# Onset of flux penetration into a type-I superconductor disk

A. V. Kuznetsov and D. V. Eremenko
*Department of Quantum Electronics, Moscow State Engineering Physics Institute, 115409 Moscow, Russia*

V. N. Trofimov
*Laboratory of Nuclear Problems, Joint Institute for Nuclear Research, 141980, Dubna, Moscow region, Russia*
(Received 5 September 1997)

Virgin magnetization of a superconducting disk in a transverse magnetic field is discussed. A field in which a metastable state arises in samples due to the geometrical barrier is calculated on the base of both the Landau theory of the intermediate state and the Hao-Clem description of the mixed state. It was found that this field agrees well with the penetration field measured for thin flat samples of type-I superconductors with weak pinning. [S0163-1829(98)02906-3]

## I. INTRODUCTION

It is well known that a uniform intermediate state is formed in an infinite plate of a type-I superconductor inserted in an uniform magnetic field $H$ applied normally to the plate.[1] The intermediate state consists of alternating normal and superconducting domains. The surface separating them, the so-called domain wall, is characterized by a positive energy described by the parameter $\Delta$ having dimension of length. The contribution of the energy of the domain walls (or surface energy) into the free energy of the intermediate state can be neglected when $\Delta$ is small in comparison with the plate thickness $d$, $\sqrt{\Delta/d}\ll 1$. In this case the plate magnetization linearly decreases with field:

$$m=H-H_c. \tag{1}$$

However, the surface energy affects the thermodynamic parameters of thin samples. For example, the field corresponding to the suppression of superconductivity in foils and films, is lower than the critical field $H_c$ of a bulk superconductor.[2]

Due to demagnetization the intermediate state arises in an infinite plate in an infinitesimally weak applied field, but this field is finite for samples of finite lateral size. The question of how the intermediate state is formed in such samples arises.

Let us consider the observed magnetization[2–13] taking as an example a disk-shaped sample with large diameter-to-thickness ratio $a$ which is called the aspect ratio. When the field increases, the sample corners come into the intermediate state in a very weak applied field due to a strong field enhancement at the corners. Being separated at the equator by the Meissner phase, the opposite normal domains in the corners expand with field.[4,6,11] This corresponds to a reversible magnetization[3,10,13] described by the dependence[3]

$$m=-2H/3(1-e)\equiv-\nu H, \tag{2}$$

$$\frac{1}{1-e}=\frac{(a^2-1)^{3/2}}{a^2\arctan\sqrt{a^2-1}-\sqrt{a^2-1}},$$

where $e$ is the demagnetization factor of an oblate spheroid with the axes ratio equal to $a$. In some field $H_p$, called the penetration field, part of the normal domains separate from the edge in the form of flux tubes, migrate towards the center, and accumulate there,[4,7] as shown in Fig. 1. Surrounded by the Meissner phase a region of the intermediate phase is formed in the inner part of the sample. If the field decreases, the Meissner phase prevents an exit of the normal domains from the inner part, therefore the magnetization becomes irreversible.[3,10,12] The inner region of the intermediate phase expands in increasing field. In some fields the magnetization again becomes reversible.[2,3,10,12]

The magnetization described above is determined by the following thermodynamic processes. In low field the main part of the sample is in the Meissner state and the intermediate phase occupies only the sample corners. In some fields the formation of an intermediate phase in the inner part of the sample becomes favorable from a thermodynamic point of view, but the transition into the intermediate state is impeded by a potential barrier arising at the sample edge.[9,11] A metastable state arises in the sample, so we call the corresponding field the metastability field $H_m$.[14] Such a barrier, called the geometrical barrier, is also observed in thin flat samples of a type-II superconductor with weak pinning.[15–18] The geometrical barrier prevents the penetration of the normal domains into the inner part of the sample when the field

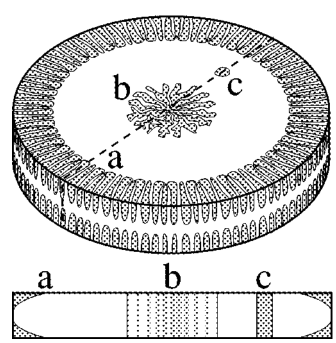


FIG. 1. Sketch of a flux distribution in a type-I superconductor disk. The bottom picture represents the cross section along the dashed line. Grey regions correspond to the normal domains and blank ones to the diamagnetic phase. (a) ''fringe'' of the normal domains at the edge, (b) central region of the intermediate state, (c) a domain tube migrating towards the center.

