PHYSICAL REVIEW B VOLUME 57, NUMBER 9 1 MARCH 1998-I

# Paramagnetic current and dissipative vortex motion in type-II superconductors

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(Received 4 June 1997)

In order to find the microscopic origin of the components of the paramagnetic current generated in a moving vortex core, we present a perturbation method to solve the Bogoliubov–de Gennes equations for a moving vortex in clean type-II superconductors. The paramagnetic current and its associated dissipation at the moving vortex core are shown to result from intrinsic concommitant effects. From the present perturbation approach involving no presupposition of the Magnus force, the long-standing problem of the validity of the two different classical models is discussed. The components of the derived paramagnetic current are found to be identical to the classical model of Noziéres and Vinen. [S0163-1829(98)03809-0]

## I. INTRODUCTION

A correct account of the normal core current in a moving vortex core is essential for understanding vortex dynamics and energy dissipation. Ignoring the contribution of the diamagnetic current in the core, we consider the normal core current only from the paramagnetic current (PC).[1] Two of the well-known classical models for the dissipative motion of vortex are one developed by Bardeen and Stephen[2] (BS) and the other by Noziéres and Vinen[3] (NV). In both models, the normal core current is determined from the force balance equation between the driving force which acts on the bulk of the vortex core and the dissipative force which results in energy dissipation to the crystal lattice. In the subsequent paper, we largely focus, on the nature of energy dissipation, the drag force, and Hall angle anomaly, based on the macroscopic classical descriptions depending on the superfluid hydrodynamics.[4]

In the present paper, we focus on the microphysical origins of the components of the normal current in order to find differences between the two well-known classical models. For this cause, we resort to the application of the Bogoliubov–de Gennes (BdG) equations[5,6] for a moving vortex. Earlier, in their pioneering works Caroli, De Gennes, and Matricon[7,8] showed that the BdG equations can be used for the microscopic descriptions of quasiparticle low-energy excitations in the core of an isolated stationary vortex line for extreme type-II superconductors. Their work became a basis of the well-known normal core model of the stationary vortex, upon which the two classical models (BS and NV) are founded. In this paper we present a generalized perturbation approach of solving the BdG equations by directly treating a moving vortex on the basis of a unitary transformation involving a Galilean transformation between the laboratory frame and the moving vortex frame. We find from this approach that the physics of a moving vortex can be naturally decomposed into a part associated with a stationary vortex and the other related to the motion of the vortex. The latter involves various concommitant effects which can be determined from the perturbation terms. From each perturbative effect linear in the velocity of the vortex, the PC components in the core of the moving vortex will be determined.

## II. BOGOLIUBOV–de GENNES EQUATIONS FOR A MOVING VORTEX

Superconductivity concerned with the order parameter $\Delta(\mathbf{r})$ prompted Bogoliubov to introduce the well-established Bogoliubov equations.[5,6] These equations are essentially a generalization of the ordinary Hartree-Fock equation for many-body systems into superconductivity by introducing the effects of the superconducting pair potential $\Delta(\mathbf{r})$, in addition to the ordinary scalar Hartree-Fock potential $U_0(\mathbf{r})$. By solving the Bogoliubov equations involving a magnetic field, that is, the BdG equations,[6,7] Caroli *et al.*[7,8] were able to describe the low-lying excited states of quasiparticles in the core of a vortex line. The Hamiltonian for the BdG equations are written as

$$\hat{H}_0=\hat{\sigma}^z\left[\frac{1}{2m}\left(\mathbf{p}-\hat{\sigma}^z\frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2-E_F\right]+\begin{pmatrix}0 & \Delta(\mathbf{r})\\ \Delta^*(\mathbf{r}) & 0\end{pmatrix}. \tag{1}$$

Here $\mathbf{p}$ and $\mathbf{A}$ are the momentum of charge carriers and the electromagnetic vector potential, respectively. $\hat{\sigma}^z$ is the $z$ component of the Pauli matrix and $E_F$ is the Fermi energy. Considering the cylindrical symmetry of the vortex structure, Caroli *et al.* chose a gauge such that the pair potential $\Delta(\mathbf{r})=|\Delta(\mathbf{r})|e^{-i\theta(\mathbf{r})}$, with the angle $\theta(\mathbf{r})$ about the center of the vortex measured from the $x$ axis as shown in Fig. 1. The solutions of the BdG equations are given by the spinor form[6,7,9] of

$$\hat{\phi}_{0\alpha}(\mathbf{r})=\begin{pmatrix}u_\alpha(\mathbf{r},z)\\ v_\alpha(\mathbf{r},z)\end{pmatrix}=e^{ik_Fz\cos\beta}e^{i(\mu-\hat{\sigma}_z/2)\theta(\mathbf{r})}\begin{pmatrix}f_\alpha^+(\mathbf{r})\\ f_\alpha^-(\mathbf{r})\end{pmatrix}, \tag{2}$$

where $k_F$ is the Fermi wave vector, $\beta$ an arbitrary angle, and $2\mu$ an odd integer. The explicit forms of the radial function $f_\alpha^+$ and $f_\alpha^-$ are given by Eqs. (4)–(8) of Ref. 7. From this solution, Caroli *et al.* found that for $\Delta_\infty^2/E_F<\epsilon\ll\Delta_\infty$ the density of states associated with the energy levels of a bound quasiparticle is the same as that in a normal region of a radius close to the coherence length $\xi$. The vortex core may be approximated by a normal region of size $\xi$. This simplified approximation of the complicated vortex core structure

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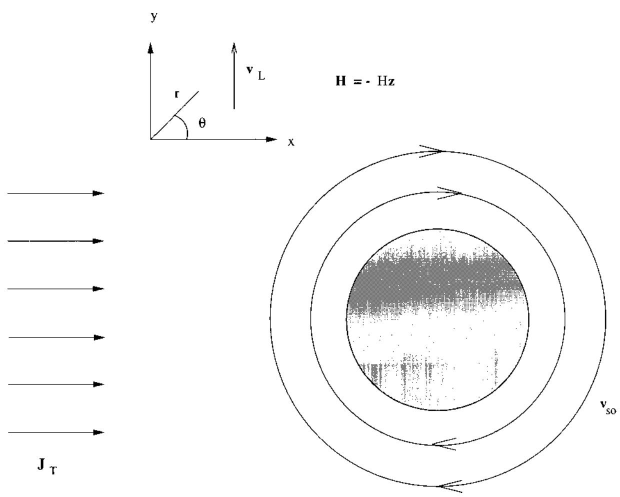


FIG. 1. Geometry used in this work.

is known as the normal core model and is widely used by various researchers (including BS and NV).

