Susceptibilities in the region of the disorder-induced crossing resonance

V. A. Ignatchenko and M. V. Erementchouk
L. V. Kirensky Institute of Physics, Krasnoyarsk, 660036, Russia

L. I. Deych
Queens College of City University of New York, Flushing, New York 11367
(Received 7 March 1997)

We study the susceptibility matrix for the system of two wave fields of different nature coupled by a random coupling parameter with zero mean value. It is shown that average properties of the system can be considered within the concept of two effective media that can be introduced in the same real material in order to describe properties of the fields. These two media have different characteristics and are independent in the sense that each of the averaged wave fields propagates through its own effective medium without interaction with the partner wave field. Diagonal components of the susceptibility matrix demonstrate resonance peculiarities caused by an interaction of the averaged wave of one nature with fluctuation waves of another nature. Relations between positions of maxima of the susceptibilities and eigenfrequencies of the system are analyzed for different values of statistical characteristics of the coupling parameter and initial relaxation parameters of the corresponding waves. [S0163-1829(98)01001-7]

I. INTRODUCTION

Crossing resonance in homogeneous systems is a well-studied effect that occurs when initial dispersion laws of two coupled wave fields cross each other at a certain resonance point. The resonance interaction creates new mixed excitations with dispersion laws that avoid crossing at the resonance point. It is said sometimes to describe this situation that a gap between different branches at the resonant point opens up, though a real spectral gap might not appear. In Ref. 1 the effect of crossing resonance was considered in systems with a random coupling parameter, where the model of the disorder-induced crossing resonance (DICR) was introduced. Within this model the coupling parameter between the wave fields was assumed to be a random zero-mean function of coordinates so that the interaction only occurred due to spatial fluctuations of this parameter. The model is the extreme of the more general situation when both the mean value and fluctuations of the coupling parameter exist in a material. It is a convenient model that describes the influence of the disorder on crossing resonances in the most prominent way, and also related to some real situations. One can mention, for example, amorphous ferromagnets with a zero-mean magnetostriction or polaritons arising due to the coupling between electromagnetic waves and vibrations that would be dipole inactive in the absence of the disorder. In Ref. 1 the reconstruction of dispersion laws caused by fluctuations of the coupling parameter as well as accompanying decay of the averaged waves have been investigated. There were found a considerable qualitative difference between DICR and a crossing resonance in homogeneous medium. Both coupled wave fields in homogeneous medium are coherent and have a joint dispersion law that consists of two branches; depending on the value of a damping in the system, the gap between these branches in the resonance point can be opened or closed. In contrast to this, the mixed excitations in the DICR model consist of the coherent part of one of the wave fields and scattered waves of the other field. In this case each averaged wave field characterizes by its own dispersion law. The situation is possible, for instance, when the dispersion law of one averaged wave field has a gap in the resonance point, whereas the dispersion curve of the other field is continuous.

The model of DICR has been used to study magnetoelastic resonance in ferromagnets with zero-mean magnetostriction[2,3] and polariton resonance with dipole inactive phonons in disordered dielectrics.[4] It was shown that main characteristics of the stochastic magnetoelastic interaction can be measured if one experimentally studies either the modified dispersion law of acoustic waves[2] or the elastic analogs of Faraday and Cotton-Mutton effects.[3] Energy oscillations of scattered waves for DICR also have been considered.[5]

In this paper we consider the averaged susceptibility matrix of the system with DICR. Components of the matrix describe amplitudes and phases of excited waves as well as the amount of energy of the coherent wave absorbed or scattered by the media. Section II of the paper plays the auxiliary role; it deals with basics results regarding susceptibilities of the simple model of two oscillators coupled by a deterministic coupling parameter. We analyze the relationship between positions of resonance maxima of different components of the susceptibility and eigenfrequencies of the waves. The results of this section will be used in the next section where the more complicated situation of DICR in the system with magnetoelastic interaction will be examined.

II. SUSCEPTIBILITY OF THE SYSTEM OF TWO COUPLED OSCILLATORS

In this auxiliary section we present basic results on susceptibilities of the system of two linear coupled oscillators for different relations between their relaxation parameters and a coupling parameter. Most of these results have been discussed in many papers. We, however, present them here using those approximations and notations that are relevant to our consideration of the more complicated situation of randomly coupled waves.

The system of two coupled oscillators corresponding to the dimensionless fields ϕ and ψ of different physical nature excited by harmonic external forces $\propto e^{i\omega t}$ is described by the following equations:

$$(\omega^2-2i\omega\Gamma_a-\omega_a^2)\phi-\eta\psi=f_a,$$

$$(\omega^2-2i\omega\Gamma_b-\omega_b^2)\psi-\eta\phi=f_b, \tag{1}$$

where $\omega_{a,b}$ and $\Gamma_{a,b}$ are the initial frequencies and damping parameters of the corresponding oscillators, $f_{a,b}$ are the forces acting upon the corresponding oscillators, and η is the coupling parameter. All the values can depend upon some parameters, such as wave number, magnetic field, etc. We assume that under some circumstances the crossing resonance between these oscillators can occur

$$\omega_a=\omega_b=\omega_r. \tag{2}$$

Assuming that $\Gamma_{a,b}\ll\omega_{a,b}$ one can simplify Eqs. (1):

$$(\omega-\omega_a-i\Gamma_a)\phi-\frac{\lambda_a}{2}\psi=\Omega_a F_a,$$

$$(\omega-\omega_b-i\Gamma_b)\psi-\frac{\lambda_b}{2}\phi=\Omega_b F_b. \tag{3}$$

Here

$$\lambda_{a,b}=\frac{2\eta}{\omega+\omega_{a,b}},\quad \Omega_{a,b}F_{a,b}=\frac{f_{a,b}}{\omega+\omega_{a,b}}. \tag{4}$$

