this controversy could be attributed to the theoretical models themselves, but other problems arise from the difficulty of comparison between theory and experiments. This difficulty is enhanced by the fact that most of the usual experimental techniques only give information about some particular aspects of the martensitic transformation (transition temperature, hysteresis, transformation latent heat, crystallographic atomic shearing, softening at the transition, etc.). This partial experimental knowledge does not allow checks of the global ability of the theoretical models to explain the experimental behavior.

Among the different experimental techniques to study the martensitic transformation, the anelastic techniques,²³ including the simultaneous measurements of the internal-friction spectrum, the dynamic modulus evolution, and the associated microdeformation²⁴ vs temperature, have been extensively used to characterize the martensitic transformation in different families of shape memory alloys: Ti-Ni (Refs. 25–31), Cu-Zn-Al (Refs. 32–39), Cu-Al-Ni (Refs. 40–43), Cu-Al-Zn-Ni-Mn (Ref. 44), Fe-Mn-Si-*X* (Refs. 45–47), and Fe-Co-Ni-Ti.⁴⁸

5684

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FIG. 1. Schematic representation of IF and its three contributions during a martensitic phase transformation. IF_{Tr} : Transistory term. IF_{PT} : Phase transition (or isothermal) term. IF_{Int} : Intrinsic term.

One of the advantages of the anelastic techniques is that they are able to provide simultaneously a very precise information about a large number of aspects involved in the martensitic transformation: (a) global behavior of the reversible transformation; (b) critical transition temperature; (c) transformation temperatures and hysteresis; (d) specific damping capacity evolution; (e) transformed volume fraction evolution; (f) softening of the modulus at the transition; (g) microdeformation associated with the atomic shearing; (h) pretransitional effects. The knowledge of all of these aspects is very useful making comparisons between theory and experiment, but we would like to remark that it is also very interesting from a technological point of view, particularly points (c) to (g), due to the applications of the shape-memory alloys.^{2–6}

However, the analysis of the experimental data obtained by anelastic techniques during a martensitic transformation requires a specific treatment in order to establish a correlation with each one of the previously indicated parameters and to give a good interpretation of them. Many studies attempted to understand the main aspects involved in this subject, see the works by R. De Batis⁴⁹ and J. Van Humbeeck^{50,51} for a review. It is well known that the internal friction (IF) spectrum obtained during a martensitic phase transformation can be decomposed into three different contributions:⁵²

$$IF(T) = IF_{Tr}(T) + IF_{PT}(T) + IF_{Int}(T).$$
(1)

 $IF_{Tr}(T)$ is the *transitory contribution* of the IF spectrum, because it appears only during heating or cooling and becomes zero when the temperature is held constant. It takes account of the transformation kinetics, and is therefore directly related to the amount of volume fraction of transformed phase per unit time. The $IF_{PT}(T)$ term is associated with the *phase transformation itself*, and is responsible for the IF peak during isothermal measurements. Finally, the *intrinsic term*, $IF_{Int}(T)$, gives the own IF contribution of each phase, martensite and austenite. In Fig. 1 the schematic representation of an IF spectrum and its different contributions during a martensitic transformation are shown. It is obvious that considerable amount of information can be obtained from the total IF spectrum, but to get a more quantitative and precise information it becomes necessary to separate the different contributions of the IF spectrum. Thus, taking into account the dependence of $IF_{Tr}(T)$ on the transformed volume fraction, n(T), it could be possible to calculate n(T) if the IF transitory term were isolated. Moreover, in order to check the different models proposed to explain the martensitic transformation itself, it is necessary to remove the transitory and the intrinsic terms, and use only the phase transformation term $IF_{PT}(T)$. That is the case of the models based on the Landau theory. Nevertheless, there is no experimental procedure to decompose the IF spectrum and to obtain the evolution of the transformed volume fraction n(T).