In order to find the origin of the PC without any presumption, we introduce the unitary operator[10]

$$\hat{U}=\exp\left(\frac{i}{\hbar}\mathbf{v}_L t\cdot\mathbf{p}-\frac{i}{\hbar}m\mathbf{v}_L\cdot\mathbf{r}\right), \tag{3}$$

which involves a Galilean transformation between the laboratory frame and the frame of the moving vortex. This transformation allows the invariance of the BdG equations. Thus, introducing the transformation $\hat{H}(\mathbf{p},\mathbf{r})=\hat{U}^{\dagger}\hat{H}_o\hat{U}$, we obtain the following effective Hamiltonian for a vortex moving with a small velocity $v_L$:

$$\hat{H}(\mathbf{p},\mathbf{r})\simeq\hat{H}_0+\hat{H}_\Delta+\hat{H}_\theta+\hat{H}_{v_L}+\hat{H}_{A1}+\hat{H}_{A2}+\hat{H}_{A_p}+O((v_Lt/\xi)^2), \tag{4}$$

where the perturbation terms linear in $\mathbf{v}_L$ are

$$\hat{H}_\Delta=\begin{pmatrix}0 & -\mathbf{v}_L t\cdot\vec{\nabla}\Delta(\mathbf{r})\\ -\mathbf{v}_L t\cdot\vec{\nabla}\Delta^*(\mathbf{r}) & 0\end{pmatrix},$$

$$\hat{H}_\theta=\begin{pmatrix}0 & \dfrac{2mi}{\hbar}\Delta(\mathbf{r})\mathbf{v}_L\cdot\mathbf{r}\\ -\dfrac{2mi}{\hbar}\Delta^*(\mathbf{r})\mathbf{v}_L\cdot\mathbf{r} & 0\end{pmatrix},$$

$$\hat{H}_{v_L}=\begin{pmatrix}-\mathbf{p}\cdot\mathbf{v}_L & 0\\ 0 & \mathbf{p}\cdot\mathbf{v}_L\end{pmatrix}, \tag{5}$$

$$\hat{H}_{A1}=\begin{pmatrix}\dfrac{e}{mc}[\mathbf{v}_L t\cdot\vec{\nabla}\mathbf{A}(\mathbf{r})]\cdot\mathbf{p} & 0\\ 0 & \dfrac{e}{mc}[\mathbf{v}_L t\cdot\vec{\nabla}\mathbf{A}(\mathbf{r})]\cdot\mathbf{p}\end{pmatrix},$$

$$\hat{H}_{A2}=\begin{pmatrix}\dfrac{e}{c}\mathbf{v}_L\cdot\mathbf{A}(\mathbf{r}) & 0\\ 0 & \dfrac{e}{c}\mathbf{v}_L\cdot\mathbf{A}(\mathbf{r})\end{pmatrix},$$

and $\hat{H}_0$ is given by Eq. (1). The perturbation terms above are valid only for the condition of $\mathbf{v}_L t\ll\xi$ for a slowly moving vortex. $\hat{H}_\Delta$ above shows the contribution of the spatial variation of the order parameter. $\hat{H}_\theta$ represents the small phase shift of the order parameter due to the slow motion of the vortex. $\hat{H}_{v_L}$ introduces the coupling of the motion of quasiparticles to the motion of the vortex along $\mathbf{v}_L$. The vortex frame is assumed to move with uniform velocity $\mathbf{v}_L$ with respect to the laboratory frame. $\hat{H}_{A1}$ and $\hat{H}_{A2}$ represent the coupling of the vortex motion with the magnetic field. The magnetic field effect was ignored in the original work of Caroli *et al.*,[7] since it is of order $H/H_{c2}\ll 1$.

## III. ANALYSIS OF PERTURBATION TERMS

We first explore the first perturbative Hamiltonian term $\hat{H}_\Delta$. In this work, we assume that the time required for relaxation of the vortex motion to a steady state is of the order of the electron scattering time $\tau$ as in the BS model. The basis $\{\hat{\phi}_{0\alpha}\}$ of $\hat{H}_0$ obtained by Caroli *et al.*[7,8] will be used for the perturbation treatment of $(\hat{H}_0+\hat{H}_\Delta)\hat{\phi}=\epsilon\hat{\phi}$. $\hat{H}_\Delta$ can be decomposed into two parts in association with the pair potential $\Delta(\mathbf{r})=|\Delta(\mathbf{r})|e^{-i\theta(\mathbf{r})}$:

$$\hat{H}_\Delta=\hat{H}_{\Delta1}+\hat{H}_{\Delta2},$$

$$\hat{H}_{\Delta1}\equiv\begin{pmatrix}0 & i\mathbf{v}_L\tau\cdot[\vec{\nabla}(\theta)]|\Delta(r)|e^{-i\theta}\\ -i\mathbf{v}_L\tau\cdot[\vec{\nabla}(\theta)]|\Delta(r)|e^{i\theta} & 0\end{pmatrix}, \tag{6}$$

$$\hat{H}_{\Delta2}\equiv\begin{pmatrix}0 & -\mathbf{v}_L\tau\cdot[\vec{\nabla}|\Delta(r)|]e^{-i\theta}\\ -\mathbf{v}_L\tau\cdot[\vec{\nabla}|\Delta(r)|]e^{i\theta} & 0\end{pmatrix}.$$

