Anomalous dissipation near T_λ under a large heat flux

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We report on thermal transport experiments in liquid ^{4}He near T_λ using heat fluxes $8 \leqslant Q \leqslant 55$ μW/cm^2. We have confirmed the presence of a region near the superfluid transition, reported by Liu and Ahlers [Phys. Rev. Lett **76**, 1300 (1996)], in which thermal dissipation is anomalously small. The temperature transients for reaching a steady state upon entering this region from the superfluid side or cooling back into the superfluid have been studied, and are found to be quite different from each other; one possible explanation for this behavior is proposed which implies that the region of anomalous dissipation has a low thermal diffusivity. We discuss the location of this region in the phase diagram of liquid ^{4}He. [S0163-1829(98)01201-6]

I. INTRODUCTION

Recently Liu and Ahlers[1,2] reported the observation of a region of dissipation close to the superfluid transition temperature for ^{4}He, T_λ, at saturated vapor pressure. Using nanoKelvin resolution thermometry and a standard thermal conductivity cell, the authors applied heat to the bottom of a superfluid helium layer and quasistatically ramped the temperature at the top of the layer, $T_{\rm top}$, from below T_λ to a temperature in the normal phase. The finite thermal conductivity of the fluid led to a substantial temperature difference across the layer once thermal dissipation set in; the local thermal conductivity of the fluid was integrated across the cell to obtain the predicted temperature drop across the layer for comparison with the experimental data. Liu and Ahlers found that their experimental data just above the onset of dissipation did not agree with those expected from the thermal conductivity measured in the limit of zero heat; furthermore, they observed the onset of dissipation at a lower temperature than predicted for $T_\lambda(Q)$, a temperature they labeled $T_c(Q)$. (In this paper, we will use the same notation.) In light of these observations, Liu and Ahlers proposed the existence of a dissipative region in the superfluid phase with a larger thermal conductivity than that of the normal phase, with a width which increases with Q and vanishes as $Q \rightarrow 0$.[2]

Stimulated by these results, we have measured the thermal resistance of a helium layer at saturated vapor pressure very close to T_λ, under an applied heat flux Q with values between 8 and 55 μW/cm^2. Our experimental procedure was designed to measure transients and relaxation times in addition to the steady-state temperature difference across the fluid.

II. EXPERIMENT

Our experimental cell consists of two OFHC (oxygen-free high conductivity) (copper) endplates separated by a stainless-steel wall, with a gap of $h = 0.108 \pm 0.004$ cm. Germanium thermometers with a nominal resolution of 0.3 μK are embedded in each of the plates (bottom and top), allowing the temperature of each plate, as well as the difference $\Delta T = T_{\rm bot} - T_{\rm top}$, to be measured. The temperature of the top plate, $T_{\rm top}$, measured by a third thermometer, is regulated to within 0.5 μK of the desired value. This cell, with a symmetric design about the center of the fluid layer, has been described previously.[3]

We begin a measurement with the entire fluid layer in the superfluid phase, and apply a constant heating power to the bottom plate. The large (nearly infinite) thermal conductivity of the superfluid ensures that the temperature of the fluid layer is uniform, although the boundary resistance R_b between copper and superfluid produces a nonzero $\Delta T = 2R_bQ$ across the cell. The temperature $T_{\rm top}$ is raised to an initial temperature $T_0(Q)$ which is within approximately 3 μK of the temperature where thermal dissipation first occurs. Once ΔT has reached an equilibrium value $\Delta T_0(Q)$, which occurs a few seconds after the top plate temperature has been fixed at T_0, $T_{\rm top}$ is increased instantaneously to $T_0 + \delta T_{\rm top}$, where $1 < \delta T_{\rm top} < 25$ μK, so that the fluid layer is now in the dissipative region [See Fig. 1(top).] (Because we only situated T_0 relative to the temperature of the dissipation onset to within 3 μK for a given data sequence, we only know the position of $\delta T_{\rm top}$ relative to that temperature to within ± 1 μK. The values for $\delta T_{\rm top}$ which appear in the figures below are shifted by varying amounts so that dissipation first occurs at $\delta T_{\rm top} = 0$.) The finite thermal conductivity κ of the fluid leads to a temperature difference $\Delta T_{\rm obs}(Q)$ across the layer, the steady-state value of which is reached with a time constant τ which is a function of $\delta T_{\rm top}$ and Q; for typical values of $\delta T_{\rm top}$ and Q used in our experiments, $\Delta T_{\rm obs}(Q) \gg \Delta T_0(Q)$. The temperature of the top plate is then returned to the initial temperature T_0 just below the transition and $\Delta T(Q)$ returns to its original value $\Delta T_0(Q)$. Figure 1 (bottom) illustrates the process described above for three values of $\delta T_{\rm top}$. Later, we will refer to steps from the superfluid phase into the region with dissipation as having $\delta T_{\rm top} > 0$, and those in the reverse direction as having $\delta T_{\rm top} < 0$.