Commonly, the experimentally measured IF spectrum is carried out at a constant cooling (or heating) rate $(T \neq 0)$, this way that spectrum includes all the contributions, and the problem arises when trying to separate each term. It is also possible, however, to get an IF spectrum at T=0 by measuring each IF point in isothermal condition. In such isothermal measurements, the IF spectrum does not show the transitory contribution and then we obtain $IF_{PT}(T) + IF_{Int}(T)$. This way, subtracting the isothermal spectrum from the normal spectrum obtained at $T \neq 0$, the $IF_{Tr}(T)$ can be separated.⁵³ In the same way, during a low-frequency IF spectrum with a high enough T, the $IF_{PT}(T)$ contribution becomes markedly lower than the $IF_{Tr}(T)$ one (less than 10%), and so the $IF_{PT}(T)$ contribution could be neglected in a first approximation. Then, depending on the experimental conditions, it becomes possible to separate or remove only one of the IF contributions, $IF_{Tr}(T)$ or $IF_{PT}(T)$, keeping the other two terms together, $IF_{PT}(T) + IF_{Int}(T)$ or $IF_{Tr}(T) + IF_{Int}(T)$, respectively.

The aim of the present work is to describe a standard method that permits the complete separation of the different contributions of the IF spectrum obtained during a martensitic transformation. Two different procedures have been developed depending if an isothermal measurement has been carried out in addition to the normal spectrum (at $T \neq 0$) or not. Taking into account this fact, the paper is organized as follows. In Sec. II an overview of the characteristics of the different IF terms and the published theoretical models that explain each IF contribution is presented. This overview shows that there is a common functional expression for the $IF_{Tr}(T)$ term. In Sec. III A the analysis method using an isothermal measurement is explained. This allows one to decompose the total IF spectrum into the three different terms. In Sec. III B a method that may be used to treat the IF spectrum when an isothermal measurement is not available, is described. It consists of an iterative process that uses the relationship between the transformed volume fraction and both the transitory and intrinsic terms. Finally, we summarize the main results obtained in this paper in Sec. IV. Along with the presentation of the analysis method, its application to a thermoelastic Cu-Al-Ni shape memory alloy is shown. This permits us to obtain results on the $IF_{PT}(T)$ contribution.

II. THEORETICAL OVERVIEW

In the previous section it has been indicated that the internal-friction spectrum can be decomposed into three dif-

