Resistance of layered superclean superconductors at low temperatures

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The low-energy excitation spectrum is found for a layered superconductor vortex with a small number of impurities inside the vortex core. All levels are found to be correlated. This leads to the strong enhancement of conductivity in superclean layered superconductors.
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I. INTRODUCTION

In superconductors with weak pinning, the I-V characteristic displays anomalous properties.[1,2] Some of them are very difficult to explain in the framework of the quasiclassical approach. In the case of the quasiclassical approach there are three limiting cases, determined by values of the three parameters: the size of the gap Δ in the single-particle excitation spectrum, the level spacing inside the vortex core $\omega_0 \sim \Delta^2/\varepsilon_F$, and the electron mean collision time $\tau_{\rm tr}$. The three limiting cases are ''dirty'' limit, for $\tau_{\rm tr}\Delta \ll 1$; ''clean'' limit for $\Delta \gg \tau_{\rm tr}^{-1} \gg \omega_0$, and superclean limit when the condition $\omega_0\tau_{\rm tr} \gg 1$ is fulfilled.

In the dirty limit at zero temperature the calculation of Gor'kov and Kopnin[3] confirms the qualitative picture of vortex motion of Bardeen and Stephen.[4] In accordance with the picture of Bardeen and Stephen the vortex core is in a ''normal'' state. Bardeen and Sherman[5] and Larkin and Ovchinnikov[6] derived conductivity in a mixed state for low temperatures and small magnetic field in the case of moderately clean superconductors. In this case compared to the previous picture a logarithmically large factor arises in conductivity. This factor is related to shrinkage of the vortex core at low temperatures $T \ll T_c$.[7] The Hall component of conductivity was found for moderately clean superconductors by Kopnin and Lopatin.[8]

The superclean case was studied in the Kopnin and Kravtsov paper.[9] It was found that the level spacing ω_0 inside a vortex core plays the same role as the cyclotron frequency $\omega_c = eH/mc$ in a normal metal. It was also found that in the superclean limit the Hall component of the conductivity tensor is the largest one $\sigma \simeq en_e/B$ (here n_e is the electron density in the conduction band, B is the magnitude of the magnetic field). The dissipative part of the conductivity tensor is smaller by the parameter $(\omega_0\tau)^{-1}$. Hence the dissipative part of the resistance tensor is the same as in moderately clean superconductors.

The quasiclassical approach is probably violated in the two-dimensional case (in layered superconductors), because the excitation spectrum in the vortex core is then discrete. Guinea and Pogorelov[10] considered the dissipation in the vortex state as a result of transitions between unperturbed levels induced by ''moving'' impurities. Such a perturbation theory approach is valid only in the high velocity limit $v \gg v_F(\Delta/\epsilon_F)^2$.

Feigel'man and Skvorzov[11] consider energy dissipation during the vortex motion as a result of Landau-Zener transitions between levels. They suppose that the level distribution inside the vortex core obeys Wigner-Dyson statistics subject to some corrections, related to specifics of superconductivity.[12] Such a treatment can probably be used in dirty and moderately clean superconductors. This method, although differing from the quasiclassical approach, nevertheless gives for the essential range of electrical fields the same expression of conductivity as the quasiclassical approach.

In this paper we consider the superclean limit. We find that in this region a new mechanism of dissipation arises. In the superclean limit no more than one impurity can be found at distances of order of the correlation length $\xi = v_F/\Delta$ from the vortex center. It will be shown that in such a case a statistical description of level positions is impossible. If an impurity is placed at a distance of order of ξ from the vortex center and is weak (Born parameter is small), then the shift of levels is also small. It is also important that levels with even and odd orbital momentum are shifted in opposite directions. The level shift increases as the impurity comes closer to the vortex center. At some distance from the impurity to the vortex center levels practically cross. It is very important, that all levels with energy $|\varepsilon| \ll \Delta$ cross simultaneously.

If we neglect the weak (of order of $\omega_0/p_F\xi$) repulsion of levels in this region, then positions of levels as a function of the distance from the vortex core to the impurity form two families of crossing straight lines. Outside of the dangerous level crossing region these lines are practically horizontal (see Fig. 1). The size of the dangerous zone, where the level lines can be considered as crossing, depends on the Landau-Zener parameter and hence on the vortex velocity. For the vortex velocity V in the range $\omega_0 \gg p_F V \gg \omega_0(\Delta/\varepsilon_F)$ the distribution function of excitations inside the core of the vortex does not change until the impurity comes into the dangerous zone. But it changes essentially when the impurity goes through the dangerous zone. Excitations, which arise when the impurity goes through this zone, determine the value of the dissipative part of the conductivity. Such a mechanism of dissipation is essential for the magnitude of the electrical field E lying in the range

$$B\frac{v_F}{c}\left(\frac{\Delta}{\varepsilon_F}\right)^2 \gg E \gg B\frac{v_F}{c}(\Delta/\varepsilon_F)^3.$$

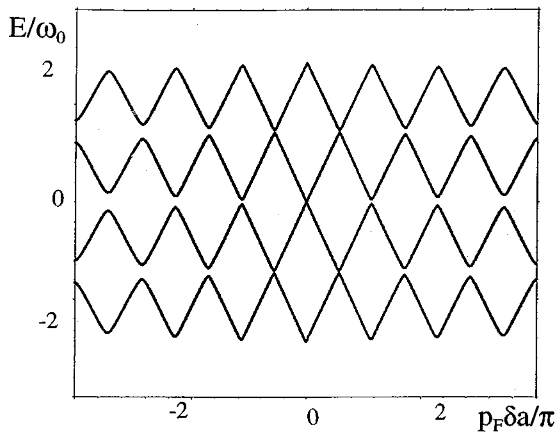

FIG. 1. The excitation spectrum as a function of the impurity distance from the vortex center. The parameter $(\pi^2/2\omega_0 p_F)(\partial I_{1,\mathrm{am}}/\partial a \ldots)$ equals 0.02. The quantity $\delta a = a - a_0$, with a_0 is given by Eq. (20).