We emphasize that Q is defined as the heat flux through the fluid layer. In the superfluid state, the power generated in the heater passes entirely through the fluid layer, since the superfluid has an infinite thermal conductivity; in the state with a finite conductivity, κ, a sizeable fraction of the total power passes through the stainless-steel wall. The heat flux Q passing through the fluid is a function of $\Delta T_{\rm obs}$ after taking into account the thermal resistance of the stainless-steel

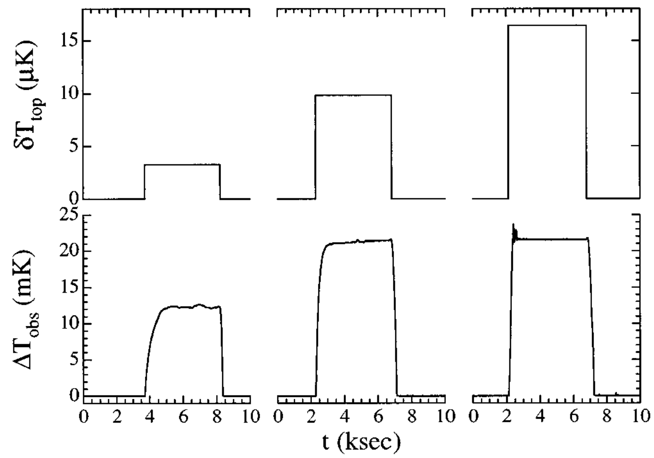

FIG. 1. Representation of data taking procedure, with results for $\delta T_{\mathrm{top}} = 3.0$, 10.0, and 16.6 μK for $Q = 45$ μW/cm^2. Top: The temperature step δT_{top} of the top thermometer, starting from $T_0(Q)$. Bottom: the observed temperature difference ΔT_{obs} versus time, showing the asymmetry of the transients. The overshoot recorded at the top of the third trace is an electronic effect due to the large temperature steps.

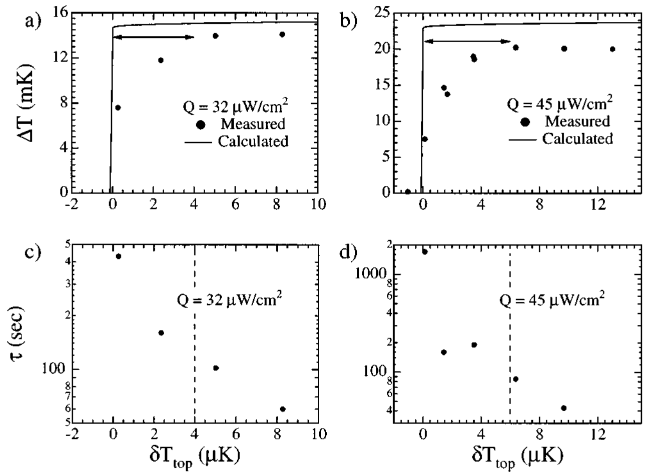

FIG. 2. (a and b): ΔT plotted versus δT_{top} for $Q \approx 32$ and 45 μW/cm^2. Symbols represent the measured data, solid lines the predicted behavior. (The arrows indicate the width of the anomalous region; see text). (c and d): The relaxation times τ versus δT_{top} for the same values of Q as in (a and b). The vertical dashed lines mark the temperature step $\delta T^*_{\mathrm{top}}$.

wall. During a sequence of steps δT_{top}, where the heater power is kept constant, Q is a function of δT_{top} until a plateau region, with ΔT_{obs} approximately constant, is reached, as described in Sec. III A. For the figures in that section, we have labeled the curves with the average value of Q.

