# Vortex correlation lengths and bundle sizes from voltage noise in  $YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>$

K. E. Gray

*Materials Sciences Division, Argonne National Laboratory, Argonne, Illinois 60439*

(Received 20 August 1997)

Voltage noise is one way to estimate flux-bundle sizes in superconductors undergoing flux creep. Modeling such data from epitaxial films of  $YBa_2Cu_3O_7$ , we show that at high temperatures, the bundle size grows with the  $c$ -axis correlation length of pancake vortices (residing in the Cu-O planes), while at low temperatures, flux creep is consistent with vortex-line bundles which are larger than a minimum size. The separation of these regimes is roughly at the vortex-lattice melting line, implying that a large *c*-axis correlation length is needed for vortex-lattice freezing.  $[$0163-1829(98)00509-8]$ 

## **INTRODUCTION**

Since its introduction by Van Gurp, $<sup>1</sup>$  voltage noise asso-</sup> ciated with flux flow has been used to estimate flux-bundle sizes in superconductors undergoing flux creep. The traditional assumption of flux jumps with a single bundle size (i.e., number of Abrikosov vortices) and time duration is attractive for its simplicity. The bundle size is proportional to  $S_v(0)/V_{dc}$ , where  $S_v(0)$  is the noise power spectrum in the limit of zero frequency and  $V_{dc}$  is the time-averaged voltage. The intervening years have seen much activity in this field. Clem addressed<sup>2</sup> the effect of changes of magnetic flux in the external measuring circuit due to flux creep in the sample. He also developed<sup>2</sup> a general theoretical framework in which flux motion was regarded in terms of rigid flux lines that reduced the dimensionality of the problem to a twodimensional  $(2D)$  one. Thompson and Joiner<sup>3</sup> introduced the concept of ''interruption of fluxoid motion by pinning centers,'' which explicitedly included averages over the distance the bundles move before being repinned, as well as their size. This,<sup>3</sup> together with Clem's works,<sup>2</sup> are the starting points for the present analysis of voltage-noise data in hightemperature superconductors (HTS's). Concerns over the limitation of Clem's theory<sup>2</sup> to 2D are addressed by Placais and co-workers,<sup>4</sup> but this may not be as relevant to HTS's. The reason is related to the long penetration length in HTS's, compared to typical intervortex distances, which avoids the large field modulations of the flux lattice found in lowtemperature superconductors. As such, vortex cores can move with little change in the field profile. As a example, vortex motion in HTS's can occur independently by pancake vortex motion<sup>5</sup> in individual Cu-O bilayers. Other significant differences in HTS's include flux-lattice melting and the greater role of thermal fluctuations.

Fairly complete data<sup>6</sup> have been obtained on epitaxial films of  $YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>$  for fields parallel to the *c* axis. Since this experiment was done in the limit of small currents, i.e., linear response, any flux-bundle correlations should closely resemble those in thermal equilibrium. The data display a noise peak vs temperature: it is field-dependent, corresponds closely to the vortex-lattice melting line<sup>7</sup> and separates two regimes. At higher temperatures, we present strong evidence that the interbilayer Josephson coupling energy, compared to  $k_B T$ , determines the noise voltage by setting the

*c*-axis correlation length of pancake vortices residing in individual bilayers. This, in turn, reinforces the *ab*-plane correlation length and thus determines the average bundle size. At low temperatures, the *c*-axis correlation length exceeds the sample size and is thus irrelevant, but there is a need to explicitly include the bundle-size dependence and bundle statistics. A sum over *all* bundle sizes adequately follows the temperature dependence of  $S_v(0)/V_{dc}$ , but experimentally,  $S_v(0)/V_{\text{dc}}^2$  exhibits a strong thermal activation that the model does not predict. We show that a consistent solution to this dilemma can be found by assuming there is a minimum bundle size that can be activated at low temperature.

