Disorder effect on melting transitions of vortex lattices with periodic pinning

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Employing molecular-dynamics simulations, we study the melting transitions in driven vortex lattices with commensurability between arrays of vortices and defects. It is shown that the largest pinning effect can be obtained for periodic pinning arrays at zero temperature, and either thermal fluctuation or disorder of pinning array plays a significant depinning role in the melting transitions of the driven vortex lattices. As the pinning sites deviate from the periodic array, the intervortex interaction is found to produce an additional depinning effect on driven vortex lattices. Furthermore, enough disorder will result in a crossover from a first-order melting transition to a continuous phase transition. [S0163-1829(98)09509-5]

Many efforts have been made to increase the critical current density J_c of high-T_c (type-II) superconductors by introducing pinning centers into superconductors.[1] An effective flux-pinning mechanism is essential in order to minimize the resistive losses through Lorentz-force-induced vortex motion. The quenched defects, such as point pinning defects, columnar defects, and twin boundaries, usually form random pinning arrays, which can pin down vortices against the Lorentz force exerted on vortices by an electric current. With increasing the Lorentz force, there is a melting transition of vortices from a solidlike pinning state to a liquidlike flow state. It was recently reported that the regular arrays of pinning sites can be produced in high-T_c superconductors by using lithographic techniques to make periodic submicron holes,[2] or by using well-controlled irradiation with high energetic heavy ions to create linear tracks in regular arrays.[3,4] Such periodic pinning arrays are found to produce higher J_c than an equal number of randomly placed pins.[5]

Experimental[3] and numerical works[6] show that the effectiveness of pinning depends strongly on the commensurability between arrays of vortices and defects, the vortices being difficult to move due to matching effect in the the commensurate cases of specific applied magnetic fields H. At $H/H_\phi=1$ where H_ϕ is the field at which the number of vortices N_v is equal to the number of pinning sites N_p, the vortex lattice locks into periodic pinning arrays and the pinning force is maximized. In real systems, however, such a matching effect may be weakened by two types of disorder: static, due to deviation from the ordered pinning lattice, and dynamic, associated with thermal fluctuation. Both the distribution of pinning centers and the thermal fluctuation are of great importance in practical applications for obtaining large J_c of high-T_c superconductors, as well as in studying the dynamic phase transitions of the vortex lattice in the mixed state. Much work has been done on dynamic phases in systems with random pinning arrays. In the case of periodic pinning, although several dynamic phase diagrams were obtained as a function of commensurability, pinning strength, and spatial order of the pinning sites,[6] the effect of thermal fluctuation on the melting transition of a matching vortex lattice has not been addressed.

In this work, using molecular-dynamics simulations, we investigate the disorder effect on the melting transitions of driven vortex lattices interacting with triangular arrays of columnar pinning centers. A rich variety of dynamical plastic flow phases have been reported for periodic pinning systems.[6] To see clearly the disorder effect on the melting transition, we focus our attention on the commensurate case of $H/H_\phi=1$, in which there exist only two phases: pinned and flowing,[7] with the onset of flow as the signature of the vortex melting transition. It is found that, for a periodic pinning system at $T=0$ (without thermal fluctuation), the critical Lorentz force for which the vortex lattice starts to melt is maximized, and the vortex melting transition from a solidlike pinning state to a liquidlike flow state is a first-order phase transition accompanied by a resistivity hysteresis. The application of either static or dynamic disorder to the periodic pinning system plays an important role in depinning the vortex lattice. We show that the critical Lorentz force decreases with increasing disorder, no matter whether it arises from the thermal fluctuation or the deviation from the ordered pinning arrays. When the degree of disorder is large enough, the resistivity hysteresis disappears and the first-order melting transition is replaced by a continuous glass phase transition. It appears that the static and dynamic disorders have the similar effect on the melting transition. The difference between them is that enough thermal fluctuation can make the vortex lattice melt, while the static disorder alone cannot. The increase of static disorder leads to a crossover from periodic pinning arrays to random pinning arrays. It is found that following such a crossover, depinning effects are enhanced due to the intervortex interaction.