Because the thickness $h = 0.108$ cm of our fluid layer did not permit the installation of an additional thermometer to measure the temperature of the superfluid layer, we were unable to determine directly $\delta T_c(Q) = T_\lambda(Q=0) - T_c(Q)$ as a function of Q. To estimate $\delta T_c(Q)$, we relied on our cell's high degree of symmetry to calculate the temperature of the fluid layer $\overline{T}$, using the temperature drop $\Delta T_0(Q)$ between the top and bottom plates, and assuming that $\overline{T} = T_0 + \Delta T_0/2$. Over the range of heat fluxes used in our experiment, $0 < Q < 55$ μW/cm^2, we obtained $\delta\overline{T_c}(Q) = \overline{T_\lambda}(Q = 0) - \overline{T_c}(Q) = 0 \pm 1$ μK, where the bar over the symbol T indicates that the quantity was not measured directly. Correcting $\delta\overline{T_c}(Q)$ to obtain $\delta T_c(Q)$, the actual depression of the transition temperature, is uncertain because of several factors, some of them difficult to estimate. For example, the cell is not perfectly symmetric about its middle. Also, there may be an asymmetry in R_b resulting from its divergence at T_λ.[4] Finally, there are small nonlinear effects from unknown sources (see Ref. 3, Fig. 3).

III. RESULTS

As mentioned before, our experiment allowed us to measure a steady-state property of the system, $\Delta T_{\mathrm{obs}}(Q, \delta T_{\mathrm{top}})$, as well as transient times for the system to reach a steady state. These transients followed an increase and, after reaching a steady state, a decrease in the temperature of the top plate by δT_{top}. The steady-state measurements, $\Delta T_{\mathrm{obs}}(Q, \delta T_{\mathrm{top}})$, can be directly compared to the results of Liu and Ahlers,[1] and will therefore be discussed first.

A. Steady-state data

Heat fluxes in our experiment ranged from 8 to 55 μW/cm^2 and led to temperature differences across the fluid of 2 to 23 mK in the dissipative phase. In Figs. 2(a) and 2(b) the observed temperature difference across the fluid layer, ΔT_{obs}, is plotted versus δT_{top} for two representative heat fluxes, $Q \approx 32$ and 45 μW/cm^2. As can be seen in the figure, ΔT_{obs} rises at first with increasing δT_{top} until it reaches a constant value when $\delta T_{\mathrm{top}} > \delta T^*_{\mathrm{top}}(Q)$; we will refer to region where ΔT_{obs} is approximately constant as being "saturated." The solid line that rises sharply for $\delta T_{\mathrm{top}} > 0$ and becomes horizontal very close to the onset of dissipation is the expected behavior, calculated from Eq. (1) (see below). Liu and Ahlers,[1,2] who slowly ramped T_{top} up and down, have already presented such results with, owing to their superior temperature control, more tightly spaced data points than in our experiments. Their results agree quantitatively with our data when compared at similar heat fluxes.

B. Transient data

Our measurement technique was designed so that we could observe the temperature difference across the fluid layer progressing towards a steady state after a temperature increase or decrease of the top plate, δT_{top}; typical examples of the transient process are shown in Fig. 1 (bottom). Two features of the relaxation curves are striking. First of all, the relaxation rates depend on the size of δT_{top}. Secondly, there is an asymmetry in the rates between steps from the dissipationless phase into the dissipative phase and those back into the superfluid.

In Figs. 2(c) and 2(d), two data sets of relaxation times τ for the same values of Q as in Figs. 2(a) and 2(b) are shown versus δT_{top}, where $\delta T_{\mathrm{top}} > 0$. In our experiment the thermal diffusivity varies greatly with position and time due to the large temperature gradients in the normal phase, and as a result the equilibration process is expected to be complex. However, we found that once the transient to the steady state

ΔT_{obs} had decreased to approximately half its original value, a simple exponential with a characteristic time τ represented the transient data quite well. For $\delta T_{\mathrm{top}}>0$, the relaxation time τ is longest for the smallest temperature step and decreases with increasing δT_{top}. As a rough guide, a steady state is reached after a time $t\approx 5\tau$.

For $\delta T_{\mathrm{top}}<0$, a steady state is reached after a time t_{back} following a change in the temperature of the top plate, which should be compared quantitatively with 5τ. The transients for steps from the dissipative phase to the superfluid phase are not exponential, but rather appear nearly linear, ending abruptly when the final ΔT_0 is reached. The time t_{back} is of the order of 3×10^2 s and, in contrast to τ, varies only slightly with small changes in δT_{top}. For a given constant heat flux, t_{back} tends to increase with the temperature difference ΔT_{obs} across the fluid layer.

IV. DISCUSSION

In this section we will discuss the observations described above, their implications, and also the location of the new regime of anomalous dissipation in relation to $T_\lambda(Q=0)$.