#### **FLUX-FLOW NOISE**

For voltage pulses of equal magnitude  $\delta V$ , with an average frequency *v*, and a Poisson distribution of pulse lengths (with average,  $\tau_0$ ), the time-averaged voltage  $V_{dc}$ , and noise power spectrum (at frequency,  $\omega$ )  $S_v(\omega)$ , are<sup>8</sup>

$$
V_{\rm dc} = \nu \, \delta V \tau_0, \tag{1}
$$

$$
S_v(\omega) = 4V_{dc}\delta V \tau_0 / (1 + \omega^2 \tau_0^2) = S_v(0) / (1 + \omega^2 \tau_0^2). \quad (2)
$$

For the motion of magnetic flux vortices in superconductors, a workable approximation to Clem's model<sup>2</sup> gives

$$
\delta V = \Phi v_{\text{ff}} / w = \Phi / \tau_0 w_{\text{ff}} / w, \qquad (3)
$$

where  $\Phi$  is the total flux moving,  $w$  is the sample width in the direction of flux flow,  $w_{\text{ff}}(\leq w)$  is the distance the bundle moves in the time  $\tau_0$  at a velocity  $v_{\text{ff}}$ . Here  $v_{\text{ff}}$  is the *instantaneous* flux-flow velocity, which could be taken to be the free-flux-flow velocity of Bardeen and Stephen,<sup>9</sup> but the results that follow do not depend on its value. Measuring both at the same current, a useful ratio is

$$
S_v(0)/V_{\text{dc}} = 4 \, \delta V \tau_0 = 4 \, \frac{w_{\text{ff}}}{w} \, \Phi = 4 \, \frac{w_{\text{ff}}}{w} \, \phi_0 n, \tag{4}
$$

since if  $\Phi$  has a single value, it is determined by this ratio, within an uncertainty of  $w_{ff}/w$ . Here, the number of Abrikosov vortex lines (of flux  $\phi_0$ ) in the moving flux bundle is  $n=\Phi/\phi_0$ . If all bundles move across the entire sample,  $\tau_0$  is constant,  $w_{\text{ff}} = w$ , and  $S_v(0)/V_{\text{dc}} = 2 \delta V \tau_0$ . Equation (4) is

0163-1829/98/57(9)/5524(5)/\$15.00 57 5524 © 1998 The American Physical Society

preferred because a distribution of pulse lengths is expected, even though it will have a cutoff at  $\tau_0 = w/v_{\rm ff}$  and is not Poisson.

Real systems can also have a distribution of bundle sizes, and this introduces a complication into the above simple interpretation of *n* from  $S_v(0)/V_{dc}$ . Bigger bundles have a larger relative effect on  $S_v(0)$  because of its quadratic dependence on *n*, since  $\Phi = n/\phi_0$  in Eqs. (1)–(3). Because thermal activation favors smaller bundles, the assumption of a constant  $\nu$  [needed for Eq. (2)] will be invalid. For the layered high-temperature superconducting cuprates, one can also decouple vortex lines along their length into shorter segments,<sup>5</sup> as small as 2D-pancake vortices in individual Cu-O layers (or more commonly in each strongly coupled Cu-O bilayer).

## **HIGH-TEMPERATURE MODEL**

At sufficiently high temperatures, the vortex lines can break up into segments that are shorter than the sample thickness. The bundle size is best described by the number of pancake vortices  $n_p$ , treating the bilayers as a single unit. Then  $n_p$  is affected by the *c*-axis correlation length  $l_c$ , of the pancake vortices and Eq.  $(3)$  is modified to

$$
\delta V = \phi_0(n_p s/d) v_{\rm ff}/w,\tag{5}
$$

where *d* is the sample thickness along the experimental field direction, i.e., perpendicular to the Cu-O bilayers and *s* is the bilayer repeat distance. Considering the probability of breaking the Josephson bond between pancakes in neighboring Cu-O bilayers,  $l_c$  should depend on the Josephson coupling energy  $E_i(T, B)$  as

$$
l_c = s \exp\{E_j(T, B)/k_B T\}.
$$
 (6)

