Nonlinear generalization of the Bardeen-Stephen model and the Hall angle anomaly

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The two classical models for the moving vortex, one suggested by Bardeen and Stephen (BS) and the other by Noziéres and Vinen (NV), are incompatible regarding the nature of normal core current upon which the dissipation characteristics depend. In order to resolve this ambiguity, we generalize the BS model by including the nonlinear convection term. This nonlinear generalization leads to an additional electric field inside and outside the core which was not accounted for in the BS model. The electric force field in the core is found to be identical to the NV model. Using this field, we determine the normal core current and the rate of energy dissipation in the core. The rate of energy dissipation allows us to determine the drag coefficients proposed by Hagen *et al.* From these coefficients, we demonstrate the appearance of the negative Hall angle if the effective pinning force is small compared to the Lorentz force. [S0163-1829(98)03609-1]

I. INTRODUCTION

A correct account of the normal core current in a moving vortex core is essential for understanding vortex dynamics and the nature of energy dissipation. Two of the widely studied classical models for the dissipative vortex dynamics are one developed by Bardeen and Stephen[1] (BS) and the other by Noziéres and Vinen[2] (NV). In both models, the normal core current is determined from the force balance equation between the driving force which acts on the bulk of the vortex core and the dissipative force which results in energy dissipation to the crystal lattice. However, these models showed disagreeable characteristics, demanding us a first-principles-type investigation.

In the BS model [Eq. (4.4) in Ref. 1], the force balance equation for the normal region of the isolated vortex can be written as

$$-\frac{1}{2}\frac{ne}{c}\mathbf{v}_L\times\boldsymbol{\phi}-\frac{nm}{\tau}\pi a^2\mathbf{v}_c+O\left(\frac{H}{H_{c2}}\right)=0, \qquad (1)$$

where n is the charge density, a is the size of the core (of order ξ, the coherent length), and m, e, and τ are the mass of charge carrier, charge, and electron-lattice collision time, respectively. $\mathbf{v}_c$ and $\mathbf{v}_L$ are the uniform velocities of the normal charge carriers and the vortex, respectively. $\boldsymbol{\phi}$ is the flux quantum $(hc/2e)$ with its direction parallel to the vortex line. The first and second terms describe the force balance in connection with the motion of the vortex and the normal charge carriers in the core accompanying momentum dissipation. As shown in Eqs. (3.8) and (3.11) of Ref. 1, the third term represents the higher-order correction due to the presence of the magnetic field in the core. According to the BS model, the electric force field inside the core is proportional to $\mathbf{v}_L\times\boldsymbol{\phi}$ and the normal core current is parallel (within the Hall angle) to the transport current. On the other hand, Nozières and Vinen found that half of the Magnus force $[(ne/c)(\mathbf{v}_{s1}-\mathbf{v}_L)\times\boldsymbol{\phi}]$ drives the normal charge carriers through the core against the dissipative force from the crystal lattice. In the NV model[2] the force balance equation for the normal region of an isolated vortex as a consequence of $H\ll H_{c2}$ is

$$\frac{1}{2}\frac{ne}{c}(\mathbf{v}_{s1}-\mathbf{v}_L)\times\boldsymbol{\phi}-\frac{nm}{\tau}\pi a^2\mathbf{v}_c+O\left(\frac{H}{H_{c2}}\right)=0, \qquad (2)$$

with $\mathbf{v}_c=\mathbf{v}_{s1}$ for a homogeneous material. Here $\mathbf{v}_{s1}$ is the local superfluid velocity field that can be set as $\mathbf{v}_{s1}=\mathbf{v}_T$ for the applied uniform current $\mathbf{J}=ne\mathbf{v}_T$. In Eq. (49) of Ref. 2, $\mathbf{v}_c$ of Eq. (2) is taken to be equal to $\mathbf{v}_{s1}$. Accordingly the normal core current ($\mathbf{v}_c$) of the NV model has an additional component proportional to $\frac{1}{2}(ne/c)\,\mathbf{v}_T\times\boldsymbol{\phi}$, which is parallel to $\mathbf{v}_L$ if the Hall angle is negligible.

Here we focus on the force field generated outside the core of a moving vortex in order to explain differences in the normal core currents of the two models mentioned above. According to the BS model, the force field around a moving vortex core is[1]

$$\mathbf{f}(\mathbf{r})=m\frac{\partial\mathbf{v}_s}{\partial t}. \qquad (3)$$

As well summarized in Ref. 3, the force field above is based on the local London theory where the local electric field is defined by the partial time derivative of $(m/n_se^2)\,\mathbf{J}_s$, where n_s is the charge density of superelectrons and $\mathbf{J}_s$ is the density of supercurrent. But the contribution to the electric field from the nonlinear convective derivative (NCD) $\mathbf{v}_s\cdot\vec{\nabla}\mathbf{v}_s$ is absent in Eq. (3) as noticed by Ref. 4 (p. 84). The NCD may be negligible if $\mathbf{v}_s$ is small or uniform. However, if we consider the dynamics of the superfluid around the vortex core, $\mathbf{v}_s(\mathbf{r})$ is neither small nor uniform (with the values of 10^6-10^7 cm/sec near the core). Thus, in order to fully account for the contribution of the NCD to the electric field and the dynamics of the superfluid, we use the method developed by BS (Ref. 1) and Vijfeijken and Niessen.[5] Using the hydrodynamic result on the flow pattern of the incompressible fluid, we express the electric field in terms of the superfluid velocity field around the moving vortex core. From this approach, we find that there exists an additional electric field originated from the NCD in addition to the dipolar force field shown in the BS model.