	$f(\dot{T}, \omega, \sigma_0)$	$IF_{\rm Tr}(T)$			
Belko <i>et al.</i> (Ref. 54)	$\frac{\dot{T}}{\omega T}$	$\frac{\mu V \varepsilon_t^2 \dot{T}}{k\omega T} \frac{dn}{dT}$	μ: shear modulus. V: volume associated to a critical germ. $ε_t$: transformation strain.		
Delorme et al. ^a (Ref. 55)	$rac{\dot{T}}{\omega}(\sigma_0 \operatorname{high})$	$\frac{K\mu\dot{T}}{\omega}\frac{dn}{dT}$	μ : shear modulus. <i>K</i> : constant. σ_0 : oscillation stress-amplitude.		
	$rac{\dot{T}}{\omega\sigma_0}(\sigma_0 \operatorname{low})$	$\frac{K\mu\dot{T}}{\omega\sigma_0}\frac{dn}{dT}$			
Gremaud <i>et al.</i> (Ref. 56)	$\frac{1 - \frac{\pi \dot{T}}{2\alpha\omega\sigma_0}}{1 + \frac{\pi \dot{T}}{2\alpha\omega\sigma_0}}\frac{\dot{T}}{\omega\sigma_0}$	$\frac{2\varepsilon_0^t}{J} \frac{1 - \frac{\pi \dot{T}}{2\alpha\omega\sigma_0}}{1 + \frac{\pi \dot{T}}{2\alpha\omega\sigma_0}} \frac{\dot{T}}{\omega\sigma_0} \frac{\partial n}{\partial T}$ if $\left(\frac{\dot{T}}{\alpha\omega\sigma_0} < \frac{2}{3\pi}\right)$	σ_0 : oscillation stress-amplitude. α : stress induced change of critical temperature (Clausius-Clapeyron coefficient). <i>J</i> : elastic compliance. ε_0^t : stress-free transformation shear strain.		
	$\left(\frac{\dot{T}}{\omega\sigma_0} + \frac{2}{\pi}\alpha\right)$	$\frac{\varepsilon_0^t}{4J} \left(\frac{\dot{T}}{\omega \sigma_0} + \frac{2}{\pi} \alpha \right) \frac{\partial n}{\partial T}$ if $\left(\frac{2}{3\pi} < \frac{\dot{T}}{\alpha \omega \sigma_0} < \frac{2}{\pi} \right)$			
	$rac{\dot{T}}{\omega\sigma_0}$	$\frac{\varepsilon_0^t}{2J} \frac{\dot{T}}{\omega \sigma_0} \frac{\partial n}{\partial T}$ if $\left(\frac{\dot{T}}{\alpha \omega \sigma_0} > \frac{2}{\pi}\right)$			
Wang <i>et al.</i> (Ref. 57)	$\left(rac{\dot{T}}{\omega} ight)^{1-r}$	$A(T)\left(\frac{\dot{T}}{\omega}\right)^{1-r}$	A(T) temperature-dependent coefficient. 0 < r < 1, (materials with elastic softening). r=0 (materials without elastic softening).		
Zhang <i>et al.</i> ^b (Ref. 58)	$rac{\dot{T}^m}{\omega^{m+2q}}$	$A(T) rac{\dot{T}^m}{\omega^{m+2q}}$	A(T) temperature-dependent function. 0 < m, q < 1		

TABLE I. The most important IF_{Tr} models can be expressed as: $IF_{Tr}(T) = K \partial n / \partial T f(\dot{T}, \omega, \sigma_0)$. The $f(\dot{T}, \omega, \sigma_0)$ and $IF_{Tr}(T)$ functions for these models are shown.

^aOther authors, W. Dejonghe *et al.* (Ref. 59), and more recently, T. Xiao (Ref. 60), have taken into account the possibility that the material transforms under the oscillating stress, and then an isothermal term is added, but the transitory contribution remains like the Delorme model to high oscillation amplitude.

^bIn the Zhang *et al.* model there is also an isothermal term that can be expressed as $B(T)\omega^{1-2q}$.

ferent terms (1). In this section we present the origins and dependences of each contribution, as well as a summary of the theoretical and empirical models proposed for each term.

The $IF_{Tr}(T)$ is a kinetic term, which only appears during heating or cooling, and is closely related to the transformed volume fraction. It shows a peak during the martensitic transformation and is strongly dependent on external variables such as the temperature rate (\dot{T}) , the oscillation frequency (ω) and the oscillation stress amplitude (σ_0). The specific variable contribution depends on the different models proposed, but in general can be expressed as

$$IF_{\rm Tr}(T) = K \frac{\partial n}{\partial T} f(\dot{T}, \omega, \sigma_0), \qquad (2)$$

where *n* is the volume fraction of transformed martensite, *K* is a constant, and $f(\dot{T}, \omega, \sigma_0)$ is a function of \dot{T} , ω , and σ_0 .

