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Electron-electron interaction in disordered mesoscopic systems: Weak localization and mesoscopic fluctuations of polarizability and capacitance

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The weak localization correction and the mesoscopic fluctuations of the polarizability and the capacitance of a small disordered sample are studied systematically in two-dimensional and three-dimensional geometries. While the grand canonical ensemble calculation gives the positive magnetopolarizability, in the canonical ensemble (appropriate for isolated samples) the sign of the effect is reversed. The magnitude of mesoscopic fluctuations for a single sample exceeds considerably the value of the weak localization correction. [S0163-1829(98)04707-9]

I. INTRODUCTION

The phenomena of weak localization (WL) and mesoscopic fluctuations in disordered systems have been intensively studied during the past 15 years, mainly in connection with transport properties of these systems.[1,2] For these phenomena the role of the electron-electron interaction is just in setting the length scale l_ϕ (phase breaking length), below which the electron wave function preserves its phase coherence. Systems with a size L less than l_ϕ are called mesoscopic systems.

In this paper we consider quantum corrections and mesoscopic fluctuations of the two other characteristics of a mesoscopic system, where the electron-electron interaction is essential: polarizability and capacitance. The former quantity can be measured by putting a sample into a capacitor, while the latter one determines the charging energy that shows up in the Coulomb blockade experiments.[3]

The first quantum calculation of the polarizability of a small metallic particle was obtained in a seminal paper by Gor'kov and Eliashberg (GE).[4] It relied on the following two assumptions concerning statistical properties of energy levels and eigenfunctions in disordered systems: (i) The single-particle energy spectrum exhibits the same statistics as the eigenvalue spectrum of random matrices from the Gaussian ensemble of the corresponding symmetry and (ii) exact single-particle eigenfunctions $\psi_k(\boldsymbol{r})$ and $\psi_l(\boldsymbol{r})$, which are close enough in energy, are correlated as

$$V^2\langle \psi_k^*(\boldsymbol{r})\psi_l(\boldsymbol{r})\psi_k(\boldsymbol{r}')\psi_l^*(\boldsymbol{r}')\rangle_{\epsilon,\omega}=\Pi_D(\boldsymbol{r},\boldsymbol{r}'). \qquad (1)$$

Here the average is defined as

$$\langle \psi_k^*(\boldsymbol{r})\psi_l(\boldsymbol{r})\psi_k(\boldsymbol{r}')\psi_l^*(\boldsymbol{r}')\rangle_{\epsilon,\omega}\equiv \frac{\left\langle \sum_{k\neq l} \psi_k^*(\boldsymbol{r})\psi_l(\boldsymbol{r})\psi_k(\boldsymbol{r}')\psi_l^*(\boldsymbol{r}')\,\delta(\epsilon-\epsilon_k)\,\delta(\epsilon+\omega-\epsilon_l)\right\rangle}{\left\langle \sum_{k\neq l} \delta(\epsilon-\epsilon_k)\,\delta(\epsilon+\omega-\epsilon_l)\right\rangle}, \qquad (2)$$

V is the sample volume, and the diffusion propagator Π_D is a solution to the diffusion equation

$$-D\nabla^2\Pi_D(\boldsymbol{r},\boldsymbol{r}')=(\pi\nu)^{-1}[\delta(\boldsymbol{r}-\boldsymbol{r}')-V^{-1}], \qquad (3)$$

with the boundary conditions $\boldsymbol{\nabla}_n\Pi_D=0$. The first of these conjectures was proved by Efetov[5] and the second by the present authors.[6] More precisely, it was shown in Ref. 6 that for the energy difference much less than the Thouless energy $\omega\ll E_c$,

$$V^2\langle \psi_k^*(\boldsymbol{r})\psi_l(\boldsymbol{r})\psi_k(\boldsymbol{r}')\psi_l^*(\boldsymbol{r}')\rangle_{\epsilon,\omega} =k_d(\boldsymbol{r}-\boldsymbol{r}')+\Pi_D(\boldsymbol{r},\boldsymbol{r}'), \qquad (4)$$

where $k_d(\boldsymbol{r})=(\pi\nu)^{-2}\langle \operatorname{Im}G^R(\boldsymbol{r})\rangle^2$ is a short-range function $[G^R(\boldsymbol{r})$ being the retarded Green's function$]$ explicitly given by

$$k_d(\boldsymbol{r})=\exp(-r/l)\times\begin{cases} J_0^2(p_Fr), & 2\mathrm{D}\\ (p_Fr)^{-2}\sin^2 p_Fr, & 3\mathrm{D}.\end{cases} \qquad (5)$$

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The short-range part $k_d(\boldsymbol{r}-\boldsymbol{r}')$ of the correlation function (4) was not taken into account by Gor'kov and Eliashberg, but it would give only a small correction to their result.[7]

Based on these conjectures, GE concluded that the polarizability for very low frequencies $\omega \ll \Delta$ $[\Delta=(\nu V)^{-1}$ is the mean single-particle level spacing and ν is the density of states per spin] is enhanced in comparison to the classical value $\alpha_0 \sim V$, the enhancement factor being of order $(\kappa R)^2$, where

$$\kappa=\begin{cases}(8\pi e^2\nu)^{1/2}, & \text{3D}\\ 4\pi e^2\nu, & \text{2D}\end{cases} \tag{6}$$

is the inverse screening radius. Although the original paper[4] gave insight into the field (called later mesoscopic physics) and had a substantial impact on the further development of the condensed-matter physics, this result for the polarizability is incorrect for the following reason. The paper by Gor'kov and Eliashberg does not take into account the effects of screening: They calculate the polarizability in response to the local field rather than to the external one.[8] As was found in Refs. 9 and 10 (see also Ref. 11), the screening restores the classical value of the polarizability, thus reducing the quantum effects to a relatively small correction. Evaluation of this correction was recently attempted by Efetov,[12] who combined the nonperturbative calculation of the polarization function[6] with the electron-electron interactions taken into account in the random phase approximation (RPA). Since the value of the quantum correction depends on the presence (or absence) of the time-reversal symmetry, it was denoted by Efetov as a ''weak localization correction to polarizability''; we are following this terminology in the present article. However, he estimated incorrectly the contribution of the short-range term in Eq. (4), which made him conclude that the weak localization correction is dominated by this term. As we show below, this is not the case if the system size exceeds considerably the mean free path. More recently, Noat, Reulet, and Bouchiat[13] presented a perturbative calculation of the weak localization correction to the polarizability in a particular geometry of a narrow two-dimensional (2D) ring. They considered both the canonical ensemble (CE) and grand-canonical ensemble (GCE) and concluded that the correction to the polarizability is parametrically suppressed in the CE. While essentially confirming their GCE result, we disagree with the above statement concerning the CE. We show below that the effect in the CE is of the same (up to a coefficient of order one) magnitude as the GCE one, but has opposite sign.

As was realized by Berkovits and Altshuler,[14] fluctuations in the polarization function lead to mesoscopic fluctuations of the polarizability of the sample. They considered a specific thin-film geometry and identified the four-diffusion diagrams giving the leading contribution to the fluctuations. We will follow their approach when studying the polarizability fluctuations in 2D and 3D geometries.

Along with the polarizability, we consider another quantity characterizing a mesoscopic system, the capacitance C. It determines the charging energy e^2/C, which manifests itself in the I-V characteristics of a quantum dot in the Coulomb blockade regime. In particular, the charging energy represents the main contribution to the threshold voltage in the excitation spectra and to the distance between adjacent conductance peaks in the addition spectra. Statistical properties of the Coulomb blockade I-V characteristics are attracting a great deal of interest, and hence research, now,[15–21] which motivated us to consider the WL correction and the mesoscopic fluctuations of the charging energy. In addition, the capacitance determines the low-frequency behavior of the impedance of mesoscopic systems.[22,23]

Therefore, the purpose of the present paper is to study *systematically* the WL effects and mesoscopic fluctuations of the polarizability and capacitance in 2D and 3D geometries. Where it is necessary, we refine the results of previous research. We show that the polarizability and the capacitance can be treated on the same physical grounds. We will also find a simple relation between the magnitude of the WL correction and that of the mesoscopic fluctuations. The electron-electron interaction is taken into account in the RPA, which works for $\kappa < p_F$, p_F being the Fermi momentum. We consider the case of low temperature $T \ll \Delta$ (thus setting $T=0$ in all formulas) and study both grand-canonical and canonical ensembles.