Here c is the velocity of light. As we prove below, this mechanism of dissipation leads to the dissipative part of the current density being equal to

$$j_x = \frac{a_0 n_{\mathrm{imp}}}{\phi_0} \frac{\varepsilon_F^{5/3}}{\Delta^{2/3}} \left(\frac{E}{v_F B} \right)^{2/3}, \tag{1}$$

where a_0 is the distance from the ''dangerous'' region to the vortex center $a_0 \sim \theta\xi$. θ is the Born parameter which is equal to the phase shift of an electron scattering off the impurity. Usually its value is of the order of one $\theta \sim 1$. Hence parameter $(p_F a_0)$ is much larger than one, and current density essentially exceeds the value obtained in the framework of the quasiclassical approximation. In the range $p_F a \gg \omega_0 \tau_{\mathrm{tr}} \gg 1$ the Hall angle is small.

II. THE LOW-ENERGY EXCITATIONS SPECTRUM FOR AN IMPURITY AT THE DISTANCE *a* FROM THE VORTEX CENTER IN THE RANGE $A \gg \xi(\Delta/\varepsilon_F)^{1/2}$

The excitations spectrum E in the vortex state can be found as a solution of the eigenvalue problem for the system of equations[13,14]

$$\begin{pmatrix} -\frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - \mu + V(\mathbf{r}) - E; & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^*; & \frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + \mu - V(\mathbf{r}) - E \end{pmatrix} \times \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = 0, \tag{2}$$

where Δ is the order parameter, μ is the chemical potential (Fermi energy $\mu = \varepsilon_F$), $V(\mathbf{r})$ is the potential of impurities. We suppose here that the magnetic field B is weak ($B \ll H_{c2}$) and omit the vector potential in Eq. (2). Below we consider the two-dimensional case. We suppose, also, that there is only one short-range impurity (with the interaction radius of order of p_F^{-1}) inside the vortex core.

In our problem the order parameter Δ in the absence of the impurity is given by the expression

$$\Delta(\mathbf{r}) = \Delta(r)\exp(i\varphi), \tag{3}$$

where φ is polar angle, $r = |\mathbf{r}|$. The low-energy excitation spectrum in the absence of the impurity was found in Ref. 13. The system (2) possesses a very important property: if E is an eigenvalue with the eigenfunction (f_1, f_2), then $-E$ is also an eigenvalue and the corresponding eigenfunction is $(f_2^*, -f_1^*)$. This property holds in a magnetic field too.

The low-energy excitation spectrum E_n^0 is given by the equation[13]

$$E_n^0 = -(n - 1/2)\omega_0, \tag{4}$$

where

$$\omega_0 = \int_0^\infty \frac{dr\Delta(r)}{p_F r} e^{-2K(r)} \bigg/ \int_0^\infty dr e^{-2K(r)},$$

$$K(r) = \int_0^r dr_1 \Delta(r_1)/v_F. \tag{5}$$

If Kramer-Pesh effect takes place, then with a logarithmic accuracy we obtain from Eqs. (4),(5)

$$\omega_0 = \frac{\Delta^2}{\varepsilon_F}\ln\left(\frac{\Delta}{T}\right), \quad \Delta = \Delta_{(\infty)}. \tag{6}$$

The eigenfunction, corresponding to the eigenvalue (4) is

$$\overline{f}_n = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_n = \tilde{c} e^{-K(r)} \begin{pmatrix} e^{in\varphi} J_n(p_F r) \\ -e^{i(n-1)\varphi} J_{n-1}(p_F r) \end{pmatrix}, \tag{7}$$

where $\tilde{c}$ is the normalization constant, $J_n(x)$ is a Bessel function, $n = 0, \pm 1, \pm 2, \ldots$.

For the excitation spectrum E in the presence of an impurity inside the vortex core, we obtain from Eq. (2) the following system of equations:

$$\det((\hat{\varepsilon} - E) + \hat{A}) = 0, \tag{8}$$

where the operator $\hat{A}$ is given by its matrix elements. In the basis (7) we have

$$A_{kn} = \left\langle \overline{f}_k^+ \begin{pmatrix} V(\mathbf{r} - \mathbf{a}); & 0 \\ 0; & -V(\mathbf{r} - \mathbf{a}) \end{pmatrix} \overline{f}_n \right\rangle, \tag{9}$$

a is the position of the impurity relative to the vortex center, and

$$\hat{\varepsilon}_{kn} = \delta_{kn} E_n^0. \tag{10}$$

In Eq. (9), essential are the values of r such that $r \gg p_F^{-1}$. So we can use an asymptotic expansion of Bessel functions to find matrix elements A_{kn}. A simple calculation gives

$$A_{kn} = e^{i(k-n)\varphi_a} \left\{ I_1(a)\cos\left(\frac{\pi(n+k)}{2}\right) - I_2(a)\sin\left(\frac{\pi(n+k)}{2}\right) \right\}, \tag{11}$$

where φ_a is the polar angle of the vector **a**, and the quantities $I_{1,2}$ are given by the equation

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \frac{C^2}{a} e^{-2K(a)} \int d^2 r V(r) \begin{bmatrix} \sin\left(2p_F\left(a+\frac{(\mathbf{ar})}{a}\right)\right) \\ \cos\left(2p_F\left(a+\frac{(\mathbf{ar})}{a}\right)\right) \end{bmatrix}. \tag{12}$$