 

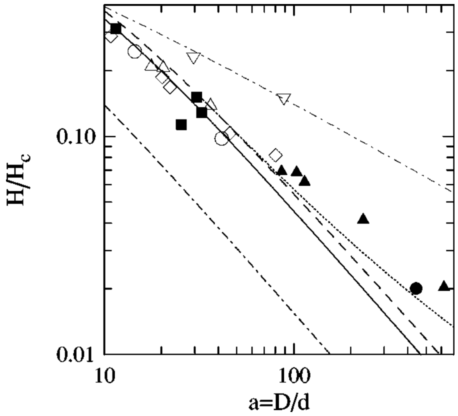


FIG. 2. Characteristic fields of a type-I flat superconductor vs the aspect ratio of sample. Open symbols are experimental data on the penetration field of bulk samples, closed symbols are those for films with $h_c \simeq 0.8$: △ ▲ (Ref. 3); ▽ (Ref. 4); ■ (Ref. 7); ◇ (Ref. 13); ○ ● (present work). Dash-dotted curves represent the $H_m(a)$ calculated by Fortini and Paumier (Ref. 11) (bottom) and $H_g(a)$ calculated by us (Ref. 18) (top). Other curves correspond to the metastability field calculated in the present work for bulk samples [solid: Eq. (3)] and films with $h_c \simeq 0.8$ [dashed: Eq. (9), dotted: Eq. (11)].

increases up to some field $H_g$ called the penetration field of the geometrical barrier.[19] Below $H_g$ a local field at the sample equator is lower than the critical field, therefore the normal domains cannot cross the equator.[11,18] Above $H_g$ the equatorial field reaches $H_c$. Flux tubes nucleating at the edge migrate towards the center since their energy is minimal there.[9,11] Formation of the intermediate phase in the central part of the sample gradually depresses the geometrical barrier and completely destroys it in some field called the irreversibility field of the geometrical barrier.[16,17]

It has been assumed that the metastable state arises when the energy of the Meissner state exceeds the minimal energy of state containing one flux tube in the inner part of the sample.[11] This energy is minimal when a flux tube is placed in the sample center. However, according to the fluxiod theorem, the normal domain cannot be spontaneously nucleated there. The flux tube should at first be nucleated at the edge and only then can it migrate towards an equilibrium position. Therefore, it has been concluded that flux penetration is impossible until in field $H_g$ the Meissner energy reaches an energy of state in which a flux tube is placed at the edge.[9,11] Nevertheless, if flux tubes can in some manner overcome the barrier, the metastable state decays. In principle, such a decay may start as soon as the metastability arises. Thus $H_m$ can be assumed as a lower limit for the penetration field and $H_g$ as an upper limit.

The metastability field $H_m \sim 1/a$ (Refs. 9 and 11) and the penetration field of the geometrical barrier $H_g \sim 1/\sqrt{a}$ (Refs. 9, 11, and 18) strongly differ for samples with a large aspect ratio. In Fig. 2, the dependences $H_m(a)$ and $H_g(a)$ are presented as well as experimental data on the penetration field tabulated by Andrew and Lock,[3] De Sorbo and Healy,[4] Huebener and co-workers,[7,20] and Kunchur and Poon.[13] As found in the above references, $H_p$ is more than $H_m$ but less than $H_g$. The latter means that flux tubes surmount the geometrical barrier. To clarify the reason of flux penetration, in this

TABLE I. Parameters of the lead samples at $T = 4.2$ K.

| $D$ mm | $d\,\mu$m | $a = D/d$ | $\mu_0 H_c$ mT | $\mu_0 H_p$ mT |
|---|---|---|---|---|
| $2.10 \pm 0.05$ | $145 \pm 5$ | $14.5 \pm 0.3$ | 53.0 | 13.0 |
| $2.08 \pm 0.05$ | $50 \pm 5$ | $42 \pm 4$ | 53.0 | 5.0 |
| $1.55 \pm 0.05$ | $3.5 \pm 0.5$ | $440 \pm 50$ | 43.0 [a] | 1.1 |

[a]Field of superconductivity suppression.

paper we investigate the magnetization of lead samples and recalculate $H_m(a)$ assuming that a uniform intermediate state is equilibrium in high field.

This paper is organized as follows. The penetration field is considered in Sec. II. Analyzing the magnetization we conclude that the metastable state arises when the energy of the Meissner state becomes equal to the energy of the uniform intermediate state. We calculate $H_m$ for bulk samples and films. In Sec. III we consider a flux tube nucleation in a field lower than $H_g$ and the influence of pinning on the penetration field. We also briefly discuss $H_p$ of a type-II superconductor disk. In Sec. IV we present a brief summary of our results. In the Appendix $H_m$ is calculated for a type-II superconductor disk.

## II. PENETRATION FIELD

### A. Disk magnetization

The measurements were performed using a home-built superconducting quantum interference device (SQUID) susceptometer.[21] The details of a low-field experimental procedure have been described elsewhere.[22] Disk-shaped samples were cut from a lead foil of 99.99% purity and etched in a water solution of nitric acid to remove surface contamination. The film was thermally deposited at $10^{-6}$ Torr pressure from the lead target of the same purity onto a glass substrate. The parameters of the samples are shown in Table I.

