Collective oscillations in superconducting thin films in the presence of vortices

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A plasma wave propagates inside an anisotropic superconducting film sandwiched between two semi-infinite nonconducting bounding dielectric media. In the presence of a c-axis-parallel magnetic field, perpendicularly applied to the film surfaces, we show how vortices, known to cause dissipation and change the penetration depth, affect the propagative mode. We obtain the complex wave number of this mode and, using $YBa_2Cu_3O_{7-x}$ at 4 K as an example, determine a region where the vortex contribution is dominant and dissipation is small. [S0163-1829(98)02406-0]

I. INTRODUCTION

At the plasma frequency, a collective oscillation of the electron gas around the positive ionic background occurs, which is fundamental to understand the electromagnetic properties of conductors. For wave frequencies below the plasma frequency, the conductor reflects the incident electromagnetic radiation but, for frequencies above the plasma one it becomes transparent thus allowing a propagative mode. For metals, the plasma frequency is typically found in the ultraviolet region ($10^{15}-10^{16}$ Hz).[1]

A widely known feature of superconductors is the existence of a gap, the energy required to break a Cooper pair in the ground state condensate. Typically for conventional superconductors the frequency associated to the gap is in the 10^{11} Hz range, whereas for high-T_c superconductors it is one order of magnitude higher.[2]

The question of whether the superconductor can support collective modes *without* inducing pair breaking effects is an old one, and has been discussed since the early days of the theory of superconductivity.[3,4] Apart from the so-called Carlson-Goldman mode, which happens under special circumstances,[5,6] any other attempt to excite collective modes in isotropic *bulk* superconductors has led to the destruction of the superconducting state. This follows the well-known argument[4] that the Coulomb interaction shifts the frequency of such oscillations, the plasma frequency, to above the gap frequency. However, it was recently shown that highly anisotropic superconductors do display plasma oscillations below the superconducting gap.[7] These oscillations, especially for the layered structure, are due to the Josephson coupling between the superconducting planes.

Plasma modes in superconductors, isotropic or not, have been recently revisited from another point of view. Plasma modes below the gap are possible without destroying the superconducting state, as long as they propagate in the interface between the superconductor and a nonconducting bounding medium of a very high dielectric constant. This is the so-called superficial plasma mode,[8,9] which is made possible by the charges located at the interface of the superconductor and the dielectric medium, responsible for the creation of an electric field mainly concentrated outside the superconductor.

In a thin film, the coupling between the two superficial plasma modes yields two possible branches, a symmetric and a antisymmetric branch, in a fusion similar to metals[10–12] and semiconductors.[13] The film thickness must be smaller than the London penetration depth in order for this coupling to be established. Oscillations between the kinetic energy of the superelectrons and the electrical field energy take place in these modes and for this reason they are called *plasma modes*. The lower frequency branch was predicted for superconductors[14–16] some time ago and observed in thin granular aluminum films, in the hundreds of MHz range,[17] and in thin $YBa_2Cu_3O_{7-x}$ films,[18] in a higher frequency range of hundreds of GHz. The highest frequency branch was predicted to be within experimental observation range for high-T_c materials.[19] For highly anisotropic superconducting materials, measurements of such upper and lower branches are expected to give information on the transverse and longitudinal London penetration depths.[19] In conclusion, plasma modes in superconducting films can be an important tool for probing many intrinsic electromagnetical properties of superconductors.

Long time ago, Gittleman and Rosenblum[20] studied the effects of an applied current at the radio and microwave frequency range on pinned vortices and obtained as a result the associated surface impedance. For an ac applied magnetic field and in the weak pinning regime, Campbell[21] showed that the effect of vortices can be described by a frequency dependent effective penetration depth, whose square is the original magnetic penetration depth squared plus a new term describing the elastic interaction between the vortices and the pinning centers. In the beginning of this decade these models were generalized to convey the effects of creep[22] and to provide a detailed description of the elastic properties of the vortex lattice near the superconductor surface.[23]

In this paper we analyze the effects of a constant uniform magnetic field, perpendicularly applied to the thin film surfaces, on the thin film propagative mode.[24] A sufficiently large magnetic field causes the thermodynamic stability of a vortex system, which influences the collective oscillations, affecting considerably the above modes. The vortices are induced into an oscillatory dissipative motion around their pinning centers. This motion couples to the electromagnetic fields resulting either in an underdamped or an overdamped regime. This paper is developed in the context of independent vortex and superelectron degrees of freedom. We understand by superelectron current, any supercurrent other than that one necessary to bring the thermodynamic equilibrium of vortices. The vortex degree of freedom is described by its position in space. In this framework, the question of whether the superelectron or the vortex contribution dominates the propagative mode behavior arises. Hereafter, by plasma mode we refer to the limit where superelectron contribution is the largest. Thus it may be evident that pure plasma modes are only found in the complete absence of an applied magnetic field. In this paper, we look for conditions that render the modes underdamped and vortex dominated. This is the most interesting case because the attenuated oscillations can be regarded as an energy alternation taking place between the vortex pinning and the electrical field energies.