The first term $\hat{H}_{\Delta1}$ represents the contribution of the diamagnetic current ($\mathbf{v}_{s0}$) around the vortex line, since the gradient of the phase $\theta$ is proportional to the superfluid velocity. The microscopic origin of the force field $\mathbf{f}=-m\mathbf{v}_L\cdot\vec{\nabla}\mathbf{v}_{s0}$ in the BS model is related to $\hat{H}_{\Delta1}$ above. Both the perturbative treatment of $\hat{H}_{\Delta1}$ and the BS model are physically valid only for $v_L\tau\ll\xi$. $\hat{H}_{\Delta2}$ is associated with the spatial variation of the amplitude of the order parameter. Here we assume that $\hat{H}_{\Delta2}$ vanishes, considering the uniform normal core. It will be of great interest to treat this term more accurately using a numerical method in the future.

In the expression (6) above, we note $\vec{\nabla}(\theta)=(1/r)\hat{\boldsymbol{\theta}}$ for the geometric configuration shown in Fig. 1. $\hat{H}_{\Delta1}$ is then written as

$$\hat{H}_{\Delta 1}=\begin{pmatrix} 0 & \frac{i}{r}v_L\tau|\Delta(r)|\cos\,\theta e^{-i\theta} \\ -\frac{i}{r}v_L\tau|\Delta(r)|\cos\,\theta e^{i\theta} & 0 \end{pmatrix}. \tag{7}$$

By using the solution of Caroli *et al.*[7] shown in Eq. (2), we obtain (see Appendix A for detailed derivation)

$$\langle\hat{\phi}_{0\alpha}|\hat{H}_{\Delta 1}|\hat{\phi}_{0\beta}\rangle \simeq \pm i\,\delta_{\mu_\alpha,\mu_\beta\pm 1}\frac{v_L\tau k_{F\perp}\Delta_\infty^2}{E_F}, \tag{8}$$

where $\Delta_\infty$ is the order parameter far away from the vortex, that is, $r\gg\xi$. A similar result in form was obtained by Šimánek[9] and Hsu,[11,12] using different approaches (detailed discussions are presented in Appendix A).

Using this matrix element and the perturbation theory, we expand $\hat{\phi}_\alpha$ in terms of $\hat{\phi}_{0\alpha}$ and obtain (see Appendix A for derivation),

$$\hat{\phi}_\alpha \simeq \hat{\phi}_{0\alpha}+A(\hat{\phi}_{0\alpha+1}+\hat{\phi}_{0\alpha-1}), \tag{9}$$

where $A=-iv_L\tau k_{F\perp}$. These eigenvectors are orthonormal to leading order,

$$\int \hat{\phi}_\alpha^*\hat{\phi}_\beta d^3r=\delta_{\alpha\beta}+O((v_Lt/\xi)^2). \tag{10}$$

From the explicit relation between the quasiparticle operators $\gamma_\alpha^\dagger(\gamma_\alpha)$ and the solutions $\hat{\phi}_\alpha$,[11,13] we obtain (see Appendix B for details)

$$\gamma_\alpha^\dagger \simeq \gamma_{0\alpha}^\dagger + A(\gamma_{0\alpha+1}^\dagger+\gamma_{0\alpha-1}^\dagger) \quad \text{for all} \quad \alpha. \tag{11}$$

These operators satisfy the required anticommutation relation to leading order,

$$\{\gamma_\alpha,\gamma_\beta^\dagger\}=\delta_{\alpha\beta}+O((v_Lt/\xi)^2). \tag{12}$$

We can determine the paramagnetic current at the core of the moving vortex, by using the expression (C4) shown in Appendix C and the relations (9)–(12) shown above. We obtain the spatially averaged PC due to the vortex motion,

$$\begin{aligned}\left\langle\int \mathbf{n}\cdot\mathbf{J}_p d^2r\right\rangle &\simeq -\frac{ie\hbar k_{F\perp}}{2m}\sum_\beta \big(e^{i\theta}\langle\gamma_\beta^\dagger\gamma_{\beta+1}\rangle \\ &\quad -e^{-i\theta}\langle\gamma_{\beta+1}^\dagger\gamma_\beta\rangle\big) \\ &\simeq -\frac{ie\hbar k_{F\perp}}{2m}\sum_\beta \{e^{i\theta}[A^*f(\epsilon_{0\beta})+Af(\epsilon_{0\beta+1})] \\ &\quad -e^{-i\theta}[Af(\epsilon_{0\beta})+A^*f(\epsilon_{0\beta+1})]\} \\ &=\frac{e\hbar k_{F\perp}^2 v_L\tau}{2m}\cos\,\theta\sum_\beta [f(\epsilon_{0\beta})-f(\epsilon_{0\beta+1})],\end{aligned} \tag{13}$$

where the Fermi distribution function $f(\epsilon_{0\beta})=1/1+e^{\epsilon_{0\beta}/(k_BT)}$ and $\mathbf{n}$ is a unit vector with an angle $\theta$ with respect to $\hat{\mathbf{x}}$. In the low-temperature limit, the summation over $\beta$ becomes 1. The dashed lines in Fig. 2 depict the computed PC due to $\hat{H}_{\Delta 1}$ for $\theta=0°$, 30°, 60°. The angular-averaged PC, represented by the solid lines in Fig. 2, is seen to be parallel with the normal core current of the BS model. The $x$ component of the normal core current, $v_{nx}$, in the BS model is proportional to $v_L$,[2] $v_{nx}=\omega_{c2}\tau v_L$, with the cyclotron frequency $\omega_{c2}=eH_{c2}/mc$. Using the relation