The magnetization curves of zero-field-cooled samples were measured at liquid helium temperature upon a step by step increase of the applied field. The magnetic moment $M$ and the remanence stored by a sample after a decrease of the applied field to zero, were registered for each $H$.

Let us consider the magnetization curves. In Fig. 3 they

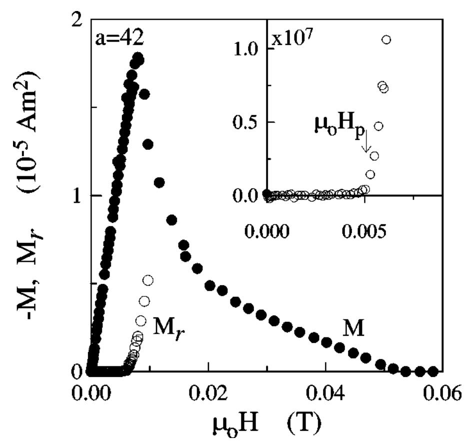


FIG. 3. Magnetization curves of a lead disk. The inset shows remanent moment $M_r$ with expanded scales. Arrow marks the penetration field.

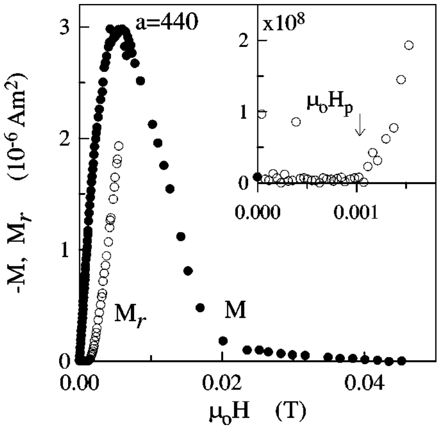


FIG. 4. Magnetization curves of a lead film. The inset shows remanent moment $M_r$, with expanded scales. Arrows mark the penetration field.

are shown for the disk with $a=42$. The $M(H)$ curve includes two linear parts separated by a sharp asymmetric maximum. Such a shape was observed earlier.[3,10,13] The low-field magnetization is well described by the dependence (2). Long before the magnetization maximum is reached, the flux begins to penetrate into the sample. In field $\mu_0 H_p = 5$ mT the slope of $M(H)$ starts to decrease, the magnetization becomes irreversible, and the remanence appears. In high field the magnetization becomes reversible again. The decrease of magnetization with field is described by the dependence (1). The measured field of superconductivity suppression is in good agreement with the value of the critical field of bulk lead at liquid helium temperature.[2]

Depicted in Fig. 4 the magnetization curve of the film is similar to that of the disk, but strong pinning leads to a prevalence of an irreversible central part on the curve. The virgin reversible part is described by the dependence (2) as well. The remanence appears in the field $\mu_0 H_p = 1.1$ mT which is five times less than the field corresponding to the magnetization maximum. In high field the magnetization is reversible again. As seen from Fig. 5, its decrease with field is close to linear. The measured field of superconductivity suppression is consistent with the experimental data on lead films.[2]

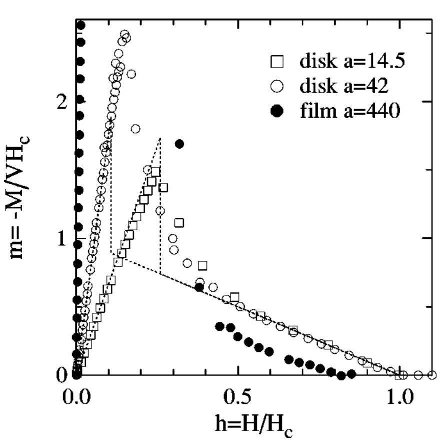


FIG. 5. Magnetization curves of lead disks and film. Magnetization and field unit is $\mu_0 H_c = 53$ mT. Dotted lines correspond to Eqs. (1), (2), and (3).

Figure 5 represents the magnetization of the sample with a relatively small aspect ratio $a=14.5$. The area under the magnetization curve is equal to the condensation energy with an accuracy of a few per cent. The linear increase of $M(H)$ is observed only for very weak field since an expansion of the intermediate phase in the sample corners results in a decrease of the slope of the magnetization curve with field. The magnetization is reversible before its maximum is reached. At $H > 0.5 H_c$ the magnetization linearly decreases with field.