In Eq. (12) the normalization constant C is equal to

$$C^2 = \left\{ 2\pi \int_0^\infty dr e^{-2K(r)} \right\}^{-1}. \tag{13}$$

If there are several impurities inside the vortex core, then the operator $\hat{A}$ in Eq. (8) is a simple sum $\hat{A}_i$ over all impurities. Therefore

$$A_{kn}^{\{a_i\}} = \sum_i A_{kn}(a_i), \tag{14}$$

where $A_{kn}(a_i)$ is given by Eq. (11). It follows from Eqs. (11),(14), that the transition-matrix elements A_{kn} are separable. That is A_{kn} can be presented as a finite sum of terms of the type $\tilde{A}_k^j \tilde{B}_n^j$

$$A_{kn} = \sum_j \tilde{A}_k^j \tilde{B}_n^j. \tag{15}$$

As a result we can obtain an expression for the excitation spectrum in an explicit form. If only one impurity is placed inside the vortex core, then we obtain from Eqs. (8), (11), and (15) the following equation for the excitation spectrum:

$$\det\begin{pmatrix} 1 + I_1 \sum_{L=-N}^{N+1} \frac{1}{\varepsilon_{2L}+E}; & I_2 \sum_{L=-N}^{N+1} \frac{1}{\varepsilon_{2L}+E} \\ I_2 \sum_{L=-N}^{N+1} \frac{1}{\varepsilon_{2L-1}+E}; & 1 - I_1 \sum_{L=-N}^{N+1} \frac{1}{\varepsilon_{2L-1}+E} \end{pmatrix} = 0. \tag{16}$$

For the linear spectrum given by Eq. (4), we obtain in the limit $N \to +\infty$

$$\sum_{L=-N}^{N+1} \frac{1}{\varepsilon_{2L}+E} = \frac{\pi}{2\omega_0} \cot\left(\pi\left(\frac{1}{4} + \frac{E}{2\omega_0}\right)\right),$$

$$\sum_{L=-N}^{N+1} \frac{1}{\varepsilon_{2L-1}+E} = -\frac{\pi}{2\omega_0} \cot\left(\pi\left(\frac{1}{4} - \frac{E}{2\omega_0}\right)\right). \tag{17}$$

With a help of Eq. (17) we reduce Eq. (16) to the form

$$1 + \frac{\pi^2[(I_1)^2 + (I_2)^2]}{4\omega_0^2} + \frac{\pi I_1}{\omega_0 \cos(\pi E/\omega_0)} = 0. \tag{18}$$

It follows from Eq. (18) that the low-energy excitation spectrum is strong correlated even in the presence of an impurity inside the vortex core. If E_0 is a spectrum point, that is if E_0 is some solution of equation for the spectrum (18), then all solutions of Eq. (18) are given by the equation

$$E = \pm E_0 + 2\omega_0 N, \quad N = 0, \pm 1, \pm 2. \tag{19}$$

Hence the discrete spectrum is given by two sets of equidistant points.

Functions $I_{1,2}$ are periodic with the period π/p_F. Both are defined by the same function with shift by a quarter of the period. The amplitude $I_{1.\mathrm{am}}$ of these functions is a smooth function of the parameter (a/ξ) and is given by Eq. (12).

With the accuracy of $\omega_0(\Delta/\varepsilon_F)$ a point a_0 exists such that

$$I_{1.\mathrm{am}}(a_0) = -\frac{2\omega_0}{\pi}; \quad I_2(a_0) = 0. \tag{20}$$

Hence at the points $a_0 + \delta a$ given by equation

$$\delta a = \left(\frac{\pi}{p_F}\right) N, \quad N = 0, \pm 1, \pm 2 \ldots, \tag{21}$$

we have

$$E_0 = \frac{\delta a}{2} \left(\frac{\partial I_{1.\mathrm{am}}}{\partial a}\right)_{a_0}. \tag{22}$$

Equation (22) means that in a vicinity of the points of the trajectory of the vortex, given by Eq. (20), there is a set of points, separated by the distance δa, where spectrum lines are practically crossing (see Fig. 1). In Fig. 1 the quantity $\delta a = a - a_0$ is shown, where a_0 is given by Eq. (20). The vicinity of such points we denote as the *dissipation region*. If the impact parameter of the trajectory is smaller than some critical value, then on such a trajectory there are two dissipation regions. When the vortex moves through these two dissipation regions many excitations are created inside the vortex core. The contribution of these excitations to the dissipative part of conductivity will be found below.

The situation of having several impurities inside the vortex core is considered in the Appendix. We prove there that the strong correlation in the level positions survives for two impurities inside the vortex core. We can make a conjecture that the strong correlation in level positions exists for many impurities inside the vortex core, while the condition $l_{\mathrm{tr}} \gg \xi$ is fulfilled. But on a vortex trajectory of a general status dissipation regions with a large number of practically crossing energy-level lines do not exist, if there are two or more impurities inside the vortex core.

III. ONE IMPURITY AT SMALL DISTANCES $[A \ll \xi(\Delta/\varepsilon_F)^{1/2}]$ FROM THE VORTEX CENTER

First of all we shall consider one impurity with a short-range (of order of p_F^{-1}) potential placed exactly at the vortex center. At distances $\rho \gg p_F^{-1}$ from the vortex center we can use for the solution of Eq. (2) the quasiclassical approximation with the first-order correction terms. Indeed these correction terms will give an expression for the spectrum. And at small distances of order of p_F^{-1} we can omit nondiagonal elements in Eq. (2). As a result we obtain the following expression for the spectrum:

$$E_n = -(n-1/2)\omega_0 + v_F \tan\left(\frac{\theta_{n-1} - \theta_n}{2}\right) \Big/ \int_0^\infty d\rho e^{-2K(\rho)}, \tag{23}$$

where θ_n is the scattering phase in a state with angular momentum n in the presence of the impurity potential $V(\mathbf{r})$. The corresponding eigenfunction is given by the expression

$$\mathbf{f}_n=\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_n=\frac{\widetilde{C}}{2}e^{-K(\rho)} \times\begin{pmatrix} e^{in\varphi}[J_n(p_F\rho+\theta_n)+J_n(p_F\rho+\theta_{n-1})] \\ -e^{i(n-1)\varphi}[J_{n-1}(p_F\rho+\theta_n)+J_{n-1}(p_F\rho+\theta_{n-1})] \end{pmatrix}, \tag{24}$$

where $\widetilde{C}$ is a normalization constant. Suppose now that the impurity is placed on a distance $\mathbf{a}$ from the vortex center, such that $|\mathbf{a}|\ll\xi(\Delta/\varepsilon_F)^{1/2}$. In Eq. (2) we make a transformation to the coordinate system with the origin at the impurity. Then we obtain

$$\begin{pmatrix} -\frac{1}{2m}\frac{\partial^2}{\partial\mathbf{r}^2}-\mu+V(\mathbf{r})-E; & |\Delta|e^{i\varphi}+\left(\mathbf{a}\frac{\partial}{\partial\mathbf{r}}\right)(|\Delta|e^{i\varphi}) \\ |\Delta|e^{-i\varphi}+\left(\mathbf{a}\frac{\partial}{\partial\mathbf{r}}\right)(|\Delta|e^{-i\varphi}); & \frac{1}{2m}\frac{\partial^2}{\partial\mathbf{r}^2}+\mu-V(\mathbf{r})-E \end{pmatrix} \times\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}=0. \tag{25}$$