The present work is done in the simplest possible theoretical framework, essentially a generalization of the Gittleman-Rosenblum approach,[25,26] in which vortices and superelectrons are independently coupled to Maxwell's electromagnetic theory. Here, we are mainly interested in a low temperature range such that we can ignore the contribution of normal electrons to the problem. Thus, it must be clear that any wave damping is only due to vortex dissipative motion and not to normal ohmic conduction.

We consider here an anisotropic superconductor with its uniaxial direction (c axis) orthogonal to the film surfaces: the two magnetic penetration depths, perpendicular ($\lambda_\perp$) and parallel ($\lambda_\parallel$) to the surfaces, give an anisotropy such that $\lambda_\perp/\lambda_\parallel > 1$. There are two dielectric constants, the nonconducting medium and the superconductor ones, $\tilde{\varepsilon}$ and ε_s, respectively. Thus we are assigning to the superconductor a frequency independent dielectric constant. We refer to the speed of light in the dielectric as $v = c/\sqrt{\tilde{\varepsilon}}$. The uniform static applied magnetic field is H_0. For each individual vortex, the viscous drag coefficient is η_0 and the elastic restoring force constant (Labusch parameter) is α_0. Their ratio, $\omega_0 \equiv \alpha_0/\eta_0$, is the so-called depinning frequency, above which dissipation becomes dominant in the vortex motion. To have coupling between the two surfaces the film thickness d must be smaller than $\lambda_\parallel$.

The choice of a nonconducting bounding medium of very high dielectric constant is crucial to lower the frequency range of the modes to below the gap frequency. For this reason we take $SrTiO_3$ as the bounding media, whose dielectric constant is known to be high up to the GHz frequency[17] at low temperatures: $\tilde{\varepsilon} \approx 2.0 \times 10^4$. Then the speed of light in the dielectric, $v = 2.1 \times 10^6$ ms^{-1}, is substantially smaller than c. Our work is restricted to identical top and bottom dielectric media, which does not imply lack of generality. Similar conclusions should also apply to the general asymmetric case.

This paper is organized as follows. In Sec. II, we introduce the major equations governing the film mode in the presence of vortices. Its dispersion relation is analytically derived under some justifiable approximations. In Sec. III, we apply our model to the high-T_c superconductor $YBa_2Cu_3O_{7-\delta}$, choosing a range of parameters such that the lowest energy film mode is mostly associated to the vortex dynamics, but yet remains underdamped. Finally, in Sec. IV, we summarize our most important results.

II. PROPAGATING MODES IN SUPERCONDUCTING FILMS WITH PERPENDICULAR MAGNETIC FIELD

In this section we introduce the basic equations governing wave propagation in a superconducting film sandwiched between two identical nonconducting dielectric media and subjected to an uniform static magnetic field perpendicularly applied to the film surface. An external electromagnetic wave of angular frequency ω and vacuum wave number $k \equiv \omega/c$ is inserted in the dielectric bounded film. We determine the dispersion relation of the lowest energy film mode, whose imaginary part reveals the attenuation behavior. Phenomenological theories, such as the present one, only describe the superconductor in a energy range much lower than the pair breaking threshold.

The electromagnetic dynamics of fields and superelectrons is described by the Maxwell's equations

$$\nabla \cdot \vec{D} = e(n_s - \overline{n_s}), \tag{1}$$

$$\nabla \cdot \vec{H} = 0, \tag{2}$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}, \tag{3}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \tag{4}$$

and consequently by the continuity equation

$$\nabla \cdot \vec{J} + e \frac{\partial n_s}{\partial t} = 0, \tag{5}$$

where n_s represents the space and time dependent charge density, $\overline{n_s}$ is its equilibrium value, and e stands for the electron charge. The distinction between n_s and $\overline{n_s}$ is necessary because, propagation through the system disrupts the neutrality, as seen from Gauss' law ($n_s - \overline{n_s} = 0$), and the local charge density is no longer constant.

As previously noted the contribution of vortices and of superelectrons are independent in the present model. The field $\vec{J}$, the superelectron current density involved in net macroscopic transport, and the field $\vec{u}$, the vortex displacement from its equilibrium position, are independent in the present model. Hence the supercurrent density $\vec{J}$ corresponds to a macroscopic average of the total superelectron motion, where the supercurrent necessary to establish each vortex averages to zero. This approximation, valid for the present purposes, cannot give any information on the supercurrent distribution surrounding each vortex line.