When the geometrical barrier is absent, a phase transition into the intermediate state must occur in field $H_m$. Therefore $H_m$ can be obtained from an equilibrium magnetization curve at the condition that the area under the curve should be equal to the condensation energy density $\mu_0 H_c^2/2$. As follows from the results described above, the magnetization of our samples is not entirely equilibrium. The area under the magnetization curve for both the disk with $a=42$ and the film noticeably exceeds the condensation energy. The presence of the remanence indicates the presence of flux pinning. However, when the magnetization is reversible, the samples are in an equilibrium state. In low field this state corresponds to the Meissner phase occupying the whole sample except small regions of the intermediate phase in the corners. In high field, when the geometrical barrier is destroyed, it is the uniform intermediate state. One can expect that in the field range in which the geometrical barrier results in the metastable state, the uniform intermediate state is also equilibrium. Taking Eqs. (2) and (1) as an equilibrium magnetization, one obtains

$$H_m = 2H_c/(1+\nu). \tag{3}$$

Following from the expression (3), the dependence $H_m(a)$ presented in Fig. 2 is in good agreement with the experimental data on the penetration field of bulk samples but disagrees with that of films. To obtain $H_m$ for foils and films a contribution of the domain walls to the energy of the intermediate state should be accounted for.

### B. Surface energy and domain structure of foils and films

Consider the free energy density of a type-I superconductor disk taking the energy of the normal state as a reference. In the Meissner state it includes condensation energy $F_c = -\mu_0 H_c^2/2$ and the magnetic energy of shielding current $F_e = -\mu_0 \mathbf{mH}/2$. The formation of the intermediate phase in the disk corners leads to the correction of the energy on an addition of the order of $1/a$ which we neglect. Let us express energy in units of condensation energy $f = F/|F_c|$ and field in units of the critical field, $h = H/H_c$. Using expression (2) one can write for the energy of the Meissner state

$$f_M = -1 + \nu h^2. \tag{4}$$

In the uniform intermediate state the energy, accounting for the contribution due to expansion of the normal domains at the surface of a flat sample, can be written as[23]

$$f_i = -(1-h)^2 + 2(\Delta/p + p\psi_L/d), \tag{5}$$

where

$$p = \sqrt{d\Delta/\psi_L} \tag{6}$$

is a period of the domain structure and

$$4\pi\psi_L=[(1+h)^4\ln(1+h)+(1-h)^4\ln(1-h)-(1+h^2)^2\ln(1+h^2)-4h^2\ln(8h)], \quad (7a)$$

$$\psi_L\simeq[h^2\ln(0.56/h)]/\pi, \quad h\leqslant 0.2, \quad (7b)$$

$$\psi_L\simeq[\ln 2h(1-h)^2]/\pi, \quad h\geqslant 0.75, \quad (7c)$$

$$\psi_L\leqslant 0.023, \quad 0\leqslant h\leqslant 1 \quad (7d)$$

is the Landau function.[1,24]

Superconductivity is suppressed in field $h_c$ which for thin samples is less than the critical field, $h_c<1$. The superconductivity suppression is described by the equation $f_i(h_c)=0$ relating $h_c$ to the usually unknown ratio $\Delta/d$. Using Eqs. (5), (6), and (7c) one obtains

$$\Delta/d=(1-h_c)^4/16\psi_L(h_c), \quad (8)$$

$$\Delta/d\simeq\pi(1-h_c)^2/16\ln 2h_c, \quad h_c\geqslant 0.75.$$

A metastable state is formed when $f_M$ becomes more than $f_i$. If $f_M<f_i$ the Meissner state is stable. If $f_M>f_i$ the uniform intermediate state is favorable but cannot be achieved because of the geometrical barrier. The field $h_m$, corresponding to the arising of the metastable state, is calculated from $f_M=f_i$. Using Eqs. (4), (5), (6), and (8) one writes this equation as

$$(1+\nu)h_m^2-2h_m-(1-h_c)^2\sqrt{\psi_L(h_m)/\psi_L(h_c)}=0. \quad (9)$$

When the contribution of the surface energy is small in comparison with the condensation energy, $h_c$ is close to unity. Using the asymptotes (7c) for $\psi_L(h_c)$ and (7d) for $\psi_L(h_m)$, one estimates the third term in Eq. (9) to be $0.3(1-h_c)\ll 1$. Therefore, in the case of a bulk superconductor Eq. (9) is reduced to the expression (3).

Figure 2 represents dependence $h_m(a)$ calculated from Eq. (9) for $h_c=0.81$ measured for our film. The data on the penetration field for films with $\Delta/d\simeq 10^{-2}$ (corresponding to this $h_c$) are also shown for comparison. As seen, the calculated $h_m(a)$ is consistent with the experimental data for $a<100$ but is less than those for $a>100$. The reason is a small value of field.