The specific $IF_{Tr}(T)$ and $f(T, \omega, \sigma_0)$ functions for the most important models are displayed in Table I. In view of this table, some general observations on the dependence of $IF_{Tr}(T)$ on the temperature rate and the frequency can be made. $IF_{Tr}(T)$ is directly proportional to \dot{T} and inversely proportional to ω , and the general relationship between these parameters can be expressed as $IF_{Tr} \propto \dot{T}^p / \omega^l$. Here *l* and *p* are two coefficients equal to one in the models of Belko *et al.*,⁵⁴ Delorme and Gobin,⁵⁵ and Gremaud and co-workers⁵⁶ (in the last model, this is an approximation, that becomes exact when the term \dot{T}/ω is high enough), whereas they are less than one in the models of Wang and co-workers⁵⁷ and Zhang and co-workers.⁵⁸

However, the most important fact is to notice the linear relationship between $IF_{Tr}(T)$ and $\partial n/\partial T$. Although in the Wang and co-workers⁵⁷ and Zhang and co-workers⁵⁸ models

		$IF_{\rm PT}(T)$
Mercier and Melton	$\frac{\lambda L^4 B \omega}{[F(\sqrt{A^{-1}})]^2}$	 λ: density of dislocations. L: average loop length. B: viscous coefficient.
(Ref. 62)		<i>F</i> : is a function that depends on the crystallographic directions of the dislocations, <i>A</i> is the anisotropic factor.
Kosimizu (Ref. 63)	$rac{\chi^2 M \omega G_0}{\omega^2 + 1/ au^2}$	G_0 : unrelaxed modulus. au: relaxation time, calculated using the Landau theory of the first-order phase transition. M: parameter depending on the kinetics of the order parameter.
Kuska (Ref. 64)	$\frac{\chi^2 M \omega G_0}{\omega^2 + 1/\tau^2 + \chi^2 M G_0 / \tau}$	χ^2 : coupling constant between the stress and the orde parameter.

TABLE II. $IF_{PT}(T)$ contribution proposed by the different models.

there is no explicit dependence of the internal friction on the volume fraction of transformed material, their calculations clearly show that a direct relationship must exist between both functions IF(T) and n(T). In addition, Zhang and co-workers⁶¹ have found experimentally that the volume fraction of the new phase is equivalent to the normalized internal friction integral.

The phase transformation $IF_{PT}(T)$ term is independent of \dot{T} and is responsible for the IF peak during isothermal measurements ($IF_{PT}(T)$ is also called the isothermal term), being the most important contribution in kHz frequency range measurements. According to the explanation given by Bidaux and co-workers,⁵² $IF_{PT}(T)$ is related to the reversible displacement of the martensite—austenite interfaces during one oscillation cycle, while the $IF_{Tr}(T)$ is associated with the displacement of interfaces over large distances, produced by the transformation kinetics. The main models that explain this contribution are schematically represented in Table II. The first one was proposed by Mercier and Melton⁶² who assumed that the origin of this term is a dislocation relaxation process. The models by Kosimizu⁶³ and Kuska⁶⁴ are based on the Landau theory of the phase transition.

The *intrinsic term* $IF_{Int}(T)$ is the sum of the separate IF contributions of each phase, martensite in the low-temperature side and austenite in the high one. It depends only on the microstructure of each phase and consequently is expressed as

$$IF_{Int}(T) = n(T)IF_m + [1 - n(T)]IF_a,$$
 (3)

where IF_m and IF_a are the martensite and the austenite IF contributions, respectively.

III. ANALYSIS METHOD OF INTERNAL-FRICTION SPECTRA

The method of analysis that we will describe subsequently has been applied successfully to IF spectra of different alloys (Fe- and Cu-based alloys). Particularly, in this paper, we use the method to analyze the martensitic transformation in a Cu-13Al-3Ni (wt. %) polycrystalline alloy. The measurements were carried out in an inverted torsion pendulum working at about 1 Hz with a temperature rate of change of 60 K/h for the nonisothermal measurement.

The starting point is the experimental IF spectrum obtained as a function of temperature IF(T) between a starting T_s and a finishing T_f . The temperature range (T_s, T_f) is assumed to be large enough to reach the background regions on both peak sides. This way during the forward (cooling) spectrum: $T_s > M_s > M_f > T_f$, where M_s and M_f are the martensite starting and the martensite finishing temperatures, respectively. And during the reverse (heating) spectrum: $T_s < A_s < A_f < T_f$, where A_s and A_f are the austenite starting and the austenite finishing temperatures respectively.