The system response to an external excitation can be described by the components of the susceptibility matrix $\hat{\chi}$

$$\phi=\chi_a F_a+\chi_{ab}F_b,\quad \psi=\chi_b F_b+\chi_{ba}F_a. \tag{5}$$

The diagonal components $\chi_{a,b}$ of the matrix describe the direct excitation of oscillators by their own forces, while off-diagonal components χ_{ab} and χ_{ba} are responsible for the indirect excitation of an oscillator by a force applied to its partner. The diagonal and off-diagonal susceptibilities can be found as

$$\chi_a=\frac{\Omega_a}{D}(\omega-\omega_b-i\Gamma_b),$$

$$\chi_b=\frac{\Omega_b}{D}(\omega-\omega_a-i\Gamma_a), \tag{6}$$

$$\chi_{ab}=\frac{\Omega_a}{D}\frac{\lambda_a}{2},\quad \chi_{ba}=\frac{\Omega_b}{D}\frac{\lambda_b}{2}.$$

where

$$D(\omega)=(\omega-\omega_a-i\Gamma_a)(\omega-\omega_b-i\Gamma_b)-\frac{\lambda^2}{4}, \tag{7}$$

and

$$\lambda^2=\lambda_a\lambda_b=\frac{4\eta^2}{(\omega+\omega_a)(\omega+\omega_b)}. \tag{8}$$

Imaginary parts of the susceptibilities determine the energy absorbed by the oscillators, absolute values of the susceptibilities are responsible for the oscillators' amplitudes. In the absence of the external forces the equation

$$D(\tilde{\omega})=0 \tag{9}$$

determines frequencies and dampings of the eigenmodes of the system. Here $\tilde{\omega}=\omega+i\xi$ is the complex frequency consisting of the eigenfrequency ω and the damping ξ. This equation was analyzed in many papers; a brief description of the results in the form relevant to the present work can be found in Ref. 1. Here we just recall that in the vicinity of the crossing resonance the form of the dispersion curves is determined by the relation between parameters λ_r and Γ, which are defined as follows:

$$\lambda_r=\frac{\eta}{\omega_r},\quad \Gamma=|\Gamma_a-\Gamma_b|. \tag{10}$$

If $\lambda_r>\Gamma$ a gap $\Delta\omega$ between different branches of the dispersion curve appears at the crossing point, and dampings for both branches $\omega_\pm$ of the spectrum become equal to each other:

$$\omega_\pm=\omega_r\pm\frac{1}{2}\Delta\omega,\quad \xi_\pm=\frac{1}{2}(\Gamma_a+\Gamma_b), \tag{11}$$

where

$$\Delta\omega=\sqrt{\lambda_r^2-\Gamma^2}. \tag{12}$$

In the opposite case $\lambda_r<\Gamma$ the branches keep crossing each other at the resonance point, while the dampings differ:

$$\omega_\pm=\omega_r,\quad \xi_\pm=\frac{1}{2}(\Gamma_a+\Gamma_b\pm\Delta\xi), \tag{13}$$

where

$$\Delta\xi=\sqrt{\Gamma^2-\lambda_r^2}. \tag{14}$$

When the ratio Γ/λ increases the dampings $\xi_\pm$ tend to their initial magnitudes Γ_a and Γ_b.

Among the diagonal components of the susceptibility we shall only consider the component χ_a, since the second one can be obtained by trivial substitution of the index $(a\leftrightarrow b)$. Let us present the imaginary part of the susceptibility in a form that is the most convenient for analyzing the relations between the positions of the maxima of the susceptibility and solutions of the dispersion equation (9):

$$\frac{\chi_a''}{\Omega_a}=\frac{1}{(\omega_+-\omega_-)^2+(\xi_+-\xi_-)^2}\times\Bigg\{[(\Gamma_b-\xi_-)(\omega_+-\omega_-)-(\omega_b-\omega_-)(\xi_+-\xi_-)]\frac{\omega-\omega_-}{(\omega-\omega_-)^2+\xi_-^2}$$
$$+[(\omega_b-\omega_-)(\omega_+-\omega_-)+(\Gamma_b-\xi_-)(\xi_+-\xi_-)]\frac{\xi_-}{(\omega-\omega_-)^2+\xi_-^2}+[(\xi_+-\Gamma_b)(\omega_+-\omega_-)-(\omega_+-\omega_b)(\xi_+-\xi_-)]$$
$$\times\frac{\omega-\omega_+}{(\omega-\omega_+)^2+\xi_+^2}+[(\omega_+-\omega_b)(\omega_+-\omega_-)+(\xi_+-\Gamma_b)(\xi_+-\xi_-)]\frac{\xi_+}{(\omega-\omega_+)^2+\xi_+^2}\Bigg\}. \tag{15}$$

Here $\omega_\pm$ and $\xi_\pm$ are real and imaginary parts of the solutions of the dispersion equation (9). The branches are designated in such a way that at $\omega_a \ll \omega_r$ signs "+" and "−" correspond to the frequencies of ψ and ϕ oscillators accordingly.

Let us consider Eq. (15) at the crossing point $\omega=\omega_r$. When the gap between branches of the dispersion curve is opened up ($\lambda_r>\Gamma$) it reads

$$\frac{2\chi_a''}{\Omega_a}=\frac{\xi}{(\omega-\omega_-)^2+\xi^2}+\frac{\Gamma_b-\Gamma_a}{\Delta\omega}\frac{\omega-\omega_-}{(\omega-\omega_-)^2+\xi^2}+\frac{\xi}{(\omega-\omega_+)^2+\xi^2}-\frac{\Gamma_b-\Gamma_a}{\Delta\omega}\frac{\omega-\omega_+}{(\omega-\omega_+)^2+\xi^2}, \tag{16}$$

where $\omega_\pm$ and $\xi=\xi_+=\xi_-$ are determined by Eqs. (11).