II. WEAK LOCALIZATION CORRECTION TO THE POLARIZABILITY OF SMALL PARTICLES

We consider an isolated disordered metallic particle Ω (3D or 2D) placed into a uniform external frequency-dependent electric field $\boldsymbol{E}(\omega)$. We assume that the system is diffusive, $l \ll L$, where l and L are the mean free path and the typical size of a particle, respectively. In the RPA the potential distribution $\Phi(\boldsymbol{r})$ and the electron density $\rho(\boldsymbol{r})$ in the particle obey the Poisson equation ($e=-|e|$ being the electron charge)

$$\Delta\Phi(\boldsymbol{r})=-4\pi e\rho(\boldsymbol{r})\,\theta_\Omega(\boldsymbol{r})\times\begin{cases}1, & \text{3D}\\ \delta(z), & \text{2D},\end{cases}$$

$$\theta_\Omega(\boldsymbol{r})=\begin{cases}1, & \boldsymbol{r}\in\Omega\\ 0 & \text{otherwise}'\end{cases} \tag{7}$$

in combination with the equation

$$\rho(\boldsymbol{r})=-2e\int_\Omega \Pi(\boldsymbol{r},\boldsymbol{r}')\Phi(\boldsymbol{r}')d\boldsymbol{r}'. \tag{8}$$

In two dimensions we use the following convention throughout the paper: $\boldsymbol{r}=(x,y)$ denotes the coordinates in plain and z is the transverse coordinate. The Laplacian Δ is always a three-dimensional operator $\Delta\equiv\Delta_3=\Delta_2+\partial_z^2$. Furthermore, Π is the polarization function (per spin), which can be readily expressed through the Matsubara Green's functions,

$$\Pi(\boldsymbol{r},\boldsymbol{r}',\omega)=-T\sum_{\epsilon_m}\langle G(\boldsymbol{r},\boldsymbol{r}',i\epsilon_m+i\omega_n)$$
$$\times G(\boldsymbol{r}',\boldsymbol{r},i\epsilon_m)\rangle|_{i\omega_n\to\omega+i0}, \tag{9}$$

or in terms of the retarded and advanced Green's functions $G^{R,A}(\boldsymbol{r},\boldsymbol{r}',\epsilon)$,

$$\Pi(\boldsymbol{r},\boldsymbol{r}',\omega)=\frac{-i}{2\pi}\int d\epsilon\{n_F(\epsilon)\langle G^R(\boldsymbol{r},\boldsymbol{r}',\epsilon+\omega)G^R(\boldsymbol{r}',\boldsymbol{r},\epsilon)\rangle$$
$$-n_F(\epsilon+\omega)\langle G^A(\boldsymbol{r},\boldsymbol{r}',\epsilon+\omega)G^A(\boldsymbol{r}',\boldsymbol{r},\epsilon)\rangle$$
$$+[n_F(\epsilon+\omega)-n_F(\epsilon)]$$
$$\times\langle G^R(\boldsymbol{r},\boldsymbol{r}',\epsilon+\omega)G^A(\boldsymbol{r}',\boldsymbol{r},\epsilon)\rangle\}$$
$$\equiv\Pi^{RR}(\boldsymbol{r},\boldsymbol{r}',\omega)+\Pi^{AA}(\boldsymbol{r},\boldsymbol{r}',\omega)+\Pi^{RA}(\boldsymbol{r},\boldsymbol{r}',\omega). \tag{10}$$

While Π^{RR} and Π^{AA} can be calculated in the usual impurity perturbation theory, the evaluation of Π^{RA} for low frequencies $\omega\lesssim\Delta$ requires a nonperturbative treatment. This was done in Ref. 6 in the framework of the supersymmetric σ-model approach and we present only the results here:

$$\Pi^{RR}(\boldsymbol{r},\boldsymbol{r}',\omega)+\Pi^{AA}(\boldsymbol{r},\boldsymbol{r}',\omega)=\frac{1}{\pi}\int_{-\infty}^{0}d\epsilon\ \mathrm{Im}\langle G^R(\boldsymbol{r},\boldsymbol{r}',\epsilon)\rangle^2$$
$$+\frac{\omega}{2\pi i}[\langle\ \mathrm{Re}G^R(\boldsymbol{r},\boldsymbol{r}',0)\rangle^2$$
$$-\langle\ \mathrm{Im}G^R(\boldsymbol{r},\boldsymbol{r}',0)\rangle^2];$$

$$\Pi^{RA}(\boldsymbol{r},\boldsymbol{r}',\omega)=-\frac{\nu}{V}-\frac{\omega}{2\pi i}\{\langle\ \mathrm{Re}G^R(\boldsymbol{r},\boldsymbol{r}',0)\rangle^2+S(\omega)$$
$$\times\langle\ \mathrm{Im}G^R(\boldsymbol{r},\boldsymbol{r}',0)\rangle^2$$
$$+(\pi\nu)^2[1+S(\omega)]\Pi_D(\boldsymbol{r},\boldsymbol{r}')\}. \tag{11}$$

Here $S(\omega)$ is a correlation function of the zero-dimensional σ-model, $S(\omega)=-\langle Q_{bb}^{11}Q_{bb}^{22}\rangle$ (in the notation of Ref. 6), given explicitly by

$$S(\omega)=1+\frac{2i\Delta^2}{\pi^2\omega^2}\exp\left(\frac{\pi i\omega}{\Delta}\right)\sin\frac{\pi\omega}{\Delta}. \tag{12}$$

It is related to the two-level correlation function $R_2(\omega)$ as $R_2(\omega)=\ \mathrm{Re}[1+S(\omega)]/2$. Now we decompose the polarization function into frequency-independent and frequency-dependent parts

$$\Pi(\boldsymbol{r},\boldsymbol{r}',\omega)=\Pi_0(\boldsymbol{r},\boldsymbol{r}')+\Pi_1(\boldsymbol{r},\boldsymbol{r}',\omega),$$

$$\Pi_0(\boldsymbol{r},\boldsymbol{r}')=\frac{1}{\pi}\ \mathrm{Im}\int_{-\infty}^{0}d\epsilon\langle G^R(\boldsymbol{r},\boldsymbol{r}',\epsilon)\rangle^2-\frac{\nu}{V}, \tag{13}$$

$$\Pi_1(\boldsymbol{r},\boldsymbol{r}',\omega)=-\frac{\omega}{2\pi i}[1+S(\omega)][\langle\ \mathrm{Im}G^R(\boldsymbol{r},\boldsymbol{r}',0)\rangle^2$$
$$+(\pi\nu)^2\Pi_D(\boldsymbol{r},\boldsymbol{r}')]$$
$$=\frac{\nu}{V}A(\omega)[k_d(\boldsymbol{r}-\boldsymbol{r}')+\Pi_D(\boldsymbol{r},\boldsymbol{r}')], \tag{14}$$

where we defined $A(\omega)=(i\pi\omega/2\Delta)[1+S(\omega)]$ and introduced the function $k_d(\boldsymbol{r}-\boldsymbol{r}')=\langle\ \mathrm{Im}G^R(\boldsymbol{r},\boldsymbol{r}',0)\rangle^2$ given explicitly by Eq. (5). Note that the formulas (11), (12), and (14) are written for the case of the unitary ensemble (broken time-reversal symmetry due to the presence of a strong enough magnetic field), which was considered in Ref. 6. Generalization to the orthogonal ensemble (unbroken time-reversal symmetry) is straightforward and results in the following modification of the factor $A(\omega)$ in Eq.(14) for Π_1:

$$A(\omega)=\frac{i\pi\omega}{2\Delta}[1+S(\omega)]-1, \tag{15}$$

where now

$$S(\omega)=1+\frac{2ie^{is}\sin s}{s^2}+2i\frac{d}{ds}\left(\frac{\sin s}{s}\right)\int_1^{\infty}\frac{e^{ist}}{t}dt \tag{16}$$

and $s=\pi\omega/\Delta$. In the limit of low frequency $\omega\ll\Delta$, the factor $A(\omega)$ is equal to

$$A(\omega\ll\Delta)=-\frac{2}{\beta}\equiv\begin{cases}-1, & \text{unitary}\\ -2, & \text{orthogonal,}\end{cases} \tag{17}$$

where β is the usual parameter equal to 1 (2) for the orthogonal (unitary) ensemble.

Now we turn to the calculation of the dipole moment. The general expressions were obtained in Ref. 12; we present here the derivation for the sake of completeness. We consider the frequency-dependent part Π_1 as a perturbation and expand functions $\rho(\boldsymbol{r})$, $\Phi(\boldsymbol{r})$, and the dipole moment

$$\boldsymbol{d}=e\int\boldsymbol{r}\rho(\boldsymbol{r})d\boldsymbol{r},$$

with respect to Π_1. In the zeroth-order approximation we obtain

$$\boldsymbol{d}_0=-2e\int_\Omega\boldsymbol{r}\Pi_0(\boldsymbol{r},\boldsymbol{r}')\Phi_0(\boldsymbol{r}')d\boldsymbol{r}d\boldsymbol{r}', \tag{18}$$

where the potential $\Phi_0(\boldsymbol{r})$ satisfies Eqs. (7) and (8) with Π replaced by Π_0.