A. Steady-state data

The steady-state measurements described in this paper cannot be directly compared to theoretical predictions. The physical quantities of interest diverge near T_λ, and the large temperature differences across the cell cause these quantities to vary strongly with position in the layer. The predicted final temperature difference, $\Delta T_{\mathrm{calc}}=T_b-T_t$, has been calculated by integrating κ measured with low heat fluxes[6–8] across the fluid layer with temperatures T_b and T_t at the bottom and top of the layer. (Note that T_b and T_t differ from T_{bot} and T_{top} in that they do not include the temperature difference due to R_b.) We used the relation

$$Qh=\int_{T_t}^{T_b}\kappa(0,T)dT, \tag{1}$$

which is Eq. (1) of Ref. 1, assuming that the transition temperature we observed corresponds to $T_\lambda(Q)$. This integration gives the horizontal dashed line in Fig. 2(b) of Ref. 1 and will be used below in the analysis of our experiments.

We have made several assumptions in using Eq. (1) to obtain ΔT_{calc}. This equation is, in principle, only applicable above $T_\lambda(Q=0)$, the superfluid transition temperature in the limit of zero heat, and it needs to be modified for the situation of a nonzero Q. Haussmann and Dohm[9] have calculated $\kappa(Q)$ in the nonlinear regime near $T_\lambda(0)$, and they show that it does not diverge at this temperature, but rather approaches a constant value which depends upon the heat flux. At present, there are no predictions for $\kappa(Q)$ below $T_\lambda(0)$. We follow Liu and Ahlers[1] in extrapolating $\kappa(Q)$ above $T_\lambda(Q=0)$ to $T_c(Q)$, the temperature of the onset of dissipation, rather than the theoretical prediction of Haussmann and Dohm.[10] This analysis assumes that both $\kappa(Q)$ and its slope are continuous at $T_\lambda(0)$, whereas it is possible that either exhibits a discontinuity at this temperature.

The results of our analysis are shown as the solid lines in Figs. 2(a) and 2(b) for two representative heat fluxes; the solid circles are the data. Comparison of the data and the calculations show two distinct regions. Where the data are constant versus δT_{top}, they parallel the calculated curves; we call this region the ''saturated'' one. Closer to the onset of dissipation, $\Delta T_{\mathrm{obs.}}$ varies strongly with δT_{top}, and the data and the calculations are qualitatively very different; this is the region identified by Liu and Ahlers as having anomalous dissipation.[1,2]

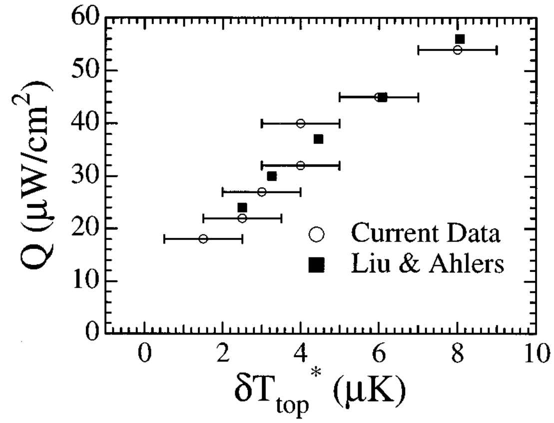

FIG. 3. The map of the region with the anomalous transport properties, expressed by a plot of $\delta T^*_{\mathrm{top}}(Q)$ versus Q. Open circles: current data. Solid squares: data by Liu and Ahlers (Ref. 2).

1. Anomalous region

The region of anomalous dissipation is characterized by its width, $\delta T^*_{\mathrm{top}}(Q)$, indicated by the arrows in Figs. 2(a) and 2(b). As Liu and Ahlers observed,[1,2] this width grows with Q, and goes to zero as the heat flux does. In Fig. 3 we plot Q versus the width of this region, along with the data measured by Liu and Ahlers in the same Q range.[2] Although the temperature control of our top plate was inferior to that used by Liu and Ahlers, the data measured in the two experiments agree quite well.

We have not repeated the fit to the data in the anomalous regime using ''model 3'' of Liu and Ahlers,[1,2] although it will be used in the discussion of our transient data.

2. ''Saturated'' region

For $Q=32\ \mu\mathrm{W/cm^2}$ there is a discrepancy of 7% between the data and the calculations in the measured ''saturated'' value of ΔT, which can be accounted for by the uncertainty in the height h of the fluid layer, as will be discussed below. The difference between our data and predictions for $Q=45\ \mu\mathrm{W/cm^2}$, however, is larger than can be attributed to the error for h.