The IF spectra and the square frequency curve (\propto modulus) for a nonisothermal measurement appear in Figs. 2(a) and 2(b), respectively. On the other hand, Fig. 3 shows an isothermal spectrum obtained during the reverse martensitic transformation stabilizing the temperature every ten degrees approximately. Comparison of the two IF spectra makes evident the difference between the IF values of the two kinds of measurements, linked to the absence of $IF_{Tr}(T)$ in the isothermal spectrum.

A. Using an isothermal IF measurement

In this section we describe the procedure to be applied when it is possible to obtain an isothermal spectrum in addition to a nonisothermal one. As pointed out in the introduction, subtracting the isothermal spectrum from a nonisothermal one (with three contributions) obtained immediately after, we get the $IF_{Tr}(T)$ contribution [see Fig. 4(a)]. Moreover, it has been shown in Sec. II that the transitory contribution of the IF spectrum could be expressed in general as

$$IF_{\rm Tr} = K \,\frac{\partial n}{\partial T} \frac{T^p}{\omega^l},\tag{4a}$$

where the *p* and *l* coefficients are equal to one, except for the models of Wang and co-workers⁵⁷ and Zhang and co-workers.⁵⁸

This way, calculating the integral function we get

$$\int_{T_s}^{T} IF_{\mathrm{Tr}} \omega^l dT = K \dot{T}^p n(T), \qquad (4b)$$



FIG. 2. Experimental internal friction (a) and square frequency (b) measurements during the forward (black circles) and reverse (open circles) martensitic transformations in Cu-13Al-3Ni (wt. %) shape-memory alloy.

and then taking into account that $n(T_s) = 0$ and $n(T_f) = 1$, we obtain the normalization condition

$$K\dot{T}^{p} = \int_{T_{s}}^{T_{f}} IF_{\mathrm{Tr}} \omega^{l} dT.$$
(4c)

Expressions (4b) and (4c) allow us to obtain the volume fraction of transformed martensite as a function of temperature:



FIG. 3. Isothermal IF spectrum during a reverse martensitic transformation. The line is the interpolated curve.



FIG. 4. Curves of the three IF contributions during a reverse martensitic transformation: (a) The transitory term, obtained subtracting the interpolated curve of Fig. 3 from the experimental curve of Fig. 2(a). (b) Intrinsic term and n(T), obtained from the $IF_{Tr}(T)$ curve and the square frequency spectrum using Eqs. (4d) and (3). (c) $IF_{PT}(T)$ contribution, obtained subtracting the $IF_{Int}(T)$ curve from the isothermal IF spectrum of Fig. 3.

$$n(T) = \int_{T_s}^{T} IF_{\mathrm{Tr}} \omega^l dT / \int_{T_s}^{T_f} IF_{\mathrm{Tr}} \omega^l dT.$$
 (4d)

Therefore, it is clear that n(T) can be easily calculated from the $IF_{Tr}(T)$ using the last equation, and n(T) so obtained is plotted in Fig. 4(b). $IF_{int}(T)$, also plotted in Fig. 4(b), shows the same functional shape as n(T), according to Eq. (3). Its numerical values can be obtained using the values of IF_a and IF_m , which can be easily calculated from the IF spectrum. Once two terms of the IF spectrum have been calculated, it is



FIG. 5. Relationship between n(T) and the normalized integral curve of $IF_{PT}(T)$.

trivial to get the third, that is, $IF_{PT}(T)$. This contribution, obtained during a reverse martensitic transformation, is plotted in Fig. 4(c); it shows a peak during the transformation.