Near  $T_c$ ,  $E_i(T,B) \sim (1-t-b)$ , where  $b = B/B_{c2}(0)$  and  $B_{c2}(T) = B_{c2}(0)$  (1-*t*<sup>2</sup>) is the clean-limit upper critical field. As  $E_i(T, B)$  becomes comparable to  $k_B T$ ,  $l_c$  can be less than the sample size and the average bundle size should be  $\eta(l_c/s)^{\zeta}$  where  $\zeta$  is of order 1–3. This is physically intuitive: the collective effect of a increasing correlation length in one direction can strengthen the correlation in other directions as well. Then,

$$
S_v(0)/V_{\text{dc}} = 4 \frac{s}{d} \frac{w_{\text{ff}}}{w} \phi_0 \eta \exp{\{\zeta E_j(T)/k_B T\}}
$$

$$
\equiv 4 \frac{s}{d} \frac{w_{\text{ff}}}{w} \phi_0 n_p^{\text{eff}}.
$$
(7)

Note that pinning at high temperatures is so weak, going as  $(1-t^2-b)^2$ , that it can be ignored compared to  $E_j(T,B)$  $\sim(1-t-b)$ : it is not a bottleneck in the process.

## **COMPARISON WITH HIGH-TEMPERATURE DATA**

The experimental data of Ref. 6 are presented in Fig. 1 in terms of the equivalent number of pancake vortices in bundles of a single size, i.e.,  $n_p^{\exp} = S_y(0)/V_{\text{dc}}/\{4s\phi_0/d\}$ , which can be compared with  $(w_{ff}/w)$   $n_p^{eff}$  from Eq. (7). The high-temperature data in Fig. 1 shows that the number of pancake vortices,  $n_p$ , rises to many times that for one vortex



FIG. 1. The experimental data of Ref. 6 are presented in terms of the equivalent number of pancake vortices in bundles of a single size, i.e.,  $n_p^{\text{exp}} = S_v(0)/V_{dc}/(4s\phi_0/d)$ . The high-temperature fit is to  $(w_{\text{ff}}/w)n_p^{\text{eff}}$  from Eq. (7). Although the small  $u_{\text{min}}$  approximation to Eq. (16), essentially  $\{t/\varepsilon_p(T)\}^2$ , fits the low-temperature data reasonably well, it fails to correctly predict the experimental  $S_v(0)/V_{\text{dc}}^2$ .

 $(K)$ 

line, i.e.,  $d/s = 130$  for this sample. Therefore the bundles clearly include in-plane nearest neighbors. The fit to Eq.  $(7)$ , shown in Fig. 1, gives a prefactor,  $\eta(w_{\text{ff}}/w)$ , whose magnitude is too highly sensitive to the values of *b* |i.e.,  $B_{c2}(0)$  | and  $T_c$  to be useful, but  $\zeta E_i(T)$  in the exponent is determined quite reliably. The fit values of  $\zeta E_i(T)/(1-t)$  are shown in Fig. 2 to decrease with *B*, consistent with Clem's model $10$  of the Josephson energy as a function of separation  $\rho$  between pancakes in neighboring bilayers. In this model, the Josephson coupling energy crosses over from  $e_i \rho^2$  for  $\rho \ll \rho_c$  to  $e_j \rho \rho_c$  for  $\rho \gg \rho_c$ , where  $\rho_c = s \lambda_c / \lambda_{ab}$  and  $\lambda$  are the penetration depths. A simple interpolation between these limits is  $e_i \rho \rho_c / [1 + (\rho_c / \rho)]$ . The maximum energy (i.e., the barrier for thermal activation) occurs<sup>11</sup> at a separation  $\rho_m$  of about one vortex spacing  $a_0$ , since for larger separations, flux cutting and realignment is energetically favorable.



FIG. 2. The field dependence of the Josephson coupling energy of pancake vortices in adjacent Cu-O bilayers. The solid line is a fit to the interpolation formula in the text between the limits of the Clem model (Ref. 10).

Thus<sup>11</sup>  $\rho_m \sim a_0 \sim \sqrt{\phi_0 / B}$ . The relatively small anisotropy of  $\lambda_c/\lambda_{ab}$  in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> leads to fairly large crossover fields, compared to the highly-anisotropic Bi and Tl-based cuprates where the  $1/B$  dependence of  $e_j \rho_m^2$  is valid<sup>11</sup> over a wide range of *B*.