From Eq. (25) we obtain the following equation for the excitation spectrum:

$$\det((\hat{\varepsilon}-E)+\hat{A})=0, \tag{26}$$

where $\hat{\varepsilon}_{kn}=E_n\delta_{nk}$ and E_n is given by Eq. (23). The operator $\hat{A}$ is given by matrix elements $\hat{A}_{kn}$ in the basis defined in Eq. (24),

$$\hat{A}_{kn}=\left\langle \overline{f_k^+}\begin{pmatrix} 0; & \left(\mathbf{a}\frac{\partial}{\partial\mathbf{r}}\right)(|\Delta|e^{i\varphi}) \\ \left(\mathbf{a}\frac{\partial}{\partial\mathbf{r}}\right)(|\Delta|e^{-i\varphi}); & 0 \end{pmatrix}\mathbf{f}_n \right\rangle . \tag{27}$$

A simple straightforward calculation making use of Eqs. (24), (27) gives

$$\widetilde{A}_{kn}=-\pi aC^2\int_0^\infty d\rho\frac{|\Delta_{(\rho)}|}{\rho}e^{-2K(\rho)} \times\left\{\delta_{k,n+1}e^{-i\varphi_a}\cos\left(\frac{\theta_{n-1}-\theta_{n+1}}{2}\right)+\delta_{k,n-1}e^{i\varphi_a}\cos\left(\frac{\theta_n-\theta_{n-2}}{2}\right)\right\}. \tag{28}$$

In Eq. (28) the constant C is given by Eq. (13). It follows from Eq. (28) that in the operator in Eq. (26), only diagonal and near-diagonal elements are nonzero. Now we define the function $B(I,E,n)$ in the following manner:

$$B(I,E,n-1)=-E_n-E-\frac{I^2\cos^2[(\theta_{n-1}-\theta_{n+1})/2]}{B(I,E,n)}, \tag{29}$$

where

$$I=\pi aC^2\int_0^\infty d\rho\frac{|\Delta|}{\rho}e^{-2K(\rho)}, \quad I/\omega_0=p_Fa/2.$$

With the help of function B we reduce Eq. (26) for the spectrum to the following simple form:

$$\det\begin{pmatrix} B(I,E,1); -Ie^{i\varphi_a}\cos\left(\frac{\theta_0-\theta_2}{2}\right);0;0 \\ -Ie^{-i\varphi_a}\cos\left(\frac{\theta_0-\theta_2}{2}\right);E_0-E;-Ie^{i\varphi_a};0 \\ 0;-Ie^{-i\varphi_a};E_1-E;-Ie^{i\varphi_a}\cos\left(\frac{\theta_0-\theta_2}{2}\right) \\ 0;0;Ie^{-i\varphi_a}\cos\left(\frac{\theta_0-\theta_2}{2}\right);B(I,-E,1) \end{pmatrix}=0. \tag{30}$$

Equation (29) means that $B(I,E,1)$ can be presented as an infinite fraction

$$B(I,E,1)=-E_2-E-\cfrac{I^2\cos^2((\theta_1-\theta_3)/2)}{-E_3-E-\cfrac{I^2\cos^2((\theta_2-\theta_4)/2)}{-E_4-E-\cfrac{I^2\cos^2((\theta_3-\theta_5)/2)}{\ldots B(I,E,n+1)}}}. \tag{31}$$

The fraction (31) converges very quickly, if for $B(I,E,n+1)$ we use the expression

$$B(I,E,n+1)=\Big[(n+1/2)\omega_0-E+\sqrt{[(n+1/2)\omega_0-E]^2-4I^2}\Big]/2, \tag{32}$$

$$(n+1/2)\omega_0\pm E\gg|I|. \tag{33}$$

Suppose now, that the impurity is of a small size, so only S scattering is essential. Suppose also, that the impurity potential is of order of the atomic one and hence the inequality takes place

$$\varepsilon_0=\Delta\ \tan\,(\theta_0/2)\gg\omega_0. \tag{34}$$

Then in the first approximation Eq. (30) for low-energy excitations uncouples to two independent branches

$$B(I,E,1)=0 \quad \text{and} \quad B(I,-E,1)=0. \tag{35}$$

Hence we obtain two independent families of spectrum lines. Of course in this approximation they will cross, and only in the next approximation with respect to the parameter $(\omega_0/\varepsilon_0)^2$ will a gap in the crossing points open.

For small values of E we have

$$B(I,E,1)=B+\alpha E, \tag{36}$$

where

$$\alpha=\frac{\partial B(I,E,1)}{\partial E}. \tag{37}$$

Inserting expression (36) into Eq. (30) we obtain the following equation for the lowest energy level near the crossing points:

$$E^2\alpha^2\varepsilon_0^2 = B^2[I\cos(\theta_0/2)]^2 + [\varepsilon_0 B + (I\cos(\theta_0/2))^2]^2. \tag{38}$$

Hence the value of the gap δ near this crossing point is equal to

$$\delta = \frac{|I\cos(\theta_0/2)|^3}{|\alpha\varepsilon_0|\sqrt{\varepsilon_0^2 + (I\cos(\theta_0/2))^2}}. \tag{39}$$