It is easy to check that the first term in the expression (13) for $\Pi_0(\boldsymbol{r},\boldsymbol{r}')$ gives ν after integration over one of the coordinates, the integral being dominated by the distances of order of the Fermi wavelength, $|\boldsymbol{r}-\boldsymbol{r}'|\sim\lambda_F$. Assuming the screening length [which sets the scale for the variation of the potential $\Phi(\boldsymbol{r})$] to be larger than λ_F, we can thus replace this term by a δ function:

$$\Pi_0(\boldsymbol{r},\boldsymbol{r}')=\nu[\delta(\boldsymbol{r}-\boldsymbol{r}')-V^{-1}]. \tag{19}$$

This approximation for the polarization function Π_0 leads to the simple relation between the potential $\Phi_0(\boldsymbol{r})$ and the excess density $\rho_0(\boldsymbol{r})$ (Thomas-Fermi approximation)

$$\Phi_0(\boldsymbol{r})=-(2e\nu)^{-1}\rho_0(\boldsymbol{r}) \tag{20}$$

and consequently to the equation for the potential $\Phi_0(\boldsymbol{r})$,

$$\Delta\Phi_0(\boldsymbol{r})=\begin{cases}\kappa^2\Phi_0(\boldsymbol{r})\theta_\Omega(\boldsymbol{r}), & 3\mathrm{D}\\ \kappa\delta(z)\Phi_0(\boldsymbol{r})\theta_\Omega(\boldsymbol{r}), & 2\mathrm{D},\end{cases} \tag{21}$$

with κ defined in Eq. (6) and the boundary condition $\Phi_0(r\to\infty)=-\boldsymbol{E}\boldsymbol{r}+$ const. In Eqs. (20) and (21) we have chosen the arbitrary additive constant in the definition of Φ_0 in such a way that $\int_\Omega\Phi_0(\boldsymbol{r})d\boldsymbol{r}=0$.

In the following we consider particular geometries of a 3D spherical sample of a radius R (to be referred as 3D), a

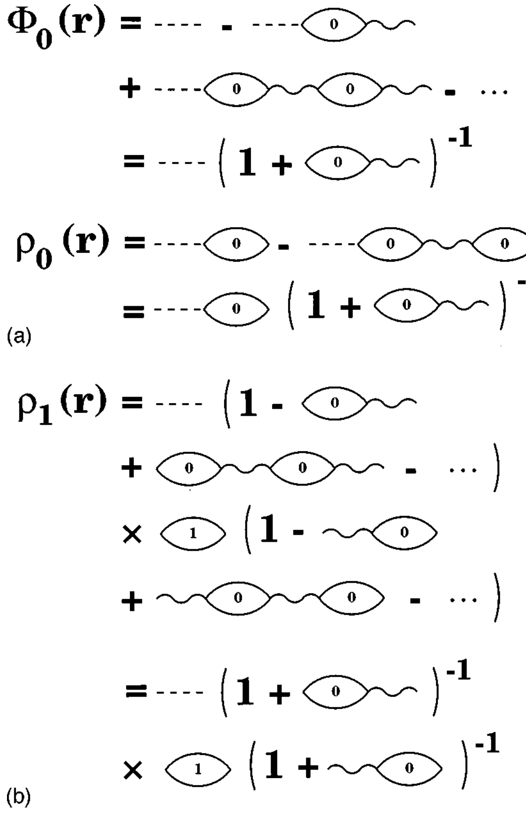

FIG. 1. Electrostatic potential and electron density in the RPA. The external dashed line is the bare potential of the electric field $-\mathbf{Er}$ and the loops with 0 and 1 denote the contributions Π_0 and Π_1 to the polarization function, respectively. The wavy line is the Coulomb interaction.

circle of a radius R in the in-plane electric field (2D), and a quasi-two-dimensional sample of a thickness h $(0<z<h)$ and an area S in the field directed along the z axis, i.e., transverse to the sample (Q2D). The polarizability tensor α_{ij} is generally defined as $\boldsymbol{d}(\omega)=\hat{\alpha}(\omega)\boldsymbol{E}(\omega)$. Equations (18) and (21) yield the classical polarizability[25] equal in the limit $\kappa R\ll 1$, $\kappa h\ll 1$ to (for all the geometries under consideration the dipole moment is directed along the field and the tensor is reduced to a scalar)

$$\alpha_0\simeq\begin{cases} R^3, & \text{3D}\\ Sh/4\pi, & \text{Q2D}\\ (4/3\pi)R^3, & \text{2D.}\end{cases} \tag{22}$$

Now we turn to the corrections due to the function Π_1. One obtains (Fig. 1)

$$\boldsymbol{d}_1\boldsymbol{E}=2e^2\int d\boldsymbol{r}d\boldsymbol{r}'\Phi_0(\boldsymbol{r})\Pi_1(\boldsymbol{r},\boldsymbol{r}')\Phi_0(\boldsymbol{r}'), \tag{23}$$

in full accordance with Ref. 12. Note that Eq. (23) depends explicitly on the symmetry of the system with respect to the time reversal and therefore constitutes the WL correction to the polarizability. As follows from Eq. (14), this correction consists of two contributions. The first one (to be referred as α_{1S}) is due to the short-range contribution to the polarization function [the first term in the second set of square brackets in Eq. (14)], and the second one (α_{1D}) is due to the diffusion contribution Π_D.

To evaluate the second term, we use the expansion of Π_D in the eigenfunctions of the Laplace operator $\phi_\alpha(\boldsymbol{r})$ with the boundary conditions $\boldsymbol{\nabla}\phi_\alpha=\boldsymbol{0}$ and the corresponding eigenvalues $-\epsilon_\alpha$,

$$\Pi_D(\boldsymbol{r},\boldsymbol{r}')=(\pi D\nu)^{-1}\sum_{\alpha\neq 0}\epsilon_\alpha^{-1}\phi_\alpha(\boldsymbol{r})\phi_\alpha(\boldsymbol{r}'). \tag{24}$$

For the purposes of estimate, one can use the expressions valid for $l\ll|\boldsymbol{r}-\boldsymbol{r}'|\ll L$ (L is a typical size of the system),

$$\Pi_D(\boldsymbol{r},\boldsymbol{r}')\approx\begin{cases}(2\pi^2\nu D)^{-1}\ln[L/|\boldsymbol{r}-\boldsymbol{r}'|], & \text{2D}\\ (4\pi^2\nu D|\boldsymbol{r}-\boldsymbol{r}'|)^{-1}, & \text{3D.}\end{cases} \tag{25}$$

Now we evaluate and compare both contributions α_{1S} and α_{1D} in 3D and 2D systems. Since the structure of the potential Φ_0 is different in 3D and 2D cases, these should be treated separately.

A. 3D geometry

For any 3D geometry with $\kappa L\gg 1$ the expression for $\Phi_0(\boldsymbol{r})$ can be written in the form

$$\Phi_0(\boldsymbol{r})=\frac{E}{\kappa}\varphi(\boldsymbol{r}_\parallel)\exp(-\kappa r_\perp), \tag{26}$$

with φ being some function of magnitude unity. We have introduced a transverse coordinate $r_\perp$ (r for the sphere, z in the case of a disk) and the vector $\boldsymbol{r}_\parallel$ of coordinates along the surface of the sample. Note that according to Eq. (20),

$$\varphi(\boldsymbol{r}_\parallel)=-4\pi e\sigma_0(\boldsymbol{r}_\parallel)/E, \tag{27}$$

where $e\sigma_0(\boldsymbol{r}_\parallel)$ is the charge density on a surface of an ideal conductor induced by the electric field E. It can be found by the methods of the classical electrostatics.[25] In the integral for α_{1S},

$$\alpha_{1S}=\frac{2e^2\nu A(\omega)}{VE^2}\int d^3\boldsymbol{r}d^3\boldsymbol{r}'\Phi_0(\boldsymbol{r})k_d(|\boldsymbol{r}-\boldsymbol{r}'|)\Phi_0(\boldsymbol{r}'), \tag{28}$$

both points $\boldsymbol{r}$ and $\boldsymbol{r}'$ lie in fact in the layer of thickness $\kappa^{-1}\ll l$ near the surface of the sample. One can then integrate over the transverse coordinates and reduce the remaining double surface integral to the integral over one coordinate only. We obtain

$$\alpha_{1S}\simeq\frac{1}{Vp_F^2\kappa^2}A(\omega)\ln(\kappa l)\int d^2\boldsymbol{r}_\parallel\varphi^2(\boldsymbol{r}_\parallel)$$

$$\sim\frac{1}{Lp_F^2\kappa^2}A(\omega)\ln(\kappa l). \tag{29}$$

In Ref. 12 the kernel in Eq. (28) was incorrectly replaced by a δ function, which led to an overestimate of the contribution α_{1S} by a factor of $\kappa l[\ln(\kappa l)]^{-1}\gg 1$.

