Long-range Coulomb interaction and frequency dependence of shot noise in mesoscopic diffusive contacts

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The frequency dependence of shot noise in mesoscopic diffusive contacts is calculated with account taken of long-range Coulomb interaction and external screening. While the low-frequency noise is 1/3 of the noise of classical Poisson process independently of the contact shape, the high-frequency noise tends to the full classical value for long and narrow contacts because of strong screening by the surrounding medium. In this case, the current fluctuations at opposite ends of the contact are completely independent. [S0163-1829(98)06407-8]

I. INTRODUCTION

Recently, the shot noise in mesoscopic contacts became a subject of extensive study.[1] In particular, much attention was given to mesoscopic diffusive contacts. One of the principal results was that in short contacts with a strong elastic scattering, the low-frequency shot noise is 1/3 of the full noise of the classical Poisson process. This result was obtained almost simultaneously by different authors using different methods and, more importantly, different physical assumptions. Beenakker and Büttiker[2] obtained this result using the multichannel scattering-matrix formalism and the assumption of quantum-coherent transport. In contrast to this, in paper[3] this result was obtained using quasiclassical kinetic equation with no assumption of quantum-coherent scattering. These theoretical predictions were experimentally confirmed in Refs. 4 and 5.

Since it was discovered that the shot noise does not vanish in contacts much longer than the elastic mean free path, it was debated how the electron-electron Coulomb interactions affect this result. This problem was qualitatively discussed in a number of papers (see, e.g., Ref. 6).

The effects of Coulomb interaction on the shot noise are most easily treated using the Boltzmann-Langevin approach. Basically, the electron-electron Coulomb interaction may be separated in two parts. First, there is the long-range part, which is associated with fluctuations of electron density in the contact. These fluctuations produce electrical fields at characteristic length scales on the order of the size of the contact and should be taken into account self-consistently. Second, there is the electron-electron scattering with the characteristic length scale about the screening length.

In the low-frequency limit, the long-range Coulomb interaction does not affect the magnitude of noise,[3,7] since the coordinate-dependent charge fluctuations are ''frozen'' in the contact and the current is conserved at any point inside it. Hence the only corrections to the noise result from short-range electron-electron scattering. The latter smear out the distribution function of electrons, which would be otherwise double-step shaped, and produce additional partially occupied states available for impurity scattering. This results in a slight decrease of the shot noise from 1/3 to $\sqrt{3}/4$ of Poisson value.[8,9] This increase was experimentally observed in Ref. 5.

Yet there still remains a question of how the long-range Coulomb interaction affects the noise at high frequencies. The problem of frequency-dependent shot noise was addressed by Büttiker[10] for the general case of multichannel quantum-coherent transport and by Altshuler *et al.*[12] for the particular case of coherent transport in diffusive contacts. However, the electron-electron interactions were not taken into account in these papers. In particular, these results were insensitive to the contact geometry provided that the transmission probabilities of the quantum channels remained unchanged, whereas the noise should depend on the possibility for the charge to pile up in the contact, i.e., on its external capacity. More recently, Büttiker extended his formalism to the case of multiterminal contacts with allowance made for capacitive coupling between the conductors.[11] However, no explicit expression for the shot noise in any particular geometry was given there.

In the present paper, we consider the effects of contact geometry on the frequency dependence of shot noise within the semiclassical approach (this suggests that the measuring frequency is much smaller than the voltage drop across the contact). We consider the case where all its dimensions are much larger than the screening length λ_0. The contact of length L is either a cylinder of circular section with a diameter $2r_0$ or a plane-parallel layer of thickness d_0 consisting of a metal with a high impurity content (see Fig. 1, upper inset). The electrodes are of the same section, yet the resistivity of their material is negligible. The contact is embedded in a perfectly conducting grounded medium, which is separated from its surface by a thin insulating film of thickness δ_0 and the dielectric constant ε_d. As will be shown below, this particular choise of contact geometry allows us to avoid solving the Poisson equation in the surrounding medium and reduces the effects of environmental screening to frequency-dependent boundary conditions. The external circuit is assumed to have a large grounding capacity, which allows accumulation of the charge in it.

Our consideration is based on the Boltzmann-Langevin approach first proposed in Ref. 13. The nonuniform extraneous currents caused by randomness of electron-impurity scattering result in local charge-density fluctuations in the bulk of the contact. These fluctuations are effectively screened by the surface charge induced at the outer surface of the contact and in the surrounding medium (environmental screening)