In Eq. (38) the quantity α should be taken at the point

$$\alpha = \frac{\partial B(I,E,1)}{\partial E}\Bigg|_{B=-\frac{\varepsilon_0[I\cos(\theta_0/2)]^2}{[I\cos(\theta_0/2)]^2+\varepsilon_0^2}}. \tag{40}$$

By the order of magnitude the value of the gap δ is given by the equation

$$\delta \sim \Delta(a/\xi)^3. \tag{41}$$

It is possible to keep in the expansion of the order parameter Δ with respect to the shift **a** in Eq. (25) terms up to the second order in **a**. Then the operator $\hat{A}$ will have the following nonzero matrix elements: A_{nn};$A_{n,n\pm1}$;$A_{n,n\pm2}$. Equation (26) in this approach enables us to determine the excitation spectrum up to the shift **a** of order of

$$a \sim \xi(\Delta/\varepsilon_F)^{1/3}. \tag{42}$$

At the boundary of this region the gap δ, given by Eq. (41), is of the order of ω_0. Hence energy levels can "cross" only in dissipation regions [Eq. (22)], or when the impurity is placed near the vortex core [Eq. (39)].

Equation (30) for the spectrum can be reduced to the form

$$\left[\frac{I^2}{E_0}\cos^2\left(\frac{\theta_0-\theta_2}{2}\right) + \frac{B(I,E,1)+B(I,-E,1)}{2}\right]^2 - \left[\frac{B(I,E,1)-B(I,-E,1)}{2} - \frac{EI^2}{E_0^2}\cos^2\left(\frac{\theta_0-\theta_2}{2}\right)\right]^2 + \frac{I^2-E^2}{E_0^2}B(I,E,1)B(I,-E,1) + \left[\frac{EI^2}{E_0^2}\cos^2\left(\frac{\theta_0-\theta_2}{2}\right)\right]^2 = 0. \tag{43}$$

Near the crossing points the last two terms are small [of order of $(I^3/E_0)^2$] and lead to the repulsion of spectrum lines. In zero approximation neglecting two last terms Eq. (43) uncouples to give two families of independent spectrum lines.

Energy levels as a function of the shift **a** (or quantity $I \sim a$) are given in Fig. 2. Inside the circle in Fig. 2 the equations for two spectrum branches are

$$E/\omega_0 - 1.073 = \delta(0.1335t \pm \sqrt{1+t^2}); \tag{44}$$

$$t = 1.343(I/\omega_0 - 2.3172)/\delta; \tag{45}$$

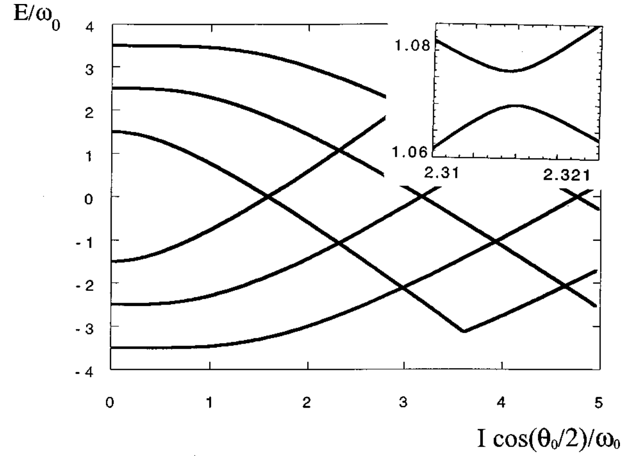

FIG. 2. The low-energy excitation spectrum at small distances a from the impurity to the vortex center; $I/\omega_0 = p_F a/2$, see Eq. (29); $\theta \ll 1$; $E_0/\omega_0 = 50$.

$$\delta = 3.2\times10^{-3} \tag{46}$$

The contribution of these crossing points to the dissipative part of the conductivity will be discussed below.

IV. LANDAU-ZENER TUNNELING NEAR THE CROSSING POINT OF SPECTRUM LINES

Near the crossing point of spectrum lines, given by Eq. (18), we can put

$$\frac{\pi I_1}{2\omega_0} = -1 + y; \quad \frac{\pi I_2}{2\omega_0} = 2p_F X; \quad X = Vt, \tag{47}$$

where V is the velocity of the vortex. For two close spectrum points $E_\pm$ we obtain from Eq. (18) the following value:

$$E_\pm = \pm\varepsilon_{(t)}\frac{\omega_0}{\pi}, \quad \varepsilon_{(t)} = \sqrt{y^2 + (2p_F X)^2}. \tag{48}$$

The usual Landau-Zener consideration leads to the following value for the probability W_{++} for a "particle" to remain on the same branch after collision[15]

$$W_{++} = 1 - \exp\left(-\frac{\omega_0 y^2}{2p_F|V|}\right). \tag{49}$$

With the help of Eq. (49) we will find the energy transmitted to the vortex at one collision with impurity.

V. DISSIPATIVE PART OF CONDUCTIVITY IN SUPERCONDUCTORS

Let us first consider the superclean limit, when only one impurity can be found inside the vortex core. Then for small values of the vortex velocity V, such that

$$V \ll v_F(\Delta/\varepsilon_F)^2, \tag{50}$$

an excitation can arise only at spectrum line crossing points. Such crossing points are located only in dissipation regions [Eqs. (20), (21)], or if the vortex center is close to the impurity [Eq. (39)].

Suppose that between two consequent collisions the system goes to the equilibrium due to inelastic-scattering processes. In such a case the dissipative part of conductivity is directly connected to the energy, stored by the vortex after one collision with impurity.