The simplest possible model that treats the response of the vortices to the presence of a supercurrent external to them is the harmonic approximation of Gittleman and Rosenblum,[20]

$$\eta_0 \frac{\partial \vec{u}}{\partial t} + \alpha_0 \vec{u} = \Phi_0 (\vec{J} \times \hat{n}), \tag{6}$$

where $\hat{n}$ is a unit vector parallel to the flux lines mean direction. From its turn, the displacements of vortices from their equilibrium positions affect the propagating electromagnetic wave. Fiory and Hebard[25] have considered this question and found that besides the kinetic inductance due to the superelectrons, the moving vortices also contribute, producing an electric field inside the superconductor:

$$\vec{E} = \mu_0 \lambda^2 \cdot \frac{\partial \vec{J}}{\partial t} - \mu_0 H_0 \left(\frac{\partial \vec{u}}{\partial t} \times \hat{n} \right). \tag{7}$$

The assumption of anisotropy yields a tensorial London penetration depth:

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_\perp & 0 & 0 \\ 0 & \lambda_\parallel & 0 \\ 0 & 0 & \lambda_\parallel \end{pmatrix} \quad \lambda_\perp = \sqrt{\frac{m_\perp}{\mu_0 \overline{n}_s e^2}} \quad \lambda_\parallel = \sqrt{\frac{m_\parallel}{\mu_0 \overline{n}_s e^2}}. \tag{8}$$

We pick a coordinate system where the two plane parallel surfaces separating the superconducting film from the dielectric medium are at $x = d/2$ and $x = -d/2$, such that $\hat{n} \equiv \hat{x}$ and propagation is along the z axis. Vortex displacement is described by a vector field parallel to the surfaces, $\vec{u} = u_y \hat{y} + u_z \hat{z}$, with no orthogonal components to them $(u_x = 0)$. According to symmetry arguments, all fields for the present geometry can be expressed as $F_i(x)\exp[-\imath(qz-\omega t)]$, where the wave number q have yet to be determined. Because vortex motion is dissipative, the wave's amplitude decays exponentially with distance, and one obtains for the fields's expression $F_i(x)\exp(q''z)\exp[-\imath(q'z-\omega t)]$. Then the wave number is a complex number, $q = q' + \imath q''$.

Solving Maxwell's equations for the chosen geometry gives two independent sets of field components, the transverse electric (TE) and the transverse magnetic (TM) propagating modes. In the former the nonzero electromagnetic field components are H_x, E_y, and H_z, the nonvanishing supercurrent is J_y, and the vortex displacement is along the direction of wave propagation (u_z). This is an extremely high frequency mode in the present theory, and so not interesting because it lies above the gap. For the latter, the nonzero electromagnetic field components are E_x, H_y, and E_z, the nonvanishing supercurrent components are J_x and J_z, and the propagating wave displaces the vortices perpendicularly to its direction of propagation (u_y). This is a very interesting mode because it supports low frequency propagating waves. The major difference between TE and TM modes is that the latter displays superficial charge densities at the film-dielectric interfaces and the former does not. Such superficial charge densities stem from the supercurrent component orthogonal to the film surface J_x, which is discontinuous at the interfaces, thus rendering a strong coupling between the superconducting film and the bounding media.

Introducing the time dependence $\exp(\imath\omega t)$ into Eqs. (6) and (7) results in a change of the penetration depth parallel to the surfaces due to the vortex contribution:[23,21]

$$\imath\omega\mu_0\lambda_\perp^2 J_x = E_x, \quad \imath\omega\mu_0\overline{\lambda_\parallel^2} J_z = E_z, \tag{9}$$

$$\overline{\lambda_\parallel^2} = \lambda_\parallel^2 + \left(\frac{B_0\Phi_0}{\mu_0\alpha_0}\right)\frac{1}{1+\imath(\omega/\omega_0)}. \tag{10}$$

This equation shows that vortices and superelectrons contribute additively to the parallel penetration depth. Notice the depinning frequency ω_0 establishes two distinct physical regions for the vortices response. For $\omega \ll \omega_0$ dissipation is weak and $\overline{\lambda_\parallel}$ is essentially a real number. For $\omega \gg \omega_0$ and a sufficiently large magnetic field, dissipation dominates the vortices response because $\overline{\lambda_\parallel}$ is complex.

The superconductor's dielectric constant is tensorial, $\vec{D} = \epsilon_0\varepsilon_s\vec{E} - \imath\vec{J}/\omega = \epsilon_0\boldsymbol{\varepsilon}\cdot\vec{E}$, and for the TM mode we have that

$$\varepsilon_x = \varepsilon_s - \frac{1}{(k\lambda_\perp)^2}, \quad \varepsilon_z = \varepsilon_s - \frac{1}{(k\overline{\lambda_\parallel})^2}, \quad k \equiv \frac{\omega}{c}. \tag{11}$$