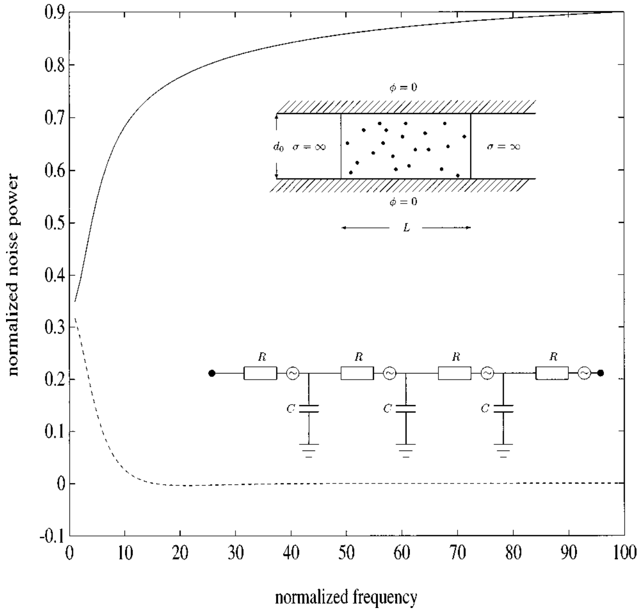

FIG. 1. Dependences of the normalized spectral density of noise at one of the contact ends $S_I^{LL}/2eI$ (solid line) and the cross-correlated spectral density $S_I^{LR}/2eI$ (dashed line) on the dimensionless frequency $\omega L^2\varepsilon_d/4\pi\delta_0 r_0$ for a long narrow contact. The upper inset shows the longitudinal cross section of the contact. The dotted rectangle is the metal with impurities, thin solid lines show the contact-electrode interfaces, thick lines show the dielectric layers of thickness δ_0, and the hatched areas show the grounded ambient medium. The lower inset shows the equivalent circuit for the noise in a long and narrow contact. Each section of the R-C line contains a generator of random current.

and at the contact-electrode interfaces (electrode screening). As will be shown below, the finite-frequency noise essentially depends on the dominating type of screening.

II. BASIC EQUATIONS

In the Boltzmann-Langevin approach, the long-range Coulomb interaction is taken into account by fluctuations of charge density $\delta\rho$ and self-consistent fluctuations of electrical field $\delta\mathbf{E}$. Consider the case of strong and purely elastic scattering. The Boltzmann-Langevin equation for fluctuations reads

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial \mathbf{r}}+e\mathbf{E}\mathbf{v}\frac{\partial}{\partial \epsilon}\right]\delta f+\delta I=-e\,\delta\mathbf{E}\mathbf{v}\frac{\partial f}{\partial \epsilon}+\delta J^{\mathrm{ext}}, \qquad (1)$$

where δJ^{ext} is the random extraneous flux. The correlation function of these fluxes is given by the expression[13]

$$\begin{aligned}&\langle \delta J^{\mathrm{ext}}(\mathbf{p},\mathbf{r},t)\,\delta J^{\mathrm{ext}}(\mathbf{p}',\mathbf{r}',t')\rangle\\ &=\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t')\Bigg(\delta_{pp'}\sum_q\{W(\mathbf{p},\mathbf{p}+\mathbf{q})f(\mathbf{p}+\mathbf{q})\\ &\quad\times[1-f(\mathbf{p})]+W(\mathbf{p}+\mathbf{q},\mathbf{p})f(\mathbf{p})[1-f(\mathbf{p}+\mathbf{q})]\}\\ &\quad-\{W(\mathbf{p}',\mathbf{p})f(\mathbf{p})[1-f(\mathbf{p}')]\\ &\quad+W(\mathbf{p},\mathbf{p}')f(p')[1-f(\mathbf{p})]\}\Bigg), \qquad (2)\end{aligned}$$

where $W(\mathbf{p},\mathbf{p}')$ is the probability of scattering from state $\mathbf{p}'$ to state $\mathbf{p}$. The fluctuation of electrical field $\delta\mathbf{E}$ in the right hand side of Eq. (1) is determined from the Maxwell equation

$$\nabla\,\delta\mathbf{E}=4\pi\,\delta\rho. \qquad (3)$$

The fluctuations of the charge and current density are given by the expressions

$$\delta\rho(\mathbf{r},t)=e\int d^3p\,\delta f(\mathbf{p},\mathbf{r},t),$$

$$\delta\mathbf{j}(\mathbf{r},t)=e\int d^3p\;\mathbf{v}\delta f(\mathbf{p},\mathbf{r},t). \qquad (4)$$