As follows from Eqs. (6) and (7b), in a weak field the ratio $d/p$ is proportional to $h$ with logarithmic accuracy. Thus the period of the domain structure should be greater than the film thickness when $h\ll 1$. At the same time, the width of the superconducting domains $p(1-h)$ is of the order of $p$. Hence, the width of the superconducting domain should be greater than its thickness. This is impossible because of a strong surface tension of the domain walls.[1] In a weak field the width of the superconducting domain should be of the order of its thickness, therefore one can estimate the period as

$$p\simeq d/(1-h). \quad (10)$$

The period described by Eq. (10) increases with field due to the expansion of the normal domains. In some field $\hat{h}$ it reaches the value given by the expression (6). Comparing the right-hand sides of Eqs. (10) and (6) and using Eq. (8) one easily calculates this field from the equation

$$4\sqrt{\psi_L(h_c)\psi_L(\hat{h})}-(1-h_c)^2(1-\hat{h})=0.$$

When $h>\hat{h}$ expression (6) describes the field dependence of the period since the width of the superconducting domain is smaller than the film thickness. When $h<\hat{h}$ the equation for $h_m$ following from Eqs. (4), (5), (8), and (10) can be written as

$$(1+\nu)h_m^2-2h_m-\frac{2\psi_L(h_m)}{1-h_m}-\frac{(1-h_c)^4(1-h_m)}{8\psi_L(h_c)}\simeq 0. \quad (11)$$

For $h_c>0.75$ and small $h_m$, the following solution can be obtained using asymptotes (7b) and (7c):

$$h_m\simeq\frac{1}{1+\nu}\left[1+\sqrt{1+\frac{\pi(1+\nu)}{8\ln 2}\,\frac{(1-h_c)^2}{h_c}}\right].$$

For a large aspect ratio, $a\geqslant 6\ln 2h_c/(1-h_c)^2$, one can see from this solution that the metastability field is inversely proportional to the square root of the aspect ratio:

$$h_m\simeq(\pi/4)(1-h_c)\sqrt{3/2\ln 2ah_c}.$$

The dotted curve plotted in Fig. 2 represents $h_m(a)$ calculated from Eq. (11) at $h_c=0.81$ corresponding to $\hat{h}=0.15$. The calculated field agrees well with our experimental data on the penetration field but it is slightly less than that by Andrew and Lock.[3] We suppose that these data, obtained as a field corresponding to the kink of the magnetization curve, overestimate the penetration field because some remanence (and, consequently, pinning) was observed in samples. For our samples with nonzero pinning the field in which maximal magnetization is reached exceeds $H_p$.

The calculations are restricted by a minimal film thickness[2] $d_c\simeq\Delta/(1-2\kappa)^2$, where $\kappa$ is the Ginzburg-Landau parameter. In thinner films the vortical state arises instead of the intermediate one.[25]

## III. DISCUSSION

As follows from results obtained in the previous section the normal domains surmount the geometrical barrier, i.e., flux tubes nucleate at the edge in an applied field lower than $H_g$. To nucleate a flux tube, two opposite normal domains in the sample corners must come into contact. When $H<H_g$, the opposite domains are separated by the diamagnetic phase at the equator as shown in Fig. 1. Let us consider in what manner a flux tube is formed in this case.

We neglected the contribution of the normal domains to the energy of the Meissner state. This contribution consists of an addition to the condensation energy due to the suppression of superconductivity inside the domains, the energy of magnetic field inside them, and the energy of the domain walls. One can write the energy per domain as $\varepsilon=\mu_0H_c^2(v+s\Delta/2)$, where $v$ is the domain volume and $s$ is the area of the domain wall. For a fixed sum $v+s\Delta/2$ the energy $\varepsilon$ is independent of domain shape. Therefore an equilibrium shape, determined by a local field at the edge, can be changed due to thermal fluctuations.

The change of the radial length of the domains leads to a change of the shielding current distribution[18] and the mag-

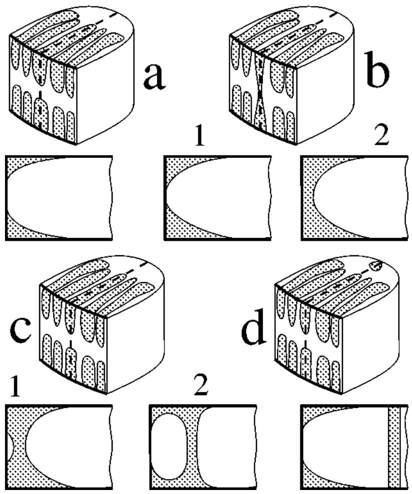


FIG. 6. Sketch of a flux tube nucleation. Part of a sample at the edge is depicted in the top pictures, the bottom ones present cross sections along the dashed line. Grey regions correspond to the normal domains and blank ones to the diamagnetic phase. (a) The opposite normal domains are separated by the diamagnetic phase; (b) two domains coming into contact (1) begin to spread towards the inner part of the sample (2); (c) the diamagnetic phase restoring at the equator (1) expands towards the inner part (2); a domain tube separates from the normal domains at the edge.

netic energy related to this current. Hence the radial length of the domains cannot vary due to fluctuations but their width and thickness vary as illustrated in Fig. 6. Some opposite domains can come into contact due to the increase of their thickness at the equator. If $H \geqslant H_m$ the intermediate phase is favorable in the inner part of the sample, therefore contacted domains begin to spread from the edge. That lowers the equatorial field below $H_c$ so the diamagnetic phase restores at the equator locking up a part of the normal domain. The locked flux, interacting with the shielding current via the Lorentz force, separates from the edge in the form of a flux tube and migrates towards the inner part.