When $\Gamma_a=\Gamma_b$ the susceptibility is merely the sum of two resonances at frequencies that coincide with the solutions of the dispersion equation. However, in the more general situation there are two additional terms, which can considerably affect the dependence of χ'' on frequency. Let us assume for the sake of concreteness, that $\Gamma_b>\Gamma_a$. Then an increase of $\Gamma=\Gamma_b-\Gamma_a$ causes the maxima of χ_a'' to become closer to the position of the eigenfrequencies, and the maxima of χ_b'' to move apart from them. It should be recalled also that the eigenfrequencies themselves move toward each other when Γ is growing. These two facts taken together lead to the possibility for $\chi_a''(\omega)$ and $\chi_b''(\omega)$ to have a different number of maxima. When

$$\lambda_r^2>\Gamma_a'^2\equiv\frac{4\Gamma_b^3}{2\Gamma_b+\Gamma_a}, \tag{17}$$

$\chi_a''(\omega)$ demonstrates two maxima. The condition that $\chi_b''(\omega)$ also has two maxima takes quite a different form

$$\lambda_r^2>\Gamma_b'^2\equiv\frac{4\Gamma_a^3}{2\Gamma_a+\Gamma_b}. \tag{18}$$

Since $\Gamma_b'<\Gamma_a'$, it is possible that $\chi_b''(\omega)$ exhibits two maxima, while $\chi_a''(\omega)$ has only one (Fig. 1). It is worth mentioning that the same conditions (17), (18) also determine the number of the maxima when the gap is closed.

Now we consider the case when the difference between Γ_b and Γ_a is greater than λ_r and eigenfrequencies coincide in the crossing-resonance point ω_r. The most interesting picture can be observed for the susceptibility χ_b'' of the oscillator with the greater initial damping. In that case Eq. (15) takes the form

$$\frac{2\chi_b''}{\Omega_b}=\frac{1}{\Delta\xi}\left\{\frac{(\Gamma+\Delta\xi)\xi_+}{(\omega-\omega_r)^2+\xi_+^2}-\frac{(\Gamma-\Delta\xi)\xi_-}{(\omega-\omega_r)^2+\xi_-^2}\right\}, \tag{19}$$

where $\xi_\pm$ and $\Delta\xi$ are determined by Eqs. (13) and (14). The first term here presents the direct response of the ψ oscillator to the external force f_b, and the second one is due to the interaction between the oscillators. Since the inequality $\Gamma\geq\Delta\xi$ is always valid, we have the sum of two resonance curves, whose frequencies coincide, but their signs as well as widths are different. The situation is best demonstrated when $\Gamma\gg\lambda$. Then $\Delta\xi$ can be presented in the form $\Delta\xi\approx\Gamma-\sigma$, where $\sigma=\lambda_r^2/2\Gamma\ll 1$, and the expression for $\chi_a''(\omega)$ can be approximated as

$$\frac{\chi_b''}{\Omega_b}=\frac{(1+\sigma/\Gamma)\xi_+}{(\omega-\omega_r)^2+\xi_+^2}-\frac{\sigma}{\Gamma}\frac{\xi_-}{(\omega-\omega_r)^2+\xi_-^2}. \tag{20}$$

Its frequency dependence can demonstrate a shape with two maxima provided that two following conditions are sat-

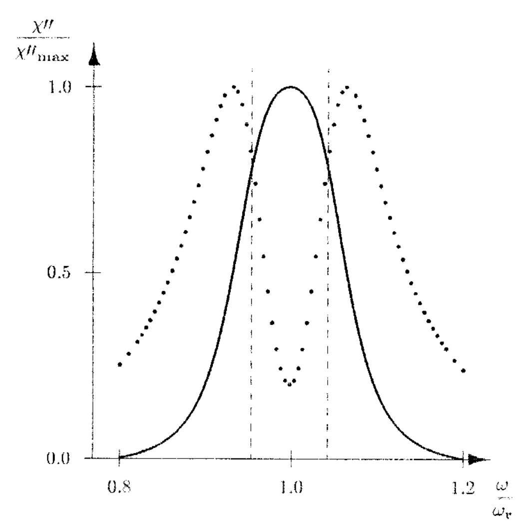

FIG. 1. Dependence of the diagonal components of the susceptibility matrix χ_a'' (solid curve) and χ_b'' (dotted curve) on the frequency in the case when the gap is open and the condition $\Gamma_b'<\lambda<\Gamma_a'$ is fulfilled. The vertical dashed lines mark the positions of the eigenfrequencies.

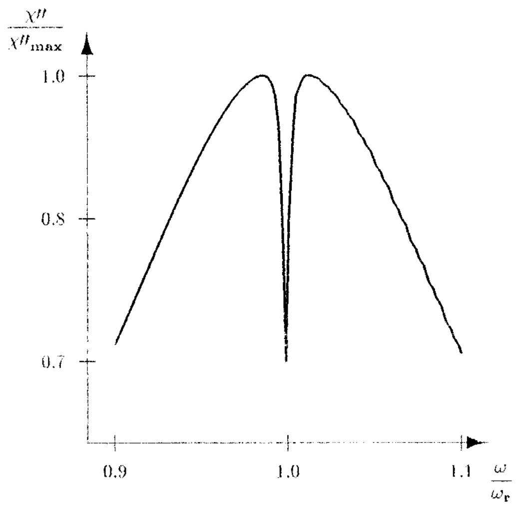

FIG. 2. Dependence of the diagonal components χ_b'' on the frequency when the gap is closed and $\Gamma_b \gg \Gamma_a$ (antiphase crossing resonance, the eigenfrequencies coincide).