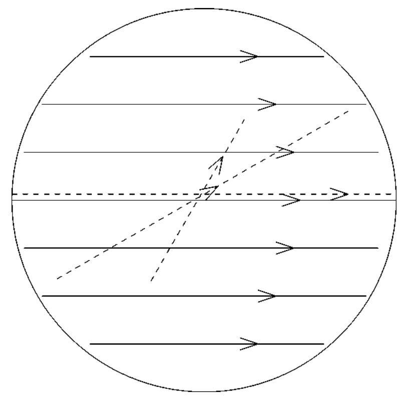

FIG. 2. The paramagnetic current due to $\hat{H}_{\Delta 1}$. Dashed lines represent only the selected values for the magnitude of the PC at $\theta=0°$, 30°, 60°, respectively. Solid lines are the angular-averaged uniform PC.

$$\frac{\hbar\tau}{m}=\frac{\tau}{m}\frac{e\phi}{\pi c}\simeq\frac{e\tau}{mc}H_{c2}\xi^2,$$

we obtain from Eq. (13) the PC in the low temperature limit,

$$\left|\left\langle\int \hat{\mathbf{x}}\cdot\mathbf{J}_p d^2r\right\rangle\right| \simeq ne(\omega_{c2}\tau v_L)\pi\xi^2 \simeq nev_{nx}\pi\xi^2. \tag{14}$$

Thus the relation between $v_{nx}$ and $v_L$, determined from our microscopic investigation of the PC, is obtained to be identical to that of the BS model.

Using the geometry shown in Fig. 1, the matrix element of $\hat{H}_\theta$ is (see Appendix A for derivation)

$$\langle\hat{\phi}_{0\alpha}|\hat{H}_\theta|\hat{\phi}_{0\beta}\rangle \simeq \delta_{\mu_\alpha,\mu_\beta\pm 1}\hbar v_L k_{F\perp}. \tag{15}$$

The perturbation $\hat{H}_\theta$ displays nonzero off-diagonal matrix elements for $\mu_\alpha=\mu_\beta\pm 1$.

The matrix elements in Eq. (15) can be used to determine the eigenvector $\hat{\phi}_\alpha(\mathbf{r})$ satisfying the equation $(\hat{H}_0+\hat{H}_\theta)\hat{\phi}=\epsilon\hat{\phi}$. We find, using a perturbation method similar to that described in Appendix A,

$$\hat{\phi}_\alpha(\mathbf{r})=\hat{\phi}_{0\alpha}-B\hat{\phi}_{0\alpha+1}+B\hat{\phi}_{0\alpha-1}+O((v_Lt/\xi)^2), \tag{16}$$

with $B=\hbar v_L k_{F\perp}E_{F\perp}/\Delta_\infty^2$. The eigenstates found above satisfy the orthonormality condition to leading order in $v_L$,

$$\int \hat{\phi}_\alpha^\dagger\hat{\phi}_\beta d^3r=\delta_{\alpha\beta}+O((v_Lt/\xi)^2). \tag{17}$$

The quasiparticle operators $\gamma_\alpha^\dagger$ and $\gamma_\alpha$ which diagonalize the Hamiltonian $\hat{H}=\hat{H}_0+\hat{H}_\theta$ can be determined from the eigenstate $\hat{\phi}_\beta$ and are expressed in terms of $\gamma_{0\alpha}^\dagger$ and $\gamma_{0\alpha}$,

$$\gamma_\alpha^\dagger=\gamma_{0\alpha}^\dagger-B\gamma_{0\alpha+1}^\dagger+B\gamma_{0\alpha-1}^\dagger+O((v_Lt/\xi)^2). \tag{18}$$

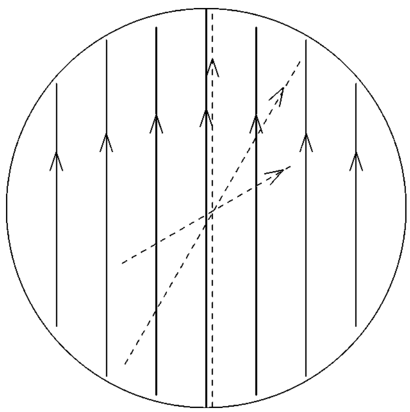

FIG. 3. The paramagnetic current due to $\hat{H}_\theta$. Dashed lines represent only the selected values for the magnitude of the PC at $\theta=0°$, 30°, 60°, respectively. Solid lines are the angular-averaged uniform PC due to $\hat{H}_\theta$.

These operators satisfy the required anticommutation relation to the leading order,

$$\{\gamma_\alpha^\dagger,\gamma_\beta\}=\delta_{\alpha\beta}+O((v_L t/\xi)^2). \tag{19}$$

The off-diagonal matrix elements of Eq. (15) cause the PC to be different from that of a stationary vortex. Similarly to the previous case of $\hat{H}_{\Delta 1}$, we obtain the PC due to $\hat{H}_\theta$,

$$\left\langle \int \mathbf{n}\cdot\mathbf{J}_p d^2r \right\rangle = \frac{e(\hbar k_{F\perp})^2 v_L E_{F\perp}}{m\Delta_\infty^2} \times \sin\theta \sum_\beta [f(\epsilon_{0\beta})-f(\epsilon_{0\beta+1})]. \tag{20}$$