The pinning of the domain walls depresses fluctuations of the normal domain shape. Therefore strong pinning, leading to a complete suppression of the fluctuations, should result in an increase of the penetration field up to $H_g$. Indeed, the data presented in Fig. 2 by DeSorbo and Healy[4] on $H_p$ of strongly defective samples punched from a cold-rolled tin sheet are of the order of $H_g$. Thus we conclude that the penetration field of a thin flat type-I superconductor varies in the range $H_m \leqslant H_p \leqslant H_g$ with pinning strength.

The magnetization described by Eqs. (1)–(3) changes at $\delta m = H_c$ at the metastability field (see Fig. 5). Since the penetration of each flux tube in fixed applied field decreases a local field at the edge,[11] the probability of further nucleation of flux tubes lowers. The more flux that enters the sample the lower the probability. Both this negative feedback and the pinning of flux tubes depress the jumpwise decrease of the magnetization when the flux begins to enter the sample. To obtain the magnetization in the field range above $H_m$ one should calculate the activation energy of the geometrical barrier, as was done, for instance, for the Bean-Livingston barrier.[26] This is beyond the scope of the present work.

In conclusion we briefly discuss flux penetration into a thin flat type-II superconductor. In the Appendix we calculated the metastability field for a disk-shaped superconductor with $\kappa \gg 1$. The following equation has been obtained:

$$(\nu - 1) h_m + [\ln(1 + \beta h_m)]/\beta - 2 = 0, \qquad (12)$$

where $\beta = \ln\kappa + 0.519$ and the field is taken in units of the lower critical field $h_m = H_m / H_{c1}$.

To nucleate a vortex which is able to migrate into the inner part of the sample in the field range $H_m \leqslant H \leqslant H_g$, an energy $\delta G \simeq \varepsilon_0 \delta l$ is required. Here $\varepsilon_0 = (\ln\kappa/\mu_0 4\pi)(\phi_0/\lambda)^2$ is the vortex line tension, $\lambda$ is the penetration depth, and $\phi_0$ is the flux quantum. A distance between the opposite regions of the mixed state in the sample corners $\delta l$ decreases with applied field down to a zero value at $H = H_g$. In the vicinity of $H_g$ when $\delta l \simeq \lambda$ one estimates $\delta G \sim 10^5 - 10^6$ K for $\lambda \simeq 1000 - 5000$ Å and $\kappa \simeq 50 - 100$. Thus the surmounting of the geometrical barrier due to thermal fluctuations can be neglected. Field $H_g$ corresponds to the penetration field.[18]

Note that, similarly to the Bean-Livingston barrier,[26] one may expect the avalanche-type[27] surmounting of the geometrical barrier by pancake vortices in a layered superconductor. To calculate the probability of this process is the subject of further study.

## IV. SUMMARY

In this paper we have considered the virgin magnetization of a thin flat type-I superconductor in a magnetic field applied normally to the sample plane. The following phenomena affect the magnetization. Due to the demagnetizing effect the magnetic energy related to the shielding current grows rapidly with field. A superconductor tends to lower the energy by means of a phase transition from the Meissner state into the uniform intermediate state. However, such a transition is prevented by the geometrical barrier. Therefore a metastable state is formed. In the case of weak pinning this barrier is surmounted by domain tubes as soon as the metastability arises. When pinning is strong, the flux enters the sample at the penetration field of the geometrical barrier. This field also corresponds to an onset of flux penetration into a thin flat type-II superconductor.

In conclusion we point out the main results obtained in the present work.

(1) A simple model based on the Landau theory of intermediate state has been developed to calculate the metastability field for thin flat samples of bulk superconductors and films.

(2) The magnetization of lead disks and films has been investigated in a wide range of the aspect ratio. A good agreement between the measured penetration field and the calculated metastability field has been found.

(3) On the basis of the mixed state description by Hao and Clem[28] the metastability field has been calculated for a type-II superconductor disk.

## ACKNOWLEDGMENTS

The authors are indebted to Professor V. A. Kashurnikov, Dr. A. A. Sinchenko, Dr. A. A. Ivanov, and K. V. Klementev for helpful discussions. We are grateful to Professor A. P. Menushenkov for his support of the present investigation.