First, we consider two-dimensional (2D) interacting vortices in the presence of triangular pinning arrays and thermal fluctuation. The quantized vortices due to penetration of the applied magnetic field are perpendicular to the 2D plane. We model the columnar pinning centers as Gaussian potential wells with a decay length $R_{\rm pin}$.[8–10] The pinning force is taken as

$$\mathbf{F}_{\rm pin}(\mathbf{r}_i)=-F_{p0}f_0\sum_k^{N_p}\frac{\mathbf{r}_i-\mathbf{R}_k}{R_{\rm pin}}\exp\left(-\left|\frac{\mathbf{r}_i-\mathbf{R}_k}{R_{\rm pin}}\right|^2\right), \qquad (1)$$

where $\mathbf{r}_i$ represents the location of the ith vortex and $\mathbf{R}_k$ stands for the location of the kth pinning site in the 2D system. $F_{p0}f_0$ denotes the intensity of the individual pinning force. In our simulations all forces are taken in units of $f_0 = \Phi_0^2/8\pi\lambda^3$ with Φ_0 the flux quantum and λ the superconducting penetration depth. The repulsive intervortex interaction has a logarithmic form in the 2D case[11] and the intervortex interacting force is given by

$$\mathbf{F}_{vv}(\mathbf{r}_i) = F_{vv0} f_0 \sum_{j \neq i}^{N_v} \frac{(\mathbf{r}_i - \mathbf{r}_j)/\lambda}{|(\mathbf{r}_i - \mathbf{r}_j)/\lambda|^2}, \qquad (2)$$

where $F_{vv0}f_0$ denotes the intensity of the intervortex interacting force and the cut length for this long-range force is taken as 4λ. The applied driven force acting on the vortices is the Lorentz force, $\mathbf{F}_L = \mathbf{J} \times \mathbf{\Phi}_0$, where $\mathbf{J}$ is the applied current. Finally, the Brownian force due to Gaussian thermal noise is taken as[9,10]

$$\mathbf{F}_{\rm th} = F_{\rm th0} f_0 \sum_j \delta(t - t_j)\Gamma(t_j)\Theta(p - q_j). \qquad (3)$$

Here $F_{\rm th0}f_0$ stands for the intensity of the thermal fluctuation force, which is proportional to the square root of temperature. $\Gamma(t_j)$ is a random number chosen from a Gaussian distribution of mean 0 and width 1, where t_j labels the jth time step. $p = \Delta/\tau$ is the probability that the noise term acts on a given vortex, where Δ is the discrete time step and τ is the mean time between two successive random noise pulses, and q_j is a random number uniformly distributed between 0 and 1. $\Theta(x)$ is the unit step function with $\Theta = 1$ for $x > 0$ and 0 for $x < 0$. As a result, the overdamped equation of the vortex motion is given by

$$\eta \mathbf{v}_i = \mathbf{F}_L + \mathbf{F}_{vv}(\mathbf{r}_i) + \mathbf{F}_{\rm pin}(\mathbf{r}_i) + \mathbf{F}_{\rm th}, \qquad (4)$$

where η is the viscosity coefficient and taken to be unity.

In our simulation the equation is solved using the discrete time step Δ in a 2D rectangular sample of the same number of vortices and pinning sites ($N_v = N_p = 320$) with periodic boundary conditions. The sample under consideration is assumed to have perfect triangular lattice of pinning sites. If the spacing between the neighboring pinning sites is taken as the unit of length, the sample size is $16 \times 10\sqrt{3}$. The other fixed lengths and forces used in the simulations are $R_{\rm pin} = 0.2$, $\lambda = 4.0$, $F_{vv0} = 0.25$, and $F_{p0} = 4.0$. The external current is applied along the x direction in the x-y plane, so the driving Lorentz force $\mathbf{F}_L$ acting on vortices is always parallel to the y axis, i.e., $F_L = F_{Ly}$. We have also employed the same numerical simulations on a sample of size $32 \times 20\sqrt{3}$ with $N_v = N_p = 1280$. The calculated result is found insensitive to the sample size provided the densities of vortices and pinning sites remain unchanged.