By using this electric field together with a proper boundary condition for the interface between inside and outside the core, we determine the electric field in the normal core of a moving vortex. The electric field is found to be identical to that of NV (Refs. 2 and 6) even if our approach differs from the NV model which is based on Bernoulli's theorem. We determine the normal core current from the force balance equation between the electric field and other necessary dissipative forces including the effective pinning force. In addition to the normal core current of BS parallel to the external current ($\mathbf{J}_T$), an additional component perpendicular to $\mathbf{J}_T$ is found to exist. We also determine the rate of energy dissipation (W_{diss}) by assuming that the normal core current is the primary source of energy and momentum dissipation related to the motion of the vortex. W_{diss} is found to depend not only on $\mathbf{v}_L$ as in the BS model but also on $\mathbf{v}_T$ and the effective pinning force.

Since the predicted energy dissipation rate differs from that of the BS model, the drag force is expected to be different. Hagen *et al.*[7,8] presented a conjecture regarding the proper form of the drag force acting on a unit length of a moving vortex. They proposed that the Hall angle anomaly can be explained if the drag force is expressed by

$$\mathbf{f}_{\mathrm{drag}} = -\eta \mathbf{v}_L - a\mathbf{v}_T . \tag{4}$$

Using our expression for W_{diss}, we determine the drag coefficients η and a. By using these results, we can explain qualitatively the Hall angle anomaly from the force balance equation on a single vortex. We find that the effective pinning force and the Hall angle anomaly are closely related. This finding differs from the theory of Wang and Ting.[9,10] According to our result, the negative Hall angle appears only if the effective pinning force is much smaller than the Lorentz force.

II. NONLINEAR GENERALIZATION OF THE BS MODEL

In order to describe the dissipation associated with the vortex motion, we need to determine the force field generated around a moving vortex. According to the two-fluid model proposed by Landau,[11–13] this force field is related to the motion of the superfluid and the normal fluid.[14] When the temperature is not low and the normal fluid density is not small, the normal fluid may strongly interact with the superfluid. In principle, we need to solve the complicated coupled equations of motion for both fluids simultaneously. However, at very low temperatures $[T \ll T_c,\ \rho_n(T) \simeq 0]$, we can assume that only the dissipationless superfluid exists outside the normal core. All the dissipation is attributed to the normal core where excited quasiparticles are confined. The classical models for the vortex dynamics by NV and BS take this simplification in the low-temperature limit. We also determine the force field governing the motion of superfluid in the low-temperature limit.

The equation of motion for the charged superfluid is

$$\frac{d\mathbf{v}_s}{dt} = -\vec{\nabla}\mu_0 + \frac{e}{m}\left[\mathbf{E} - \frac{1}{c}(\mathbf{v}_s \times \mathbf{H})\right]. \tag{5}$$

Here μ_0 is the chemical potential per unit mass in the absence of currents and fields and is set to be a constant[15] in this work. Using the identity

$$\frac{d\mathbf{v}_s}{dt} = \frac{\partial \mathbf{v}_s}{\partial t} + \mathbf{v}_s \cdot \vec{\nabla}\mathbf{v}_s = \frac{\partial \mathbf{v}_s}{\partial t} + \vec{\nabla}\frac{v_s^2}{2} - \mathbf{v}_s \times \vec{\nabla} \times \mathbf{v}_s \tag{6}$$

and the London equation, Eq. (5) can be rewritten as

$$\frac{\partial \mathbf{v}_s}{\partial t} + \vec{\nabla}\left[\frac{v_s^2}{2}\right] = \frac{e}{m}\mathbf{E}. \tag{7}$$

This equation can also be derived from the Navier-Stokes equations describing the dynamics of two fluids by taking the low-temperature limit $[\rho_n(T \simeq 0) = 0]$.[16] In the BS model the linear London equation $\partial \mathbf{v}_s/\partial t = (e/m)\mathbf{E}$ (Ref. 3) is used. Thus the effect of the NCD is neglected in association with the electric field, even if $\mathbf{v}_s(\mathbf{r})$ around a vortex is not uniform in general. On the other hand, recently researchers[17] have paid more attention to the effect of the NCD for various problems of vortex dynamics.