On the other hand, for the term due to the diffusion,

$$\alpha_{1D}=\frac{2e^2\nu}{VE^2}A(\omega)\int d\boldsymbol{r}d\boldsymbol{r}'\Phi_0(\boldsymbol{r})\Pi_D(\boldsymbol{r},\boldsymbol{r}')\times\Phi_0(\boldsymbol{r}'), \tag{30}$$

we obtain, using the estimate (25),

$$\alpha_{1D}\sim\frac{1}{\nu D\kappa^2}A(\omega)\sim\alpha_{1S}\frac{L}{l\ln(\kappa l)}. \tag{31}$$

We see that in a diffusive system of a size $L\gg l\ln(\kappa l)$, the diffusion contribution α_{1D} dominates, in contrast to the conclusion of Ref. 12. At the same time, if the sample size L is comparable to the mean free path l (which happens, e.g., in ballistic systems with surface scattering), the short-range contribution is parametrically of the same order (in fact, even larger by a logarithmic factor) as the diffusive one. As expected,[9,10,12] the WL correction is small in comparison to the classical polarizability α_0,

$$\alpha_1/\alpha_0\sim\frac{1}{g(\kappa L)^2}A(\omega), \tag{32}$$

$g\sim 2\pi\nu DL$ being the dimensionless conductance. The calculation of the numerical coefficient for the WL correction to the polarizability requires the exact expansion (24). For the particular spherical geometry the potential Φ_0 has the form

$$\Phi_0(\boldsymbol{r})=-\frac{3ER}{\pi\kappa r}\exp[-\kappa(R-r)]\cos\theta,\quad R-r\ll R, \tag{33}$$

and we obtain

$$\alpha_1=\frac{1.36}{(p_F\kappa)^2 l}A(\omega). \tag{34}$$

According to Eq. (17), the WL correction to the polarizability is negative. The value of the correction in the presence of a strong magnetic field (unitary symmetry) is smaller (twice as small for zero frequency) than that without the magnetic field (orthogonal symmetry). The experimentally measured magnetopolarizability α_B, defined as

$$\alpha_B=\alpha(B)-\alpha(0), \tag{35}$$

is therefore positive, in agreement with Ref. 12.

B. Quasi-2D geometry (transverse field)

We consider now a quasi-two-dimensional sample of a thickness $h\gg\kappa^{-1}$ and an area $S\gg h^2$ with the electric field directed transverse to the sample plane. Then Eq. (26) for the potential reduces to

$$\Phi_0(\mathbf{r})=\frac{E}{\kappa}(-e^{-\kappa z}+e^{-\kappa(h-z)}). \tag{36}$$

If the sample is relatively thick $h\gg l$, the same consideration as for the case of a spherical shape yields

$$\alpha_{1S}=A(\omega)\frac{\ln(\kappa l)}{h(p_F\kappa)^2},\quad \alpha_{1D}=\frac{3}{2}A(\omega)\frac{1}{l(p_F\kappa)^2} \tag{37}$$

and the diffusion term dominates for $h>l\ln(\kappa l)$. In the opposite case of a thin sample the short-range contribution is the leading one. In particular, for $h<l$ we find

$$\alpha_1\simeq\alpha_{1S}=A(\omega)\frac{\ln(\kappa h)}{h(p_F\kappa)^2}. \tag{38}$$

As is seen from the above formulas, the relative magnitude of the weak localization correction is rather low for both 3D and quasi-2D (with the field direction normal to the plane) geometries, so that the experimental observation of the effect in these cases may be problematic. The effect is much more pronounced in the 2D case, which we consider below.

C. 2D geometry (in-plane field)

In contrast to the 3D case, the potential Φ_0 in the case of a 2D sample in the in-plane electric field is a smooth function of coordinates, with the characteristic scale set by the sample size R. Therefore the kernel in the integral (28), which has a support of order l, can be replaced by a δ function,

$$(p_F r)^{-1}\exp(-r/l)\approx 2\pi l p_F^{-1}\delta(\boldsymbol{r}).$$

This gives an estimate

$$\alpha_{1S}\sim l(p_F\kappa)^{-1}A(\omega)\ln(R/l).$$

On the other hand, for the diffusive term (30) the estimate (25) implies

$$\alpha_{1D}\sim R^2(\kappa g)^{-1}A(\omega)\sim\alpha_{1S}(R/l)^2[\ln(R/l)]^{-1}.$$

Similarly to the 3D case, the diffusion term α_{1S} dominates for $R\gg l$. The relative magnitude of the quantum correction can thus be estimated as

$$\alpha_1/\alpha_0\sim 1/g\kappa R,$$

with $g=2\pi\nu D=k_F l/2$.

For the particular case of a circular geometry, the potential Φ_0 is given in the polar coordinates (r,θ) by[25]

$$\Phi_0(\boldsymbol{r})=-2E(\pi\kappa)^{-1}r\cos\theta(R^2-r^2)^{-1/2}. \tag{39}$$

An exact calculation gives the value of the quantum correction

$$\alpha_1(\omega)=1.53R^2(\kappa p_F l)^{-1}A(\omega)$$

and the relative magnitude of the correction is

$$\alpha_1/\alpha_0=3.6\times\frac{1}{\kappa p_F l R}A(\omega).$$

In a recent paper Noat, Reulet, and Bouchiat[13] (NRB) proposed a geometry of a narrow 2D ring (radius R, width $W\ll R$) as more favorable for observation of the effect. In the in-plane electric field the ring becomes polarized with the one-dimensional (i.e., integrated over the ring cross section) charge density

$$\rho(\theta)=\frac{ER}{e\ln(R/W)}\cos\theta$$

and the classical polarizability given by

$$\alpha_0=\frac{\pi R^3}{\ln(R/W)}.$$

Calculating the quantum correction, we find again that for a diffusive ring $R \gg l$, the contribution α_{1D} dominates and gives

$$\alpha_1=\frac{R^4}{\nu D W^2 \kappa \ln^2(R/W)} A(\omega). \tag{40}$$

The relative magnitude of the correction is

$$\alpha_1/\alpha_0=\frac{1}{\pi g W \kappa \ln(R/W)} A(\omega), \tag{41}$$

where g is now the quasi-one-dimensional conductance $g=\nu DW/R$. These results for the ring geometry are by and large in agreement with those found by NRB.[13] Actually, NRB express the polarizability in terms of the exact eigenfunctions of electrons, conceptually similarly to the original GE calculation, and then perform the impurity averaging using the semiclassical expression for the correlation of the exact single-particle eigenfunctions (see, e.g., Refs. 1 and 24). This calculation yields correct results for the following reasons. First, the short-ranged terms in Eq. (1), omitted in this calculation, turn out to be unimportant for the WL correction to the polarizability. Furthermore, the *exact* expression for the long-ranged (diffusive) part of the eigenfunction correlator (1) coincides with the semiclassical result even for $\omega \ll \Delta$, where the latter generally is not expected to be true. This has been proved and discussed previously by the authors in Ref. 6.

Similarly, we can consider a quasi-one-dimensional strip of width W and length $L \gg W$ oriented along the electric-field direction (which we choose to be the z axis). Again, the sample polarization is described by the one-dimensional charge density

$$\rho(z)=\frac{Ez}{e\ln(L/W)},$$

yielding the classical polarizability

$$\alpha_0=\frac{L^3}{12\ln(L/W)}.$$

The quantum correction is now equal to

$$\alpha_1 \simeq \alpha_{1D}=\frac{\pi L^3}{30 \kappa W g \ln^2(L/\omega)} A(\omega), \tag{42}$$

where $g=2\pi\nu DW/L$ is the dimensionless conductance. We obtain

$$\alpha_1/\alpha_0=\frac{2\pi}{5}\,\frac{1}{\kappa W g \ln(L/W)} A(\omega). \tag{43}$$

Thus, we have found that the WL correction to the polarizability can be quite appreciable in 2D (circle) and especially in quasi-1D (ring or strip) geometries, which gives a possibility of its experimental observation. These conclusions are in full agreement with those of NRB.[13]

D. Canonical ensemble

The results obtained above were derived for the grand-canonical ensemble, where the chemical potential is fixed by an external reservoir. In Appendix A we present the calculations for the canonical ensemble, which is more appropriate for the problem in question.[26] We show [Eq. (A1)] that the CE magnetopolarizability differs from the GCE result by the coefficient -2.75. This means that although the magnitude of the CE effect is the same as in the GCE, the sign is opposite in the CE case: The magnetic field suppresses the polarizability.