Now we proceed to the hydrodynamic approach and obtain a closed set of equations for macroscopic quantities $\delta\mathbf{j}$ and $\delta\rho$. As the impurity scattering is strong, we can split the fluctuation of distribution function into symmetric and antisymmetric parts in the momentum space. Then we separate the antisymmetric part of Eq. (1) from its symmetric part. Note that the extraneous flux contains only the antisymmetric part because the electron-impurity scattering does not change the total number of electrons at a given point with a given energy. Multiply the antisymmetric part of Eq. (1) by $e\mathbf{v}$, integrate it with respect to d^3p, and then multiply both its parts by the elastic scattering time τ. If the characteristic times considered are much larger than τ, one obtains

$$\delta\mathbf{j}=-D\frac{\partial}{\partial\mathbf{r}}\delta\rho+\sigma\,\delta\mathbf{E}+\delta\mathbf{j}^{\mathrm{ext}}, \qquad (5)$$

where $D=v^2\tau/3$ is the diffusion coefficient, $\sigma=e^2N_FD$ is the conductivity of metal, and $\delta\mathbf{j}^{\mathrm{ext}}=e\tau\int d^3p\mathbf{v}\delta J^{\mathrm{ext}}$. Integrating the symmetric part of Eq. (1) with respect to momentum, one obtains just the current-conservation law

$$\frac{\partial}{\partial t}\delta\rho+\nabla\,\delta\mathbf{j}=0. \qquad (6)$$

Applying the operator ∇ to both parts of Eq. (5) and making use of Eqs. (6) and (3), one obtains a closed equation for fluctuations of charge density in the form

$$\left(\frac{\partial}{\partial t}-D\nabla^2+4\pi\sigma\right)\delta\rho=-\nabla\,\delta\mathbf{j}^{\mathrm{ext}}. \qquad (7)$$

In the left-hand side of this equation, the second term describes diffusion of electrons, and the third term describes the Coulomb screening of fluctuations. In principle, Eq. (7) may be solved for each particular distribution of $\delta\mathbf{j}^{\mathrm{ext}}$, and then $\delta\mathbf{E}$ and $\delta\mathbf{j}$ may be determined from Eqs. (3) and (5), respectively. In the static limit, Eq. (7) describes the screening of an extraneous charge with the standard length λ_0 given by

$$\lambda_0^{-2}=\frac{4\pi\sigma}{D}=4\pi e^2N_F\,.$$

To complete the derivation, we must obtain the correlation function of extraneous currents $\delta\mathbf{j}^{\mathrm{ext}}$. As we are restricted to the case of strong impurity scattering, the distribution function may be considered as isotropic in the momentum space and dependent only on the coordinate $\mathbf{r}$ and energy ϵ. Multiply Eq. (2) by $e\tau v_\alpha$ and $e\tau v'_\beta$, where α and β label vector components, and integrate it with respect to d^3p and d^3p'. As a result, one obtains the spectral density of extraneous currents in the form

$$\langle \delta j_\alpha^{\rm ext}(\mathbf{r})\delta j_\beta^{\rm ext}(\mathbf{r}')\rangle_\omega = 4\sigma\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')\int d\epsilon\, f(\epsilon,\mathbf{r}) \times[1-f(\epsilon,\mathbf{r})]. \quad (8)$$

Because of smallness of λ_0, the relationship between the extraneous currents and fluctuations of charge density in the bulk of the sample may be considered as local. Taking the Fourier transform of Eq. (7) with respect to time, integrating it over the space, and making use of the Gauss theorem, one obtains

$$\delta\rho = -(-i\omega+4\pi\sigma)^{-1}\nabla\,\delta\mathbf{j}^{\rm ext}. \quad (9)$$

Note that the quasineutrality condition does not hold for fluctuations. We introduce the fluctuating potential $\delta\phi$ that satisfies the Poisson equation

$$\nabla^2\delta\phi = -4\pi\delta\rho. \quad (10)$$

Consider the boundary conditions for $\delta\phi$ at the outer insulated surface of the contact. The normal derivatives of $\delta\phi$ in the dielectric layer and inside the metal are related by the expression

$$\varepsilon_d \frac{\partial\delta\phi}{\partial n}\bigg|_d - \frac{\partial\delta\phi}{\partial n}\bigg|_s = -4\pi\delta\sigma_s, \quad (11)$$

where $\delta\sigma_s$ is fluctuating surface charge density induced by the extraneous currents. On the other hand, this charge density satisfies the charge-balance equation

$$-i\omega\delta\sigma_s = -\sigma\frac{\partial\delta\phi}{\partial n}\bigg|_s. \quad (12)$$

As the thickness of the dielectric layer is much smaller than the size of the contact, the electric field across it may be considered uniform so that $\partial\delta\phi/\partial n|_d = -\delta_0^{-1}\delta\phi|_s$. With this condition, Eqs. (11) and (12) give the boundary condition for $\delta\phi$ in the form

$$\left[-i\omega\varepsilon_d\delta_0^{-1}\delta\phi + (-i\omega+4\pi\sigma)\frac{\partial\delta\phi}{\partial n}\right]\bigg|_s = 0. \quad (13)$$

It is easily seen that at $\omega=0$, Eq. (13) takes the form $\partial\delta\phi/\partial n|_s=0$, while at $\omega\to\infty$, it takes the form $\delta\phi|_s=0$.