In analyzing our data, we must consider the effect of convection. Liquid helium at saturated vapor pressure has a minimum in its molar volume at a temperature $T_{\mathrm{min}}\approx T_\lambda+6$ mK, so that the thermal-expansion coefficient, α_p, is negative below this temperature and positive above it.[11] When the fluid layer is heated from below, convection cannot take place so long as the temperature of the entire fluid layer is less than 6 mK above T_λ. In our experiment, the temperature at the top of the fluid layer is within a few microdegrees of T_λ. Assuming that the zero-heat thermal conductivity data $\kappa(0)$ can be used, and recognizing that in the

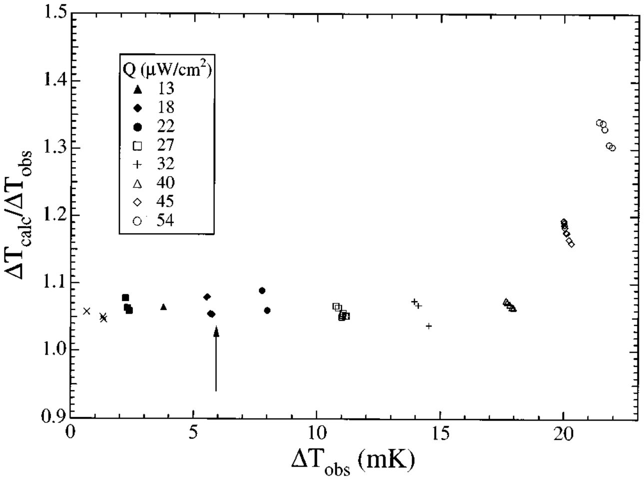

FIG. 4. $\Delta T_{\rm calc}/\Delta T_{\rm obs}$ plotted versus $\Delta T_{\rm obs}$, showing the absence of convection for $\Delta T_{\rm obs} \leqslant 18$ mK and its onset above 18 mK (or $Q > 40$ μW/cm^2). Data points shown by the same symbols were taken for different values of $\delta T_{\rm top}$.

presence of a heat flow κ is a function of position in the fluid layer, we calculate that for $Q \geqslant 20$ μW/cm^2 the temperature at the bottom of the fluid layer will rise above $T_{\rm min}$. However, larger heat flows than ≈ 20 μW/cm^2 can be used before the onset of convection. A fluid layer with a uniform, positive thermal-expansion coefficient will be stable so long as a critical temperature difference is not exceeded, and in our experiment only a small portion of the cell will have a positive thermal-expansion coefficient when the temperature of the bottom of the cell is just above $T_{\rm min}$. Our experiment then determines the critical value Q_c at which convection first occurs, where the temperature at the top of the fluid layer is kept a few microdegrees above T_λ.

In order to get a more comprehensive picture of the ''saturated'' regime, we plot in Fig. 4 the ratio $R \equiv \Delta T_{\rm calc}(Q)/\Delta T_{\rm obs}(Q)$ versus $\Delta T_{\rm obs}(Q)$ for several values of Q; the vertical arrow indicates the temperature beyond which α_p at the bottom of the layer becomes positive. The spatial extent of the fluid region at the bottom of the cell with $\alpha_p > 0$ increases with $\Delta T_{\rm obs}$. For $\Delta T_{\rm obs} \leqslant 17$ mK, the ratio R is constant and approximately equal to 1.07, indicating that convection is absent, while the progressive increase of R for $\Delta T_{\rm obs} > 17$ mK signals the onset of convection, which corresponds to a critical heat flow $Q_{\rm c} = 40$ μW/cm^2. The spread of data points at a given Q in the convective regime shown in Fig. 4 occurs because $\Delta T_{\rm obs}$ decreases slightly with increasing $\delta T_{\rm top}$. We suspect that this effect is the result of the increasing fraction of the fluid layer with a positive thermal-expansion coefficient; i.e., a greater portion of the fluid layer convects as $\delta T_{\rm top}$ increases. (The expected value of R in the nonconvecting region is 1.00, and can be obtained by adjusting the cell height h by an amount within its experimental uncertainty. Upon decreasing h by 3% from its nominal value of 0.108 cm, we obtain $R = 1.00$. We note that $\Delta T_{\rm calc}$ depends on h in a nonlinear fashion.)

B. Transient data

We now return to the relaxation time data upon increasing the temperature from the superfluid phase to the dissipative phase, presented in Figs. 2(c) and 2(d). The vertical dashed lines indicate the boundaries between the anomalous and the normal regime estimated from the steady-state measurements for these two values of Q. The relaxation times begin to increase at approximately the same point at which the anomalous region begins, suggesting that the large values of τ are associated with this region. (The times required for reaching a steady state when decreasing the temperature of the fluid layer from the dissipative phase to the superfluid phase were much shorter than that of the reverse and did not depend strongly upon $\delta T_{\rm top}$, as mentioned above.)