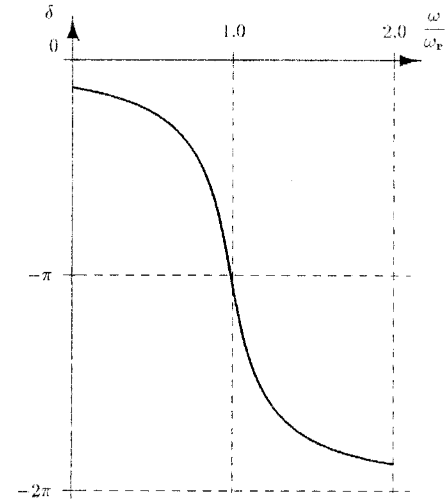

FIG. 3. Frequency dependence of the phase difference δ between the oscillations excited directly by the external force and the coupling caused oscillations in the crossing-resonance point.

isfied simultaneously: $\Gamma > \lambda_r$, but $\lambda_r > \Gamma_b'$. This can happen if $\Gamma_b > 2\Gamma_a$. Though such a form of the frequency dependence resembles the case when the gap is opened, in the considered situation it has an absolutely different origin. These two maxima appear here due to interplay between positive and negative terms in Eq. (20), both of which have the prominent resonance form with the same resonance frequency, but different signs and widths of the peaks (see Fig. 2). This phenomenon has been predicted in Ref. 6 and observed in Ref. 7 in the frequency dependence of the magnetic susceptibility of a ferromagnet in the region of the coincidence of the NMR and FMR frequencies and has been named electron-nuclear magnetic resonance (ENMR).

Let us consider χ_b in a more general case when ω_a can differ from ω_b. If $\lambda^2 \ll \Gamma_a \Gamma_b$, we can use the approximation suggested in Ref. 8, where the more general analysis of the ENMR phenomena has been done. The susceptibility χ_b can be approximately represented as a sum of two terms with resonancelike behavior

$$\chi_b \approx \chi_{b0} + \frac{\lambda^2}{4\Omega_a \Omega_b} \chi_{b1}. \tag{21}$$

The first term here is the initial susceptibility χ_{b0} and describes resonance at its ''own'' frequency ω_b, while the second one is caused by coupling and represents the reflection of the resonance behavior of the ψ oscillator in the response of the ϕ oscillator. The second term can be presented in the form

$$\chi_{b1}' = (\chi_{b0}'^2 - \chi_{b0}''^2)\chi_{a0}' - 2\chi_{b0}'\chi_{b0}''\chi_{a0}'', \tag{22}$$

$$\chi_{b1}'' = (\chi_{b0}'^2 - \chi_{b0}''^2)\chi_{a0}'' + 2\chi_{b0}'\chi_{b0}''\chi_{a0}',$$

where χ_{a0} and χ_{b0} are the initial susceptibilities of the fields ϕ and ψ, respectively

$$\chi_{a0} = \frac{\Omega_a}{\omega - \omega_a - i\Gamma_a}, \quad \chi_{b0} = \frac{\Omega_b}{\omega - \omega_b - i\Gamma_b}. \tag{23}$$

When the relation $\Gamma_b \gg \Gamma_a$ takes place the form of χ_b'' in a vicinity of the frequency ω_b is mainly determined by a high and wide resonance peak ($\chi_b'' \approx \chi_{b0}''$). Parameters of the peak practically do not depend on the distance to the crossing-resonance point. But the form of the weak narrow peak, which is situated near ω_a and is proportional to λ^2, changes dramatically depending on the difference $\omega_b - \omega_a$. In the case $|\omega_b - \omega_a| \gg \Gamma_b$ one has $\chi_{b0}' \gg \chi_{b0}''$ and the imaginary part χ_{b1}'' of the coupling-caused correction to χ_b becomes proportional to χ_{a0}''. When the difference $|\omega_b - \omega_a|$ decreases the term proportional to χ_{a0}' begins to play the main role. When $|\omega_b - \omega_a|$ reaches the value of Γ_b (in this case $\chi_{b0}' \approx \chi_{b0}''$) the form of the narrow peak situated near ω_a becomes mainly determined by χ_{b0}', i.e., it has approximately equal positive and negative components. Further decreasing of $|\omega_a - \omega_b|$ leads to the full redistribution of the magnitudes of the components and in the region of the crossing resonance the second peak becomes negative

$$\chi_b'' \approx \chi_{b0}'' - \frac{\lambda^2}{4\Omega_a \Omega_b} \chi_{b0}''^2 \chi_{a0}''. \tag{24}$$

To clarify a nature of the phenomenon it is instructive to consider the phases of the oscillations associated with the ''own'' and the coupling-caused terms in the susceptibility. Let us denote them by δ_{b0} and δ_{b1}, respectively:

$$\tan\delta_{b0} = \frac{\chi_{b0}''}{\chi_{b0}'}, \quad \tan\delta_{b1} = \frac{\chi_{b1}''}{\chi_{b1}'}. \tag{25}$$

For the phase differences $\delta = \delta_{b1} - \delta_{b0}$ one can derive from Eqs. (25) and (23)

$$\tan\delta = \frac{\omega(\Gamma_a + \Gamma_b) - (\omega_a \Gamma_b + \omega_b \Gamma_a)}{(\omega - \omega_a)(\omega - \omega_b) - \Gamma_a \Gamma_b}. \tag{26}$$

In Fig. 3 the dependence δ on ω is shown for the case of the crossing resonance ($\omega_a = \omega_b = \omega_r$). One can see that $\delta = -\pi$ when $\omega = \omega_r$. It means that the oscillations associated with the ''own'' resonance and the coupling-caused one are in the opposite phases. Therefore, it makes sense to call the phenomenon by the antiphase crossing resonance. It must appear on the diagonal component of the susceptibility of the oscillator with the greater initial damping, when both conditions $\Gamma > \lambda$ and Eq. (18) are fulfilled.

The diagonal component χ_a of the oscillator with the smaller initial damping in this case exhibits only a single narrow resonance peak on the wide weak background resulting from the contribution of the second partner oscillator. This situation can be described by the same Eq. (20) if one interchanges indexes $a \leftrightarrow b$.