The TM field equations for the dielectric medium, $(x \geq d/2$ and $x \leq -d/2)$, are

$$E_x = \imath\frac{q}{\tilde{\tau}^2}\frac{\partial E_z}{\partial x}, \quad H_y = \imath\epsilon_0\frac{\omega\tilde{\varepsilon}}{\tilde{\tau}^2}\frac{\partial E_z}{\partial x},$$

$$\frac{\partial^2 E_z}{\partial x^2} - \tilde{\tau}^2 E_z = 0, \quad \tilde{\tau}^2 = q^2 - k^2\tilde{\varepsilon}, \tag{12}$$

and the ones for the superconducting film $(-d/2 \leq x \leq d/2)$ follow:

$$E_x = \imath\frac{q\varepsilon_z}{\tau^2\varepsilon_x}\frac{\partial E_z}{\partial x}, \quad H_y = \imath\epsilon_0\frac{\omega\varepsilon_z}{\tau^2}\frac{\partial E_z}{\partial x},$$

$$\frac{\partial^2 E_z}{\partial x^2} - \tau^2 E_z = 0 \quad \tau^2 = \frac{\varepsilon_z}{\varepsilon_x}q^2 - k^2\varepsilon_z. \tag{13}$$

The dispersion relations follow from the continuity of the ratio H_y/E_z at a single interface, say $x = d/2$, once assumed the longitudinal field E_z has a definite symmetry. It happens in this way because, the superconductor film is bounded by the same dielectric medium in both sides. Solving Eq. (12) one gets that above the film $(x \geq d/2)$,

$$E_z = \tilde{E}_0\exp(-\tilde{\tau}x) \quad \text{and} \quad \left.\frac{\tilde{H}_y}{\tilde{E}_z}\right|_{x=d/2} = -\imath\frac{\omega\epsilon_0\tilde{\varepsilon}}{\tilde{\tau}}. \tag{14}$$

From Eq. (13) we learn that for the superconducting film $(-d/2 \leq x \leq d/2)$ there are two possible states, symmetrical and antisymmetrical, where the longitudinal field is expressed by $E_z = E_0\cosh(\tau x)$ and $E_z = E_0\sinh(\tau x)$, respectively. As discussed earlier, we shall only consider the symmetric branch, the lowest mode in energy. So the ratio of the tangential fields becomes $H_y/E_z|_{x=d/2} = \imath\omega\epsilon_0\tilde{\varepsilon}_z \tanh(\tilde{\tau}d/2)/\tilde{\tau}$. Continuity of this ratio across the interface gives the following implicit relation:

$$\frac{\tau\tilde{\varepsilon}}{\tilde{\tau}\varepsilon_z} = -\tanh\left(\tau\frac{d}{2}\right). \tag{15}$$

To find the dispersion relation we must solve Eq. (15). Here we use an approximate method to analytically solve it. This approximation amounts to replace the function (tanhz)/z in Eq. (15), by another function $1/\sqrt{1+(2/3)z^2}$, which has an extremely close behavior. For $z \ll 1$ both functions coincide up to the second order term in the Taylor series expansion $1-(1/3)z^2+\cdots$. As all our results are derived in the range $z \ll 1$ thus, we replace Eq. (15) by the following approximate dispersion relation:

$$\frac{\tilde{\varepsilon}}{\tilde{\tau}} \approx -\frac{d\varepsilon_z}{2}\frac{1}{\sqrt{1+\frac{2}{3}(\tau d/2)^2}}. \tag{16}$$

Squaring the above expression, one obtains a linear equation for q^2:

$$q^2 = \left(\frac{\omega}{v}\right)^2 \frac{1+(2\omega\overline{\lambda_\parallel}/dv)^2[\overline{\lambda_\parallel^2}+d^2/6]}{1-2/3(\omega/v)^4\overline{\lambda_\parallel^2}\lambda_\perp^2}. \tag{17}$$

The term proportional to $d^2/6$ in the numerator is irrelevant, assuming the film much thinner than the penetration depth ($\lambda_\parallel \gg d$). We restrict the present study to frequencies much below the asymptotic frequency $[(\omega/v)^4\overline{\lambda_\parallel^2}\lambda_\perp^2 \ll 1]$, thus obtaining the following dispersion relation:

$$q^2 = \left(\frac{\omega}{v}\right)^2\left[1+\left(\frac{2\omega\overline{\lambda_\parallel^2}}{dv}\right)^2\right]. \tag{18}$$

In the absence of an applied uniform magnetic field ($H_0 = 0$), consequently with no vortices, there is no dissipation and $q''=0$. In this case we retrieve the well-known dispersion relation of plasma modes taking into account the retardation effect.[16,17]

Next we study two different behaviors of the dispersion relation in the presence of vortices.