We now study the influence of thermal fluctuations on the dynamic phase transition of the driven vortex lattice in the commensurate case of $H/H_\phi = 1$. In the present simulation the initial distribution configuration of the vortices is the same as that of the pinning sites, as shown in the left top of Fig. 1 where each vortex is attached to one pin. Figure 1 shows the resistivities as functions of the driving Lorentz force F_{Ly} for different values of $F_{\rm th0}$. The ρ vs F_{Ly} curves exhibit several remarkable features. At low driving F_{Ly} the

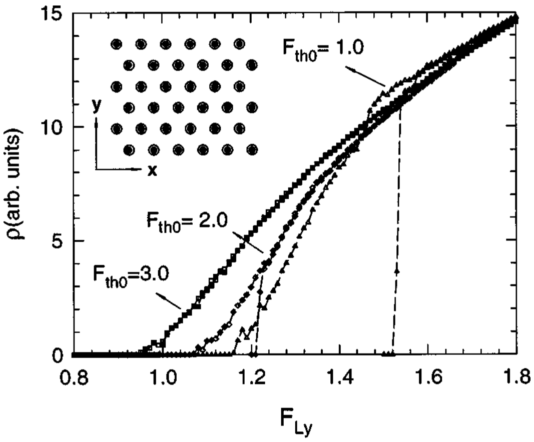

FIG. 1. Resistivity vs driving Lorentz force characteristics at various thermal fluctuation forces for increasing and decreasing F_{Ly} as indicated by solid and open symbols, respectively. Distributions of pinning sites (●) and vortices (○) in the solidlike pinning state are shown in the left top.

vortices remain pinned, forming a solidlike pinning phase. The absence of ρ reflects the pinned nature of the vortex solid. As F_{Ly} is increased beyond a threshold value F_c, the vortices depin and there is a sharp jump up in ρ as seen in Fig. 1. The dissipation arises from flux flow of the depinned vortex lattice. It is interesting to note that as the temperature is increased (increasing $F_{\rm th0}$), the jump of ρ occurs at lower values of F_c and the magnitude of jump also decreases. Another salient feature is the resistivity hysteresis which appears at low thermal fluctuation ($F_{\rm th0} \leqslant 3.0$). The increasing and descreasing branches of the hysteresis exhibit asymmetric behavior: a sharp jump in the former and a smooth decrease in the latter. The width of the hysteresis is maximum at $F_{\rm th0} = 0$, it decreases gradually with increasing $F_{\rm th0}$ and vanishes at $F_{\rm th0} \simeq 3.0$.

The sharp jumps in ρ and the resistivity hysteresis observed in type-II superconductors have been considered as the characteristic of the first-order melting phase transition of the vortex lattice.[12,13] The present calculated results indicate that in the weak dynamic disorder limit ($F_{\rm th0} \leqslant 3.0$), the phase transition is a first-order melting transition of the vortex lattice, while in the strong dynamic disorder limit ($F_{\rm th0} > 3.0$), the transition is a continuous vortex-glass transition.[14] It then follows that there should be a tricritical point that separates the first-order transition from the continuous one. In Fig. 2 we show the dynamic phase diagram of the driving Lorentz force vs the thermal noise force, in which the tricritical point (solid square) lies between the phase boundaries of the first-order and continuous transitions. With the increase of $F_{\rm th0}$, the threshold driving force F_c for the onset of the vortex motion decreases, indicating that the thermal fluctuations are favorable to the depinning and melting of the vortex lattice. Such a simulated result in a system with periodic pinning arrays is qualitatively consistent with the theoretical[15] and experimental study[13] on systems with random pinning arrays. It is seen from Fig. 2 that as the temperature is high enough, ($F_{\rm th0}$ is increased beyond a critical value), the vortex lattice may undergo a continuous glass transition in the absence of any external Lorentz force. In the present simulation this critical force ($F_{\rm th0} \approx 6.3$) is