In Fig. 2, the solid curve fits  $\zeta E_i(T,B)/(1-t)$  to the interpolation formula with  $\rho = \rho_m$ . The first parameter,  $\rho_c$ , gives  $\lambda_c / \lambda_{ab} \sim 19$ , which is larger than the commonly accepted value<sup>12</sup> of  $\sim$  7 – 8 for fully oxygenated YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>. This discrepancy could result from a highly distorted vortex lattice due to strong pinning, for which  $\rho_m < a_0$ , and/or from the films being oxygen deficient  $(T_c = 89.6 \text{ K})$  is below optimal doping) which increases  $\lambda_c / \lambda_{ab}$ . The fit also gives a field-*independent* Josephson energy per area of  $\zeta e_i(T)/(1$  $(t-t) \sim 1800 \mu J/m^2$ . Other measurements of  $e_j(T)/(1-t)$  indicate values<sup>13</sup> of  $\sim$  2000  $\mu$ J/m<sup>2</sup> for a moderate oxygen deficiency with  $T_c \sim 74$  K, while for largish oxygen deficiencies<sup>13,14</sup> ( $T_c \sim 60 \text{ K}$ ), it is  $\sim 35-100 \ \mu \text{J/m}^2$ . Thus the fit is consistent with  $\zeta \sim 1$ . The excellent fits to this model in Figs. 1 and 2, with reasonable  $\lambda_c / \lambda_{ab}$  and  $e_j(T)$ , indicate that it correctly describes the high-temperature vortex dynamics.

#### **LOW-TEMPERATURE MODEL**

To go beyond the single-bundle-size model at low temperatures introduces two new unknown parameters:  $\mathcal{D}(n)$ , the density-of-states of bundles of size *n*, and the flux-jump attempt frequency,  $v_0$ . In summing both  $V_{dc}$  and  $S_v(0)$  over bundle sizes, the main complication is arriving at an effective  $D(n)$ . Some assumptions are required to make progress since little is known about  $\mathcal{D}(n)$  and  $\nu_0$  or their temperature and field dependences, while it is the systematics of these noise-voltage dependences which are the most intriguing and potentially insightful. Expressions are derived below for  $S_v(0)$  and  $V_{dc}$ , and contributions to these have maxima as a function of bundle size because of the cutoff imposed by thermal activation. Assuming a reasonably simple functional form for  $\mathcal{D}(n)$ , the results are virtually the same by integrating over *all* values of *n* or by using the value of *n* at the maxima. These results adequately follow the temperature dependence of  $S_v(0)/V_{dc}$ , but experimentally,  $S_v(0)/V_{dc}^2$  exhibits a strong thermal activation that would require an unreasonable thermal activation of  $v_0$ . We find a consistent solution to this dilemma by assuming that at low temperatures, the integration is cut off at a minimum bundle size that can be activated.

Before addressing those issues, some useful expressions are delineated. First,

$$
\nu = 1/(\tau + \tau_0),\tag{8}
$$

where  $\tau$  is a waiting time between pulses that depends on thermal activation over some potential energy barrier that increases with bundle size. In principle, the energy barrier includes pinning and vortex-vortex interactions (both inplane due to circulating currents and interbilayer due to Josephson coupling<sup>10</sup>). At low temperatures, Josephson coupling is strong and vortex lines prevail. Due to the random spatial distribution of pinning centers, the statistical sum gives a pinning activation energy, divided by  $k_B T$ , to be of a  $(random-walk average)$  form:<sup>15</sup>

$$
u(n,T) = \sqrt{n} \varepsilon_p(T)/t,\tag{9}
$$

where the bundle has *n* vortex lines with an average pinning energy, normalized to  $k_B T_c$ , of  $\varepsilon_p(T)$  and  $t \equiv T/T_c$ . Assuming the defects are smaller than the coherence length  $\xi$ , then the average pinning energy for a vortex line is defined by

$$
\varepsilon_p(T) = \varepsilon_p(0)(1 - t^2 - b)^2. \tag{10}
$$

Because the in-plane interaction energy of the bundle (for shear deformations) is proportional to the condensation energy<sup>16</sup> times its perimeter, i.e.,  $\sqrt{n}(1-t^2-b)^2$ , it will just rescale  $\varepsilon_p(0)$ . Finally, at sufficiently low temperatures,

$$
\tau = (1/\nu_0) \exp\{u(n,T)\}.
$$
 (11)