III. WEAK LOCALIZATION EFFECTS IN THE CAPACITANCE

A. Definitions

A natural definition of the capacitance in an open system is

$$C_\mu = e\,dQ/d\mu, \tag{44}$$

where Q is the total charge of the system. This capacitance determining the low-frequency transport properties of the system was studied thoroughly by Büttiker and co-workers in Refs. 22 and 27 (where it was called ''electrochemical capacitance''). An explicit calculation[22] yields

$$\frac{e^2}{C_\mu}=\frac{e^2}{C_g}+\frac{1}{V\nu(E_F)}, \tag{45}$$

where C_g is the geometrical capacitance, determined from the equations of the classical electrostatics[25] with corrections due to the screening effects, and $\nu(E_F)$ is the density of states at the Fermi level. The average of the second term in the right-hand side of Eq. (45) is the mean level spacing Δ, which is usually much less than the average of the first term. However, the fluctuations of the second term are important.[23]

Being formally applied to the closed system, Eq. (44) yields infinite fluctuations of the charging energy. Indeed, in Ref. 19, where an attempt to calculate the fluctuations of the compressibility of a closed system has been made, the integral over energies diverged and the authors could get a finite result only by cutting it off at energies of order Δ.

A proper generalization of the definition (44) for a closed system is its ''discrete'' version

$$e^2/C_\mu=\mu(N+1)-\mu(N), \tag{46}$$

where $\mu(N)$ is the chemical potential of a *closed* system of N electrons. The quantity (46) has an important physical meaning: It is equal to the spacing between two consecutive peaks in the addition spectrum of a quantum dot in the Coulomb blockade regime. Statistical properties of these spacings were studied experimentally in Refs. 16–18 and theoretically in Refs. 20 and 21.

Similarly to the case of an open system [Eq. (45)], the peak spacing (46) can be decomposed into two parts: level spacing Δ_N and (usually much larger) contribution associated with Coulomb interaction (denoted E_1 in Ref. 20). The main contribution to the latter (and thus to the Coulomb blockade peak spacing) is given by the charging energy E_C

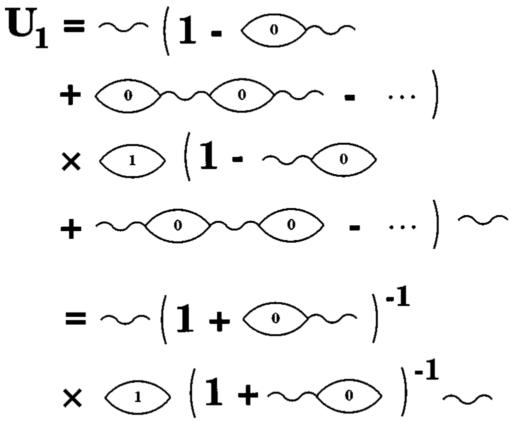

FIG. 2. RPA for the two-particle potential U. The potential U_0 is given by the same sequence of diagrams as the potential Φ_0 in Fig. 1 provided the external dashed line is replaced by the Coulomb interaction (the wavy line).

defined[20] as a constant part of the effective two-particle interaction potential $U(\boldsymbol{r},\boldsymbol{r}')$ of the electrons in the sample:

$$E_C \equiv e^2/C \equiv V^{-2}\int_\Omega d\boldsymbol{r}d\boldsymbol{r}'U(\boldsymbol{r},\boldsymbol{r}'). \tag{47}$$

In particular, the WL correction to the charging energy calculated below is the dominating term for the WL in the Coulomb blockade peak spacing and can be measured, in principle, as the magnetic-field dependence of the peak spacing. In contrast, fluctuations of the charging energy do not give the dominant contribution to the fluctuations of the peak spacing; see Secs. IV and V and Ref. 20.

B. Weak localization effects

The potential $U(\boldsymbol{r},\boldsymbol{r}')$ can be found in the RPA[20] (see Fig. 2). As in the case of the polarizability, it is convenient to split the polarization function into two parts (19) and (14). Assuming the low-frequency limit $\omega \ll \Delta$, we replace the function $A(\omega)$ by its zero-frequency value (17). In the zeroth order in Π_1 one should solve an equation

$$U_0(\boldsymbol{r},\boldsymbol{r}') = V_0(\boldsymbol{r}-\boldsymbol{r}') - 2\int_\Omega d\boldsymbol{r}_1 d\boldsymbol{r}_2 V_0(\boldsymbol{r}-\boldsymbol{r}_1) \times \Pi_0(\boldsymbol{r}_1,\boldsymbol{r}_2)U_0(\boldsymbol{r}_2,\boldsymbol{r}'), \quad V_0(r) = e^2/r. \tag{48}$$

This equation was solved for an arbitrary closed system in Ref. 20. The result is

$$U_0(\boldsymbol{r},\boldsymbol{r}') = \overline{U} + \tilde{\Phi}_0(\boldsymbol{r}) + \tilde{\Phi}_0(\boldsymbol{r}') + U_\kappa(\boldsymbol{r},\boldsymbol{r}'). \tag{49}$$

Here $\overline{U} \equiv (e^2/C)_0$ is a constant, corresponding to the charging energy calculated in the Thomas-Fermi approximation; U_κ is the usual screened Coulomb potential, shifted by a constant so that $\int d\boldsymbol{r}\, U_\kappa(\boldsymbol{r},\boldsymbol{r}') = 0$, while $\tilde{\Phi}_0$ is the contribution due to the excess positive charge, moved towards the boundary of the system after an extra electron is added to the system. For the sphere (3D) and circle (2D) geometries this potential has an explicit form[20]

$$\tilde{\Phi}_0(\boldsymbol{r}) = \text{const} - \begin{cases} e^2(\kappa R^2)^{-1}\exp[-\kappa(R-r)], & \text{3D} \\ e^2(2\kappa R)^{-1}(R^2-r^2)^{-1/2}, & \text{2D}, \end{cases}$$

where the constant is chosen in such a way that $\int d\boldsymbol{r}\, \tilde{\Phi}_0(\boldsymbol{r}) = 0$.

In the first order in Π_1 we obtain (Fig. 2)

$$U_1(\boldsymbol{r},\boldsymbol{r}') = 2\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 U_0(\boldsymbol{r},\boldsymbol{r}_1)\Pi_1(\boldsymbol{r}_1,\boldsymbol{r}_2)U_0(\boldsymbol{r}_2,\boldsymbol{r}'),$$

and, taking into account that the integral of Π_1 over any of the coordinates is zero, we write the corresponding contribution to the charging energy (which constitutes the WL correction) in the form

$$\left(\frac{e^2}{C}\right)_1 = 2\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \tilde{\Phi}_0(\boldsymbol{r}_1)\Pi_1(\boldsymbol{r}_1,\boldsymbol{r}_2)\tilde{\Phi}_0(\boldsymbol{r}_2). \tag{50}$$

Due to the structure of the function Π_1 [Eq. (8)] there are two contributions to the WL correction: One comes from the short-ranged term and the other one is related to the diffusion. A comparison of these contributions can be carried out exactly in the same way as was done for the polarizability, and it turns out that the diffusive term dominates if the sample size exceeds considerably the mean free path. Thus we obtain finally

$$\left(\frac{e^2}{C}\right)_1 = -\frac{4\nu}{\beta V}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \tilde{\Phi}_0(\boldsymbol{r}_1)\Pi_D(\boldsymbol{r}_1,\boldsymbol{r}_2)\tilde{\Phi}_0(\boldsymbol{r}_2). \tag{51}$$

The calculation for the particular geometries of a sphere (3D) and disk (2D) gives

$$\left(\frac{e^2}{C}\right)_1 = \begin{cases} -1.32\beta^{-1}\tau^{-1}(p_F R)^{-4}, & \text{3D} \\ -0.010\beta^{-1}\tau^{-1}(p_F R)^{-2}, & \text{2D}. \end{cases} \tag{52}$$

The small coefficient in front of the 2D expression is an artifact of the specific circle geometry.

The weak localization correction suppresses the charging energy, i.e., enhances the capacitance. The magnetic field suppresses the capacitance. In both 2D and 3D cases, the WL correction to the charging energy can be estimated as

$$\left(\frac{e^2}{C}\right)_1 \sim \frac{\Delta}{g}. \tag{53}$$

The WL correction can be in principle extracted from the measurements of the magnetic-field dependence of the capacitance, though its rather small value may make such a measurement problematic.

As for the polarizability, the CE calculation yields an additional factor -2.75 in Eqs. (51) and (52). Thus, in the CE the magnetic field suppresses the charging energy, enhancing the capacitance.