We solve this equation by a perturbation method similar to that used by BS (Ref. 1) and Vijfeijken and Niessen.[5] We divide Eq. (7) into a contribution for a stationary vortex (represented by the subindex 0) and a leading-order correction due to the vortex motion and the transport current (represented by the subindex 1),

$$\mathbf{v}_s = \mathbf{v}_{s0} + \mathbf{v}_{s1}, \quad \mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1 . \tag{8}$$

Here, $\mathbf{v}_{s0}$ and $\mathbf{E}_0$ denote the velocity field of the circulating diamagnetic current and the electric field, respectively, of an isolated stationary vortex. $\mathbf{v}_{s1}$ and $\mathbf{E}_1$ stand for the leading corrections due to the vortex motion. Accordingly we can rewrite Eq. (7) as

$$\frac{\partial \mathbf{v}_{s0}}{\partial t} + \frac{\partial \mathbf{v}_{s1}}{\partial t} + \vec{\nabla}\left(\frac{v_{s0}^2}{2} + \mathbf{v}_{s0}\cdot\mathbf{v}_{s1}\right) + O(v_{s1}^2) = \frac{e}{m}(\mathbf{E}_0 + \mathbf{E}_1). \tag{9}$$

This can be separated into two equations, a zeroth-order equation for the stationary vortex and another for the leading-order correction terms.

The zeroth-order equation describing the relation between the superfluid flow pattern and the local electric field in the stationary vortex is

$$\frac{\partial \mathbf{v}_{s0}}{\partial t} + \vec{\nabla}\left(\frac{v_{s0}^2}{2}\right) = \frac{e}{m}\mathbf{E}_0 . \tag{10}$$

From the fact that $\mathbf{v}_{s0}$ is constant in time, we obtain

$$e\mathbf{E}_0 = \frac{m}{2}\vec{\nabla}v_{s0}^2 . \tag{11}$$

A result similar to this, only different by the factor n_s/n on the right-hand side, was derived by Vijfeijken and Stass[15] who used Navier-Stokes equations.

The equation for the leading order correction is

$$\frac{\partial \mathbf{v}_{s1}}{\partial t} + \vec{\nabla}(\mathbf{v}_{s0}\cdot\mathbf{v}_{s1}) = \frac{e}{m}\mathbf{E}_1 . \tag{12}$$

TABLE I. Estimations of V_T and V_{Ly}.

Sample T_c	J_T (A/cm^2) V_T (cm/sec)	ρ_{xx} ($\mu\Omega$ cm) V_{Ly} (cm/sec)	Reference
$Tl_2Ba_2CaCu_2O_8$	$\sim 10^3$	~ 0.1 at $T=80-95$ K	
104 K	~ 10	1	7
$YBa_2Cu_2O_7$	10^2-10^4	~ 10 at $T=86$ K	
89 K	1–100	$10-10^3$	8
2H-$NbSe_2$	$\sim 10^2$	~ 1 at $T=4.2$ K	
7.2 K	~ 1	1	19
$Tl_2Ba_2Ca_2Cu_3O_{10}$	~ 10	~ 10 at $T=100$ K	
107 K	~ 0.1	1	26
$YBa_2Cu_3O_{7-\delta}$	$\sim 10^3$	~ 10 at $T=85$ K	
93 K	~ 10	100	27

This equation provides a relationship between the electric field $\mathbf{E}_1$ and the superfluid flow pattern as a result of the leading-order correction. From the information on $\mathbf{v}_{s1}$, this equation determines $\mathbf{E}_1$. The classical models[1,2] treat $\mathbf{v}_{s1}$ by using the well-known hydrodynamical result for the flow pattern of the incompressible fluid outside the rigid cylindrical core of radius a. According to these models, we write the total velocity field of a vortex in the vortex frame as

$$\mathbf{v}_{s0}^{\mathrm{tot}}=\mathbf{v}_{s0}+\mathbf{v}_T+\mathbf{v}_{B0}, \qquad \mathbf{v}_{B0}=\vec{\nabla}\left[(\mathbf{v}_T-\mathbf{v}_{c0})\cdot\frac{a^2}{r^2}\mathbf{r}\right], \tag{13}$$

where $\mathbf{v}_{B0}$ and $\mathbf{v}_{c0}$ represent the velocity fields of the backflow and the normal core current in the vortex frame, respectively. At this point, $\mathbf{v}_{B0}$ and $\mathbf{v}_{c0}$ are to be determined. The superfluid velocity field in the laboratory frame can be obtained from $\mathbf{v}_{s0}^{\mathrm{tot}}$ and the principle of Galilean invariance as

$$\mathbf{v}_s(\mathbf{r},t)=\mathbf{v}_{s0}^{\mathrm{tot}}(\mathbf{r}-\mathbf{v}_L t). \tag{14}$$

From this, we write $\mathbf{v}_{s1}$ as

$$\mathbf{v}_{s1}=\mathbf{v}_T+\mathbf{v}_{B0}-\mathbf{v}_L t\cdot\vec{\nabla}\mathbf{v}_{s0}+O[(v_L t)^2,v_L v_T,v_L v_{c0}]. \tag{15}$$

This expression is meaningful only when the expansion parameter of $\mathbf{v}_s^{\mathrm{tot}}$ is small, that is, $v_L t \ll \xi$ as in the BS model.