Optical mode. At low frequencies the mode is, in leading order, a plane wave traveling in the dielectric medium $q' \approx \omega/v$, with no attenuation along the direction of propagation ($q'' \approx 0$). Perpendicularly to the film, the amplitude shows no attenuation, because $\tilde{\tau} \approx 0$, according to Eq. (14). We obtain, from the Taylor expansion of Eq. (17), the lowest order corrections in ω to the above description of the optical regime:

$$q' = \frac{\omega}{v}\left\{1+\frac{1}{2}\left[\frac{2\omega(\lambda_\parallel^2+B_0\Phi_0/\mu_0\alpha_0)}{dv}\right]^2\right\}+\cdots, \tag{19}$$

$$q'' = -\frac{4\omega^4\frac{B_0\Phi_0}{\mu_0\alpha_0}(\lambda_\parallel^2+B_0\Phi_0/\mu_0\alpha_0)}{v^3d^2\omega_0}+\cdots. \tag{20}$$

Coupled mode. For sufficiently large frequencies, Eq. (18) no longer describes a linear response. In this range the superconducting film and the dielectric media are effectively coupled, which implies in a reduction of the mode propagation speed $[(\omega/q')/v \ll 1]$. This is the most interesting regime since the film and dielectric produce a low energy mode.

Far away from the linear regime, and provided that the asymptotic frequency is still out of range, Eq. (18) is approximately described by its second term, resulting in the dispersion relation $q \approx (2/d)[\omega/(v\overline{\lambda_\parallel})]^2$. From this, we obtain its wave vector and attenuation:

$$q'(\omega) = \frac{2\omega^2}{dv^2}\left[\lambda_\parallel^2+\frac{B_0\Phi_0}{\mu_0\alpha_0}\frac{1}{1+(\omega/\omega_0)^2}\right], \tag{21}$$

$$q''(\omega) = -\frac{B_0\Phi_0}{\mu_0\alpha_0}\frac{\omega^3/(\omega_0v^2d)}{1+(\omega/\omega_0)^2}. \tag{22}$$

In the frequency range where the above dispersion relation is a valid approximation, the ratio between the real and the imaginary parts of the London penetration depth determines whether the mode is overdamped or underdamped: $q'/q'' = -\mathrm{Re}(\overline{\lambda_\parallel^2})/\mathrm{Im}(\overline{\lambda_\parallel^2})$.

The crossover magnetic field

$$B_1 \equiv \frac{\lambda_\parallel^2\mu_0\alpha_0}{\Phi_0} \tag{23}$$

splits the regimes of superelectron ($B_0 \ll B_1$), and vortex ($B_0 \gg B_1$) dominance. In these limits Eq. (10) can be replaced by approximated expressions $\overline{\lambda_\parallel^2} \approx \lambda_\parallel^2$, and $\overline{\lambda_\parallel^2} \approx (B_0\Phi_0/\mu_0\alpha_0)/[1+\imath(\omega/\omega_0)]$, respectively. Recall the assumption of the present model that the superelectron contribution is never dissipative. If in addition to an applied magnetic field much larger than B_1, we choose a frequency range $\omega < \omega_0$, then $q'' < q'$ and the mode is underdamped. The dispersion relation follows a square root dependence, and becomes

$$\omega^2 \approx \frac{dv^2\mu_0\alpha_0}{2B_0\phi_0}q'. \tag{24}$$

In this interesting limit the mode energy shows many oscillations between vortex and electric field energies before dissipation dominates. At higher frequency ($\omega > \omega_0$) this is no longer possible, since the mode becomes overdamped due to the large dissipation of vortices above the depinning frequency. The frequency ω_0 coarsely defines a crossover region between the underdamped and the overdamped regimes.

In the next section, using experimental parameters measured on $YBa_2Cu_3O_{7-x}$ (YBCO), we search for favorable conditions in frequency and magnetic field to observe underdamped coupled modes on a thin film.

III. YBCO THIN FILM

In this section a $YBa_2Cu_3O_{7-\delta}$ thin film is taken, as an example, to determine a frequency and magnetic field window where the mode is coupled, underdamped and vortex dominated. The wave must be underdamped in order to

TABLE I. Properties of the high-T_c material $YBa_2Cu_3O_{7-\delta}$ at $T=4$ K.

α_0 (N/m^2)	η_0 $(N\ s/m^2)$	$\omega_0=\dfrac{\alpha_0}{\eta_0}$ $(10^9$ rad/s)	$\lambda_\parallel$ (μm)	B_1 (T)
3.0×10^5	1.2×10^{-6}	250	0.15	4.1

travel over many wavelengths before its amplitude is completely attenuated. For this high-T_c superconductor the anisotropy $(\lambda_\perp/\lambda_\parallel=5)$ and the zero-temperature London penetration depth along the CuO_2 planes are well known.[27,28] At very low temperature several experiments[29] have determined the viscosity and the Labusch constant, all giving the same numbers, which are summarized in Table I. Such parameters have a temperature dependence,[30] not taken into account here because we only consider a fixed low temperature; namely, 4 K. The magnetic field dependence of the Labusch constant, known to exist for high-T_c materials[31] and low-T_c ones,[32] is not considered either. For this discussion we choose the film thickness $d=10$ nm.