Consider first processes happening in dissipation regions. Suppose that ρ_0 is the impact parameter. Then near the point a_0 [Eq. (20)] we have

$$\frac{\pi I_{1.\mathrm{am}}}{2\omega_0}=1+y,\quad y=\frac{\pi}{2\omega_0}\left(\frac{\partial I_{1.\mathrm{am}}}{\partial\rho}\right)_{a_0} x\sqrt{1-(\rho_0/a_0)^2}. \tag{51}$$

Periodicity δx of a crossing point with respect to x is

$$\delta x=\frac{\pi}{p_F\sqrt{1-(\rho_0/a_0)^2}}. \tag{52}$$

Hence the full number N of crossing points in a dissipation region, with the effective transition of a particle to the other branch, is

$$N=\left(\frac{2x}{\delta x}\right)^{2/3}=\frac{p_F(V\omega_0)^{1/3}}{(\partial I_{1.\mathrm{am}}/\partial\rho)_{a_0}^{2/3}}. \tag{53}$$

To obtain Eq. (53) we use Eq. (49). Inside of this region the energy of excitations increases linearly with the shift. After that the diffusion processes of excitations on the energy axis start to be essential. The transitions of excitations to the other branch happen in an essentially larger region $N^*\sim N^{3/2}$ but are not important. We get with the help of Eq. (53), that the vortex, after passing through two dissipation regions (one collision with impurity), stores the energy δE being equal to

$$\delta E=\omega_0 N^2=\frac{\omega_0^{5/3}p_F^2V^{2/3}}{(\partial I_{1.\mathrm{am}}/\partial\rho)_{a_0}^{4/3}}. \tag{54}$$

As a result, we can estimate the contribution of transitions in dissipation regions to the dissipative current $j_x^{(1)}$

$$j_x^{(1)}=\frac{a_0 n_{\mathrm{imp}}}{\phi_0}\frac{\omega_0^{5/3}p_F^2}{(\partial I_{1.\mathrm{am}}/\partial\rho)_{a_0}^{4/3}}\left(\frac{E}{B}\right)^{2/3}, \tag{55}$$

where $\phi_0=\pi/e$ is the flux quantum, B is the magnetic field value. Consider now the energy, dissipated as an impurity passes near the vortex core. From Eqs. (30) and (39) we obtain the following expression for the excitation spectrum near the crossing points:

$$E=\pm\left(\frac{I^6}{\varepsilon_0^4\gamma_2^2}+[\omega_0 p_F(\delta a)/2]^2\right)^{1/2}, \tag{56}$$

where

$$\gamma_2=\frac{\partial B(I,E,1)}{\partial I};\quad I=\omega_0 p_F a/2, \tag{57}$$

(δa) is a shift from a crossing point, and the parameter γ_2 is of the order of unity. According to Eq. (49), the probability W_{++} for the particle to remain on the same branch after a collision is equal to

$$W_{++}=1-\exp\left\{-\frac{\pi a^6(\omega_0 p_F/2)^5}{\varepsilon_0^4\gamma_2^2|V|}\right\}. \tag{58}$$

Hence the number of excitations N that arise in the vortex core, when the impurity passes through the vortex near its center, is

$$N=\left(\frac{2\tilde{a}p_F}{\pi}\right)^{6/7},\quad \tilde{a}=\left(\frac{V\varepsilon_0^4\gamma_2^2}{\pi(\omega_0 p_F/2)^5}\right)^{1/6}. \tag{59}$$

These transitions give the following contribution to the energy dissipation δE per volume and time unit:

$$\delta E=2\tilde{a}V\omega_0 N^2 n_{\mathrm{imp}}(B/\phi_0). \tag{60}$$

Equations (55), (60) completely determine the dissipative part of the current

$$j_x=j_x^{(1)}+\frac{\omega_0 n_{\mathrm{imp}}}{\phi_0}p_F^{12/7}\left(\frac{\varepsilon_0^4\gamma_2^2}{\pi(\omega_0 p_F/2)^5}\right)^{19/42}\left(\frac{E}{B}\right)^{19/42}. \tag{61}$$

By the order of magnitude we have

$$\left(\frac{\partial I_{1.\mathrm{am}}}{\partial\rho}\right)_{a_0}\sim\frac{2\omega_0}{\pi\xi}. \tag{62}$$

Hence, in the range of velocities V, such that

$$V/v_F>(\Delta/\varepsilon_F)^{40/9} \tag{63}$$

the second term in Eq. (61) is smaller than the first one. By the large parameter

$$\frac{n_{\mathrm{imp}}\xi^2}{\tau_{\mathrm{tr}}\Delta}\left(\frac{\varepsilon_F}{\Delta}\right)^{2/3}\left(\frac{v_F}{V}\right)^{1/3} \tag{64}$$

the dissipative part of the current, given in Eq. (61), exceeds the quasiclassical value for the current in the two-dimensional case. Expression (55) for conductivity was obtained under the assumption that the number of crossing points in a dissipation region Eq. (53) is large. This condition gives the same restriction on the value of the velocity as given in Eq. (63).

VI. CONCLUSION

In the two-dimensional case the excitation spectrum in vortex core is discrete. This results in the strong increase of the dissipative part of the conductivity σ_{xx} [Eq. (1)] compared to its value obtained by the quasiclassical method. It is very probable that the strong increase of conductivity σ_{xx} at low temperatures, obtained in experimental papers,[1,2] is related to the phenomena considered in this paper. For a detailed comparison to the experiment both experimental and theoretical investigations are necessary.

Consider now the applicability region of Eqs. (55), (61), and (64). We do not see any restriction for the temperature T in the framework of the model used except $T\ll T_c$. In the range $T\sim T_c$ excitations with the energy $\varepsilon\sim\Delta$ are essential.

In this region the excitation spectrum is not equidistant. Probably it is more essential that in our model the main mechanism of dissipation is the scattering of excitations off impurities. In real high-T_c compounds such an assumption holds only in the low temperature region. The predicted effect should vanish in the strong magnetic field region $H \sim H_{c2}$.

More complicated are the restrictions for the electrical-field magnitude or for the vortex velocity. The restriction $v_F(\Delta/\varepsilon_F)^3 \ll V \ll v_F(\Delta/\varepsilon_F)^2$, given in the introduction, holds in our approximation. For the vortex velocity $V \gg v_F(\Delta/\varepsilon_F)^2$ the adiabatic consideration is inapplicable. An impurity on the distances $a<\xi$ from the vortex center leads to the transition of quasiparticles to highly excited states. In the region $V \ll v_F(\Delta/\varepsilon_F)^3$ the small gap in the excitation spectrum of the order of $\omega_0(\Delta/\varepsilon_F)^3$ leads to the adiabaticity of motion in dissipation regions. In this case the impurities that pass at small distances of the order of $\xi(\Delta/\varepsilon_F)^{1/2}$ from the vortex center [second term in Eq. (61)], give the main contribution to the energy dissipation. It is essential that even in the frame of our approach the I-V characteristic changes its form for different ranges of the velocity.