## APPENDIX

Consider a disk-shaped pinning-free type-II superconductor with $\kappa \gg 1$ and $a \gg 1$. The magnetic field is applied normally to the disk plane. In low field the disk is in the Meissner state except for small regions in the corners which contribute to the free energy we neglect. The geometrical barrier causes the metastable state of the sample when energies of the Meissner and the uniform mixed state $f_{\rm mix}$ become equal. Thus the corresponding field $H_m$ can be obtained from the equation $f_M = f_{\rm mix}$.

$f_{\rm mix}$ was calculated by Hao and Clem.[28] We keep their original notation and units. The numerated equations from Ref. 28 are referred to as Eq. $\{N\}$. The energy is expressed in units $\mu_0 H_c^2$ and the magnetic field in units $\sqrt{2}H_c$ where $H_c$ is the thermodynamic critical field. In these units one rewrites Eq. (4) as

$$f_M = -1/2 + \nu \mathsf{h}^2, \tag{A1}$$

where $\mathsf{h} = H/\sqrt{2}H_c$. The energy of the mixed state,

$$f_{\rm mix} = -1/2 + F_c + F_{\rm em} + F_{kg}, \tag{A2}$$

includes the condensation energy [the first term on the right-hand side of Eq. (A2)],[29] an addition to the condensation energy related to the arising of vortices $F_c$, the electromagnetic energy $F_{\rm em}$, and the kinetic energy $F_{kg}$. All these terms depend on $\kappa$, the averaged flux density inside a superconductor $B$, the radius of the vortex core $\xi_v$, and parameter $f_\infty$, which represents the depression of the order parameter due to overlapping of vortices.

Because of demagnetization $H_m$ is less than the lower critical field and much less than the thermodynamic critical field $\mathsf{h}_m < 1/\kappa \ll 1$. Due to the continuity of the normal component of the field on the disk flat surface one has the sample $B = \mathsf{h}$ inside. Therefore in the vicinity of $H_m$ one can expand $f_{\rm mix}$ in powers of $B$. Taking only the first terms, which are proportional to $B^2$, $B/\kappa$, and $1/\kappa^2$ and substituting consistent with this accuracy $f_\infty \simeq 1$ and $\xi_v \simeq \xi_{v0} \simeq \sqrt{2}/\kappa$ (Eqs. {24} and {25}, respectively), one calculates $F_{\rm em} \simeq B^2 + (B/\kappa)[\ln(\kappa/\sqrt{1/2 + B\kappa}) - \gamma]$ from Eq. {14},[30] $F_c \simeq B/2\kappa$ from Eq. {15}, and $F_{kg} \simeq B/4\kappa$ from Eq. {16}, where $\gamma = 0.57721\cdots$ is the Euler constant.

Comparing the right-hand sides of Eqs. (A1) and (A2) at $B = \mathsf{h}$, we obtain by elementary calculus the equation for the metastability field:

$$(\nu - 1)\kappa \mathsf{h}_m + \ln\sqrt{1 + 2\kappa \mathsf{h}_m} - \beta = 0, \tag{A3}$$

where $\beta = \ln(\sqrt{2}\kappa) + 3/4 - \gamma \simeq \ln\kappa + 0.519$.

It is more convenient to use the lower critical field $H_{c1}$ as a field unit. Equation {22},

$$\frac{H_{c1}}{\sqrt{2}H_c} = \left( \frac{\kappa \xi_{v0}^2}{8} + \frac{1}{8\kappa} + \frac{K_0(\xi_{v0})}{2\kappa \xi_{v0} K_1(\xi_{v0})} \right) \simeq \frac{\beta}{2\kappa},$$

allows one to change the normalizing. $K_0$ and $K_1$ are the modified Bessel's functions of zero and first order.[30,31] For $h_m = 2\mathsf{h}_m \kappa/\beta$ Eq. (A3) can be rewritten as

$$(\nu - 1)h_m + [\ln(1 + \beta h_m)]/\beta - 2 = 0. \tag{A4}$$


[1] L. D. Landau and E. M. Lifshits, *Electrodynamics of Continuous Media* (Pergamon, London, 1982), Vol. VI.57.

[2] G. D. Gody and R. E. Miller, Phys. Rev. Lett. **16**, 697 (1966).

[3] E. R. Andrew and J. M. Lock, Proc. Phys. Soc. London, Sect. A **63**, 13 (1950).

[4] W. De Sorbo and W. A. Healy, Cryogenics **4**, 257 (1964).

[5] D. C. Baird, Can. J. Phys. **42**, 1682 (1964); D. C. Baird and L. S. Wright, J. Low Temp. Phys. **8**, 177 (1972).

[6] R. Olafsson and J. R. Allen, J. Phys. F **2**, 123 (1972).