Using Eq. (15), the expression (12) can be rewritten as

$$-\mathbf{v}_L\cdot\vec{\nabla}\mathbf{v}_{s0}+\vec{\nabla}(\mathbf{v}_{s0}\cdot\mathbf{v}_{s1})=\frac{e}{m}\mathbf{E}_1. \tag{16}$$

This equation shows that $(e/m)\,\mathbf{E}_1$ arises from two contributions. The first term on the left-hand side is identical to the dipolar force field of the BS model.[1] On the other hand, the second term, originating from the NCD and absent in the BS model, allows the modification of the electric field if the superfluid flow pattern changes from $\mathbf{v}_{s0}$ to $\mathbf{v}_{s0}+\mathbf{v}_{s1}$. Realizing from Eq. (4.14) of Ref. 1, which can be rewritten as $v_T=\omega_{c2}\tau v_L$ (ω_{c2} is the cyclotron frequency at the upper critical field H_{c2}), together with the fact that $\omega_{c2}\tau \simeq 0.1$–$0.01$ for most of type-II superconductors except in the clean limit, we notice the fact that BS considered only the case with $v_L \gg v_T$. However, as shown in Table I, from estimations for v_T and v_L in some experimentally observed cases, we find that v_T is not always negligible compared with v_L. In such cases, the physical effect of $\vec{\nabla}(\mathbf{v}_{s0}\cdot\mathbf{v}_{s1})$ needs to be properly taken into account by using Eq. (16) above.

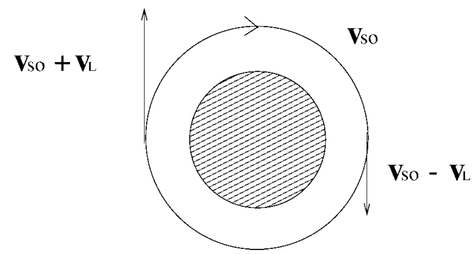

FIG. 1. The schematic description of the origin for BS's force field.

The physical origins of the two terms on the left-hand side of Eq. (16) can be understood by considering the isotropy of the superfluid's kinetic energy. The kinetic energy of the superfluid changes from the isotropic distribution $[\propto v_{s0}^2(r)]$ for a stationary vortex to an anisotropic one for a moving vortex. The motion of a vortex makes the kinetic energy of the superfluid anisotropic as schematically described in Fig. 1. It results from the first term of Eq. (16) above. As shown in Fig. 2, the change of the velocity field ($\mathbf{v}_{s1}$) results in additional anisotropy [the second term in Eq. (16)].

III. NORMAL CORE CURRENT

In the previous section, we expressed the electric field outside the normal core, $\mathbf{E}=\mathbf{E}_0+\mathbf{E}_1$, with the consideration of both $\mathbf{v}_{s0}$ and $\mathbf{v}_{s1}$. $\mathbf{E}_1$, combined with a proper boundary condition for the interface between inside and outside the normal core, can be used to determine the electric field inside the normal core of a moving vortex.[1] Several different boundary conditions have been adopted in flux flow models.[1,2,9] For example, BS assumed that the total chemical potential is continuous at the core boundary. However, as clearly shown by NV,[2] BS's condition implies that there exists a contact force at the boundary. Equivalently, it implies that the Lorentz force $[(ne/c)\,\mathbf{v}_T\times\boldsymbol{\phi}]$ is dissipated at the core boundary. Currently no dissipation mechanism confined to the boundary surface is reported, and it is hard to accept BS's condition as a general one. Here we will use a continuity condition resulted from the Maxwell's equation

$$\vec{\nabla}\times\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=0.$$

This equation implies that the tangential components of electric fields are continuous across any boundary if $\partial\mathbf{B}/\partial t$ is bounded. The boundary condition used in the NV model, the continuity of the electric potential, implies also that the tangential components of electric fields are equal at the boundary.

Using Eq. (16) and our boundary condition, we determine the electric field inside the normal core (see Appendix A for details),

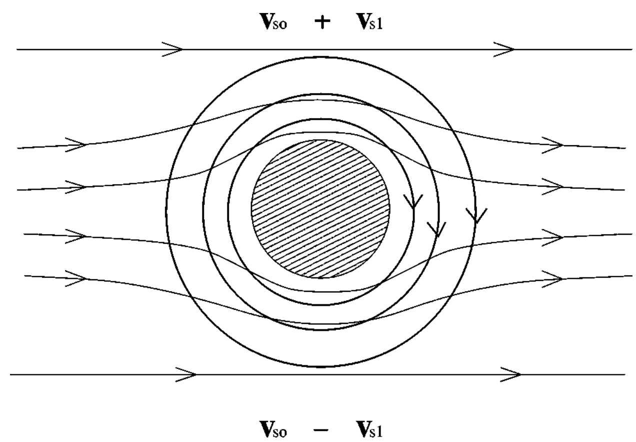

FIG. 2. The schematic description of the origin for the additional force field due to $\mathbf{J}_T$.