Figure 1 provides a pictorial intuitive view of the wave propagation inside the superconducting film for the TM symmetric propagating mode. The dimensions are out of proportion in order to enhance some of the most relevant features. Only the electric field lines inside the superconducting film are shown. The superficial charges are also shown and represent the sources of this propagating electric field. The electric field lines show a very important feature of this wave,[19] namely, the supercurrent component along the wave propagation direction J_z is dominant over J_x. A magnetic field perpendicularly applied to the film surfaces produces vortices, pictorially represented at the top surface. The oscillatory displacement suffered by vortices, because of the driving Lorentz force caused by J_z [Eq. (6)], is also shown in this figure.

As previously discussed, the adequate choice of frequency and magnetic field windows is fundamental to observe the

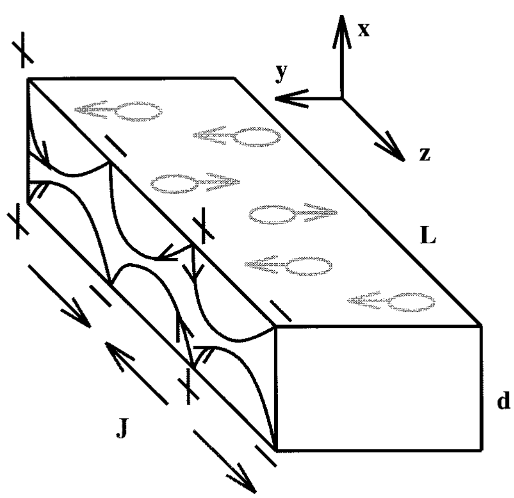

FIG. 1. A pictorial view of wave propagation in a superconducting film surrounded by identical nonconducting media in both sides. The scales are out of proportion in order to enhance some of the features. The instantaneous electric field is shown here only inside the film. The superficial charge densities and the motion of the vortex lines are also sketched here.

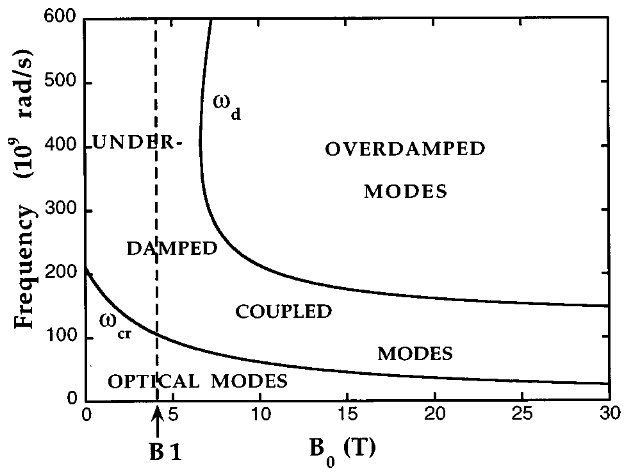

FIG. 2. The diagram B vs ω for a very thin YBCO superconducting film, $d=10$ nm thick, surrounded by the dielectric material $SrTiO_3$ shows three regions: optical, underdamped coupled, and overdamped modes. The dashed line separates the superelectron (below) to the vortex (above) dominated regime.

lower energy mode. We can distinguish several different regions within the B_0 vs ω diagram. Figure 2 shows such a region for YBCO, according to the above parameters. Two crossover lines separate this diagram in three different regions: the optical regime, the underdamped coupled regime, and the overdamped coupled regime.

The lower line in Fig. 2, called ω_{cr}, separates the optical region from the coupled regions. This crossover line is defined through Eq. (19), using as condition that the second term becomes a non-negligible fraction χ_1 of the first term and so can no longer be ignored:

$$\omega_{cr}=\sqrt{\frac{\chi_1}{2}}\frac{dv}{\lambda_\parallel^2+B_0\Phi_0/\mu_0\alpha_0}. \qquad (25)$$

We have arbitrarily chosen 10% $(\chi_1=0.1)$ as our criterion for the optical mode boundary.