Off-diagonal components of the susceptibility determine the exchange of energy between the oscillators. They have more simple frequency dependence than the diagonal components. When the gap is closed their imaginary parts have only one maximum, and when the gap is open they can have two maxima if an additional condition holds:

$$\lambda_r^2 > 2(\Gamma_a^2 + \Gamma_b^2). \tag{27}$$

III. SUSCEPTIBILITIES FOR MAGNETOELASTIC DICR

Excitations in a magnetoelastic medium are governed by the system of Landau-Lifshitz's equations for magnetization and equations for elastic displacements

$$\dot{\mathbf{M}} = -g\left[\mathbf{M}\times\left(-\frac{\partial H}{\partial \mathbf{M}} + \frac{\partial}{\partial \mathbf{x}}\frac{\partial H}{\partial(\partial \mathbf{M}/\partial \mathbf{x})}\right)\right],$$

$$G\ddot{u}_i = \frac{\partial}{\partial x_i}\frac{\partial H}{\partial u_{ij}}, \tag{28}$$

where $\mathbf{M}$ is the magnetization, $u_{ij} = 1/2(\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ is the matrix of elastic deformations, g is the gyromagnetic parameter, and G is the density of the medium.

We consider that our system is an elastically isotropic ferromagnet with single axis magnetic symmetry axis, so that the corresponding magnetoelastic potential energy H takes the form

$$H = \frac{1}{2}\alpha(\nabla\mathbf{M})^2 - \frac{1}{2}\beta(\mathbf{Mn})^2 - \mathbf{HM} + \frac{1}{2}d_1 u_{ii}^2 + \frac{1}{2}d_2(u_{ij}u_{ij} + u_{ij}u_{ji}) + \frac{1}{2}P(\mathbf{x})M_iM_ju_{ij}. \tag{29}$$

Here α is the exchange parameter, β and $\mathbf{n}$ are the magnitude and the direction of the axis of the magnetic anisotropy, respectively, d_1 and d_2 are the elastic Lamé coefficients, $\mathbf{H}$ is the magnetic field, and $P(\mathbf{x})$ is the magnetoelastic parameter. Magnetoelastic resonance in the homogeneous medium with $P = \text{const}$ is well studied.[9,10] According to the model of DICR we assume that the mean value of the magnetoelastic parameter is equal to zero, and the parameter can be presented in the following form:

$$P(\mathbf{x}) = \gamma\rho(\mathbf{x}), \tag{30}$$

where γ is the rms fluctuation of the magnetoelastic parameter, and $\rho(\mathbf{x})$ is the centered $[\langle\rho(\mathbf{x})\rangle = 0]$ and normalized $[\langle\rho^2(\mathbf{x})\rangle = 1]$ random function. Stochastic properties of the function are characterized by the normalized correlation function

$$K(\mathbf{r}) = \langle\rho(\mathbf{x})\rho(\mathbf{x}+\mathbf{r})\rangle. \tag{31}$$

Let the external dc magnetic field and the axis of anisotropy be directed along the z axis of the coordinate system. The equilibrium direction of the magnetization, then, also coincides with the z axis. We consider the excitation of the medium by bulk forces f_a and f_b with the first of them affecting the elastic subsystem and the second one influencing the magnetic subsystem. We assume that these forces are perpendicular to the z axis, therefore only x and y components of them have nonzero values. Linearizing the system (28) with respect to the small deviations $\mathbf{m}(\mathbf{x},t)$ from the equilibrium direction of the magnetization, one can obtain the following integral equations for Fourier transforms of circular components $m_\pm = m_x \pm i m_y$ and $u_\pm = u_x \pm i u_y$:

$$(\omega - \epsilon_k)m_+ - \frac{i\gamma g M^2}{2}\left[\int [k_{1_z}u_+(\mathbf{k}_1) + k_{1_+}u_z(\mathbf{k}_1)]\rho(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1\right] = \omega_m h_+,$$

$$(\omega + \epsilon_k)m_- + \frac{i\gamma g M^2}{2}\left[\int [k_{1_z}u_-(\mathbf{k}_1) + k_{1_-}u_z(\mathbf{k}_1)]\rho(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1\right] = \omega_m h_-,$$

$$(\omega^2 - \omega_k^{t\,2})u_+ + \frac{i\gamma M k_z}{2G}\left[\cos^2\theta_k + \sin^2\theta_k\frac{\omega^2-\omega_k^{t\,2}}{\omega^2-\omega_k^{l\,2}}\right]\Phi_+ + \frac{i\gamma M k_z}{2G}\sin^2\theta_k e^{2i\phi_k}\left[1 - \frac{\omega^2-\omega_k^{t\,2}}{\omega^2-\omega_k^{l\,2}}\right]\Phi_- = \Omega_u^2 f_+,$$

$$(\omega^2 - \omega_k^{t\,2})u_- + \frac{i\gamma M k_z}{2G}\left[\cos^2\theta_k + \sin^2\theta_k\frac{\omega^2-\omega_k^{t\,2}}{\omega^2-\omega_k^{l\,2}}\right]\Phi_- + \frac{i\gamma M k_z}{2G}\sin^2\theta_k e^{-2i\phi_k}\left[1 - \frac{\omega^2-\omega_k^{t\,2}}{\omega^2-\omega_k^{l\,2}}\right]\Phi_+ = \Omega_u^2 f_-,$$

$$(\omega^2 - \omega_k^{l\,2})u_z + \frac{i\gamma M k_z}{4G}\left[\sin 2\theta_k\cos\theta_k - \sin\,\theta_k\cos 2\theta_k\frac{\omega^2-\omega_k^{l\,2}}{\omega^2-\omega_k^{t\,2}}\right](e^{-i\phi_k}\Phi_+ + e^{i\phi_k}\Phi_-) = 0, \tag{32}$$

where $\Phi_\pm = \int m_\pm(\mathbf{k}_1)\rho(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1$.

Here ϵ_k, ω_k^t, and ω_k^l are the initial dispersion laws of the spin waves, transversal, and longitudinal elastic waves accordingly:

$$\epsilon_k=\omega_0+\alpha\omega_M k^2,$$
$$\omega_k^{t,l}=v_{t,l}k, \tag{33}$$

where $\omega_0=g(H+\beta M)$, $\omega_M=gM$, $v_t^2=d_1/G$, and $v_l^2=(d_2+d_1)/G$; v_t and v_l are the speeds of the transversal and longitudinal elastic waves, respectively.