There are also some other mechanisms that can change the $I-V$ characteristic. At small values of vortex velocities pinning can start to be essential. It can reduce dissipation. The pinning force is strongly dependent on the interaction between vortices and hence on the magnetic-field magnitude and anisotropy factor. In superclean superconductors pinning can be considered weak.

The tunneling of excitations to the neighboring planes can reduce dissipation at large velocity. If the anisotropy factor ε is small then the spectrum near the crossing points does not change much and the Landau-Zener tunneling probability does not change. But near the points where the energy of a quasiparticle is close to the unperturbed position of a level from the neighboring plane, a notable probability arises for tunneling of a quasiparticle to the neighboring plane. As a result, the quasiparticle energy does not increase any more and dissipation will be smaller than given by Eq. (54). The energy relaxation of quasiparticles can change I-V characteristics both for large values of vortex velocity and for small values. If the energy relaxation time is large enough, then effective heating of excitations inside of the vortex core takes place. The effective temperature reaches a value of the order of T_c.[16] The value of σ_{xx} decreases in this case. On the other hand, strong energy relaxation can lead to the equilibrium inside of one act of tunneling. This phenomenon will decrease the σ_{xx} value in the range of small vortex velocities. Now the physical reason for the energy relaxation in high-T_c superconductors at low temperatures is not clear, and we cannot make any quantitative estimation of this effect. Equations (55), (61) are valid in the superclean limit, if at distances of the order of ξ from the vortex center there is no more than one impurity. If at a distance of the order of ξ from the vortex center there are two impurities, then the probability of a level crossing decreases strongly. As a result, the value of σ_{xx} decreases too.

σ_{xx} as a function of the impurity concentration has a maximum at $\omega_0\tau_{\rm tr}$ of the order of one. The question about the value of the impurity concentration, for which the matrix can be considered as random, demands an additional investigation.

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APPENDIX: A LARGE NUMBER OF IMPURITIES INSIDE OF THE VORTEX CORE

If inside the vortex core a large number of impurities is placed, then transition-matrix elements $A_{kn}\{\mathbf{a_i}\}$ ($\mathbf{a_i}$ is the position of ith impurity), are given by Eq. (14). The equation for the spectrum in such a case is

$$\det(\hat{1}+\hat{C})=0 \tag{A1}$$

with the matrix elements of the operator $\hat{C}$ being equal to

$$\hat{C}_{jj_1}=\sum_k \frac{\tilde{B}_k^j\tilde{A}_k^{j_1}}{E_k-E}, \quad \hat{1}_{nm}=\delta_{nm}. \tag{A2}$$

If M impurities are placed inside the vortex core, then the size of the matrix in Eq. (A1) is (4M×4M). Due to symmetry properties of the elements A_{kn} [Eq. (11)], this matrix can be easily reduced to the size (2M×2M). The structure of this matrix $\hat{M}$ is simple. On the diagonal are placed blocks (2×2), defined only by one impurity. The second type of blocks are blocks (2×2), that give the interference contribution of a pair of impurities (ij).

To clarify the structure of the matrix $\hat{M}$ in the general case, we give below the explicit expression of the matrix $\hat{M}$ for two impurities inside the vortex core:

$$\hat{M}=\begin{pmatrix} 1+\frac{\pi I_1^1}{2\omega_0}\cot\left(\frac{\pi}{4}+\frac{\pi E}{2\omega_0}\right);\frac{\pi I_2^1}{2\omega_0}\cot\left(\frac{\pi}{4}+\frac{\pi E}{2\omega_0}\right);\frac{I_1^2}{2\omega_0}z_1\ ;\frac{I_2^2}{2\omega_0}z_1 \\ -\frac{\pi I_2^1}{2\omega_0}\cot\left(\frac{\pi}{4}-\frac{\pi E}{2\omega_0}\right);1+\frac{\pi I_1^1}{2\omega_0}\cot\left(\frac{\pi}{4}-\frac{\pi E}{2\omega_0}\right);\frac{I_2^2}{2\omega_0}z;-\frac{I_1^2}{2\omega_0}z \\ \frac{I_1^1}{2\omega_0}z_1^*;\frac{I_2^1}{2\omega_0}z_1^*;1+\frac{\pi I_1^2}{2\omega_0}\cot\left(\frac{\pi}{4}+\frac{\pi E}{2\omega_0}\right);\frac{\pi I_2^2}{2\omega_0}\cot\left(\frac{\pi}{4}+\frac{\pi E}{2\omega_0}\right) \\ \frac{I_2^1}{2\omega_0}z^*;-\frac{I_1^1}{2\omega_0}z^*;-\frac{\pi I_2^2}{2\omega_0}\cot\left(\frac{\pi}{4}-\frac{\pi E}{2\omega_0}\right);1+\frac{\pi I_1^2}{2\omega_0}\cot\left(\frac{\pi}{4}-\frac{\pi E}{2\omega_0}\right) \end{pmatrix}. \tag{A3}$$

In Eq. (A3) the upper index in I_i^j means the number of impurities ($j=1,\ldots M$) and the lower index i can be 1 or 2. The quantities z,z_1 are defined by the equations

$$z=\sum_{L=-\infty}^{\infty}\frac{e^{2iL(\varphi_{a_1}-\varphi_{a_2})}}{L-1/4+E/2\omega_0},$$

$$z_1=\sum_{L=-\infty}^{\infty}\frac{e^{i(2L+1)(\varphi_{a_1}-\varphi_{a_2})}}{L+1/4+E/2\omega_0}. \tag{A4}$$