As the voltage drop across the contact is held constant, fluctuations of potential are zero at the contact-electrode interfaces:

$$\delta\phi|_i = 0. \quad (14)$$

As the electrodes are perfect conductors, $\partial\phi/\partial n=0$ inside them. Equation (11) holds for contact-electrode interfaces, but the charge-balance equation takes now the form

$$-i\omega\delta\sigma_s = -\sigma\frac{\partial\delta\phi}{\partial n}\bigg|_i - \delta j_n, \quad (15)$$

where δj_n is the fluctuation of current flowing into the electrodes from the contact. From Eqs. (11) and (15), it follows that the density of outgoing current is given by

$$\delta j_n = \left(\frac{i\omega}{4\pi} - \sigma\right)\frac{\partial\delta\phi}{\partial n}\bigg|_i. \quad (16)$$

From the standpoint of average current, the problem is purely one dimensional, so the average distribution function $f(\epsilon,x)$ obeys the one-dimensional diffusion equation, its boundary values being zero-temperature Fermi distribution functions shifted in energy by eV with respect to each other. As the contact is much shorter than the characteristic inelastic length,

$$f(\epsilon,x) = \begin{cases} 0, & \epsilon > eV/2 \\ 1-x/L, & eV/2 > \epsilon > -eV/2 \\ 1, & \epsilon < -eV/2. \end{cases} \quad (17)$$

With this distribution function, the expression for the spectral density of extraneous currents (8) takes the form

$$\langle \delta j_\alpha^{\rm ext}(\mathbf{r})\delta j_\beta^{\rm ext}(\mathbf{r}')\rangle_\omega = 4\sigma\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')\frac{x}{L}\left(1-\frac{x}{L}\right). \quad (18)$$

III. CIRCULAR-SECTION CONTACT: ANALYTICAL RESULTS

Consider the Poisson equation with the boundary conditions (13). As the system is axially symmetric, all the quantities may be considered independent of the azimuthal angle and dependent only on the longitudinal coordinate x and radius r. In this case, the boundary condition (13) takes the form

$$\left(r_0\frac{\partial\delta\phi}{\partial r} + \mu\delta\phi\right)\bigg|_{r=r_0} = 0, \qquad \mu \equiv \frac{-i\omega}{-i\omega+4\pi\sigma}\frac{\varepsilon_d r_0}{\delta_0}. \quad (19)$$

Suppose first that μ is real and positive. Then one may introduce a system of normalized eigenfunctions ψ_n satisfying the equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\psi_n(r)\right) + k_n^2\psi_n(r) = 0 \quad (20)$$

with the boundary conditions (19). These functions are given by

$$\psi_n(r) = \frac{1}{\pi^{1/2}r_0}\frac{J_0(k_n r)}{\sqrt{J_0^2(k_n r_0)+J_1^2(k_n r_0)}}, \quad (21)$$

where J_0 and J_1 are the Bessel functions of zeroth and first order and the eigenvalues k_n are determined from the equation

$$k_n r_0\frac{J_1(k_n r_0)}{J_0(k_n r_0)} = \mu. \quad (22)$$

Since functions ψ_n form an orthogonal basis, for an arbitrary charge-density fluctuation $\delta\rho$, the solution of Poisson equation (10) with the boundary condition (19) is given by the expression

$$\phi(x,r) = -4\pi \sum_{n=0}^{\infty} \psi_n(x,r) \int_0^L dx' \; g_n(x,x') \times \int dS' \; \psi_n(r') \rho(x',r'), \tag{23}$$

where $dS' = 2\pi r' dr'$ and $g_n(x,x')$ is the Green's function of the equation

$$\left(\frac{d^2}{dx^2} - k_n^2 \right) g_n(x,x') = \delta(x-x') \tag{24}$$

with the boundary conditions $g_n(0,x') = g_n(L,x') = 0$, which is given by the expression

$$g_n(x<x') = -\frac{\sinh(k_n x)\sinh[k_n(L-x')]}{k_n \sinh(k_n L)},$$
$$g_n(x>x') = -\frac{\sinh(k_n x')\sinh[k_n(L-x)]}{k_n \sinh(k_n L)}. \tag{25}$$

Equation (23) may be analytically continued to complex values of μ given by Eq. (19). Substituting Eq. (9) for $\delta\rho$ into Eq. (23) for $\delta\phi$ and then Eq. (23) into Eq. (16), one obtains the expression for the fluctuation of the current flowing through the left end of the contact in the form

$$\delta I(0) = S_0 \sum_{n=0}^{\infty} \overline{\psi_n} \int_0^L dx' \int dS' \left[\frac{\partial^2 g_n(x,x')}{\partial x \partial x'} \right|_{x=0} \times \psi_n(r') \, \delta j_x^{\text{ext}} + \left. \frac{\partial g_n(x,x')}{\partial x} \right|_{x=0} \frac{\partial \psi_n}{\partial r'} \delta j_r^{\text{ext}}, \tag{26}$$

where $S_0 = \pi r_0^2$ and $\overline{\psi_n}$ is ψ_n averaged over the cross section of the contact:

$$\overline{\psi_n} = \frac{1}{S_0} \int dS \; \psi_n(r) = \frac{2J_1(k_n r_0)}{\pi^{1/2} r_0^2 k_n \sqrt{J_0^2(k_n r_0) + J_1^2(k_n r_0)}}. \tag{27}$$

To obtain the fluctuation of the current flowing through the right end of the contact, $\delta I(L)$, one must substitute $x = L$ for $x = 0$ in Eq. (26).