$$e\mathbf{E}_c = \frac{e}{2\pi a^2 c}(2\mathbf{v}_T - \mathbf{v}_L - \mathbf{v}_{c0} - \alpha\hat{\mathbf{z}}\times\mathbf{v}_L)\times\boldsymbol{\phi}, \qquad (17)$$

where $\alpha = \hbar\tau/2ma^2$. In the expression of α, we replace t by the relaxation time τ by making use of the assumption made by BS that the normal core is in the steady state via the charge carrier's relaxation to the crystal. We find from a simple vector algebra (see Appendix A for details) that $\mathbf{v}_{c0}$ and $\alpha\hat{\mathbf{z}}\times\mathbf{v}_L$ are related to the velocity of the core current observed in the laboratory frame, $\mathbf{v}_c$, as

$$\mathbf{v}_c = \mathbf{v}_{c0} + \alpha\hat{\mathbf{z}}\times\mathbf{v}_L. \qquad (18)$$

Thus Eq. (17) can be written in terms of vectors in the laboratory frame as

$$e\mathbf{E}_c = \frac{e}{2\pi a^2 c}(2\mathbf{v}_T - \mathbf{v}_L - \mathbf{v}_c)\times\boldsymbol{\phi}. \qquad (19)$$

This result is a generalization of the BS's core field $e\mathbf{E}_c^{\mathrm{BS}} = -(e/2\pi a^2 c)\,\mathbf{v}_L\times\boldsymbol{\phi}$ with additional contributions from $\mathbf{v}_T$ and $\mathbf{v}_c$ to $e\mathbf{E}_c$. When v_L is much greater than v_T and v_c, Eq. (19) reduces to $e\mathbf{E}_c^{\mathrm{BS}}$. Vinen and Warren derived an identical result as (19) using a Bernoulli equation.[6] However, their electric field in the core is determined in the vortex frame, while ours in the laboratory frame. The equivalence of the two results for the core electric field, one observed in the vortex frame and the other in the laboratory frame, is a manifestation of the Galilean invariance.

Now $\mathbf{v}_c$ can be determined from Eq. (19) by setting up a force balance equation to be satisfied by the normal charge carriers in the core. In general, we can establish the following force balance equation for each charge carrier per unit length in the z direction:

$$e\mathbf{E}_c + \mathbf{f}_p - \frac{m}{\tau}\mathbf{v}_c = 0. \qquad (20)$$

Here $\mathbf{f}_p$ is the effective pinning force field acting on the normal charge carriers, and the third term denotes the density of the momentum dissipated inside the normal core per unit time. $\mathbf{f}_p$ was first introduced by NV (Ref. 2) and widely used in vortex dynamics.[9,10,18,19] The expression (20) is a generalization of Eq. (4.4) of Ref. 1 which is valid only for samples without pinning. Considering the fact that the effective pinning force prevents the motion of vortex,[2,9,10,20] $F_p = -\gamma\mathbf{v}_L$, we write $\mathbf{f}_p$ as

$$\mathbf{f}_p = -\frac{1}{\pi a^2}\gamma\mathbf{v}_L = -\gamma'\mathbf{v}_L. \qquad (21)$$

Here γ and γ' are introduced in order to denote the intensity of the effective pinning. From the use of Eqs. (19) and (21), we rewrite

$$e\mathbf{E}_c + \mathbf{f}_p = \frac{e}{2\pi a^2 c}\beta(2\mathbf{v}_T\times\boldsymbol{\phi}) - \frac{e}{2\pi a^2 c}(\mathbf{v}_L + \mathbf{v}_c)\times\boldsymbol{\phi}, \qquad (22)$$

where we introduced the phenomenological constant

$$\beta = 1 - \frac{2\pi a^2 c}{e}\frac{\gamma' v_L}{2 v_T \phi} = 1 - \frac{f_p}{f_{\mathrm{Lorentz}}}.$$

Thus rewriting Eq. (20) above, we obtain

$$\frac{e}{2\pi a^2 c}(2\beta\mathbf{v}_T - \mathbf{v}_L - \mathbf{v}_c)\times\boldsymbol{\phi} - \frac{m}{\tau}\mathbf{v}_c = 0, \qquad (23)$$

and $\mathbf{v}_c$ is determined as (see Appendix B for details)

$$\mathbf{v}_c = v_{cx}\hat{\mathbf{x}} + v_{cy}\hat{\mathbf{y}},$$

$$v_{cx} = \tfrac{1}{2}\,\omega_{c2}\tau v_L. \qquad (24)$$

$$v_{cy} = \tfrac{1}{2}\,\beta\omega_{c2}\tau(2v_T) + O((\omega_{c2}\tau)^2),$$

According to Eqs. (24), β and v_{cy} become small if f_p and the magnitude of the Lorentz force field, f_{Lorentz}, are comparable.