From Eq. (A4) it follows that

$$z_1(E)=-z^*(-E)e^{i(\varphi_{a_1}-\varphi_{a_2})}. \tag{A5}$$

A straightforward calculation gives for the quantity $z(E)$ the following expression:

$$z(E)=-\frac{\pi}{\sin(\pi/4-\pi E/2\omega_0)}e^{-i\pi(1/4-E/2\omega_0)+iX_{a_1a_2}(1/2-E/\omega_0)}, \tag{A6}$$

where

$$X_{a_1a_2}=(\varphi_{a_1}-\varphi_{a_2})/\bmod \pi>0. \tag{A7}$$

From Eq. (A3) we obtain

$$\det\hat{M}=\left(1+J_1+\frac{\pi I_1^1}{\omega_0\cos(\pi E/\omega_0)}\right)\left(1+J_2+\frac{\pi I_1^2}{\omega_0\cos(\pi E/\omega_0)}\right)+\frac{I_2^1I_2^2}{4\omega_0^2}(z_1z^*+zz_1^*)+\frac{|zz_1|^2}{16\omega_0^4}J_1J_2$$
$$-\frac{|z|^2}{4\omega_0^2}\left[I_1^1+\frac{2\omega_0}{\pi}J_1\cot\left(\frac{\pi}{4}+\frac{\pi E}{2\omega_0}\right)\right]\left[I_1^2+\frac{2\omega_0}{\pi}J_2\cot\left(\frac{\pi}{4}+\frac{\pi E}{2\omega_0}\right)\right]$$
$$-\frac{|z_1|^2}{4\omega_0^2}\left[I_1^1+\frac{2\omega_0}{\pi}J_1\cot\left(\frac{\pi}{4}-\frac{\pi E}{2\omega_0}\right)\right]\left[I_1^2+\frac{2\omega_0}{\pi}J_2\cot\left(\frac{\pi}{4}-\frac{\pi E}{2\omega_0}\right)\right], \tag{A8}$$

where

$$J_i=\frac{\pi^2}{4\omega_0^2}[(I_1^i)^2+(I_2^i)^2]. \tag{A9}$$

Note that the quantity zz_1^* is purely imaginary. Hence the only term in Eq. (A8) that has a periodical dependence with respect to the energy E with the period, differing from $2\omega_0$, drops out. As a result, the expression for the spectrum (19) holds also in the case if there are two impurities inside the vortex. Our conjecture is: for the low-energy excitation spectrum Eq. (19) is correct in the clean limit ($\tau\Delta\gg1$), even if there are a lot of impurities inside the vortex core.

With the help of Eqs. (A5), (A6), (A8) we obtain the following equation for the excitation spectrum:

$$\det\hat{M}=0,$$

$$\det\hat{M}=\left(1+J_1+\frac{\pi I_1^1}{\omega_0\cos(\pi E/\omega_0)}\right)\left(1+J_2+\frac{\pi I_1^2}{\omega_0\cos(\pi E/\omega_0)}\right)+\frac{4}{\cos(\pi E/\omega_0)}J_1J_2$$
$$-\frac{\pi^2}{4\omega_0^2\sin^2(\pi/4-\pi E/2\omega_0)}\left[I_1^1+\frac{2\omega_0}{\pi}J_1\cot\left(\frac{\pi}{4}+\frac{\pi E}{2\omega_0}\right)\right]\left[I_1^2+\frac{2\omega_0}{\pi}J_2\cot\left(\frac{\pi}{4}+\frac{\pi E}{2\omega_0}\right)\right]$$
$$-\frac{\pi^2}{4\omega_0^2\sin^2(\pi/4+\pi E/2\omega_0)}\left[I_1^1+\frac{2\omega_0}{\pi}J_1\cot\left(\frac{\pi}{4}-\frac{\pi E}{2\omega_0}\right)\right]\left[I_1^2+\frac{2\omega_0}{\pi}J_2\cot\left(\frac{\pi}{4}-\frac{\pi E}{2\omega_0}\right)\right]. \tag{A10}$$

For the I-V characteristic in the mixed state, decisive is the value (or the existence) of the gap in the excitation spectrum. For this reason we will calculate the value of det $\hat{M}$ for $E=0$. Simple calculations with the help of Eq. (A10) give

$$\det\hat{M}(E=0)=\frac{\pi I_1^2}{\omega_0}(1-J_1)+\frac{\pi I_1^1}{\omega_0}(1-J_2)+(1+J_1)(1+J_2). \tag{A11}$$

Expression (A11) is nonnegative. It can be written as a sum of nonnegative terms

$$\det\hat{M}(E=0)=\left(1+\frac{\pi I_1^1}{2\omega_0}+\frac{\pi I_1^2}{2\omega_0}-\frac{\pi I_1^1}{2\omega_0}\frac{\pi I_1^2}{2\omega_0}\right)^2+\left(\frac{\pi I_2^1}{2\omega_0}\right)^2\left(\frac{\pi I_2^2}{2\omega_0}\right)^2+\left(\frac{\pi I_2^1}{2\omega_0}\right)^2\left(1-\frac{\pi I_1^2}{2\omega_0}\right)^2+\left(\frac{\pi I_2^2}{2\omega_0}\right)^2\left(1-\frac{\pi I_1^1}{2\omega_0}\right)^2. \tag{A12}$$

It means that on the vortex trajectory of a general status, dissipation regions with a large number of practically crossing energy levels lines do not exist. Now we can formulate the following conjecture. In the clean limit ($\tau\Delta\gg 1$), if several impurities are placed inside the vortex core, the quantity $\det\hat{M}(E=0)$ is a sum of nonnegative terms of the form (A12).

Now, for small values of $E\ll\omega_0$, we obtain from Eq. (A10)

$$\det\hat{M}(E)=\det\hat{M}(E=0)-\left(\frac{\pi E}{\omega_0}\right)^2\cdot\left\{2J_1J_2-\frac{\pi I_1^1}{2\omega_0}(1-J_2)-\frac{\pi I_1^2}{2\omega_0}(1-J_1)\right\}. \tag{A13}$$

Equations (A11), (A12), and (A13) completely determine the excitation spectrum near the crossing points of the energy levels.

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