A bulk force exerting upon the magnetic subsystem is the magnetic component h of the electromagnetic field. An elastic exciting force is written in the form of $\Omega_a^2 f$, where the parameter Ω_a is chosen to have the dimension of the frequency, then the amplitude of the force f is measured in units of displacements u. When these forces are absent Eqs. (32) turn into the equations for eigenfrequencies. The eigenfrequencies of the system have been considered earlier.[2] [Note that there are misprints in the equations for eigenfrequencies in Ref. 2: the imaginary units i have been missed in front of integral terms. The correct form of these equations which has been considered in Ref. 2 can be obtained from Eqs. (32) at $h=f=0$.] In what follows we use the scalar approximation for elastic waves assuming that $v_t=v_l=v$ and neglect terms describing nonresonant interaction between the elastic and the left-polarized spin waves. Within these approximations the coupled equations for elastic waves and right-polarized spin waves read as

$$[(\omega-i\Gamma_u)^2-\omega_k^2]u_{\mathbf{k}} - \frac{\gamma^2 M^2\omega_M k_z}{4G}\int \frac{k_{2z}u_{\mathbf{k}_2}\rho(\mathbf{k}_1-\mathbf{k}_2)\rho(\mathbf{k}-\mathbf{k}_1)}{\omega-i\Gamma_s-\epsilon_{k_1}}d\mathbf{k}_1 d\mathbf{k}_2 = \frac{i\gamma M\omega_M k_z}{4G}\int \frac{h_{\mathbf{k}_1}\rho(\mathbf{k}-\mathbf{k}_1)}{\omega-i\Gamma_s-\epsilon_{k_1}}d\mathbf{k}_1+\Omega_u^2 f_{\mathbf{k}},$$

$$(\omega-i\Gamma_s-\epsilon_k)m_{\mathbf{k}}-\frac{\gamma^2M^2\omega_M}{8G}\times\int \frac{(k_1^2+k_{1_z}^2)m_{\mathbf{k}_2}\rho(\mathbf{k}_1-\mathbf{k}_2)\rho(\mathbf{k}-\mathbf{k}_1)}{(\omega-i\Gamma_u)^2-\omega_{k_1}{}^2}d\mathbf{k}_1 d\mathbf{k}_2 = \frac{i}{2}\gamma M^2\omega_M\Omega_u^2\int \frac{k_{1z}V_{\mathbf{k}_1}\rho(\mathbf{k}-\mathbf{k}_1)}{(\omega-i\Gamma_u)^2-\omega_{k_1}{}^2}d\mathbf{k}_1+\omega_M h_{\mathbf{k}}. \tag{34}$$

Here we omitted the index ''+'' at the variables m_k, u_k, h_k, and f_k and added parameters Γ_s and Γ_u in order to model the initial damping of spin and elastic waves, respectively.

Equations (34) are to be solved by means of the usual perturbation theory with respect to the coupling parameter ρ. Off-diagonal components of the susceptibility matrix in this case contain only the products of odd numbers of the function $\rho(\mathbf{k})$ in any order of the perturbation theory. In the third order one can obtain, for instance, an expression of the form

$$\gamma^3\int \frac{k_{z_2}k_{z_1}h_{\mathbf{k}_3}\rho(\mathbf{k}_1-\mathbf{k}_2)\rho(\mathbf{k}_2-\mathbf{k}_3)\rho(\mathbf{k}-\mathbf{k}_1)}{(\omega-i\Gamma_s-\epsilon_{k_1})(\omega-i\Gamma_s-\epsilon_{k_3})[(\omega-i\Gamma_u)^2-\omega_{k_2}]}\times d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3.$$

All these terms are averaged to zero for regular inhomogeneities characterized by symmetric distribution function. One can conclude, hence, that off-diagonal components of the averaged susceptibility matrix are equal to zero in the case of DICR unlike the usual crossing resonance in homogeneous materials:

$$\chi_{su}\propto\frac{\langle m_k\rangle}{V_k}=0,\quad \chi_{us}\propto\frac{\langle u_k\rangle}{h}=0. \tag{35}$$

The diagonal components of the susceptibilities can be found in terms of the averaged Greens functions of Eqs. (34)

$$\chi_u=\frac{\langle u_k\rangle}{V_k}=\frac{\Omega_u^2}{(\omega-i\Gamma_u)^2-\omega_k^2-2\omega_k Q_u(\omega,k)},$$
$$\chi_s=\frac{\langle m_k\rangle}{h_k}=\frac{\omega_M}{\omega-i\Gamma_s-\epsilon_k-Q_s(\omega,k)}. \tag{36}$$

The mass operators Q_s and Q_u describe the influence of the inhomogeneities on the waves. Calculating these operators in the first nonvanishing order of the perturbation theory (Bourret approximation[11]), we find

$$Q_u=\frac{\lambda^2}{4}\int \frac{S(\mathbf{k}-\mathbf{k}_1)}{\omega-i\Gamma_s-\epsilon_{k1}}d\mathbf{k}_1,$$
$$Q_s=\frac{\lambda^2 v}{4k}\int \frac{(k_1^2+k_{1z}^2)S(\mathbf{k}-\mathbf{k}_1)}{(\omega-i\Gamma_u)^2-\omega_{k1}^2}d\mathbf{k}_1, \tag{37}$$

where $\lambda^2=\gamma^2M_0^2\omega_M\omega_k/(Gv^2)$. The function $S(k)$ in Eqs. (37) is a spectral density of the inhomogeneities, which is the Fourier transform of the corresponding correlation function $K(r)$.