It is important to notice that the normalized integral curve of $IF_{PT}(T)$ term looks like the transformed fraction curve n(T) of Fig. 4(b). Moreover, it can be seen in Fig. 5 that the linear correlation between n(T) and the integral curve of $IF_{PT}(T)$ is qualitatively good. This means that there is a clear dependence of $IF_{PT}(T)$ on n(T), that has not been noticed previously. Nevertheless, the models proposed to explain the $IF_{PT}(T)$ term,^{62–64} do not take into account any explicit relationship between this IF contribution and the volume fraction of transformed martensite (see Table II). Then, in the light of this new result, it is clear that these models should be revised to include explicitly this dependence of $IF_{PT}(T)$ on n(T).

Once the three IF contributions have been separated, it is obvious that they will be useful to determine the different interesting parameters of the martensitic transformation from a technological point of view, or alternatively to compare the experimental behavior with the theoretical models. In addition, important new results can be obtained from the study of the different contributions, such as the relationship between $IF_{PT}(T)$ and n(T) that has been mentioned before.

B. Using only a nonisothermal IF measurement

Section III A shows how to decompose the IF spectrum into the three terms and how to obtain n(T) making use of an isothermal internal friction measurement. However, getting an isothermal IF spectrum is difficult and tedious because it is necessary to stabilize the temperature for each data point. Then, the problem to solve now is how to analyze a normal IF(T) spectrum including the three terms, without the support of an isothermal spectrum.

1. Relationship between the internal-friction integral and the volume fraction of transformed martensite

Equation (4d) is not immediately useful in this case because $IF_{Tr}(T)$ can not be previously calculated. In order to overcome this problem we propose the following procedure. First, we define the integral J(T) as

$$J(T) = \int_{T_s}^T (IF - IF_{\text{Int}}) \omega^l dT = \int_{T_s}^T IF_{\text{Tr}} \omega^l dT + \int_{T_s}^T IF_{\text{PT}} \omega^l dT,$$
(5a)

and consequently,

$$J(T_f) = \int_{T_s}^{T_f} (IF - IF_{\text{Int}}) \omega^l dT = \int_{T_s}^{T_f} IF_{\text{Tr}} \omega^l dT + \int_{T_s}^{T_f} IF_{\text{PT}} \omega^l dT.$$
(5b)

Applying Eq. (4d), we obtain

$$J(T) = n(T) \int_{T_s}^{T_f} IF_{\mathrm{Tr}} \omega^l dT + \int_{T_s}^T IF_{\mathrm{PT}} \omega^l dT.$$
 (5c)

From these equations we reach the result

$$\frac{J(T)}{J(T_f)} = n(T) + \Delta n(T), \tag{6}$$

where

$$\Delta n(T) = \frac{\int_{T_s}^{T} IF_{\rm PT} \omega^l dT - \frac{J(T)}{J(T_f)} \int_{T_s}^{T_f} IF_{\rm PT} \omega^l dT}{\int_{T_s}^{T_f} IF_{\rm Tr} \omega^l dT}.$$
 (7)

It will be shown later that $\Delta n(T)$ can be neglected during low frequency measurements and high enough \dot{T} , and in these cases, there is a simple relationship between J(T) and n(T):

$$\frac{J(T)}{J(T_f)} \approx n(T). \tag{8}$$

This means that the n(T) curve can be calculated from Eq. (8), with no need to separate previously the transitory contribution $IF_{Tr}(T)$. The way to get the J(T) integral from the experimental IF spectrum is described in the next section.

2. Iterative process

Taking into account the relationship between the J(T) integral and the n(T) obtained in the Sec. III B 1, it is possible to determine the $IF_{Int}(T)$ contribution and consequently n(T) using the following iterative process. The process begins with the proposal of an arbitrary initial function for the intrinsic term $[IF_{Int}(T)]_{initial}$. This function is subject only to the condition $IF_m \ge [IF_{Int}(T)]_{initial} \ge IF_a$, where IF_a and IF_m are the austenite and martensite internal friction values, respectively, which are easily calculated from the IF spectrum.