IV. ENERGY DISSIPATION AND DRAG FORCE

From the expression of the normal core current Eq. (24), we can determine the rate of energy dissipation. We assume that the dissipation is via the relaxation of the normal charge carriers to the crystal with relaxation time τ. We find from the use of Eq. (24)

$$W_{\mathrm{diss}} = \pi a^2 n e\mathbf{v}_c\cdot\mathbf{E}_c = \frac{n\pi\hbar^2\tau}{4ma^2}[v_L^2 + (2\beta v_T)^2]. \qquad (25)$$

Comparing with the energy dissipation rate of the BS model, $W_{\mathrm{diss}}^{\mathrm{BS}} = n\pi\hbar^2\tau v_L^2/2ma^2$, we find that the numerical coefficient of v_L^2 in Eq. (25) is different only by factor of $\frac{1}{2}$. And there exists an additional dissipation ($\propto v_T^2$), which becomes negligible if f_p is comparable with f_{Lorentz}.

The expression (25) can be used to determine the drag force preventing the motion of vortex. BS assumed that the drag force was a viscous force $-\eta\mathbf{v}_L$ from $W_{\mathrm{diss}}^{\mathrm{BS}}$. On the other hand, our generalized result in (25), including the contribution from the NCD and the effective pinning, depends additionally on v_T^2. In the theory of superfluidity, a more generalized drag force $\mathbf{f}_{\mathrm{drag}} = -\eta(\mathbf{v}_n - \mathbf{v}_L) - \eta'\hat{\mathbf{z}}\times(\mathbf{v}_n - \mathbf{v}_L)$, where $\mathbf{v}_n$ is the local normal fluid velocity, has been proposed and investigated.[21–25] Based on this theory, Hagen *et al.*[7,8] made a conjecture that the drag force acting on vortices in the type-II superconductors can be described by

$$\mathbf{f}_{\rm drag}=-\eta\mathbf{v}_L-a\mathbf{v}_T \tag{26}$$

in order to explain the Hall angle anomaly.[7,8,26,27] In support of the conjecture of Hagen *et al.*, Ferrell proposed a microscopic theory[29] showing a possible dissipative mechanism leading to $-a\mathbf{v}_T$. His theory showed that thermally excited quasiparticles can interact with vortices via the Andreev reflection by a screening current. Apart from Ferrell's theory, details of the damping force $-a\mathbf{v}_T$ are not fully understood yet.

Here we determine η and a of Eq. (26) in a consistent way, by assuming that the relaxation of the normal charge carriers in the core is the only energy dissipation process in a moving vortex. By treating v_L and v_T as independent variables, we obtain from $W_{\rm diss}$

$$-\frac{\partial W_{\rm diss}}{\partial \mathbf{v}_L}=-\frac{n\pi\hbar^2\tau}{2ma^2}\mathbf{v}_L,\quad -\frac{\partial W_{\rm diss}}{\partial \mathbf{v}_T}=-\frac{n\pi\hbar^2\tau(2\beta)^2}{2ma^2}\mathbf{v}_T. \tag{27}$$

Comparing Eqs. (27) with Eq. (26), we deduce η and write it in a more familiar form

$$\eta=\frac{n\pi\hbar^2\tau}{2ma^2}\simeq n\pi\hbar\omega_{c2}\tau\simeq\frac{1}{2}\frac{hc}{2e}\frac{H_{c2}}{c^2}\frac{ne^2\tau}{m}\simeq\frac{1}{2}\frac{\phi H_{c2}(0)}{\rho_n c^2}. \tag{28}$$

This is identical to the result of BS [Eq. (4.12) of Ref. 1] except for the constant factor of $\frac{1}{2}$. We also deduce $a=\eta(2\beta)^2$. This result shows that the longitudinal drag force $-a\mathbf{v}_T$ depends on both f_p and $f_{\rm Lorentz}$.

V. HALL ANGLE ANOMALY

For a unit length of the single vortex, Hagen *et al.*[7,8] set up the force balance equation between the Magnus force as the driving force and the drag force (26),

$$\frac{n_s e}{c}(\mathbf{v}_T-\mathbf{v}_L)\times\boldsymbol{\phi}-\eta\mathbf{v}_L-a\mathbf{v}_T=0. \tag{29}$$

They obtained the following condition for the negative Hall angle from the above force balance equation:

$$\left(\frac{1}{c}n_s e\phi\right)^2<\eta a. \tag{30}$$

Inserting the value of

$$\eta=\frac{\phi H_{c2}(0)}{\rho_n c^2}\simeq\frac{ne\phi}{c}\omega_{c2}\tau$$

derived by BS and their own estimation of $a\simeq n_s e\phi/c\omega_{c2}\tau$, Hagen *et al.* concluded that the negative Hall angle is possible for $n_s(T)<\frac{1}{2}n$. One noticeable problem in the result of Hagen *et al.* is that $a\gg\eta$ in most of superconductors except the extremely pure samples of $\omega_{c2}\tau\gg 1$.