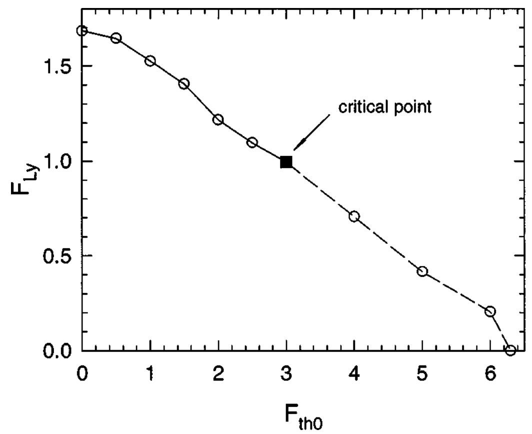

FIG. 2. Dynamic phase diagram with a triangular pinning lattice. F_{Ly} is the Lorentz force and F_{th0} is the thermal fluctuation strength. ◇ denotes the tricritical point separating the first-order melting transition (solid line) from the continuous glass transition (dashed line).

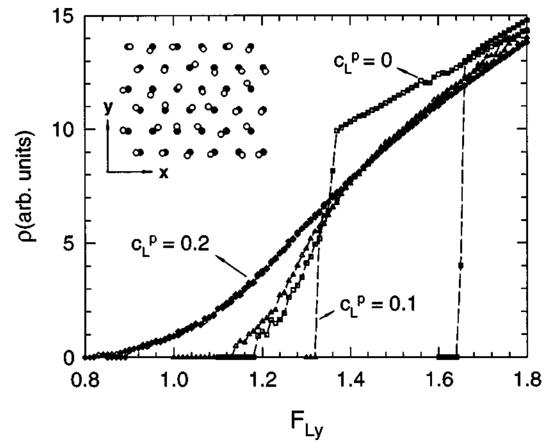

FIG. 3. Resistivity vs driving Lorentz force characteristics at various c_L^p for increasing and decreasing F_{Ly} as indicated by solid and open symbols, respectively. Distributions of periodic pinning sites (●) and disordered ones (○) for $c_L^p=2.0$ are shown in the left top.

found greater than that for a system with random pinning arrays. It may be understood by the fact that the intervortex interaction always impels vortices to have a homogeneous distribution; for a periodic pinning system, this trend will partly counteract the depinning and melting effect due to thermal fluctuation. This indicates that the periodic pinning arrays have more effective pinning effect so that they may increase the critical current of high-T_c superconductors.

In what follows we study influence of the disorder in pinning distribution on the melting transitions of driven vortex lattices. Particular attention is paid to the crossover from a first-order melting transition to a continuous phase transition with increasing disorder of the pinning sites. In order to describe the disorder degree of the pinning distribution we introduce a dimensionless parameter c_L^p defined as

$$c_L^{p\,2}=\langle|\mathbf{R}_i-\mathbf{R}_{i0}|^2\rangle. \tag{5}$$

Here R_i is the location of the ith pinning site in the real system and R_{i0} is the position of corresponding site on a perfect triangular pinning lattice, as shown in the left top of Fig. 3. Both $\mathbf{R}_i$ and $\mathbf{R}_{i0}$ are taken in units of the lattice constant of the triangular lattice, and the random differences $(\mathbf{R}_i-\mathbf{R}_{i0})$ are assumed to satisfy the Gaussian distribution. Evidently, c_L^p characterizes the magnitude of deviation of the real pinning distribution from the triangular array. We employ molecular-dynamics simulations at zero temperature by using the overdamped equation $\mathbf{v}_i=\mathbf{F}_L+\mathbf{F}_{vv}(\mathbf{r}_i)+\mathbf{F}_{\rm pin}(\mathbf{r}_i)$, where the thermal noise force has been neglected.