Let us choose the standard exponential form of the correlation function to characterize the inhomogeneities:

$$K(r)=e^{-k_c r},\quad S(k)=\frac{1}{\pi^2}\frac{k_c}{(k_c^2+k^2)^2}. \tag{38}$$

Here r_c and k_c are the correlation radius and the correlation wave number of the inhomogeneities, respectively, which are related to each other as $k_c\approx r_c^{-1}$. Integrals (37) with the spectral density given by Eq. (38) have been calculated in the previous investigations of the dispersion laws.[2] Particularly, for Q_u at $\omega>\omega_0$ one has

$$Q_u=\frac{\lambda^2/4}{\omega-i\Gamma_s-\epsilon_k-\kappa_m-2i\sqrt{\kappa_m(\omega-i\Gamma_s-\omega_0)}}, \tag{39}$$

where $\kappa_m=\omega_M\alpha k_c^2$. The term with $\sqrt{\kappa_m}$ describes the contribution of the scattering into the total attenuation of the averaged wave. Assuming that this contribution is small compared to the eigenfrequency, Eq. (39) can be reduced to the form

$$Q_u\approx\frac{\lambda^2}{4}\frac{1}{\omega-i\Gamma_s^*-\epsilon_k}, \tag{40}$$

where the effective relaxation parameter Γ_s^* is a sum of the initial damping in the spin subsystem and the relaxation due to the scattering

$$\Gamma_s^* \approx \Gamma_s + 2k_c\sqrt{\alpha\omega_M(\omega-\omega_0)}, \quad (\omega>\omega_0). \tag{41}$$

The expression for the second mass operator Q_s has a cumbersome form:

$$Q_s = \frac{\lambda^2}{2\omega_k}\left\{\frac{\omega_k^2+\kappa_p^2+2i\kappa_p(\omega-i\Gamma_u)}{(\omega-i\Gamma_u)^2-\omega_k^2-\kappa_p^2-2i\kappa_p(\omega-i\Gamma_u)} - \frac{\kappa_p}{2\omega_k^2}[\kappa_p - i(\omega-i\Gamma_u)] + \frac{\kappa_p}{4\omega_k^3}[(\omega-i\Gamma_u)^2+\omega_k^2+\kappa_p^2]\right.$$
$$\left.\times\left[\arctan\frac{2\kappa_p\omega_k}{(\omega-i\Gamma_u)^2-\omega_k^2+\kappa_p^2} + \frac{i}{2}\ln\frac{(\omega-i\Gamma_u+\omega_k)^2+\kappa_p^2}{(\omega-i\Gamma_u-\omega_k)^2+\kappa_p^2}\right]\right\}, \tag{42}$$

where $\kappa_p = vk_c$. However, assuming that the attenuation is small enough it can be simplified near the resonance frequency to take the form similar to Eq. (40)

$$Q_s \approx \frac{\lambda^2}{4}\frac{1}{\omega - i\Gamma_u^* - \omega_k}, \tag{43}$$

where the effective relaxation parameter Γ_u^* is

$$\Gamma_u^* \approx \Gamma_u + vk_c. \tag{44}$$

Within the described approximations the diagonal components of the susceptibility take the form

$$\chi_u = \frac{\langle u_k\rangle}{f_k} \approx \frac{\Omega_u^2}{2\omega_k}\frac{\omega-\epsilon_k-i\Gamma_s^*}{D_u(\omega,k)},$$

$$\chi_s = \frac{\langle m_k\rangle}{h_k} \approx \omega_M\frac{\omega-\omega_k-i\Gamma_u^*}{D_s(\omega,k)}, \tag{45}$$

where

$$D_u = (\omega-\omega_k-i\Gamma_u)(\omega-\epsilon_k-i\Gamma_s^*) - \frac{\lambda^2}{4}, \tag{46}$$

$$D_s = (\omega-\epsilon_k-i\Gamma_s)(\omega-\omega_k-i\Gamma_u^*) - \frac{\lambda^2}{4}. \tag{47}$$

Though these expressions have a form similar to that of expressions (6) and (7) obtained for the model of deterministic oscillators with damping, there is a remarkable difference between them. For oscillators with a deterministic coupling parameter the denominators of the different components of the susceptibility coincide. At the same time the denominators in Eqs. (45) are different. This difference is caused by the fact that D_u and D_s contain different pairs of the relaxation parameters, Γ_u, Γ_s^* and Γ_s, Γ_u^*, respectively. In accordance with this fact the dispersion laws of averaged elastic waves and the averaged spin waves are also determined by different dispersion equations:

$$D_u(\tilde{\omega},k)=0, \quad D_s(\tilde{\omega},k)=0. \tag{48}$$

The shape of the dispersion curves following from Eqs. (48) can be also considerably different for the elastic and spin waves. Particularly, the conditions for the gap between different branches to appear at the crossing point have a different form for the elastic and spin waves, respectively,

$$\lambda_r^2 > (\Gamma_s^* - \Gamma_u)^2, \quad \lambda_r^2 > (\Gamma_s - \Gamma_u^*)^2. \tag{49}$$

These expressions generalize the corresponding results derived in Refs. 1, 2 to include initial relaxations Γ_u and Γ_s. One can see that taking into account the initial relaxation does not change the result of Refs. 1, 2 concerning the possibility for elastic waves to have the opened gap while the gap in the spectrum of the spin waves remains closed.

To clarify the physical sense of the result obtained we recall a more simple situation of the one wave field propagating in an inhomogeneous medium. Plane waves are not the eigenexcitations in this case because of scattering from the inhomogeneities. However, an effective homogeneous medium can be introduced, where plane waves are eigenmodes, which correspond to propagation of averaged waves. These modes have a modified dispersion law and a finite time of life as a result of interaction with scattered waves, which do not present in this picture explicitly. The averaging of the stochastic integral equation in the Fourier harmonics of the propagating field is one of the methods to introduce such an effective medium.

One can see from the analysis made above that for the DICR model the averaging produces in the inhomogeneous medium two effective media, which are characterized by different parameters. The averaged elastic wave propagates in the medium characterized by the parameters Γ_u and Γ_s^*, so that its dispersion law, relaxation and the susceptibility depends upon these parameters. The effective medium for spin waves and, hence, all characteristics of averaged spin waves depend on other parameters Γ_s and Γ_u^*. Propagating in their own effective media, these averaged waves do not interact with each other.