In Fig. 3 we show the angular-averaged PC values of the expression (20) which are parallel with $\mathbf{v}_L$. In addition to the component of the normal core current parallel with $\mathbf{J}_T$ that we obtained earlier, this result shows that there exists an additional PC component parallel with $\mathbf{v}_L$ as in the NV model. Using the relations $n=k_{F\perp}^2/2\pi$ and $\Delta_\infty^2/E_{F\perp}=\hbar^2/m\xi^2$ the angular averaged PC from the expression (20) is reduced to, in the low-temperature limit,

$$\left|\left\langle \int \hat{\mathbf{y}}\cdot\mathbf{J}_p d^2r \right\rangle\right| \simeq nev_L\pi\xi^2. \tag{21}$$

The physical origin of the $\mathbf{v}_L$ component here, with characteristics different from the BS model, lies in the phase shift of the order parameter due to the motion of the vortex with velocity $\mathbf{v}_L$ as discussed in Sec. II. Thus $\hat{H}_\theta$ is shown to allow an additional velocity component of charge carriers induced by the motion of vortex. The Galilean boost approach of the quasiparticle wave function considered by Hsu[11] yielded an identical component of $\mathbf{v}_L$ to the one shown in Eq. (21). In the present study we solved the effective Hamiltonian for the moving vortex [Eq. (4)] in the laboratory frame, while Cleary[14] discussed the core current in a frame at rest with the vortex by the Green's function method. As pointed out by Cleary the relative velocity of scatterers due to the vortex motion is neglected in his study. We found that the phase shift of the order parameter is responsible for the component of the core current parallel to the vortex motion.

We also analyze other perturbation terms $\hat{H}_{v_L}$, $\hat{H}_{A1}$, and $\hat{H}_{A2}$ in order to assess their contribution to the PC. As shown in Appendix D, the off-diagonal matrix elements of these perturbations are negligible compared with other terms described above.

Assuming that the interaction between the PC and the crystal lattice is the only source of momentum loss by a moving vortex, we define the drag force preventing the motion of vortex to be $\mathbf{f}_{\rm drag}=-\int(nm/\tau)\mathbf{v}_{nc}d^2r$.[3] From Eqs. (14) and (21) for the $x$ and $y$ components of the orientation-averaged PC, we find the drag force,

$$\mathbf{f}_{\rm drag}=-\int \frac{nm}{\tau}\mathbf{v}_{nc}d^2r=-\frac{nm}{\tau}(v_{nx}\hat{\mathbf{x}}+v_{ny}\hat{\mathbf{y}})\pi\xi^2 \simeq -\frac{n|e|\phi}{c}v_L\hat{\mathbf{x}}-\frac{n|e|\phi}{c\omega_{c2}\tau}v_L\hat{\mathbf{y}}. \tag{22}$$

This result is qualitatively (but not quantitatively) consistent with the conjecture of Hagen *et al.*[15,16] regarding the Hall angle anomaly.[17–23] They suggested that, if the drag force has components along $\mathbf{v}_L$ and $\mathbf{v}_L\times\hat{\mathbf{z}}$ analogous to the theory of the superfluid,[24–27] the Hall angle inversion can then be explained simply from the force balance equation for a single vortex, assuming that the NV result for a very-low-temperature region is valid for the high-temperature region near $T_c$. Contrary to the Ferrell's theory[28] which assumes that there exists a drag force due to the Andreev reflection of quasiparticles by the diamagnetic current outside the vortex core, our theory, based on no presumption, shows that dissipation due to the drag force occurs in the normal core alone. The core contributions to the damping term $\eta_c$ and the Hall force coefficient $\gamma_c$ were obtained from the Green's function approach.[29] We find that, in the limit $\omega_{c2}\tau\gg 1$,[30] $\eta_c$ and $\gamma_c$ of Otterlo *et al.* become identical to our drag coefficients as shown in Eq. (22).

## IV. DISCUSSION

In order to study the dissipation mechanism in type-II superconductors, we examined the PC generated in a moving vortex core based on a first-principles microscopic approach. For such study we solved the Bogoliubov–de Gennes equations for a moving vortex via a perturbative approach. We find from this perturbation method that the PC and its associated dissipation at the vortex core result from the following microscopic concomitant effects: (1) The PC component along $\mathbf{v}_T$ was shown to originate from the spatial variation of the phase in the order parameter, and (2) the additional component of the PC along the direction of the vortex motion, $\mathbf{v}_L$, is caused by the phase shift of the order parameter due to the motion of vortex. Thus this finding clarifies the cause of the paramagnetic current as a result of vortex motion. From the present investigation involving no presumption, we obtained an additional component of the PC along the direction of the vortex motion, $\mathbf{v}_L$, which was absent in the BS model. The PC was found to be qualitatively equivalent to the classical NV model but not in their magnitudes. In addition we found a drag force which validates the conjecture of Hagen *et al.*

In this work, we used a time-independent perturbation theory for the study of the paramagnetic core current. However, a time-dependent perturbation approach is needed to include dynamic effects on normal current, vortex motion, and other properties such as the Hall anomaly and drag coefficients. The time-dependent BdG equations can be used in order to study the reactive effects of core fermion excitations on the inertial mass of a vortex and to predict anomalous velocity-dependent inertial mass.[31]

## ACKNOWLEDGMENTS

One of us (S.H.S.S.) is grateful to the Korean Ministry of Education (Grant No. BSRI96092) POSTECH BSRI program, and the Center for Molecular Sciences at Korea Advanced Institute of Science and Technology for financial support. We have benefited from discussions with G.F. Giuliani.