IV. MESOSCOPIC FLUCTUATIONS

Mesoscopic fluctuations of the polarizability and the capacitance can be calculated in a similar way. In addition to the average polarization function (13) and (14) there is also a random part $\Pi_r(\boldsymbol{r},\boldsymbol{r}')$ with the zero average, giving rise to the fluctuations of these quantities. Since the integral of Π_r

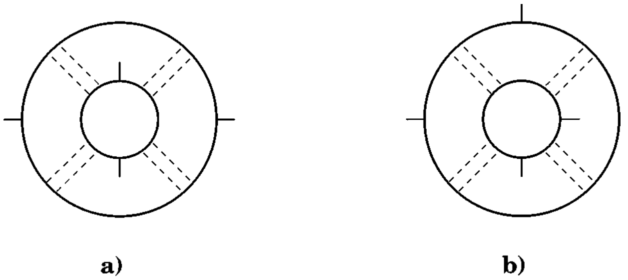

FIG. 3. Diagrams for the fluctuations of the polarization function. The double dashed lines denote the diffusion propagators. The counting factors are (a) 2 and (b) 4 for the unitary symmetry and (a) 4 and (b) 8 for the orthogonal one.

over each coordinate is zero, we immediately arrive at the expressions for the random parts of the static[28] polarizability α_r and the charging energy $(e^2/C)_r$ in precisely the same way as Eqs. (23) and (50) were obtained:

$$\alpha_r = \frac{2e^2}{E^2}\int d\boldsymbol{r}d\boldsymbol{r}'\Phi_0(\boldsymbol{r})\Pi_r(\boldsymbol{r},\boldsymbol{r}')\Phi_0(\boldsymbol{r}') \tag{54}$$

and

$$\left(\frac{e^2}{C}\right)_r = 2\int d\boldsymbol{r}d\boldsymbol{r}'\tilde{\Phi}_0(\boldsymbol{r})\Pi_r(\boldsymbol{r},\boldsymbol{r}')\tilde{\Phi}_0(\boldsymbol{r}'). \tag{55}$$

Thus the mesoscopic fluctuations of both quantities are determined by the fluctuations of the polarizability. To calculate them, we perform the perturbative calculation. As we will see, in the case of fluctuations the whole range of energies $\Delta \lesssim \epsilon \lesssim E_c$ contributes and this fact justifies the perturbative calculation (in contrast to the weak localization correction, which is determined by low energies). Following Berkovits and Altshuler,[14] we identify the four-diffusion diagrams (Fig. 3) as giving the main contribution to fluctuations of the polarization function.

We obtain

$$\langle \Pi_r(\boldsymbol{r}_1,\boldsymbol{r}_3)\Pi_r(\boldsymbol{r}_2,\boldsymbol{r}_4)\rangle = -\frac{(12/\beta)}{2\pi^2}\,\mathrm{Re}\int_0^\infty \epsilon\, d\epsilon\, D_\epsilon(\boldsymbol{r}_1,\boldsymbol{r}_2) \times D_\epsilon(\boldsymbol{r}_2,\boldsymbol{r}_3)D_\epsilon(\boldsymbol{r}_3,\boldsymbol{r}_4)D_\epsilon(\boldsymbol{r}_4,\boldsymbol{r}_1). \tag{56}$$

Here $12/\beta$ is a combinatorial factor (a number of four-diffusion diagrams); the function D_ϵ is given by the expression

$$D_\epsilon(\boldsymbol{r},\boldsymbol{r}') = -\frac{1}{i\epsilon V} + \pi\nu\Pi_D(\boldsymbol{r},\boldsymbol{r}'), \quad \Delta \lesssim \epsilon \lesssim E_c, \tag{57}$$

and decreases for $\epsilon \gg E_c$. If the integral over ϵ in Eq. (56) diverges, the cutoff at energies of order of the mean level spacing Δ, where the perturbative expression (56) ceases to be valid, should be introduced.

A naive estimate suggests that the leading contribution to the fluctuations of the polarizability as well as of the capacitance can be found by substituting the zero-mode contribution [the first term on the right-hand side (rhs) of Eq. (57)] for all the four functions D_ϵ. However, since the integrals of the potentials Φ_0 and $\tilde{\Phi}_0$ are zero, this term vanishes. For typical geometries the main contribution is given by the term where two of the functions D_ϵ are replaced by the zero-mode result, while two others are represented by the diffusion propagator [the second term on rhs of Eq. (57)]. Cutting off the logarithmically divergent integral at Δ from below and at the Thouless energy E_c from above, we obtain

$$\langle \alpha_r^2\rangle = \frac{12e^4}{\beta E^4}\nu^2 \ln g \times\left[\frac{2}{V}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\Phi_0(\boldsymbol{r}_1)\Pi_D(\boldsymbol{r}_1,\boldsymbol{r}_2)\Phi_0(\boldsymbol{r}_2)\right]^2 \tag{58}$$

and

$$\left\langle\left(\frac{e^2}{C}\right)_r^2\right\rangle = \frac{12}{\beta}\nu^2 \ln g \times\left[\frac{2}{V}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\tilde{\Phi}_0(\boldsymbol{r}_1)\Pi_D(\boldsymbol{r}_1,\boldsymbol{r}_2)\tilde{\Phi}_0(\boldsymbol{r}_2)\right]^2. \tag{59}$$

Comparing these results with the expression for the WL corrections (30) and (51), we obtain a general formula relating WL corrections (calculated within the GCE) to the mesoscopic fluctuations of the same quantity ($\mathcal{R}$ denotes root mean square),

$$\mathcal{R}(\alpha) = (3\beta \ln g)^{1/2}|\alpha_1| \tag{60}$$

and

$$\mathcal{R}\left(\frac{e^2}{C}\right) = (3\beta \ln g)^{1/2}\left|\left(\frac{e^2}{C}\right)_1\right|. \tag{61}$$

Note that in contrast to the WL corrections, the mesoscopic fluctuations are not expected to be sensitive to the GCE/CE difference since the integral (56) is determined equally by all energies $\Delta \lesssim \epsilon \lesssim E_c$. For particular geometries we obtain

$$\mathcal{R}(\alpha) = \left(\frac{\ln g}{\beta}\right)^{1/2}\begin{cases} 4.71 l^{-1}(p_F\kappa)^{-2}, & \text{3D} \\ 5.30R^2(\kappa p_F l)^{-1}, & \text{2D}\end{cases}$$

and

$$\mathcal{R}\left(\frac{e^2}{C}\right) = \left(\frac{\ln g}{\beta}\right)^{1/2}\begin{cases} 2.29\tau^{-1}(p_F R)^{-4}, & \text{3D} \\ 0.017\tau^{-1}(p_F R)^{-2}, & \text{2D.}\end{cases}$$

The remaining terms in Eq. (56), with all the four functions D_ϵ replaced by the diffusion propagator, can also be easily estimated. Their contribution is of the same order as Eqs. (60) and (61), but without the logarithmic factor in the numerator. Thus, for the rms of the charging energy in addition to the term of order $\sim\Delta(\ln g)^{1/2}/g$ (two zero modes) we obtain a correction of order $\sim\Delta/g$ (no zero modes).

Amazingly, the relations between the weak localization correction and the amplitude of mesoscopic fluctuations, (60) and (61) have a universal form, the same for 2D and 3D systems. We should note, however, that these results are not applicable for the case of polarizability fluctuations of a quasi-two-dimensional sample of the thickness $h \gg l$ and area

$S \gg h^2$ in the transverse field (see Sec. II B) calculated previously by Berkovits and Altshuler.[14] In this case, the contribution from the terms with two zero-modes, Eq. (58), which can be easily calculated with the use of Eq. (36), yields the rms value of the polarizability fluctuations,

$$\mathcal{R}(\alpha^{(2)}) = \frac{(3\beta^{-1}\ln g)^{1/2}}{\pi g}\frac{h}{\kappa^2},$$

where $g = 2\pi\nu Dh$. On the other hand, the contribution where all the four functions D_ϵ are replaced by the diffusion propagators is[14]

$$\mathcal{R}(\alpha^{(4)}) = \frac{8\beta^{-1}\pi^{3/2}}{g}\frac{S^{1/2}}{\kappa^2},$$

i.e., it is larger by a factor $\sim S^{1/2}/[h(\ln g)^{1/2}]$, and for this particular geometry represents the leading contribution to the fluctuations of the polarizability.

V. CONCLUSIONS

In this paper we have calculated the weak localization correction to the polarizability and the capacitance of a disordered sample and the mesoscopic fluctuations of these quantities. The WL corrections originate from the G_RG_A term in the polarization function, which depends on the presence or absence of the time-reversal symmetry. A change of the polarization function influences the screening and, consequently, the polarizability and the capacitance. We find that in the grand-canonical ensemble, switching on the magnetic field leads to a positive correction to the polarizability and negative one to the capacitance. In the canonical ensemble the magnitude of the effect is the same (up to a numerical coefficient ~ 2.75), however, the sign is reversed.