The long relaxation times are unlikely to be the result of thermal relaxation processes in the normal fluid. For comparison, in the limit of small Q the relaxation time in the normal phase of ^{4}He for a cell with a similar fluid layer height h is of the order of 20 s when $(T - T_\lambda) \approx 10$ mK (see Fig. 19 of Ref. 5). Calculating the equilibration time based on the total amount of heat required to raise the temperature from near $T_\lambda(Q=0)$ to the final temperature difference across the layer $\Delta T_{\rm obs}$, is complicated. As a rough estimate, we assume a constant specific heat of 70 J/mole K corresponding to the measured value 1 μK above $T_\lambda(Q=0)$;[12] this choice was based on the resolution of our temperature control. From the known dimensions of the fluid layer, the time required to raise the temperature of the entire fluid layer by 14 mK above $T_\lambda(0)$ is approximately 120 s if a heat flux $Q = 32$ μW/cm^2 is applied to the layer; this time is shorter than those shown in Figs. 2(c) and 2(d). Our calculation also overestimates the time, as most of the fluid layer has a smaller heat capacity than what is assumed above, and only the bottom of the fluid layer is raised to the final temperature difference.

Note added in proof. One of us (D.M.) has recently simulated the time-dependent response of a layer of normal fluid helium to a large heat flux, including nonlinear effects due to the divergence of the thermal conductivity and heat capacity near T_λ. The temperature of the fluid layer is initially uniform and equal to $T_\lambda + 1$ nK. At a time $t = 0$, the temperature of one side of the layer is raised to a fixed value of a few μK above T_λ and a heat flux $Q = 45$ μW/cm^2 is applied to the other side of the layer. We find that the transient response of the temperature difference across the fluid layer is exponential, and that the characteristic relaxation time τ is approximately 19 s for $h = 0.108$ cm. The relaxation time depends only weakly on the temperature at which the cold side of the layer is fixed, in contrast to the extreme sensitivity of τ, observed in our experiments, to small variations of $\delta T_{\rm top}$. We therefore conclude that the long relaxation times we observed are not associated with thermal relaxation processes in the normal phase of ^{4}He.

In an attempt to understand the asymmetry in the temporal response and the large values of τ, we calculated the steady-state temperature profile in the cell for $Q = 45$ μW/cm^2 using a model developed by Liu and Ahlers[1,2] for different values of $\delta T_{\rm top}$, and the results are shown in Fig. 5. The model used is that expressed in Eq. (3) of Refs. 1,2 with the parameters $\lambda_0(Q) = 5 \times 10^{-4}$ W/cmK (taken from Fig. 1 of Ref. 1) and the exponent $y = 0.44$.

On the basis of the temperature profiles in Fig. 5, we interpret this transient asymmetry as follows: Immediately after a step $\delta T_{\rm top} > 0$, the fluid at the top of the layer passes

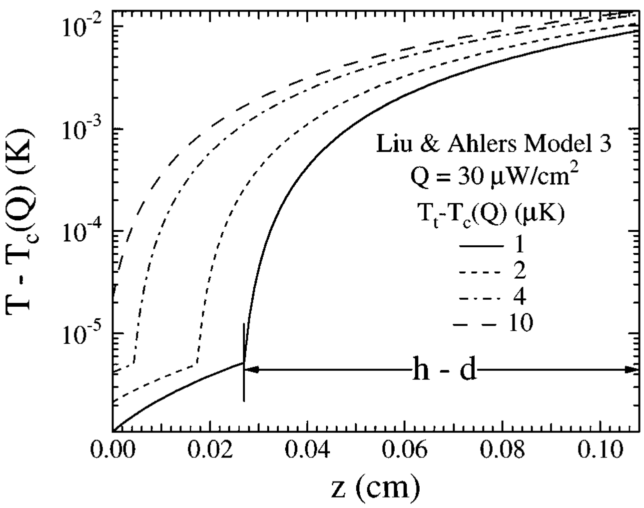

FIG. 5. The calculated steady-state vertical temperature profile in the fluid layer for $Q=32\ \mu\mathrm{W/cm}^2$ and for several values of δT_{top}. Here $z=0$ is the top (colder side) of the fluid layer. The calculations are based on the results obtained by Liu and Ahlers for a cell with similar height as used in the present experiment. The lengths d and $(h-d)$ define the anomalous and the normal fluid regions.