At $\omega = 0$, all transverse modes with $n \neq 0$ have vanishing cross-sectional averages, $\overline{\psi_n} = 0$, and the corresponding longitudinal factors g_n exponentially decay at $|x - x'| > r_0$. This is quite natural because the electrical field produced by a charge inside the contact cannot penetrate through its outer surface and is uniformly distributed over the contact cross section at large distances from the source. Hence the only contribution to Eq. (26) will be given by the lowest transverse mode with $k_0 = 0$ and $\psi_0(r) = \pi^{-1/2} r_0^{-1}$. In this case, Eq. (26) takes the form

$$\delta I(0) = \frac{1}{L} \int_0^L dx \int dS \;\; \delta j_x^{\text{ext}}. \tag{28}$$

This is just the result obtained in Ref. 3.

Consider now the case where the contact length L is much larger than its diameter $2r_0$ and the frequencies are sufficiently low, i.e., $\omega \ll 4\pi\sigma\delta_0/\varepsilon_d r_0$. In this case, the corrections to the zero-frequency eigenfunctions ψ_n, as well as the corrections to the products $k_n r_0$ with $n \neq 0$, are proportional to μ and therefore small; hence the contributions to δI (26) from the modes with $n \neq 0$ remain insignificant. However, the lowest eigenvalue is given by

$$k_0 = r_0^{-1} (2\mu)^{1/2}, \tag{29}$$

and the product $k_0 L$ may be sufficiently large. Therefore, the contribution from the lowest mode governed by $g_0(0,x)$ may change significantly. In view of this, the expression for δI takes the form

$$\delta I(0) = \int_0^L dx \; k_0 \frac{\cosh[k_0(L-x)]}{\sinh(k_0 L)} \int dS \;\; \delta j_x^{\text{ext}}, \tag{30}$$

where k_0 is given by Eq. (29). Physically, this implies that the contact is represented as an alternating series of resistors with generators of random current and grounding capacities connecting the electrodes (see Fig. 1, lower inset). Note that $\delta I(0)$ is phase shifted with respect to the extraneous current inducing it. Multiplying Eq. (30) by its complex conjugate, substituting the spectral density of extraneous currents (18) into the product, and performing the integration with respect to x, one obtains

$$S_I^{LL}(\omega) = 2eI \left[1 - \frac{1}{\gamma_\omega L} \frac{\sinh(2\gamma_\omega L) - \sin(2\gamma_\omega L)}{\cosh(2\gamma_\omega L) - \cos(2\gamma_\omega L)} \right], \tag{31}$$

where

$$\gamma_\omega = \frac{1}{2} \sqrt{\frac{\omega \varepsilon_d}{\pi \sigma \delta_0 r_0}}. \tag{32}$$

The frequency dependence of the shot noise is shown in Fig. 1. At zero frequency, we rederive the well-known result $S_I^{LL} = \frac{2}{3} eI$. However, at frequencies about $\sigma\delta_0 r_0/\varepsilon_d L^2$, the spectral density sharply rises and tends to the full value of classical shot noise, $S_I^{LL} = 2eI$. This suggests that the corresponding correlation function is negative at $t \neq t'$. The anticorrelation between current fluctuations is the consequence of the Coulomb repulsion of electrons: an entrance of an electron into the contact decreases for some time the probability for another electron to enter it, similarly to the case of a single-electron transistor.[15,14]

Along with the spectral density of noise at one end of the contact, one may also consider the cross-correlated spectral density

$$S_I^{LR} \equiv \tfrac{1}{2} \langle \delta I(0,\omega) \delta I(L,-\omega) + \delta I(0,-\omega) \delta I(L,\omega) \rangle,$$

which describes the correlation between the currents flowing through the opposite ends of the contact. Multiplying Eq.

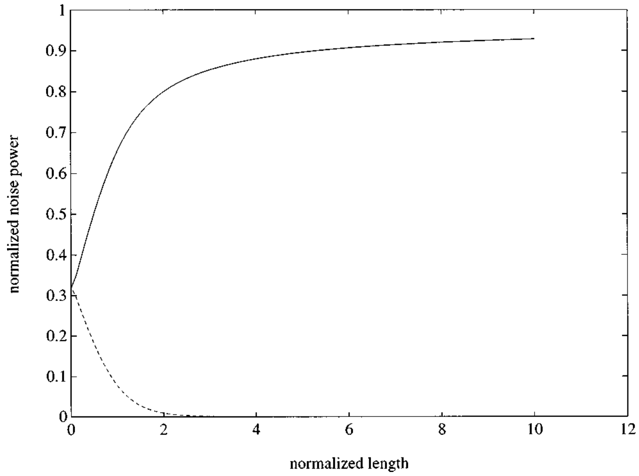

FIG. 2. Dependences of the normalized spectral density of noise at one of the contact ends $S_I^{LL}/2eI$ (solid line) and cross-correlated spectral density $S_I^{LR}/2eI$ (dashed line) on the length-to-radius ratio L/r_0 in the high-frequency limit.