Calculating the mesoscopic fluctuations of the polarizability, we find that for typical geometries they are related to the value α_1 of the WL correction as [see Eq. (60)]:

$$\mathcal{R}(\alpha) = (3\beta\ln g)^{1/2}|\alpha_1|.$$

The same conclusion is valid for the capacitance; see Eq. (61). Therefore, the magnitude of fluctuations exceeds considerably the value of the WL correction. This should be contrasted with the relation of the corresponding quantities for the case of the conductance:

$$\mathcal{R}(g) \sim 1,$$

$$|g_1| \sim \begin{cases} 1, & \text{quasi-1D} \\ \ln(L/l), & \text{2D} \\ L/l, & \text{3D}, \end{cases}$$

so that $|g_1| \gg \mathcal{R}(g)$ in 2D and 3D geometries and $|g_1| \sim \mathcal{R}(g)$ in the quasi-1D geometry. As our results show, an experimental observation of the magnetopolarizability of mesoscopic samples requires an experimental setup with large number of such samples, which would reduce the fluctuations.

Mesoscopic fluctuations of the charging energy contribute to the fluctuations of the conductance peak spacings in the addition spectra of quantum dots in the Coulomb blockade regime.[15–18] However, as follows from Eqs. (53) and (61), the magnitude of these fluctuations is much smaller [by a factor $\sim (\ln g)^{1/2}/g$] than the level spacing Δ. Therefore, the contribution of the charging energy fluctuations to the fluctuations of the peak spacings is parametrically smaller than the effect of electron level fluctuations, which is given by the random matrix theory and is of order of Δ. The charging energy fluctuations represent one (but not the only one) of the contributions to the enhancement of the peak spacing fluctuations as compared to the random matrix theory. This problem was considered in detail in Ref. 20.

Finally, we would like to mention once more that we assumed the screening length to be much larger than the wavelength or, in other words, that $r_s \ll 1$, where $r_s = e^2/\epsilon v_F$. In the opposite case $r_s > 1$ (but still below the Wigner crystallization threshold), one can roughly estimate the result assuming the screening length to be approximately given by the distance between electrons. This leads to an enhancement of the above results for the WL correction and the rms amplitude of fluctuations by a factor of order of r_s and r_s^2 for the polarizability and capacitance, respectively.

ACKNOWLEDGMENTS

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APPENDIX A: MAGNETOPOLARIZABILITY IN THE CANONICAL ENSEMBLE

In this appendix we calculate the magnetopolarizability α_B defined by Eq. (35) in the canonical ensemble. For this purpose we first rewrite our derivation of the Sec. II in terms of exact eigenfunctions and energy eigenvalues, in the same manner it was done originally by GE (Ref. 4) and later by NRB.[13] Using the results, derived for the correlation of eigenfunctions in Refs. 6 and 29 (unitary ensemble) and Appendix B (orthogonal ensemble), we first obtain the weak localization correction in the grand-canonical ensemble. It is in full agreement with the results obtained in Sec. II. Then we generalize the derivation to the CE case.

Our conclusion is that the CE magnetopolarizability α_B^{CE} can be easily obtained from the GCE value α_B^{GCE} as

$$\alpha_B^{CE} = -A_{CE}\alpha_B^{GCE}, \quad A_{CE} = 2.75. \tag{A1}$$

We should stress here that the derivation given below uses only the linear-response formalism and the properties of the eigenfunction and eigenvalue statistics in disordered systems. It relies on the fact that that the WL correction to the polarizability is determined by the energy range where the level correlation is important. It can be repeated for the case of the capacitance, where one obtains a relation analogous to Eq. (A1). In contrast to this, the fluctuations of both quantities are determined by the energies $\Delta \ll \epsilon \ll E_c$, where the correlation of levels does not play any role and the difference between CE and GCE values is not expected.

1. Magnetopolarizability in the GCE: Derivation in the manner of Gor'kov and Eliashberg

We start from Eq. (23), which can be rewritten in terms of the exact single-particle states as

$$\alpha_B(\omega)=\frac{2e^2}{E^2}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\Phi_0(\boldsymbol{r}_1)\boldsymbol{\delta}\Pi(\boldsymbol{r}_1,\boldsymbol{r}_2)\Phi_0(\boldsymbol{r}_2). \tag{A2}$$

Here we introduced the symbol $\boldsymbol{\delta}$ denoting the difference between the quantities in unitary and orthogonal ensembles

$$\boldsymbol{\delta}(\ )=(\ )_{GUE}-(\ )_{GOE}$$

and the polarization function Π is expressed as

$$\Pi(\boldsymbol{r}_1,\boldsymbol{r}_2)=\sum_{m\neq n}\psi_m^*(\boldsymbol{r}_1)\psi_n(\boldsymbol{r}_1)\psi_n^*(\boldsymbol{r}_2)\psi_m(\boldsymbol{r}_2) \times\frac{n_F(\epsilon_m)-n_F(\epsilon_n)}{\omega-\epsilon_m+\epsilon_n+i0}, \tag{A3}$$

m and n being the exact single-particle states. Thus, for $\omega\ll\Delta$ we obtain an expression valid in both GCE and CE,

$$\alpha_B(\omega)=\frac{4e^2}{E^2}\boldsymbol{\delta}\left\langle\sum_{\epsilon_n<\epsilon_F<\epsilon_m}\frac{1}{\epsilon_m-\epsilon_n}|(\Phi_0)_{mn}|^2\right\rangle. \tag{A4}$$

In the GCE the position of the Fermi level can be arbitrary and we replace the sum in Eq. (A4) by an integral with the level correlation function $R_2(\epsilon)$:

$$\sum_{\epsilon_n<\epsilon_F<\epsilon_m}(\ )=\Delta^{-2}\int_0^\infty\epsilon\, d\epsilon\, R_2(\epsilon)(\ ). \tag{A5}$$

Using the sum rule for the eigenfunctions

$$\left\langle\sum_n\psi_m^*(\boldsymbol{r}_1)\psi_n(\boldsymbol{r}_1)\psi_n^*(\boldsymbol{r}_2)\psi_m(\boldsymbol{r}_2)\right\rangle=V^{-1}\delta(\boldsymbol{r}_1-\boldsymbol{r}_2),$$

we obtain for the magnetopolarizability

$$\alpha_B=-\frac{2e^2}{E^2\Delta}\boldsymbol{\delta}\langle|(\Phi_0)_{mm}|^2\rangle. \tag{A6}$$

The above derivation of Eq. (A6) is essentially equivalent to that of Refs. 4 and 13. Now, using Eq. (B8) for the case of the orthogonal symmetry and Ref. 29 for the unitary one, we write ($r=|\boldsymbol{r}_1-\boldsymbol{r}_2|$)

$$V^2\langle|\psi_m(\boldsymbol{r}_1)\psi_m(\boldsymbol{r}_2)|^2\rangle_\epsilon =\begin{cases}[1+2\Pi_D(\boldsymbol{r}_1,\boldsymbol{r}_2)][1+2k_d(r)], & \text{GOE}\\ [1+\Pi_D(\boldsymbol{r}_1,\boldsymbol{r}_2)][1+k_d(r)], & \text{GUE.}\end{cases}$$

Separating the dominating diffusion term, we obtain for the magnetopolarizability in the grand-canonical ensemble

$$\alpha_B=\frac{2e^2}{V^2E^2\Delta}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\Phi_0(\boldsymbol{r}_1)\Pi_D(\boldsymbol{r}_1,\boldsymbol{r}_2)\Phi_0(\boldsymbol{r}_2), \tag{A7}$$

which coincides with Eq. (30).

2. Canonical ensemble

To realize the canonical ensemble, we apply the method previously developed in Refs. 30 and 31. Namely, we fix the number of electrons to an integer in each individual sample, but allow it to fluctuate slightly from sample to sample. This type of ensemble is realized by pinning the Fermi level to one of the single-particle levels ϵ_k: $\epsilon_F=\epsilon_k+0$.