Combining these with  $D(n)$  results in

$$
V_{\text{dc}} = \frac{v_{\text{ff}}}{w} \phi_0 \sum_n \frac{\mathcal{D}(n)n}{1 + (\nu_0 \tau_0)^{-1} \exp\{u(n,T)\}},\qquad(12)
$$

$$
S_v(0) = 4 \left(\frac{v_{\rm ff}}{w} \phi_0\right)^2 \sum_n \frac{\tau_0 \mathcal{D}(n) n^2}{1 + (v_0 \tau_0)^{-1} \exp\{u(n,T)\}},\tag{13}
$$

recalling that  $\tau_0 = w_{\text{ff}} / v_{\text{ff}}$ . For a single bundle size, we recover Eq.  $(4)$ .

To proceed further we need  $\mathcal{D}(n)$ . Note that for Eq. (12) to recover the Bardeen-Stephen $9$  result for free flux flow (in which the denominator equals one), requires a normalization condition of  $\sum_{n}D(n)n=N$ , where *N* is the total number of vortex lines in the sample, i.e.,  $l_v \, w \, B/\phi_0$ , where  $l_v$  is the sample length between voltage contacts. It is unclear how this result affects the thermally activated case. The limit of constant  $D(n)$  may be unrealistic, and we will rather assume that  $D(n) = D(1)/n$  (from  $n=1$  up to the limit of the sample size). Fortunately, our ignorance about  $\mathcal{D}(1)$  *and*  $\nu_0$  can be mitigated by again taking the ratio  $S_v(0)/V_{dc}$ , as can be seen in what follows. Converting the sums in Eqs.  $(12)$  and  $(13)$  to integrals gives elementary forms in the low-temperature limit of thermal activation, i.e., if the one in their denominators of can be neglected. This is justifiable except very near  $T_c$ , where  $\varepsilon_p(T)$  goes to zero {but then the exponential form for thermal activation [Eq.  $(11)$ ] is dubious}. Presuming that  $w_{\text{ff}}$ and  $v_0$  are, at most, weakly dependent on  $n$ , one finds

$$
V_{\text{dc}} = \frac{w_{\text{ff}}}{w} \phi_0 \nu_0 \mathcal{D}(1) \int_{u_{\text{min}}}^{u_{\text{max}}} dn \, \exp(-u) \tag{14}
$$

$$
S_v(0) = 4 \left(\frac{w_{\rm ff}}{w} \phi_0\right)^2 \nu_0 \mathcal{D}(1) \int_{u_{\rm min}}^{u_{\rm max}} dn \ n \ \exp(-u). \tag{15}
$$

From Eq. (9),  $dn = \{t/\varepsilon_p(T)\}^2$  2*u du*, and the integrals of Eqs. (14) and (15) are, respectively,  $-2\{t/\varepsilon_p(T)\}^2(1)$  $(u) \exp(-u)$  and  $-2\{t/\varepsilon_p(T)\}^4(6+6u+3u^2+u^3)\exp(-u)$ , both to be evaluated between  $u_{\min}$  and  $u_{\max}$ .