From the chosen $[IF_{Int}(T)]_{initial}$ function, we estimate $IF(T) - [IF_{Int}(T)]_{initial}$, and then we calculate the integral $[J(T)]_{initial}$. This is directly related to the transformed volume fraction [Eq. (8)], thus an expression for n(T) is obtained. Introducing this expression in Eq. (3) and using the values of IF_a and IF_m previously calculated, a new expression for $IF_{Int}(T)$ can be found $[IF_{Int}(T)]_1$.



FIG. 6. Three possible initial functions for the IF intrinsic contribution used to check the iterative process.

Using this new $[IF_{Int}(T)]_1$ function, instead of $[IF_{Int}(T)]_{initial}$, at the beginning of the process we perform an iteration. Several iterative cycles must be done until the difference between the input function $[IF_{Int}(T)]_{j-1}$ and the output function $[IF_{Int}(T)]_j$ in an iteration cycle becomes smaller than a prefixed value and thus a final $IF_{Int}(T)$ is reached. This process can be programmed easily for a computer, and then the IF intrinsic contribution may be quickly and precisely evaluated.

3. Checking the iterative process

In order to check the influence of the initial function $[IF_{Int}(T)]_{initial}$ chosen to begin with the iterative process we will use three different $[IF_{Int}(T)]_{initial}$ that are displayed in Fig. 6. In our opinion, these are the most intuitive functions one can choose, but we remark that it may be whichever with the only condition that it takes values between IF_a and IF_m . Once the iterative process is applied, three important facts can be remarked:

(a) In order to calculate the J(T) integral, the *l* coefficient has been chosen to be one, as it is assumed in most of the models. Nevertheless, the real *l* value could be calculated experimentally making measurements as a function of the frequency. However, several proofs have been done with different *l* and only slight differences have been observed.

(b) The process converges very quickly, where the square root of the quadratic differences between consecutive cycles has been used as parameter to check the convergence rate. In Table III, the evolution of this parameter during the application of the iterative process is shown. Columns *A*, *B*, and *C* correspond to the initial functions shown in Figs. 6(a)-6(c), respectively. After four iteration cycles the values of the convergence parameter become lower than the experimental er-

TABLE III. The evolution of the square root of the quadratic differences between consecutive cycles during the iterative process is shown in order to estimate the convergence rate. Columns *A*, *B*, and *C* give the evolution for the three different $[IF_{Int}(T)]_{initial}$ functions shown in Figs. 6(a)–6(c), respectively.

Iteration cycle	Α	В	С
1	1.63×10^{-2}	2.07×10^{-2}	1.64×10^{-2}
2	2.55×10^{-4}	2.11×10^{-3}	1.44×10^{-3}
3	2.40×10^{-6}	1.19×10^{-4}	3.64×10^{-5}
4	8.37×10^{-8}	6.12×10^{-6}	1.42×10^{-6}
5	6.95×10^{-9}	5.43×10^{-7}	1.22×10^{-7}
6	4.00×10^{-9}	1.74×10^{-8}	5.94×10^{-9}
7	1.70×10^{-9}	3.04×10^{-9}	2.41×10^{-9}
8	0.00	8.07×10^{-10}	6.59×10^{-10}
9	0.00	0.00	0.00

ror of the IF data, that is, 10^{-5} . In any case, to show the mathematical convergence, results for ten iteration cycles are presented in Table III. A stationary function is reached after eight iteration cycles.

(c) The final $IF_{Int}(T)$ function does not depend on the initial one and the three different $[IF_{Int}(T)]_{initial}$ shown in Fig. 6 give the same final result. The final $IF_{Int}(T)$ obtained from the IF spectra of Fig. 2(a) are plotted in Fig. 7(a).