Since we obtained reasonable expressions for η and a (unlike the estimation of Hagen *et al.*, our a is not larger than η) in a systematic way, we try to determine the condition for the negative Hall angle within the frame work of Hagen *et al.* We set up the force balance equations corresponding to Eq. (29) as

$$\frac{n_s e}{c}(\mathbf{v}_T-\mathbf{v}_L)\times\boldsymbol{\phi}-(\eta+\gamma)\mathbf{v}_L-a\mathbf{v}_T=0. \tag{31}$$

The driving force in this equation is assumed to be the Magnus force. We use Eq. (27) for the coefficients of the drag force. To be consistent with Eq. (20) where we considered the effective pinning force, an additional force term $-\gamma\mathbf{v}_L$ exists in Eq. (31). After a simple vector algebra together with η and a in Eq. (27), we find that the negative Hall angle occurs if

$$\frac{n_s}{n}<\omega_{c2}\tau\beta\left(1+\frac{\gamma}{\eta}\right)^{1/2}. \tag{32}$$

This condition shows clearly that the effective pinning force, represented by γ and β, is a key factor for the occurrence of the negative Hall angle. The expression (32) implies that the negative Hall angle cannot occur in the BS limit where the effective pinning force balances the Lorentz force, making $\beta\simeq 0$. On the other hand, the inequality (32) holds for certain $n_s(T)$ if the effective pinning force becomes small compared with the Lorentz force, resulting in $\beta\simeq 1$. In the flux flow limit where $\beta=1$, expression (32) implies that the negative Hall angle occurs if the temperature-dependent superelectron density satisfies the inequality $n_s(T)<\omega_{c2}\tau n$. This qualitative argument supports the experimental fact that the negative Hall angle appears near T_c where the pinning is known to be ineffective.

Wang and Ting[9,10] related the Hall angle anomaly with pinning effects. By generalizing the NV theory, they proposed that the negative Hall angle appears if the effective pinning force density is larger than the Lorentz force density. Contrary to the conclusion of Wang and Ting, our theory predicts that the negative Hall angle appears if the effective pinning force is much smaller than the Lorentz force density. Two measurements on the Hall angle in the mixed state, one by Budhani *et al.*[26] and the other by Kunchur *et al.*,[27] support our theory. Budhani *et al.* tried to find correlation between the Hall angle anomaly and the number of defects induced by heavy ion irradiation. They found that the Hall angle anomaly diminishes as the concentration of defects increases. The experiment of Kunchur *et al.* showed that the increment of the applied current $\mathbf{J}_T$ induces an enhancement of the Hall angle anomaly.

VI. DISCUSSION

We generalized the BS model by incorporating the effect of transport current ($\mathbf{J}_T$), that is, the nonlinear contribution of convection to the force field generated outside the normal core. We then determined the electric field in the core and found that it is identical to that suggested by the NV model. This electric field implies that the normal core current in a moving vortex core is not necessarily perpendicular to $\mathbf{v}_L$, contrary to the BS model. Another component of the normal core current, originated from the NCD, is found to be parallel to $\mathbf{v}_L$. The present study deals with a classical approach based on the superfluid hydrodynamics. In the previous paper[28] we studied the normal core current in the moving vortex core by using the microscopic Bogoliubov–de Gennes transformation method. In this microscopic work, in qualitative agreement we also showed that the normal core current has an other than parallel component to the vortex motion.

In order to explain the Hall angle anomaly, Hagen *et al.* suggested that the drag force has both components $\mathbf{v}_L$ and $\mathbf{v}_T$. By using the drag force suggested by Hagen *et al.* and assuming that coupling between the normal charge carriers and the lattice is the only source of momentum loss by a moving vortex, we determined the coefficients η and a in a systematic way. If these coefficients for the drag force are used, the Hall angle anomaly can be explained in a self-consistent manner. Our calculation shows that the Hall angle anomaly appears if the effective pinning force is smaller than the Lorentz force. This prediction is well supported by the experiments by Budhani *et al.*[26] and Kunchur *et al.*[27] Contrary to the theory of Ferrell[29] which introduces vortex-quasiparticle interaction, our theory is in agreement with the conjecture of Hagen *et al.* by considering the energy dissipation at the normal core alone.