In the low-temperature case considered here,  $\varepsilon_p(T)$  is large, so  $u_{\text{max}} \geq 1$ , and



FIG. 3. Thermal activation of  $S_v(0)/V_{dc}^2$ . The abcissa also accounts for the expected temperature dependence of  $\varepsilon_p(T)$  [see Eqs.  $(9)$  and  $(10)$ ].

$$
S_v(0)/V_{dc} = 4 \frac{w_{ff}}{w} \phi_0 \{t/\varepsilon_p(T)\}^2 \frac{6 + 6u + 3u^2 + u^3}{1 + u}
$$
  

$$
= 4 \frac{w_{ff}}{w} \phi_0 n^{\text{eff}},
$$
 (16)

with  $u = u_{\text{min}}$ . To lowest order in  $u_{\text{min}}$ , the effective bundle size  $n^{\text{eff}}$  is found to be  $6\{t/\varepsilon_p(T)\}^2 + 3n^{\text{min}}$ , so that two undetermined parameters are needed. To overcome this, note that  $S_v(0)/\bar{V}_{dc}^2$  only involves  $u_{\min}$ :

$$
S_v(0)/V_{\text{dc}}^2 = \frac{2}{v_0 \mathcal{D}(1)} \left[ \frac{6 + 6u + 3u^2 + u^3}{(1+u)^2} \exp(u) \right], \tag{17}
$$

since at low temperature both  $S_v(0)$  and  $V_{dc}$  are evaluated at  $u = u_{\min}$ . Fitting  $u_{\min}$  to the *temperature dependence*, eliminates the unknown prefactor in Eq.  $(17)$ 

### **COMPARISON WITH LOW-TEMPERATURE DATA**

The experimental data of Fig. 1 gives the equivalent number of pancake vortices in bundles of a single size, i.e.,  $n_p^{\text{exp}} = S_v(0)/V_{\text{dc}}/4s\phi_0/d$ , which can be compared with  $(dw_{\text{ff}}/sw)$  *n*<sup>eff</sup> from Eq. (16) at low temperatures. It is tempting to simply pass to the limit of very small  $u_{\min}$ , i.e., one vortex line, in Eq. (16), especially since the fit in Fig. 1 shows that  $\{t/\varepsilon_p(T)\}^2$  gives a good representation of the temperature dependence of  $S_v(0)/V_{dc}$ . However, as can be seen in Fig. 3, the experimental  $S_v(0)/V_{dc}^2$  are strongly thermally activated, so consistency with Eq.  $(17)$  would require<sup>17</sup> an unreasonable thermal activation for  $\nu_0\mathcal{D}(1)$ . Instead, assuming that  $\nu_0$  and  $\mathcal{D}(1)$  are, at most, weakly temperature dependent, the solid lines in Fig.  $3$  are fits to Eq.  $(17)$  of the *temperature dependence* of  $S_v(0)/V_{dc}^2$  and the resulting parameter is plotted as the squares in Fig. 4. As an important consistency check, the same is done for  $V_{dc}$ , for which Eq.  $(14)$  gives

$$
V_{\text{dc}} = 2 \frac{w_{\text{ff}}}{w} \phi_0 \nu_0 \mathcal{D}(1) n^{\min} \bigg[ \frac{1+u}{u^2} \exp(-u) \bigg], \qquad (18)
$$



FIG. 4. The field dependence of the average pinning energy scale times the square root of the minimum bundle size, obtained from  $S_v(0)/V_{\text{dc}}^2$  in Eq. (17) (squares) and from  $V_{\text{dc}}$  in Eq. (18) (solid circles). The lines are least-squares fits to the power laws.

again, to be evaluated at  $u = u_{min}$ . Then assuming  $n_{min}$  and  $w_{\text{ff}}$  are also, at most, weakly temperature dependent, Eqs.  $(17)$  and  $(18)$  are fit [recalling Eqs.  $(9)$  and  $(10)$ ], respectively, by  $\sqrt{n_{\min}} \varepsilon_p(0)$  ~ 170 and 260. That this difference is less than a factor of 2 is encouraging. Since  $n_{\min}$  and  $\varepsilon_p(0)$ are separable in Eq.  $(16)$ , an estimate is possible by fitting  $n^{\text{eff}}$  to the magnitude of  $(d/s)$   $n^{\text{exp}}_p$  with a suitable  $u = u_{\text{min}}$ , and in this case, an average of the above, i.e.,  $\sqrt{n_{\text{min}}}\varepsilon_p(0)$ ~215, is used. Then the value of  $\{t/\varepsilon_p(T)\}$  from the fit to Eq. (16) gives  $n_{\text{min}} \sim 28 \frac{w}{w_{\text{ff}}}$  vortex lines. This indicates an average pinning strength *per bilayer pancake* of  $\sqrt{s/dE_p(0)} \sim (320 \text{ K}) \sqrt{w_f}/w.$ 