(30) by its complex conjugate for $\delta I(L)$ and performing the integration with the spectral density of extraneous currents (18), one obtains

$$S_I^{LR}(\omega)=\frac{4eI}{\gamma_\omega L}\,\frac{\cosh(\gamma_\omega L)\sin(\gamma_\omega L)-\cos(\gamma_\omega L)\sinh(\gamma_\omega L)}{\cosh(2\gamma_\omega L)-\cos(2\gamma_\omega L)}. \tag{33}$$

The frequency dependence of S_I^{LR} is also shown in Fig. 1. At $\omega=0$, it also equals $\frac{2}{3}eI$. However, in contrast to $S_I^{LL}(\omega)$, it sharply decreases with increasing frequency and tends to zero in an oscillatory way with further increase of frequency.

Consider now the high-frequency limit. In this case, the boundary condition (13) takes the form $\psi_n(r_0)=0$, so that the quantities $k_n r_0$ are the zeros of zero-order Bessel function. In this case, functions ψ_n are real and form an orthogonal system. Owing to the orthonormality conditions, the expression for the spectral density of noise may be written in the form

$$S_I^{LL}(\infty)=4S_0^2 eV\sigma\sum_{n=1}^{\infty}\overline{\psi_n}^2\int_0^L dx'\,\frac{x'}{L}\left(1-\frac{x'}{L}\right)\times\left\{\left[\left.\frac{\partial^2 g_n(x,x')}{\partial x\partial x'}\right|_{x=0}\right]^2+k_n^2\left[\left.\frac{\partial g_n(x,x')}{\partial x}\right|_{x=0}\right]^2\right\}. \tag{34}$$

As $J_0(k_n r_0)=0$, Eq. (27) reduces to $\overline{\psi_n}=2\pi^{-1/2}r_0^{-2}k_n^{-1}$. Substituting the explicit expressions for g_n (25) into Eq. (34), one obtains

$$S_I^{LL}(\infty)=8eI\sum_{n=1}^{\infty}\frac{1}{(k_n r_0)^2}\coth(k_n L)\left[\coth(k_n L)-\frac{1}{k_n L}\right], \tag{35}$$

Similarly, one obtains for the cross-correlated spectral density:

$$S_I^{LR}(\infty)=8eI\sum_{n=1}^{\infty}\frac{1}{(k_n r_0)^2}\,\frac{1}{\sinh(k_n L)}\left[\coth(k_n L)-\frac{1}{k_n L}\right]. \tag{36}$$

In the limiting case of $r_0\gg L$, both expressions take the form

$$S_I^{LL}(\infty)=S_I^{LR}(\infty)=\frac{8}{3}eI\sum_{n=1}^{\infty}\frac{1}{(k_n r_0)^2}=\frac{2}{3}eI. \tag{37}$$

In the opposite limiting case of $r_0\ll L$, Eq. (35) takes the form

$$S_I^{LL}(\infty)=8eI\sum_{n=1}^{\infty}\frac{1}{(k_n r_0)^2}=2eI, \tag{38}$$

whereas $S_I^{LR}(\infty)$ (36) tends to zero according to the exponential law. The L/r_0 dependences of $S_I^{LL}(\infty)$ and $S_I^{LR}(\infty)$ are shown in Fig. 2.

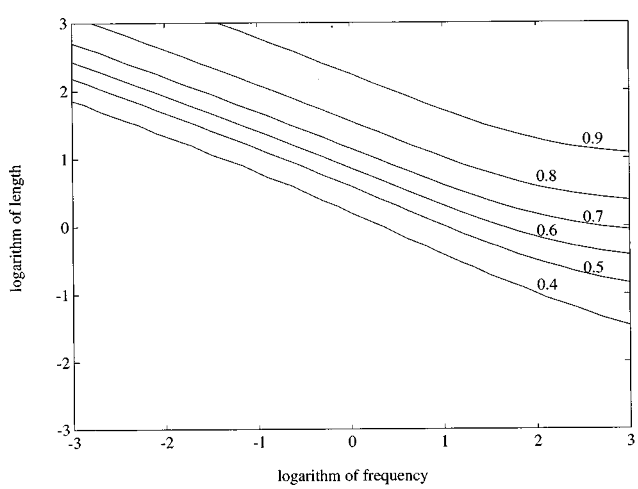

FIG. 3. Contour plots of S_I^{LL} vs logarithms of normalized frequency $\Omega=\omega\varepsilon_d d_0/4\pi\sigma\delta_0$ and normalized length L/d_0 for the planar contact.