## APPENDIX A

The matrix element $\langle\hat{\phi}_{0\alpha}|\hat{H}_{\Delta 1}|\hat{\phi}_{0\beta}\rangle$ can be calculated using the analytic expressions for $\hat{\phi}_{0\alpha}$ of Caroli *et al.*[7] Using Eqs. (2) and (7), we write

$$\begin{aligned}\langle\hat{\phi}_{0\alpha}|\hat{H}_{\Delta 1}|\hat{\phi}_{0\beta}\rangle &= iv_L t\int\left[u_\alpha^*(r)v_\beta(r)|\Delta(r)|\frac{1}{r}\cos\theta e^{-i\theta}\right.\\ &\quad\left.-u_\beta(r)v_\alpha^*(r)|\Delta(r)|\frac{1}{r}\cos\theta e^{i\theta}\right]d^3r\\ &= iv_L t L_z\frac{1}{2}\int[e^{i(\mu_\beta-\mu_\alpha+1)\theta}\\ &\quad+e^{i(\mu_\beta-\mu_\alpha-1)\theta}]d\theta\int|\Delta(r)|\\ &\quad\times[-f_\alpha^-(r)f_\beta^+(r)+f_\alpha^+(r)f_\beta^-(r)]dr,\end{aligned} \tag{A1}$$

where $L_z$ is the length of vortex. The angular integration part gives the selection rule $\delta_{\mu_\alpha,\mu_\beta\pm1}$. In order to perform the analytic integration of the radial part, we need a simplified model for the order parameter:

$$|\Delta(r)|=\begin{cases}0 & \text{if } r<\xi,\\ \Delta_\infty & \text{if } r>\xi,\end{cases} \tag{A2}$$

and we obtain

$$\begin{aligned}\langle\hat{\phi}_{0\alpha}|\hat{H}_{\Delta 1}|\hat{\phi}_{0\beta}\rangle &\simeq \pm i\,\delta_{\mu_\alpha,\mu_\beta\pm1}\frac{Cv_L t k_{F\perp}\Delta_\infty^2}{E_F}\\ &\simeq \pm i\,\delta_{\mu_\alpha,\mu_\beta\pm1}\frac{Cv_L t\Delta_\infty}{\xi}.\end{aligned} \tag{A3}$$

Here $C$ is a numerical constant depending on the parameters of the system like $k_{F\perp}$, $\xi$, $\Delta_\infty$, and $E_{F\perp}$. Using typical values for the high-temperature superconductors, e.g., $\Delta_\infty/E_{F\perp}=0.1$ and $k_{F\perp}\xi=10$, we obtain $C\simeq0.1$. Šimánek[9] also used the above step pair potential (A2) (Ref. 13) and found $C=0.35$.[9] Hsu found $C=0.5$ by using a method based on the integration by parts. For the radial integration, Hsu took Bessel functions for $f_\alpha^\pm(r)$,[12] while the solution obtained by Caroli *et al.*[7] was used in our and Šimánek's approach.

The matrix element $\langle\hat{\phi}_{0\alpha}|\hat{H}_\theta|\hat{\phi}_{0\beta}\rangle$ can be calculated similarly. We write

$$\begin{aligned}\langle\hat{\phi}_{0\alpha}|\hat{H}_\theta|\hat{\phi}_{0\beta}\rangle &= \frac{i2mv_L}{\hbar}\int[u_\alpha^*(r)v_\beta(r)|\Delta(r)|r\sin\theta e^{-i\theta}\\ &\quad-u_\beta(r)v_\alpha^*(r)|\Delta(r)|r\sin\theta e^{i\theta}]d^3r\\ &= \frac{mv_L}{\hbar}L_z\int[e^{i(\mu_\beta-\mu_\alpha+1)\theta}\\ &\quad-e^{i(\mu_\beta-\mu_\alpha-1)\theta}]d\theta\int|\Delta(r)|\\ &\quad\times[-f_\alpha^-(r)f_\beta^+(r)+f_\alpha^+(r)f_\beta^-(r)]r^2dr.\end{aligned} \tag{A4}$$

We perform the radial integration using the above step pair potential (A2) and obtain

$$\langle\hat{\phi}_{0\alpha}|\hat{H}_{v_L}|\hat{\phi}_{0\beta}\rangle\simeq\delta_{\mu_\alpha,\mu_\beta\pm1}\hbar v_L k_{F\perp}. \tag{A5}$$

Here we prove the relation (9) in Sec. II. Defining the projection operators,

$$P=|\hat{\phi}_{0\beta}\rangle\langle\hat{\phi}_{0\beta}|,\quad Q=\sum_{\alpha\neq\beta}|\hat{\phi}_{0\alpha}\rangle\langle\hat{\phi}_{0\alpha}|, \tag{A6}$$

and using

$$|\hat{\phi}_\beta\rangle=\frac{1}{\epsilon_\beta-H_0}H_{\Delta1}|\hat{\phi}_\beta\rangle, \tag{A7}$$

we write

$$\begin{aligned}|\hat{\phi}_\beta\rangle &= (P+Q)|\hat{\phi}_\beta\rangle=|\hat{\phi}_{0\beta}\rangle\langle\hat{\phi}_{0\beta}|\hat{\phi}_\beta\rangle\\ &\quad+\sum_{\alpha\neq\beta}\frac{|\hat{\phi}_{0\alpha}\rangle\langle\hat{\phi}_{0\alpha}|H_{\Delta1}||\hat{\phi}_{0\alpha}\rangle}{\epsilon_\beta-\epsilon_{0\alpha}}+O((H_{\Delta1})^2).\end{aligned} \tag{A8}$$

Using the matrix element (A3) above and setting $\langle\hat{\phi}_{0\beta}|\hat{\phi}_\beta\rangle\simeq1$, we obtain,

$$\hat{\phi}_\beta=\hat{\phi}_{0\beta}+A(\hat{\phi}_{0\beta+1}+\hat{\phi}_{0\beta-1})+O((H_{\Delta1})^2), \tag{A9}$$

where $A=-iv_L t k_{F\perp}$. In the expression above, we used the following approximation[7,9]

$$\epsilon_\beta-\epsilon_{0\beta-1}\simeq\epsilon_{0\beta}-\epsilon_{0\beta-1}\simeq\frac{\Delta_\infty^2}{E_F}. \tag{A10}$$