The presumption of this procedure is that the temperature dependence is mostly set by exponential thermal activation. Therefore, the parameters, for which fixed values were derived by fitting over a range of temperatures, should not be interpreted as being constant over that temperature interval. Instead these fixed values should be regarded as an approximate average, over that temperature interval, of the weaklytemperature-dependent physical quantities they represent.

The same analysis is repeated at the other fields where  $S_v(0)$  and  $V_{dc}$  were measured (2, 4, and 8 T). The values of  $\sqrt{n^{\min}}\varepsilon_p(0)$ , from Eqs. (17) and (18), are shown in Fig. 3. The average drops with field as  $\sim B^{-0.8}$ , implying smaller minimum bundle size at higher fields. At first, this seems inconsistent with the expectation that the stronger in-plane vortexvortex interactions at higher fields would increase the bundle size. However, it is the standard result found<sup>1,18</sup> in lowtemperature superconductors. Given the complexities associated with the configurations of flux lines in a system of highly-disordered pinning sites, it is unlikely that a simple understanding of this result is presently possible.

## **SENSITIVITY TO**  $\mathcal{D}(n)$

It is important to understand how these results depend on our assumption for the unknown density of states,  $\mathcal{D}(n)$ . If  $\mathcal{D}(n)$  is assumed to be constant in Eqs. (12) and (13), rather than  $D(1)/n$ , then the integral in Eq. (15) is the appropriate one for  $V_{dc}$  and the integral of  $n^2$ , which is appropriate for  $S_v(0)$ , is  $-2\{t/\varepsilon_p(T)\}^6(120+120u+24u^2+6u^3+3u^4)$  $(u^5)$ exp( $-u$ ). There are only minor changes in the above analysis, and from Eqs. (17) and (18),  $\sqrt{n_{\text{min}}\epsilon_p(0)}$  are both

about twice as big. The truth may lie somewhere between these simple forms, but the overall consistency of the model does not depend strongly on our choice for  $\mathcal{D}(n)$ .

## **SUMMARY**

Perhaps the most significant implication of this work comes from the potential connection of the high-temperature results to flux-lattice melting.<sup>7,12</sup> Since the experiment was done in the limit of small currents, i.e., linear response, the bundle sizes found closely resemble the correlations in thermal equilibrium. The increase of the correlation volume of pancake vortices is seen to be controlled by the *c*-axis correlation length through the Josephson coupling energy. The collective reinforcement of this correlation throughout the *ab* planes can then lead to an ordered solid. The melting entropy found<sup>19</sup> in low-pinning single crystals is too large for melting into a liquid of vortex lines, but may be consistent with melting into pancake vortices.<sup>20</sup> A possibility presented here is that vortex-lattice freezing only occurs for a sufficiently long *c*-axis correlation length of pancake bundles.

This idea is implicit in the first-principles, densityfunctional theory<sup>21</sup> of flux-lattice melting. Although the calculations were explicitedly for  $Bi_2Sr_2CaCu_2O_8$  and Josephson interlayer coupling was ignored, a qualitative, field-and temperature-dependent increase in the *c*-axis correlation length *l<sub>c</sub>* was found *in the liquid phase*. For example, at 30

Another interesting conclusion is that a minimum fluxbundle size exists for low-temperature flux creep. The strongly activated behavior of  $S_v(0)/V_{dc}^2$  seems to dictate this conclusion. It is somewhat surprising since in the very simple picture presented above the interaction and pinning energies both scale as  $\sqrt{n}(1-t^2-b)^2$ . Intuitively one might expect that the detailed situation is more complicated and the data seem to support this. It may be that both the random spatial distribution of pointlike pinning defects and the occasional occurance of extended defects conspire to disfavor a significant number of flux jumps by small bundles.