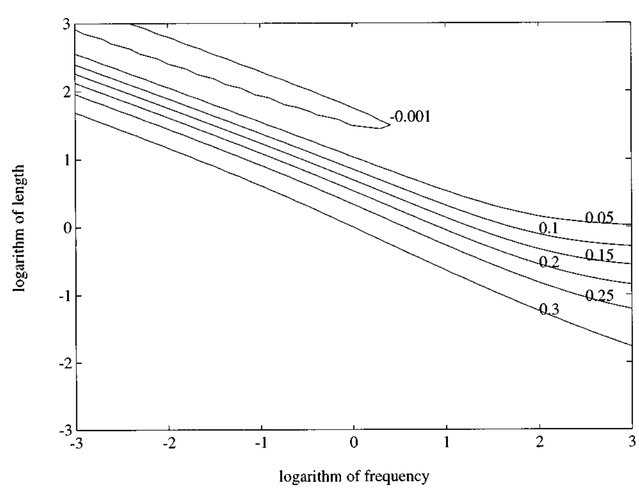

FIG. 4. Contour plots of S_I^{LR} vs logarithms of normalized frequency $\Omega=\omega\varepsilon_d d_0/4\pi\sigma\delta_0$ and normalized length L/d_0 for the planar contact.

IV. PLANAR CONTACT: NUMERICAL RESULTS

Consider now a planar contact in the shape of a layer of thickness d_0 in the y direction ($0<y<d_0$) and of width W [$W \gg \max(d_0,L)$] in the z direction, the average current flowing in the x direction. Because of large W, the effects of boundaries in the z direction may be neglected and all the quantities may be considered as independent of z. Introduce an orthonormal system of functions

$$\varphi_n(x)=\sqrt{\frac{2}{L}}\sin(q_n x),\qquad q_n=\frac{\pi n}{L}, \tag{39}$$

which obey the boundary conditions $\varphi_n(0)=\varphi_n(L)=0$. For an arbitrary charge-density fluctuation $\delta\rho$, the potential fluctuation $\delta\phi$ induced by it may be presented in the form

$$\delta\phi(x,y)=-4\pi\sum_{n=1}^{\infty}\varphi_n(x)\int_0^L dx'\,\varphi(x')\times\int_0^{d_0}dy'\,Q_n(y,y')\,\delta\rho(x',y'), \tag{40}$$

where $Q_n(y,y')$ satisfies the equation

$$\left(\frac{\partial^2}{\partial y^2}-q_n^2\right)Q_n(y,y')=\delta(y-y') \tag{41}$$

with the boundary conditions

$$\left.\left(\frac{-i\omega\varepsilon_d}{4\pi\sigma\delta_0}Q_n+\frac{\partial Q_n}{\partial y}\right)\right|_{y=d_0}=0,$$

$$\left.\left(\frac{i\omega\varepsilon_d}{4\pi\sigma\delta_0}Q_n+\frac{\partial Q_n}{\partial y}\right)\right|_{y=0}=0. \tag{42}$$

Explicitly, Q_n for $y>y'$ is given by the expression

$$Q_n(y,y')=-\frac{1}{2q_n}\,\frac{q_nd_0\cosh[q_n(d_0-y)]-i\Omega\sinh[q_n(d_0-y)]}{q_nd_0\cosh(q_nd_0/2)-i\Omega\sinh(q_nd_0/2)}\,\frac{q_nd_0\cosh(q_ny')-i\Omega\sinh(q_ny')}{q_nd_0\sinh(q_nd_0/2)-i\Omega\cosh(q_nd_0/2)}, \tag{43}$$

where $\Omega=\omega\varepsilon_d d_0/4\pi\sigma\delta_0$ is the dimensionless frequency. The corresponding expression for $y<y'$ is obtained from Eq. (43) by interchanging y and y'. Substituting Eq. (9) for $\delta\rho$ into Eq. (40) and then substituting Eq. (40) into Eq. (16), one obtains the expression for the fluctuation of current flowing through the left end of the contact in the form

$$\delta I(0)=W\sum_{n=1}^{\infty}\left.\frac{d\varphi_n}{dx}\right|_{x=0}\int_0^L dx'\int_0^{d_0}dy\int_0^{d_0}dy'\left[\frac{d\varphi_n(x')}{dx'}Q_n(y,y')\,\delta j_x^{\text{ext}}+\varphi_n(x')\frac{\partial Q_n(y,y')}{\partial y'}\,\delta j_y^{\text{ext}}\right]. \tag{44}$$