Once we get the final $IF_{Int}(T)$ function, making use of the expression (3) the volume fraction of transformed martensite, $n(T) = (IF_{Int}(T) - IF_a)/(IF_m - IF_a)$, is obtained. The technical transformation temperatures can be easily calculated from the n(T) curve. It is usual in technological applications to determine these temperatures at a specific percentage of transformation. The transformation temperatures displayed in Fig. 7(a) are taken at 5% and 95%.



FIG. 7. Final results of the iterative process for the IF spectra shown in Fig. 2 (black and open circles stand for forward and reverse transformations, respectively): (a) curve of the intrinsic term and the n(T). (b) curve of the transitory plus phase transformation contributions.



FIG. 8. Linear correlation between the n(T) curve obtained using the isothermal spectrum and the $n(T) \approx J(T)/J(T_f)$ curve obtained using the iterative process during a reverse martensitic transformation.

Subtracting the calculated $IF_{Int}(T)$ from the total IF spectrum one gets the two contributions directly related to the martensitic transformation: $IF_{Tr}(T)$ and $IF_{PT}(T)$. This curve is plotted in Fig. 7(b).

In order to check the accuracy of the iterative method we have compared the n(T) curve obtained from the IF isothermal spectrum [see Fig. 4(b)] to the approximate one [Eq. (8)] obtained by direct application of the iterative method to the three contributions spectrum. Figure 8 shows that the linear correlation between the two n(T) curves is very good. The deviation from the plotted straight line represents the difference between the n(T) and the approximated function $J(T)/J(T_f)$, that is $\Delta n(T)$. This difference remains always below 1.5% of transformed material. Consequently, the $\Delta n(T)$ term in Eq. (7) can be neglected, and the approximate relationship (8) turns out to be correct, as it was previously supposed.

IV. SUMMARY AND CONCLUSIONS

An overview of the theoretical models published to interpret the IF spectrum during martensitic transformation has been done. This overview shows a common functional expression for the $IF_{Tr}(T)$ term. Taking into account this functional form, a standard method that permits a systematic and objective treatment of the IF spectrum during a martensitic phase transformation has been developed. Thus, the three contributions of the IF spectrum are obtained and in addition the curve of transformed volume fraction of martensite is calculated. In Fig. 9 the general schema of the analysis method is shown.

The left branch describes the procedure to be applied

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FIG. 9. General schema of the analysis method. Dashed lines show the different terms obtained by the process. Using a nonisothermal spectrum plus an isothermal one the total spectrum can be decomposed, obtaining the three IF terms and n(T) (left branch). From a nonisothermal spectrum by means of the iterative process $IF_{Int}(T)$, $IF_{Tr}(T) + IF_{PT}(T) \approx IF_{Tr}(T)$ and n(T) can be calculated (right branch).

when it is possible to use an isothermal spectrum in addition to the normal IF spectrum (nonisothermal). This way, the total decomposition of the spectrum and the n(T) curve are obtained. This is an interesting fact for the experimental checking of the theoretical models proposed to explain the martensitic phase transformation, like the models based of the Landau theory, for which the $IF_{PT}(T)$ must be isolated. Thus, the dependence of this term on n(T) has been found that allows us to reject the proposed theories^{62–64} to explain this term.

The right branch in Fig. 9 displays the procedure to be applied when only the nonisothermal spectrum is available. This way it is possible to obtain the intrinsic term $IF_{Int}(T)$, the n(T) curve and also the $IF_{Tr}(T) + IF_{PT}(T)$ curve that in the low frequency and \dot{T} high enough ranges is approximately just the $IF_{Tr}(T)$ contribution. It has been pointed out that a total decomposition of the spectrum is possible only with the aid of an isothermal spectrum. Nevertheless, from nonisothermal measurements, in spite of the impossibility of a total decomposition, there can be obtained information of technological interest such as the n(T) curve and the damping capacity evolution during the transformation. From the n(T) curve the characteristic transformation temperatures can be accurately determined.

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