## APPENDIX B

The quasiparticle operators $\gamma_{\alpha\uparrow}^\dagger$ and $\gamma_{\alpha\downarrow}^\dagger$ (Refs. 11 and 13) are related to $\hat{\phi}_\alpha$ in Eq. (2) in Sec. II,

$$\begin{pmatrix}\gamma_{\alpha\uparrow}^\dagger\\ \gamma_{\alpha\downarrow}^\dagger\end{pmatrix}=\int d\mathbf{r}\begin{pmatrix}\psi_\uparrow^\dagger(\mathbf{r}) & \psi_\downarrow(\mathbf{r})\\ \psi_\downarrow^\dagger(\mathbf{r}) & -\psi_\uparrow(\mathbf{r})\end{pmatrix}\hat{\phi}_\alpha \tag{B1}$$

where $\psi^{\dagger}_{\uparrow,\downarrow}, \psi_{\uparrow,\downarrow}$ are the field operators. Using Eq. (9) of Sec. II, we obtain

$$\gamma^{\dagger}_{\alpha\uparrow} \simeq \gamma^{\dagger}_{0\alpha\uparrow} + A(\gamma^{\dagger}_{0\alpha+1\uparrow} + \gamma^{\dagger}_{0\alpha-1\uparrow}),$$
$$\gamma^{\dagger}_{\alpha\downarrow} \simeq \gamma^{\dagger}_{0\alpha\downarrow} + A(\gamma^{\dagger}_{0\alpha+1\downarrow} + \gamma^{\dagger}_{0\alpha-1\downarrow}). \quad \text{(B2)}$$

The conventional notation defines a basis from the set $\hat{\phi}_\alpha$ with positive eigenvalues $\epsilon_\alpha$.[13] Accordingly the quasiparticle creation operators are defined only for the positive energy states. Hsu introduced a compact notation,[11] where he used a basis including both positive and negative eigenstates, in order to eliminate the spin degeneracy. According to Hsu's notation, for $\epsilon_\alpha > 0$,

$$\gamma^{\dagger}_{\alpha} \equiv \gamma^{\dagger}_{\alpha\uparrow} \quad \text{(B3)}$$

and, for $\epsilon_\alpha < 0$,

$$\gamma_{\alpha} \equiv \gamma^{\dagger}_{-\alpha\downarrow}. \quad \text{(B4)}$$

The inverse transformation of Eq. (B1) is

$$\psi^{\dagger}_{\uparrow}(\mathbf{r}) = \sum_{\alpha} \gamma^{\dagger}_{\alpha} u^{*}_{\alpha}(\mathbf{r}), \quad \psi^{\dagger}_{\downarrow}(\mathbf{r}) = \sum_{\alpha} \gamma_{\alpha} v_{\alpha}(\mathbf{r}). \quad \text{(B5)}$$

Then Eq. (B2) can be rewritten as

$$\gamma^{\dagger}_{\alpha} \simeq \gamma^{\dagger}_{0\alpha} + A(\gamma^{\dagger}_{0\alpha+1} + \gamma^{\dagger}_{0\alpha-1}). \quad \text{(B6)}$$

## APPENDIX C

The paramagnetic current at the moving vortex core can be expressed in terms of the field operator as

$$\mathbf{J}_p(\mathbf{r}) = -\frac{ie\hbar}{2m}\sum_{\beta} \psi^{\dagger}_{\sigma}(\mathbf{r})\vec{\nabla}\psi_{\sigma}(\mathbf{r}) + \text{H.c.} \quad \text{(C1)}$$

The projection of this current on the unit vector $\mathbf{n}$ is

$$\mathbf{n}\cdot\mathbf{J}_p(\mathbf{r}) = -\frac{ie\hbar}{2m}\sum_{\beta} \psi^{\dagger}_{\sigma}(\mathbf{r})\mathbf{n}\cdot\vec{\nabla}\psi_{\sigma}(\mathbf{r}) + \text{H.c.} \quad \text{(C2)}$$

Using the relation (B5), we obtain

$$\psi^{\dagger}_{\uparrow}(\mathbf{r})\mathbf{n}\cdot\vec{\nabla}\psi_{\uparrow}(\mathbf{r}) = \frac{k_{F\perp}}{2}\sum_{\alpha\beta} \gamma^{\dagger}_{\alpha}\gamma_{\beta}[e^{i\theta}u^{*}_{\alpha}u_{\alpha-1} - e^{-i\theta}u^{*}_{\alpha}u_{\alpha+1}],$$
$$\psi^{\dagger}_{\downarrow}(\mathbf{r})\mathbf{n}\cdot\vec{\nabla}\psi_{\downarrow}(\mathbf{r}) = \frac{k_{F\perp}}{2}\sum_{\alpha\beta} \gamma_{\alpha}\gamma^{\dagger}_{\beta}[e^{-i\theta}v_{\alpha}v^{*}_{\alpha-1} - e^{i\theta}v_{\alpha}v^{*}_{\alpha+1}]. \quad \text{(C3)}$$

Here, $\theta$ is the angle of the unit vector $\mathbf{n}$ with respect to the $x$ axis. By inserting Eq. (C3) into Eq. (C2), we obtain, in terms of $\gamma^{\dagger}_{\alpha}$ and $\gamma_\alpha$,

$$\int \mathbf{n}\cdot\mathbf{J}_p d^2r = -\frac{ie\hbar k_{F\perp}}{2m}\sum_{\beta} (e^{i\theta}\gamma^{\dagger}_{\beta}\gamma_{\beta+1} - e^{-i\theta}\gamma^{\dagger}_{\beta+1}\gamma_{\beta}). \quad \text{(C4)}$$