The upper line in Fig. 2, called $\omega_{d\pm}$, is related to the dissipation and separates the underdamped to the overdamped regimes. The criterion for dissipation is the ratio q'/q'', which for the coupled regime, is approximately given by the ratio between the real and the imaginary part of the squared penetration depth $\overline{\lambda_\parallel^2}$ [Eq. (10)], according to Eq. (22). Thus our second crossover line is defined by $\mathrm{Im}(\overline{\lambda_\parallel^2})$ $=\chi_2\mathrm{Re}(\overline{\lambda_\parallel^2})$ where χ_2 is an arbitrary factor. This condition gives a second degree equation for ω/ω_0, $\chi_2\lambda_\parallel^2(\omega/\omega_0)^2-(B_0\Phi_0/\alpha_0\mu_0)(\omega/\omega_0)+\chi_2(\lambda_\parallel^2+B_0\Phi_0/\alpha_0\mu_0)=0$, whose solutions, $\omega_{d\pm}(B_0)$, form the upper and lower branches of a single curve that encircles the overdamped regime area:

$$\frac{\omega_{d\pm}}{\omega_0}=\frac{B_0}{2\chi_2B_1}\pm\sqrt{\left(\frac{B_0}{2\chi_2B_1}\right)^2-\frac{B_0}{B_1}-1}. \qquad (26)$$

Therefore, the dissipative region demands a minimum applied field B_2 to exist, defined by the vanishing of the above square root:

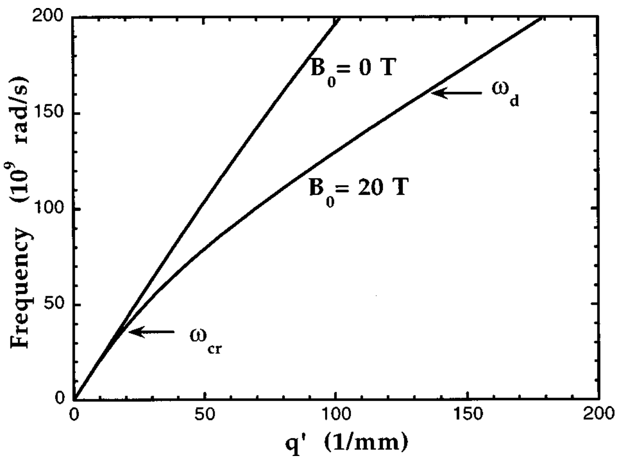

FIG. 3. Dispersion relation ω versus q' for a 10 nm YBCO film. In this frequency range and for zero magnetic field the dispersion relation is purely optical. For $B_0=20$ T, the modes are associated to the vortex dynamic and are underdamped until the frequency ω_d is reached.

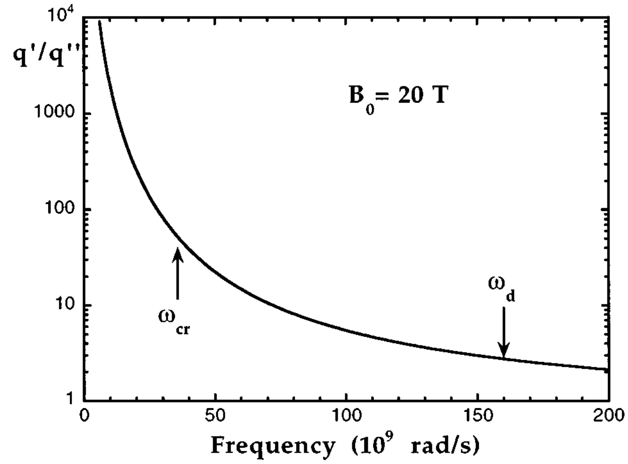

FIG. 4. The ratio q'/q'' is displayed here versus ω showing the mode damping for the same frequency range of Fig. 3. The ratio, although undergoes a dramatic change in this range, is always larger than one, thus signaling underdamped behavior.

$$B_2=2\chi_2(\chi_2+\sqrt{1+\chi_2^2})B_1. \tag{27}$$

Hence the two curves ω_+ and ω_- have a common start at (B_2,ω_2), where $\omega_2=(\chi_2+\sqrt{1+\chi_2^2})\omega_0$, and approach the asymptotic lines $(\omega_0/\chi_2)(B_0/B_1)$ and $\omega_0\chi_2$, respectively. For the diagram in Fig. 2, we have taken $\chi_2=0.5$ thus, obtaining that $B_2\approx 6.64$ T and $\omega_2\approx 4{,}05\times 10^{11}$ rad/s. The asymptotic lines become $\omega_{d+}/\omega_0\rightarrow 2(B_0/B_1)$ and $\omega_{d-}/\omega_0 \rightarrow 0.5$.

As indicated by the B_0 vs ω Fig. 2 diagram, the modes are optical for frequencies below the ω_{cr} line where they are weakly affected by the superconductor properties and the vortex dynamics. In this region and for $B_0 \ll B_1$ the superelectron dominates over the vortex response and, effectively, there are plasma modes. For $B_0 \gg B_1$, the ω_{cr} line decreases inversely proportional to B_0. Above the ω_{d-} line, and, at large magnetic fields, $B>B_2$, the modes become overdamped. Thus the interesting region lies above the ω_{cr} line and below the ω_{d-} line, where the modes are underdamped coupled and vortex dominated. In this intermediate region, dissipation should be small enough ($q'>q''$) to allow wave propagation over some wavelengths before attenuation sets in.