### **ACKNOWLEDGMENTS**

The author thanks David Steel, Jeff Hettinger, and D. Ertas for helpful discussions and insights. This research is supported by the U.S. Department of Energy, Basic Energy Sciences, Materials Sciences, Energy Efficiency and Renewable Energy, as part of a program to develop electric power technology, under Contract No. W-31-109-ENG-38, and the National Science Foundation, Office of Science and Technology Centers under Contract No. DMR 91-20000.

- <sup>1</sup>G. J. Van Gurp, Phys. Rev. **166**, 436 (1968); **178**, 650 (1969).
- <sup>2</sup> J. R. Clem, Phys. Rev. B 1, 2140 (1970); Phys. Rep. **75**, 1 (1981).
- $3$  J. D. Thompson and W. C. H. Joiner, Phys. Rev. B  $20$ , 91 (1979); J. Low Temp. Phys. 37, 13 (1979).
- ${}^{4}$ B. Placais, P. Mathieu, and Y. Simon (unpublished).
- $5$  J. R. Clem, Phys. Rev. B 43, 7837  $(1991)$ .
- 6D. G. Steel, D. H. Kim, K. E. Gray, S. E. Pfanstiel, J. H. Kang, and J. Talvacchio, Physica C 248, 55 (1995).
- ${}^{7}$ H. Safar, P. L. Gammel, D. A. Huse, D. J. Bishop, J. P. Rice, and D. M. Ginsberg, Phys. Rev. Lett. **69**, 824 (1992).
- <sup>8</sup>D. K. C. MacDonald, *Noise and Fluctuations* (Wiley, New York, 1962).
- $9$  J. Bardeen and M. J. Stephen, Phys. Rev.  $140$ , A1197 (1965).
- <sup>10</sup> J. R. Clem, Physica A **200**, 118 (1993).
- <sup>11</sup>D. H. Kim, K. E. Gray, R. T. Kampwirth, J. C. Smith, D. S. Richeson, T. J. Marks, J. H. Kang, J. Talvacchio, and M. Eddy, Physica C 177, 431 (1991).
- 12A. Schilling, R. A. Fisher, N. E. Phillips, U. Welp, W. K. Kwok, and G. W. Crabtree, Phys. Rev. Lett. **78**, 4833 (1997).
- $13$  M. Rapp, A. Murk, R. Semerad, and W. Prusseit, Phys. Rev. Lett.

77, 928 (1996).

- <sup>14</sup> J. D. Hettinger, K. E. Gray, B. W. Veal, A. P. Paulikas, P. Kostic, B. R. Washburn, W. C. Tonjes, and A. C. Flewelling, Phys. Rev. Lett. **74**, 4726 (1995).
- <sup>15</sup> A. I. Larkin, Zh. Eksp. Teor. Fiz. **58**, 1466 (1970) [Sov. Phys. JETP 31, 784 (1970)].
- <sup>16</sup>R. Labusch, Phys. Status Solidi **32**, 439 (1969).
- <sup>17</sup>Note that substituting  $u = u_{\text{min}} \le 1$  removes any temperature dependence in the bracket of Eq.  $(14)$  and  $exp(u)=1$ .
- <sup>18</sup> E.g., see W. J. Yeh and Y. H. Kao, Phys. Rev. B 44, 360 (1991).
- 19R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. Lett. **76**, 835 (1996); U. Welp, J. Fendrich, W. K. Kwok, G. W. Crabtree, and B. W. Veal, *ibid.* **76**, 4809 (1996); M. Roulin, A. Junod, and E. Walker, Science 273, 1210 (1996).
- $^{20}$ L. I. Glazman and A. E. Koshelev, Phys. Rev. B 43, 2835 (1991).
- <sup>21</sup> S. Sengupta, C. Dasgupta, H. R. Krishnamurthy, G. I. Menon, and T. V. Ramakrishnan, Phys. Rev. Lett. 67, 3444 (1991); G. I. Menon, C. Dasgupta, H. R. Krishnamurthy, T. V. Ramakrishnan, and S. Sengupta, Phys. Rev. B 54, 16 192 (1996).