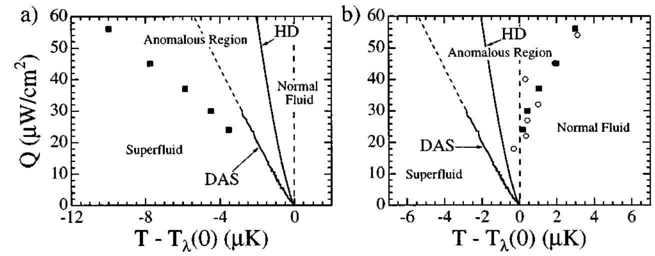

FIG. 6. $Q-T$ phase diagram of liquid ^{4}He near $T_\lambda(0)$. (a) Diagram as proposed by Liu and Ahlers (Ref. 2). Solid squares: width $\delta T^*_{\mathrm{top}}(Q)$ measured by Liu and Ahlers subtracted from the calculated curve by Haussmann and Dohm (HD). The curve labeled DAS represents the measurements of $T_c(Q)$ by Duncan, Ahlers, and Steinberg up to $Q=30\ \mu\mathrm{W/cm}^2$ and the dashed line is a linear extrapolation to higher Q. (b) Alternative suggestion for phase diagram, where the $\delta T^*_{\mathrm{top}}(Q)$ has been added to the DAS curve. Solid squares: width $\delta T^*_{\mathrm{top}}(Q)$ measured by Liu and Ahlers; open circles, present data.

into a ''low dissipation'' regime, which, in spite of a large thermal conductivity, appears to have a small diffusivity. The amount of fluid in this regime has a width d in a steady-state, while the rest of the layer, of thickness $(h-d)$, behaves like ''ordinary'' normal fluid. We speculate that the region of width d is responsible for the slow relaxation rate close to T_λ. As δT_{top} increases, d in the steady-state decreases, and therefore the relaxation rate increases. When δT_{top} approaches the critical value $\delta T^*_{\mathrm{top}}$, d tends to 0, and the relaxation rate of the layer approaches — but remains larger than — that estimated for the thermal diffusivity of the normal phase, ≈ 20 s. In the reverse operation (i.e., $\delta T_{\mathrm{top}} < 0$) the top of the fluid layer is converted back into the superfluid phase immediately after the step; the width of the superfluid layer expands rapidly because its equilibration time is very short. Recent theories by Haussmann and Dohm[13,14] and by Chui *et al.*[15] predict a strong dynamical divergence of the heat capacity as the transition $T_\lambda(Q)$ is reached from the superfluid side. This might account[4] for the surprisingly low diffusivity for small steps δT_{top} into the dissipative region.

C. Location of new region

The question arises as to where the region of low dissipation and anomalous relaxation times is located with respect to the superfluid transition T_λ and the temperature of the onset of dissipation, $T_c(Q)$. Theories of the depression of T_λ under heat flow were developed by Onuki[16] and by Haussmann and Dohm.[10]

As in the present experiment, the absolute temperature of the onset of dissipation was not measured directly by Liu and Ahlers, since they did not use a midplane thermometer.[1,2] Instead, they relied upon previous direct measurements of $T_c(Q)$ by Duncan, Ahlers, and Steinberg[17] (DAS) who did use a midplane thermometer. These measurements of $T_c(Q)$ have subsequently been confirmed by Moeur *et al.*[18] for $Q < 6.5\ \mu\mathrm{W/cm}^2$, using a very different experimental procedure from that of DAS. Liu and Ahlers[2] have proposed that the new region is located between the transition curve experimentally determined by DAS and the spinodal-like line, calculated by Haussmann and Dohm,[10] and have reached this conclusion based on their results at heat fluxes $Q < 10\ \mu\mathrm{W/cm}^2$ where the width of the anomalous region is smaller than $\delta T_c = T_\lambda(Q=0) - T_c(Q)$, the measured depression of the transition temperature.[17] In Fig. 6(a) we present their proposed phase diagram, together with the transition curve measured by DAS with data up to 30 $\mu\mathrm{W/cm}^2$ (obtained from the thesis of Duncan[19]), and the spinodal-like line predicted by Haussmann and Dohm[10] labeled HD. The data points for the phase diagram of Liu and Ahlers are calculated by subtracting the width of the anomalous region from the theoretical predictions of Haussmann and Dohm. This choice was justified by the agreement between their calculation and the measured values of $T_c(Q)$ by Duncan *et al.* for $Q < 10\ \mu\mathrm{W/cm}^2$.[1] Figure 6(a) indicates that this agreement breaks down at larger values of Q.