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APPENDIX A

In order to derive Eq. (17), we insert Eq. (15) into Eq. (16) and obtain

$$\frac{e}{m}\mathbf{E}_1=-\mathbf{v}_L\cdot\vec{\nabla}\mathbf{v}_{s0}+\mathbf{v}_T\cdot\vec{\nabla}\mathbf{v}_{s0}+\vec{\nabla}[\mathbf{v}_{s0}\cdot(\mathbf{v}_{B0}-\mathbf{v}_L t\cdot\vec{\nabla}\mathbf{v}_{s0})]. \tag{A1}$$

By inserting $\mathbf{v}_{s0}=(-\hbar/2mr)\hat{\boldsymbol{\theta}}$ and $\mathbf{v}_{B0}=\vec{\nabla}[(\mathbf{v}_T-\mathbf{v}_{c0})\cdot(a^2/r^2)\,\mathbf{r}]$ into Eq. (A1), we obtain (with $\mathbf{H}=-H\hat{\mathbf{Z}}$, $\mathbf{v}_T=v_T\hat{\mathbf{x}}$, $\mathbf{v}_{c0}=v_{c0,x}\hat{\mathbf{x}}+v_{c0,y}\hat{\mathbf{y}}$, and $\mathbf{v}_L=v_L\hat{\mathbf{y}}$)

$$\frac{e}{m}\mathbf{E}_1=\vec{\nabla}\left[\frac{\hbar v_L}{2mr}\cos\theta+\frac{\hbar v_T}{2mr}\sin\theta+\frac{\hbar a^2}{2mr^3}[(v_T-v_{c0,x})\sin\theta+v_{c0,y}\cos\theta]+\frac{\hbar^2 v_L t}{4m^2r^3}\sin\theta\right]. \tag{A2}$$

As discussed in Sec. III, the electric force field outside the core $e\mathbf{E}_1$ and the core force field $e\mathbf{E}_c$, which is assumed to be uniform, are related by the following boundary condition

$$e\mathbf{E}_1(a^+)\cdot\hat{\boldsymbol{\theta}}=e\mathbf{E}_c(a^-)\cdot\hat{\boldsymbol{\theta}}, \tag{A3}$$

where $r=a^{\pm}=a-0^{\pm}$. Using Eq. (A2) together with the above boundary condition, we obtain Eq. (17).

The expression (18) can be obtained by writing the total current observed in the laboratory frame $[\mathbf{v}_s(\mathbf{r},t)]$ in different ways. As shown in Eqs. (14) and (15), we can express $\mathbf{v}_s(\mathbf{r},t)$ in terms of the backflow in the vortex frame ($\mathbf{v}_{B0}$),

$$\mathbf{v}_s(\mathbf{r},t)=\mathbf{v}_{s0}^{\text{tot}}(\mathbf{r}-\mathbf{v}_L t)\simeq\mathbf{v}_{s0}+\mathbf{v}_T+\mathbf{v}_{B0}-\mathbf{v}_L t\cdot\vec{\nabla}\mathbf{v}_{s0}. \tag{A4}$$

If we write $\mathbf{v}_s(\mathbf{r},t)$ in terms of the backflow defined in the laboratory frame, $\mathbf{v}_B=\vec{\nabla}[(\mathbf{v}_T-\mathbf{v}_c)\cdot(a^2/r^2)\,\mathbf{r}]$ with $\mathbf{v}_c$, the normal core current defined in the laboratory frame, $\mathbf{v}_s(\mathbf{r},t)$ is then

$$\mathbf{v}_s(\mathbf{r},t)=\mathbf{v}_{s0}+\mathbf{v}_T+\mathbf{v}_B. \tag{A5}$$

By comparing Eqs. (A4) and (A5), we obtain

$$\mathbf{v}_{B0}-\mathbf{v}_L t\cdot\vec{\nabla}\mathbf{v}_{s0}=\mathbf{v}_B. \tag{A6}$$

The expression (18) results from the above equation.

APPENDIX B

The expression (23) can be written as

$$\mathbf{v}_c=\frac{e}{2\pi a^2 c}\frac{\tau}{m}(2\beta\mathbf{v}_T-\mathbf{v}_L-\mathbf{v}_c)\times\boldsymbol{\phi}$$
$$=\frac{1}{2}\omega_{c2}\tau(2\beta\mathbf{v}_T-\mathbf{v}_L-\mathbf{v}_c)\times(-\hat{\mathbf{z}}). \tag{B1}$$

This expression is separated into two parts

$$v_{cx}=\tfrac{1}{2}\,\omega_{c2}\tau(v_{Lx}+vcy),\quad v_{cy}=\tfrac{1}{2}\,\omega_{c2}\tau(2\beta v_T-v_{Lx}+v_{cx}), \tag{B2}$$

and v_{cx} and v_{cy} are

$$v_{cx}=\frac{\omega_{c2}\tau/2}{1+(\omega_{c2}\tau)^2}v_{Ly}+\frac{(\omega_{c2}\tau/2)^2}{1+(\omega_{c2}\tau)^2}(2\beta v_T-v_{Lx}), \tag{B3}$$

$$v_{cy}=\frac{\omega_{c2}\tau/2}{1+(\omega_{c2}\tau)^2}(2\beta v_T-v_{Lx})-\frac{(\omega_{c2}\tau/2)^2}{1+(\omega_{c2}\tau)^2}v_{Ly}.$$

For the case of negligibly small Hall angle ($v_{Lx}\ll v_{Ly}\equiv v_L$) and small parameter $\omega_{c2}\tau$, we obtain Eq. (24).

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