In Fig. 3, we show the resistivity vs the driving force for $c_L^p=0$, 0.1, and 0.2. In the case of periodic pinning, i.e., $c_L^p=0$, the threshold driving force at which the vortices start to move along the direction of $\mathbf{F}_L$ is the largest. With increasing c_L^p, the threshold value F_c becomes gradually small. The sharp jump in ρ at F_c and the hysteresis of ρ shown in Fig. 3 suggest that the melting transition of the vortex lattice is of the first order. For $c_L^p=0.2$, however, there is only a very small jump in ρ and the resistivity hysteresis tends to vanish. It implies that with further increasing c_L^p, both the resistivity jump and hysteresis will disappear and the first-order melting transition will give way to a continuous phase transition. The tricritical point is found at $c_L^p\simeq 0.25$, as shown in Fig. 4. It separates the first-order melting transition (solid line) from the continuous glass transition (dashed line). As seen in Fig. 4, the slope of the phase boundary for the former ($c_L^p<0.25$) is much greater than that for the latter ($c_L^p>0.25$). When c_L^p is large enough, the pinning sites can be regarded as being randomly distributed[6] so that the threshold driving force tends to that in a system with random pinning array. The simulated result shows that the regular array of the pinning sites has the strongest pinning effect, especially in the commensurate cases. This conclusion is in good agreement with the experiments.[3]

An important difference between Figs. 2 and 4 is that the threshold driving force can be decreased to zero in Fig. 2, while it is saturated to a finite value in Fig. 4. This difference stems from the fact that the thermal fluctuation alone can

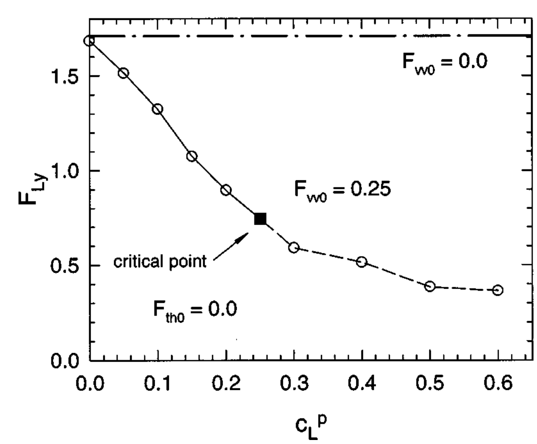

FIG. 4. Dynamic phase diagram with a triangular pinning lattice. F_{Ly} is the Lorentz force and c_L^p stands for the disorder degree of pinning sites deviating from the ordered lattice position. ◇ denotes the tricritical point separating the first-order melting transition (solid line) from the continuous glass transition (dashed line). The dot-dashed line stands for the phase boundary in the absence of intervortex interaction.

melt the vortex lattice, but the disorder in pinning distribution cannot. In fact, the latter does not play a direct depinning role and its effect is to transform the periodic pinning array into a random pinning array. For a system with random pinning array, the vortex melting still requires a threshold Lorentz force which must overcome the pinning force exerted on vortices by random pinning centers. In the present simulation this threshold value is about 0.35.[16]

We wish to point out here that the intervortex interaction plays an important role in depinning the vortex lattice. To see clearly this point, we numerically integrate the overdamped equation of motion by neglecting the intervortex interaction (taking $F_{vv0}=0$). It is found that in the absence of F_{vv} the threshold Lorentz force remains unchanged with the periodic pinning array being changed into a random one, as shown by the dot-dashed line in Fig. 4. It strongly suggests that the intervortex interaction is the origin of the enhancement of the depinning effect due to the disorder of pinning array. This can explain why the pinning effect of a periodic pinning array is the strongest and that of a random pinning array is relatively weaker.

In summary, we have shown that either the thermal fluctuation or the disorder of the pinning distribution plays a significant depinning role in the melting transitions of driven vortex lattices with periodic pinning. Unlike the thermal fluctuation, the disorder of pinning array does not directly produce a depinning force. However, we find that in this disorder case the intervortex interaction provides an additional depinning force exerted on the vortices. This gives a reasonable explanation why the driven vortex lattices with periodic pinning have the strongest pinning effects. The dynamic phase diagrams obtained indicate that with increasing disorder, regardless of being static or dynamic, the threshold driving force decreases gradually and the first-order melting transition is replaced by the continuous glass transition at a tricritical point.

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