We now examine the data of DAS for $Q > 10\ \mu\mathrm{W/cm}^2$ in relation to those by Liu and Ahlers. At higher heat fluxes, Duncan *et al.* report that in the superfluid phase there was a small bulk temperature gradient, possibly due to mutual friction. This effect was held responsible for the deviation of $\delta T_c(Q)$ from the power law $\delta T_c \propto Q^{0.81}$. (see Fig. 5-4 and related text in Ref. 19). Between $Q=10$ and 30 $\mu\mathrm{W/cm}^2$, the exponent increased and the dependence of T_c on Q became nearly linear. The dashed line in Fig. 6(a) is simply an extrapolation of this linear plot. It is unlikely that mutual friction effects at $Q > 30\ \mu\mathrm{W/cm}^2$ would bend this line to the left, i.e., towards lower temperatures.[4] Duncan *et al.* claim that their data for the transition overestimate, rather than underestimate, the depression of $T_c(Q)$ on account of mutual friction.[17] If this analysis is correct, our extrapolation for $T_c(Q)$ would be shifted higher in temperature, not lower. Moreover, the difference between $T_c(Q)$ calculated by Liu and Ahlers and the value measured by DAS is approximately 1.5 μK at $Q=30\ \mu\mathrm{W/cm}^2$, whereas the effect of mutual friction at this heat flux is approximately 0.1 μK.[17] We therefore conclude that the curve $T_c(Q)$ as proposed by Liu and Ahlers[2] is located significantly lower in temperature than the transition line measured by DAS for Q above $\approx 10\ \mu\mathrm{W/cm}^2$.

We propose an alternative phase diagram to that of Liu and Ahlers. Rather than combining theoretical predictions with experimental data, our analysis relies solely on the latter. We also assume that the transition temperature measured by DAS corresponds to the onset of the low-dissipation phase. However, to obtain the transition from the low-dissipation phase to the normal phase, we add the width of the anomalous region, $\delta T^*_{\mathrm{top}}(Q)$, to the experimental data of DAS over the entire range of Q. The results are shown in Fig. 6(b). The open circles are calculated from our data for $\delta T^*_{\mathrm{top}}$, while the solid squares are again from the data of Liu and Ahlers.[2] When $Q<30\ \mu\mathrm{W/cm}^2$, where there are direct measurements by DAS, $[T_c(Q)+\delta T^*_{\mathrm{top}}]$ is very close to $T_\lambda(Q=0)$, and higher in temperature than the spinodal-like line predicted by HD. Where the DAS curve has been extrapolated beyond $Q>30\ \mu\mathrm{W/cm}^2$, this scheme suggests that the region of anomalous dissipation extends into the normal phase. It is possible that an extension of measurements of $T_c(Q)$ to larger heat fluxes will show that our extrapolation is incorrect, and for this reason such measurements are very much needed to determine the location of the region of anomalous dissipation.

Our alternate scheme for the location of the low dissipation regime creates several problems, however. First is the location of $T_c(Q)$ in the normal phase as mentioned above. Second, in the normal phase, the large heat current should lead to a smaller thermal conductivity in the nonlinear region, as has been calculated in detail by Haussmann and Dohm.[9] The anomalous region that is the subject of this paper has a larger thermal conductivity than does the normal phase of ^{4}He. Haussmann's and Dohm's calculations do not contradict the alternate phase diagram for $Q\leqslant 25\ \mu\mathrm{W/cm}^2$, but they are inconsistent with it at larger heat fluxes where direct measurements of $T_c(Q)$ are lacking. Second sound measurements, as suggested by Duncan,[4] would be the most convicing probe for determining whether the low dissipation region is in the superfluid phase.

V. CONCLUSION

We have confirmed the existence of a region of anomalous dissipation in ^{4}He under a heat flux Q, observed by Liu and Ahlers. For sufficiently large values of Q we have observed the onset of convection in the normal phase. Long relaxation times τ were observed when the anomalous region is entered from the dissipationless superfluid side, while the reverse process is completed abruptly in a sharply defined time t_{back} which is much shorter than the transient time $(\approx 5\tau)$ for entering the anomalous regime. We have proposed a qualitative explanation of this asymmetry in the transients, which relies on the region of anomalous dissipation having a very small thermal diffusivity.

We also have discussed the location of this region of anomalous behavior. An alternative suggestion to that made by Liu and Ahlers has been presented, because their phase diagram is inconsistent with experiments by Duncan *et al.* at heat fluxes above $\approx 10\ \mu\mathrm{W/cm}^2$. In this alternative scheme the anomalous dissipation region extends from the transition curve $T_c(Q)$, determined experimentally by Duncan *et al.*, towards higher temperatures. It appears to extend to $T_\lambda(Q=0)$, and possibly even into the normal phase, which would be in conflict with theories of the normal fluid under a heat flow. Resolution of this issue requires further direct measurements of the absolute temperature of transition curve $T_c(Q)$ at large heat fluxes for comparison with the width of the new dissipative region.

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