The fluctuation of current flowing through the right end of the contact may be obtained by substituting $x=L$ for $x=0$ in Eq. (44). Using the correlator of extraneous currents (18), one obtains the expressions for the spectral densities S_I^{LL} and S_I^{LR} in the form

$$S_I^{LL}=8eId_0\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}k_mk_n(M_{mn}P''_{mn}+M''_{mn}P_{mn}), \tag{45}$$

$$S_I^{LR}=4eId_0\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}[(-1)^m+(-1)^n]k_mk_n(M_{mn}P''_{mn}+M''_{mn}P_{mn}). \tag{46}$$

In these expressions, we used the notation

$$M_{mn}=\frac{1}{d_0^2}\int_0^{d_0}dy\int_0^{d_0}dy_1\int_0^{d_0}dy_2\,Q_m(y_1,y)Q_n^*(y_2,y), \tag{47}$$

$$M''_{mn}=\frac{1}{d_0^2}\int_0^{d_0}dy\int_0^{d_0}dy_1\int_0^{d_0}dy_2\,\frac{\partial Q_m(y_1,y)}{\partial y}\,\frac{\partial Q_n^*(y_2,y)}{\partial y}, \tag{48}$$

$$P_{mn}=\int_0^L dx\;\varphi_m(x)\varphi_n(x)\frac{x}{L}\left(1-\frac{x}{L}\right), \tag{49}$$

$$P''_{mn}=\int_0^L dx\;\frac{\partial\varphi_m}{\partial x}\,\frac{\partial\varphi_n}{\partial x}\left(1-\frac{x}{L}\right). \tag{50}$$

Using the notation $t_m = \tanh(k_m d_0/2)$ and $D_m = (k_m^2 d_0^2 t_m^2 + \Omega^2)^{-1}$ and performing partial summation over the internal index, one may bring Eqs. (45) and (46) to the form

$$S_I^{LL}(\omega) = \frac{2}{3}eI + 4eI\Omega^2 \sum_{m=1}^{\infty} \left\{ \left(\frac{2}{3} - 8\frac{1+(-1)^m}{k_m^2 L^2} \right) \frac{D_m t_m}{k_m d_0} - \frac{D_m}{k_m^2 L^2}(1-t_m^2) - \frac{8}{L^2} \sum_{n=1}^{\infty}{}' \, [1+(-1)^{m+n}] \frac{D_m D_n}{d_0(k_m^2 - k_n^2)^2} \times (d_0^2 k_m t_m k_n t_n + \Omega^2)(k_m t_m + k_n t_n) \right\}, \tag{51}$$

$$S_I^{LR}(\omega) = \frac{2}{3}eI + 4eI\Omega^2 \sum_{m=1}^{\infty} (-1)^m \left\{ \left(\frac{2}{3} - 8\frac{1+(-1)^m}{k_m^2 L^2} \right) \frac{D_m t_m}{k_m d_0} - \frac{D_m}{k_m^2 L^2}(1-t_m^2) - \frac{8}{L^2} \sum_{n=1}^{\infty}{}' \, [1+(-1)^{m+n}] \frac{D_m D_n}{d_0(k_m^2 - k_n^2)^2} (d_0^2 k_m t_m k_n t_n + \Omega^2)(k_m t_m + k_n t_n) \right\}, \tag{52}$$

where the primes by the sums over n show that $n \neq m$.

The contour plots of S_I^{LL} and S_I^{LR} versus logarithms of frequency and contact length are shown in Figs. 3 and 4. Qualitatively, their behavior is similar to that in the case of a circular-section contact. At low frequencies and small contact lengths, both quantities tend to $\frac{2}{3}eI$. At high frequencies and large contact lengths, S_I^{LL} and S_I^{LR} tend to $2eI$ and zero, respectively. It is also clearly seen that at $L/d_0 \geqslant 2.86$, the frequency dependences of S_I^{LR} exhibit negative portions.

V. CONCLUSION

Both circular and planar contacts exhibit qualitatively similar noise properties. At small length-to-width ratios, when the screening of charge fluctuations by the electrodes is more efficient than the screening by the ambient medium and pileup of the charge in the contact is forbidden, the effects of long-range Coulomb interaction reduce to averaging the extraneous currents over the volume of the contact at arbitrary frequencies. The situation is different, however, for long and narrow contacts, where the charge fluctuations are mostly screened by the ambient medium and pileup of charge in the contact is allowed. At sufficiently high frequencies, the correlation length of fluctuations becomes smaller than the length of the contact. In this case, the fluctuations of current at the ends of the contact, which are observed in the external circuit, are dominated by extraneous currents in the narrow adjacent layers. The corresponding spectral densities are equal to that of the classical shot noise, $2eI$, while the fluctuations at different contact ends are completely independent.

To the best of our knowledge, the only measurements of high-frequency shot noise in mesoscopic diffusive contacts were performed in [16]. However, the authors focused on quantum suppression of shot noise at small voltages, which is beyond this quasiclassical approach. So their results are difficult to interpret in terms of Coulomb interactions.

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