All figures discussed below were obtained using Eq. (18) expression. The complex wave number q is then easily derived as a function of ω.

Figure 3 shows the dispersion relation ω vs q' for $B_0=0$ and 20 T. In the case of zero magnetic field, the frequency window considered in this figure is below the zero magnetic field optical-coupled crossover ($\omega_{cr}\approx 2{,}10\times 10^{11}$ rad/s). Indeed, the B_0 equal to zero mode shows a quasilinear dependence. However for a magnetic field $B_0=20$ T, the presence of vortices changes dramatically the dispersion relation. The frequency ω_{cr} has dropped substantially, according to this figure. Below the mode is optical, similarly to the zero magnetic field case, and above the mode is slow in comparison to the zero field one. This effect clearly comes from the vortex's overwhelming contribution at this large magnetic field value. In this frequency window, the mode is underdamped until the frequency ω_{d-} is reached. Above it turns out to be overdamped. In order to better estimate the attenuation, we have plotted the ratio q'/q'' for the same frequency window (Fig. 4). Notice that q'/q'', obtained from Eq. (18) and shown here, gives directly the mode attenuation, whereas Eq. (22) just provides an approximate criterion, used to define the dissipative curve $\omega_{d\pm}$ of Fig. 2. Figure 4 shows that for $\omega_{cr}<\omega<\omega_{d-}$ the mode propagates over various wavelengths before its amplitude goes to zero. According to Fig. 4 q'/q'' diverges for low frequencies within the optical regime. This behavior is explained recalling that all losses are caused by vortices and disappear at zero frequency.

The reduced speed, defined as the ratio between the phase velocity and the speed of light in the dielectric $(\omega/q')/v$ is plotted in Fig. 5. In the optical regime this ratio is essentially equal to 1. This is quite verifiable at zero magnetic field but not at 20 T, where the modes are strongly slowed by the presence of vortices.

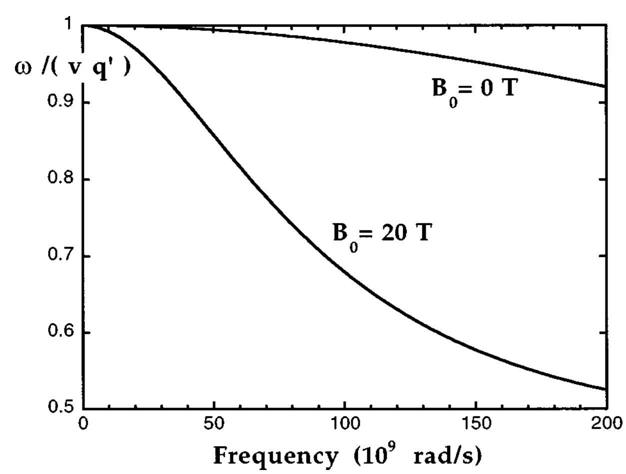

FIG. 5. The retardation ratio $(\omega/q')/v$ is shown for the frequency range of Fig. 4. For zero applied field this ratio is near one showing that mode is essentially optical. This is not case for $B_0=20$ T whose strong deviation from one signals coupling between the dielectric and the superconducting film due to the presence of vortices.

IV. CONCLUSION

In this paper, we have studied superficial coupled modes in a superconducting film, bounded by two identical dielectric media, in the presence of an applied magnetic field perpendicular to the film surfaces. The superconductor is anisotropic and its uniaxial direction (c axis) is perpendicular to the superconductor-dielectric interfaces. The choice of a nonconducting media of high dielectric constant helps to lower the propagating wave frequency far below the gap frequency range. The considered static magnetic field is greater than the lowest critical field, which allows the existence of a vortex system. The vortices are considered pinned and their constrained movement is associated with dissipation. In the present approach superelectrons and vortices contribute additively to the film surface impedance. Vortices and superelectrons interact with each other through the Lorentz force and the electric field, created by vortex motion and the superelectrons acceleration. Here we have studied how the lowest energy branch, the TM symmetric mode, is affected by vortices. Under justifiable approximations, we obtained an analytical expression for its dispersion relation, which can describe simultaneously the three different possible behaviors for a propagating mode in a superconducting film subjected to an uniform magnetic field; namely, the optical regime, underdamped coupled regime, and overdamped coupled regime.

We found that under very high magnetic fields, vortex motion and not the supercurrent dominates the film electromagnetic response. The modes are well described by the vortex oscillations around their pinning centers, where their energy oscillates between the pinning energy and the electrical one. We have studied the B_0 vs ω diagram for a very thin superconducting film, made of the high-T_c compound YBCO. We found three regions of distinct propagative behavior: optical, underdamped coupled, and overdamped modes. Nested between the optical and the overdamped regions, and above a certain threshold magnetic field, is the region of interest. There exist, in this frequency and magnetic field window, underdamped propagative modes, whose behavior is determined by the vortex system response to the external excitation.

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