[7] R. P. Huebener, R. T. Kampwirth, and J. R. Clem, J. Low Temp. Phys. **6**, 275 (1972).

[8] H. Kirchner and A. Kiendl, Phys. Lett. **39A**, 293 (1972).

[9] J. R. Clem, R. P. Huebener, and D. E. Gallus, J. Low Temp. Phys. **12**, 449 (1973).

[10] J. Provost, E. Paumier, and A. Fortini, J. Phys. F **4**, 439 (1974); A. Fortini, E. Paumier, J. Provost, and J. -P. Girard, J. Phys. (Paris) **8**, 427 (1973).

[11] A. Fortini and E. Paumier, Phys. Rev. B **14**, 55 (1976); A. Fortini, A. Hairie, and E. Paumier, *ibid.* **21**, 5065 (1980); A. Fortini, A. Hairie, and J.-P. Girard, J. Math. Phys. **20**, 2139 (1979).

[12] A. K. Grover, Ravi Kumar, P. Chaddah, C. K. Subramanian, and V. Sankaranaraynan, Physica C **170**, 431 (1990).

[13] M. N. Kunchur and S. J. Poon, Phys. Rev. B **43**, 2916 (1991).


[14] This field was named as the thermodynamic transition field by Fortini and Paumier (Ref. 11) while Clem *et al.* (Ref. 9) designated it as $H_{c1}^*$. Since, on one hand, a transition into an equilibrium state does not occur at this field and, on the other hand, the only critical field $H_c$ is inherent in a type-I superconductor, we renamed this field to avoid confusion.


[15] M. V. Indenbom, H. Kronmüller, T. W. Li, P. H. Kes, and A. A. Menovsky, Physica C **222**, 203 (1994).

[16] E. Zeldov, A. I. Larkin, V. B. Geshkenbein, M. Konczykowski, D. Majer, B. Khaykovich, V. M. Vinokur, and H. Shtrikman, Phys. Rev. Lett. **73**, 1428 (1994).

[17] M. Benkraouda and J. R. Clem, Phys. Rev. B **53**, 5716 (1996).

[18] A. V. Kuznetsov, D. V. Eremenko, and V. N. Trofimov, Phys. Rev. B **56**, 9064 (1997).


[19] The present name is commonly used now (Refs. 16–18). This field was called the migration threshold by Fortini and Paumier (Ref. 11) and the critical entry field by Clem *et al.* (Ref. 9).

[20] An effective aspect ratio of rectangular samples was determined in a way given in Ref. 18.


[21] V. N. Trofimov, Cryogenics **32**, 513 (1992).

[22] A. V. Kuznetsov, A. A. Ivanov, D. V. Eremenko, and V. Trofimov, Phys. Rev. B **52**, 9637 (1995).

[23] A. Fortini and E. Paumier, Phys. Rev. B **5**, 1850 (1972).


[24] The accuracy of Eqs. (7a) and (7b) is better than 2.5 and 6.4 %, respectively.


[25] M. Tinkham, Phys. Rev. **129**, 2413 (1963).

[26] B. V. Petukhov and V. R. Chechetkin, Zh. Eksp. Teor. Fiz. **65**, 1653 (1973) [Sov. Phys. JETP **38**, 827 (1974)]; L. Burlachkov, V. B. Geshkenbein, A. E. Koshelev, A. I. Larkin, and V. M.

Vinokur, Phys. Rev. B **50**, 16 770 (1994).

[27] R. G. Mints and I. B. Snapiro, Phys. Rev. B **47**, 3273 (1993).

[28] Z. Hao, J. R. Clem, M. W. McElfresh, L. Civale, A. P. Malozemoff, and F. Holtzberg, Phys. Rev. B **43**, 2844 (1991).

[29] $f_{\rm mix}$ was considered by Hao *et al.* (Ref. 28) with respect to energy of the Meissner state including the condensation energy. Since we measure the energy with respect to the normal state, we add the condensation energy to $f_{\rm mix}$.

[30] The expression for the electromagnetic energy $F_{\rm em} \simeq B^2 + BK_0(\sqrt{2+4B\kappa}/\kappa)/\sqrt{2}K_1(\sqrt{2}/\kappa)$, following from Eq. (14), contains the modified Bessel's functions of zero and first order. Their asymptotes at small argument (Ref. 31) $K_0 \simeq (1+x^2/4) \times [1-\gamma+\ln(2/x)]-1$, and $2xK_1 \simeq 2-x^2[1-\gamma+\ln(2/x)]$, have been used to expand $F_{\rm em}$.

[31] *Handbook of Mathematical Functions With Formulas, Graphs and Mathematical Tables*, edited by M. Abramowitz and I. A. Stegun (National Bureau of Standards, U.S. GPO, Washington, 1964), pp. 9.6.13, 9.6.11.