## APPENDIX D

The matrix element of $\hat{H}_{v_L}$ can be calculated using the eigenstates $\hat{\phi}_{0\beta}$. For the geometric configuration shown in Fig. 1, we have

$$\langle\hat{\phi}_{0\alpha}|\hat{H}_{v_L}|\hat{\phi}_{0\beta}\rangle = \langle\hat{\phi}_{0\alpha}| -\hat{\sigma}^z\mathbf{p}\cdot\mathbf{v}_L|\hat{\phi}_{0\beta}\rangle = i\hbar v_L \int (u^{*}_{\alpha}, v^{*}_{\alpha}) \times \begin{pmatrix} \hat{\mathbf{y}}\cdot\vec{\nabla} & 0 \\ 0 & -\hat{\mathbf{y}}\cdot\vec{\nabla} \end{pmatrix}\begin{pmatrix} u_\beta \\ v_\beta \end{pmatrix} d^2r. \quad \text{(D1)}$$

For the step pair potential defined in Appendix A, we find that the differential operator $\vec{\nabla}$ acting on the eigenstate $u_\beta \propto f^{+}_{\beta}(r) \propto J_{\mu_\beta - 1/2}(k_{F\perp}r)$, $v_\beta \propto f^{-}_{\beta}(r) \propto J_{\mu_\beta + 1/2}(k_{F\perp}r)$ gives

$$\hat{\mathbf{y}}\cdot\vec{\nabla}u_\beta(\mathbf{r}) = \frac{k_{F\perp}}{2}\sin\theta(e^{i\theta}u_{\beta-1} - e^{-i\theta}u_{\beta+1}) - \frac{k_{F\perp}}{2i}\cos\theta(e^{i\theta}u_{\beta-1} + e^{-i\theta}u_{\beta+1}) + O\left(\frac{\Delta_\infty}{E_{F\perp}}\right). \quad \text{(D2)}$$

Here the Bessel function identity $J'_\beta(r) = \frac{1}{2}[J_{\beta-1}(r) - J_{\beta+1}(r)]$ has been used to derive the term proportional to $\sin\theta$. Another Bessel function identity $(\beta/r)J_\beta(r) = \frac{1}{2}[J_{\beta-1}(r) + J_{\beta+1}(r)]$ is used to derive the term proportional to $\cos\theta$. The effect of the differential operator acting on $v_\beta$ can be obtained simply by replacing $u_\beta$ by $v_\beta$ in the above result. This relation was first derived by Hsu,[12] who assumed the quasiparticle distribution functions $u_\alpha$ and $v_\alpha$ to be Bessel functions through the whole range of $r$. Using the relation in Eq. (D2), we find the matrix element

$$\langle\hat{\phi}_{0\alpha}| -\hat{\sigma}^z\mathbf{v}_L\cdot\mathbf{p}|\hat{\phi}_{0\beta}\rangle \propto \delta_{\mu_\alpha,\mu_\beta\pm 1}\int \{[f^{+}_{\beta\pm 1}(r)]^2 - [f^{-}_{\beta\pm 1}(r)]^2\} r\,dr \simeq 0. \quad \text{(D3)}$$

By using similar method, we also determine

$$\langle\hat{\phi}_{0\alpha}|\mathbf{v}_L\cdot\mathbf{p}|\hat{\phi}_{0\beta}\rangle \simeq \delta_{\mu_\alpha,\mu_\beta\pm 1}\,\tfrac{1}{2}\,\hbar v_L k_{F\perp}. \quad \text{(D4)}$$

This matrix element is identical to that of $\hat{H}_\theta$ except for the numerical coefficient $\frac{1}{2}$.

The matrix element of $\hat{H}_{A1}$ can be rewritten for an isolated vortex with $\mathbf{A}(\mathbf{r}) = -\frac{1}{2}rH\hat{\boldsymbol{\theta}}$,

$$\hat{H}_{A1} = \frac{1}{2}\omega_c t v_L \begin{pmatrix} \hat{\mathbf{x}}\cdot\mathbf{p} & 0 \\ 0 & \hat{\mathbf{x}}\cdot\mathbf{p} \end{pmatrix}, \quad \text{(D5)}$$

where $\omega_c$ is the cyclotron frequency at $H$, the magnetic field at the core. From Eq. (D4), we find that $\langle\hat{\phi}_{0\alpha}|\hat{H}_{A1}|\hat{\phi}_{0\beta}\rangle$ is smaller than $\langle\hat{\phi}_{0\alpha}|\hat{H}_\theta|\hat{\phi}_{0\beta}\rangle$ by a factor of $\omega_c\tau$ if $t$ is replaced by the proper relaxation time $\tau$.

Using $\mathbf{v}_L\cdot\mathbf{A}=\frac{1}{2}Hv_L r\cos\theta$ for the magnetic vector field of the isolated vortex, the matrix element $\langle\hat{\phi}_{0\alpha}|\hat{H}_{A2}|\hat{\phi}_{0\beta}\rangle$ can be written as

$$\langle\hat{\phi}_{0\alpha}|\hat{H}_{A2}|\hat{\phi}_{0\beta}\rangle=\frac{eHv_L}{2c}\int[u_\alpha^*(r)u_\beta(r)r\cos\theta+v_\alpha^*(r)v_\beta(r)r\cos\theta]d^2r. \quad \text{(D6)}$$

From the angular integration about $\theta$, we obtain the selection rule $\delta_{\mu_\alpha,\mu_\beta\pm1}$. But the radial integration makes $\langle\hat{\phi}_{0\alpha}|\hat{H}_{A2}|\hat{\phi}_{0\beta}\rangle$ zero since

$$\langle\hat{\phi}_{0\alpha}|\hat{H}_{A2}|\hat{\phi}_{0\beta}\rangle\propto\int[f^+_{\beta+1}(r)f^+_\beta(r)+f^-_{\beta+1}(r)f^-_\beta(r)]r^2dr\simeq0. \quad \text{(D7)}$$


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