The concept of two effective media allows us to use all the expressions obtained in the previous section for deterministic oscillators in order to analyze the model of DICR. One has to discard only the expressions for off-diagonal components, which are equal to zero in our case. When considering the susceptibility, dispersion law, and damping of the elastic waves one has to use $\Gamma_a = \Gamma_u$ and $\Gamma_b = \Gamma_s^*$ in the expressions of the previous section. When analyzing spin waves one has to accept for Γ_a and Γ_b expressions Γ_u^* and Γ_s, respectively. For instance, Eqs. (17) and (18), which determine the conditions, when two maxima on the curves $\chi_u''(\omega)$ and $\chi_s''(\omega)$ appear, take the form

$$\lambda_r^2 > \frac{4\Gamma_s^{*3}}{2\Gamma_s^* + \Gamma_u}, \quad \lambda_r^2 > \frac{4\Gamma_u^{*3}}{2\Gamma_u^* + \Gamma_s}. \tag{50}$$

As another example let us consider the expression for the susceptibility, when the gap is closed for both the elastic and spin waves. This situation seems to be more likely for standard zero-mean-magnetostrictive amorphous ferromagnets, though one can also find materials where the gap in the spectrum of elastic waves could appear. Under the natural assumptions $\Gamma_s \gg \Gamma_u^*$ we obtain for the elastic susceptibility

$$\frac{\chi_u''}{\Omega_u} \approx \left(1 + \frac{\sigma_u}{\Gamma_s^* - \Gamma_u}\right) \frac{\xi_-}{(\omega - \omega_r)^2 + \xi_-^2} - \frac{\sigma_u}{\Gamma_s^* - \Gamma_u} \frac{\xi_+}{(\omega - \omega_r)^2 + \xi_+^2}, \tag{51}$$

where $\xi_+ = \Gamma_s^* - \sigma_u$, $\xi_- = \Gamma_u + \sigma_u$, $\sigma_u = \lambda^2/4(\Gamma_s^* - \Gamma_u)$. The expression for the spin component of the susceptibility takes the form

$$\frac{\chi_s''}{\omega_M} \approx \left(1 + \frac{\sigma_s}{\Gamma_s - \Gamma_u^*}\right) \frac{\xi_+'}{(\omega - \omega_r)^2 + \xi_+'^2} - \frac{\sigma_s}{\Gamma_s - \Gamma_u^*} \frac{\xi_-'}{(\omega - \omega_r)^2 + \xi_-'^2}, \tag{52}$$

where $\xi_+' = \Gamma_s - \sigma_s$, $\xi_-' = \Gamma_u^* + \sigma_s$, $\sigma_s = \lambda^2/4(\Gamma_s - \Gamma_u^*)$.

These expressions show that each of the susceptibilities is a difference between two resonancelike terms, resonance frequencies of which coincide, but amplitudes and widths differ. The most considerable contribution into χ_u'' is due to the first term, which has the shape of a narrow and tall peak describing resonance absorption of the elastic energy. The second term in the considered situation is almost negligible. In contrast to that, the spin-wave susceptibility is determined by both terms in Eq. (52). Their frequency dependence manifests the typical structure of the antiphase resonance with a weak but narrow reverse elastic peak on the background of wide and strong magnetic resonance (Fig. 2). It is clear from Eqs. (51) and (52) that experimental study of both the elastic and spin-wave susceptibilities allows us to determine the major stochastic characteristics of the magnetostriction parameter — the rms fluctuation γ and the correlation radius r_c.

We would like also to note that though DICR has been studied in this paper for the case of magnetoelastic resonance, the results obtained can be applied to crossing resonances of any nature if the mean value of a coupling parameter is equal to zero, and the conditions of the Bourret approximation are fulfilled.

ACKNOWLEDGMENTS

This work was supported by the NATO Science Program and Cooperation Partner Linkage Grant No. HTECH 960919, and the Krasnoyarsk Region Science Foundation Grant No. 6F0006.

[1] V. A. Ignatchenko and L. I. Deych, Phys. Rev. B **50**, 16 364 (1994).
[2] L. I. Deych and V. A. Ignatchenko, Zh. Eksp. Teor. Fiz. **107**, 842 (1995) [JETP **80**, 478 (1995)].
[3] L. I. Deych, M. V. Erementchouk, and V. A. Ignatchenko, Zh. Eksp. Teor. Fiz. **109**, 1370 (1996) [JETP **82**, 739 (1996)].
[4] L. I. Deych and A. A. Lisyansky, Phys. Lett. A **220**, 125 (1996).
[5] L. I. Deych, V. A. Ignatchenko, and A. A. Lisyansky, Phys. Rev. B **55**, 11 287 (1997).
[6] V. A. Ignatchenko and V. I. Tsifrinovich, Zh. Eksp. Teor. Fiz. **68**, 672 (1975) [JETP **41**, 33 (1975)].
[7] V. A. Ignatchenko, N. M. Salansky, V. K. Maltsev, and V. I. Tsifrinovich, Pisma Zh. Eksp. Teor. Fiz. **21**, 472 (1975) [JETP Lett. **21**, 218 (1975)].
[8] V. A. Ignatchenko and V. I. Tsifrinovich, Usp. Fiz. Nauk **131**, 75 (1981) [Sov. Phys. Usp. **24**, 42 (1981)].
[9] A. I. Akhiezer, V. G. Baryakhtar, and S. V. Peletminsky, Zh. Eksp. Teor. Fiz. **35**, 228 (1958) [Sov. Phys. JETP **8**, 157 (1959)].
[10] A. I. Akhiezer, V. G. Baryakhtar, and S. V. Peletminsky, *Spinovye volny (Spin Waves)* (Nauka, Moscow, 1967).
[11] R. Bourret, Physica (Amsterdam) **54**, 623 (1971).