Now, instead of Eq. (A5), one should split the sum over energy levels in Eq. (A4) into two. The first contribution consists of the terms with $n=k$ and can be transformed to the integral with the use of the two-level correlation function R_2. The rest of the sum requires the three-level correlator $R_3(0,\epsilon,\epsilon_1)$, which corresponds to the probability to find three levels with energies $0,-\epsilon_1$, and $\epsilon-\epsilon_1$ (counted from the Fermi surface). Thus we obtain

$$\sum_{\epsilon_n<\epsilon_F<\epsilon_m}(\ )=\Delta^{-1}\int_0^\infty d\epsilon\, R_2(\epsilon)(\ ) +\Delta^{-2}\int_0^\infty d\epsilon\int_0^\epsilon d\epsilon_1 R_3(0,\epsilon_1,\epsilon)(\ ), \tag{A8}$$

and for the magnetopolarizability

$$\alpha_B=\frac{4e^2}{E^2\Delta^2}\boldsymbol{\delta}\left\{\int_0^\infty\frac{d\epsilon}{\epsilon}\left[R_2(\epsilon)\Delta+\int_0^\epsilon d\epsilon_1 R_3(0,\epsilon,\epsilon_1)\right] \times\langle|(\Phi_0)_{mn}|^2\rangle_\epsilon\right\}. \tag{A9}$$

Since the integral over ϵ converges for $\epsilon\sim\omega$, we can replace the matrix element by its low-frequency limit. For $\epsilon\ll E_c$ we obtain for the correlation of the wave functions with $m\neq n$ [see Eq. (B11) for the orthogonal symmetry and Ref. 29 for the unitary one]

$$V^2\langle\psi_m^*(\boldsymbol{r}_1)\psi_n(\boldsymbol{r}_1)\psi_m(\boldsymbol{r}_2)\psi_n^*(\boldsymbol{r}_2)\rangle_{\epsilon_0,\epsilon} =\begin{cases}k_d(r)+[1+k_d(r)]\Pi_D(\boldsymbol{r}_1,\boldsymbol{r}_2), & \text{GOE}\\ k_d(r)+\Pi_D(\boldsymbol{r}_1,\boldsymbol{r}_2), & \text{GUE.}\end{cases}$$

In the cases when the diffusion dominates,[32] one can replace this eigenfunction correlator for either symmetry by Π_D. As a result, taking into account Eq. (A7), we obtain Eq. (A1), with the coefficient A_{CE} expressed in terms of the level correlation functions

$$A_{CE}=-1-2\int_0^\infty\frac{ds}{s} \times\boldsymbol{\delta}\left(R_2(s)+\int_0^s ds_1[R_3(0,s_1,s)-R_2(s)]\right), \tag{A10}$$

where we have made a change of variables $s=\pi\epsilon/\Delta$. Note that Eq. (A10) differs from the similar expression derived by NRB (Ref. 13) by the second term on the rhs.

In the leading approximation the correlation function R_2 and R_3 may be taken from the random matrix theory.[33] The first integral on the rhs of Eq. (A10) can be calculated analytically and is given by

$$I_1=\int_0^\infty\frac{ds}{s}\boldsymbol{\delta}R_2(s)=\int_0^\infty\frac{ds}{s}g(s)h(s)=\frac{1}{4}-\frac{\pi^2}{16}$$

$$\simeq-0.367,$$

in contrast to the statement of NRB that it is equal to $-1/2$. We have defined the functions

$$f(s)=\frac{\sin s}{s},\quad g(s)=\frac{d}{ds}f(s),\quad h(s)=\int_s^\infty f(s_1)ds_1.$$

The second term on the rhs of Eq. (A10) after a lengthy algebra can be expressed as follows:

$$\begin{aligned}I_2&=\int_0^\infty\frac{ds}{s}\int_0^s ds_1\boldsymbol{\delta}(R_3(0,s_1,s)-R_2(s))\\&=2\int_0^\infty\frac{ds}{s}\int_0^s ds_1\{g(s_1)h(s_1)-f(s)g(s_1)h(s-s_1)\\&\quad+g(s)f(s_1)h(s-s_1)+h(s)f(s_1)g(s-s_1)\}.\end{aligned}$$

Calculating this numerically, we find $I_2=-1.509$ and thus $A_{CE}=-1-2(I_1+I_2)=2.753$.

Note that in the above derivation we neglected the contribution of the so-called Debye processes (relaxation to the instantaneous equilibrium distribution due to coupling with phonons or other possible inelastic processes).[34,35,13] These processes do not exist in a closed sample in the limit of zero temperature ($T\ll\Delta$) that we are considering.[36]

APPENDIX B: CORRELATIONS OF EIGENFUNCTIONS IN DISORDERED SYSTEMS: ORTHOGONAL ENSEMBLE

In this appendix we derive the expressions for the correlations of the eigenfunctions in the orthogonal ensemble in the same way as these were obtained in Ref. 29 for the unitary ensemble. We restrict ourselves to the terms of order g^{-1}.

Following Ref. 29, we define the eigenfunctions correlators [see Eq. (2)],

$$\begin{aligned}\eta(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon)&=\langle|\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)|^2\rangle_\epsilon\\&\equiv\frac{\left\langle\sum_k\left|\psi_k(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\right|^2\delta(\epsilon-\epsilon_k)\right\rangle}{\left\langle\sum_k\delta(\epsilon-\epsilon_k)\right\rangle},\end{aligned}\qquad\text{(B1)}$$

$$\beta(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)=\langle|\psi_k(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_2)|^2\rangle_{\epsilon,\omega},\quad k\neq l,$$

and

$$\gamma(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)=\langle\psi_k^*(\boldsymbol{r}_1)\psi_l(\boldsymbol{r}_1)\psi_k(\boldsymbol{r}_2)\psi_l^*(\boldsymbol{r}_2)\rangle_{\epsilon,\omega},\quad k\neq l.$$

The quantities η and β are related as

$$\begin{aligned}B&\equiv\eta(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon)\Delta^{-1}\delta(\omega)+\beta(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)\Delta^{-2}R_2(\omega)\\&=\nu^2+(2\pi^2)^{-1}\mathrm{Re}\{\langle G^R(\boldsymbol{r}_1,\boldsymbol{r}_1,\epsilon)G^A(\boldsymbol{r}_2,\boldsymbol{r}_2,\epsilon+\omega)\rangle\\&\quad-\langle G^R(\boldsymbol{r}_1,\boldsymbol{r}_1,\epsilon)\rangle\langle G^A(\boldsymbol{r}_2,\boldsymbol{r}_2,\epsilon+\omega)\rangle\};\end{aligned}\qquad\text{(B2)}$$

here the two-level correlation function,

$$R_2(\omega)=\Delta^2\left\langle\sum_{k\neq l}\delta(\epsilon-\epsilon_k)\delta(\epsilon+\omega-\epsilon_l)\right\rangle,\qquad\text{(B3)}$$

is introduced.

The right-hand side of expression (B2) can be directly calculated with the use of the supersymmetry technique. For the case of preserved time-reversal symmetry (orthogonal ensemble) one obtains

$$\begin{aligned}B(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)&=-(2\pi^2)^{-1}\mathrm{Re}\{\langle g^{11}_{b1,b1}(\boldsymbol{r}_1,\boldsymbol{r}_1)g^{22}_{b1,b1}(\boldsymbol{r}_2,\boldsymbol{r}_2)\\&\quad+g^{12}_{b1,b1}(\boldsymbol{r}_1,\boldsymbol{r}_2)g^{21}_{b1,b1}(\boldsymbol{r}_2,\boldsymbol{r}_1)\rangle_F\\&\quad-\langle g^{11}_{b1,b1}(\boldsymbol{r}_1,\boldsymbol{r}_1)\rangle_F\langle g^{22}_{b1,b1}(\boldsymbol{r}_2,\boldsymbol{r}_2)\rangle_F\}.\end{aligned}\qquad\text{(B4)}$$

Here $\langle\ \rangle_F$ denotes the averaging with the action of the supermatrix σ-model $F[Q]$:

$$\langle\ \rangle_F=\int DQ(\ )\exp(-F[Q]),$$

$$F[Q]=-\frac{\pi\nu}{8}\int d\boldsymbol{r}\,\mathrm{Str}[D(\boldsymbol{\nabla}Q)^2+2i(\omega+i0)\Lambda Q],\qquad\text{(B5)}$$

where D is the diffusion coefficient, $Q=T^{-1}\Lambda T$ is a 8×8 supermatrix, $\Lambda=\mathrm{diag}(1,1,1,1,-1,-1,-1,-1)$, and T belongs to the supercoset space $\mathrm{U}(2,2|4)/\mathrm{U}(2|2)\times\mathrm{U}(2|2)$. The symbol Str denotes the supertrace (trace over bosonic degrees of freedom minus that over fermionic ones). The upper matrix indices correspond to the retarded-advanced decomposition, while the lower indices denote the boson-fermion one (here we need only the index $b1$, which denotes one of two bosonic components of a supervector). The Green's function g in Eq. (B4) is the solution to the matrix equation

$$\left[-i\left(\epsilon+\frac{\omega}{2}-\hat{H}_0\right)-\frac{i}{2}(\omega+i0)\Lambda+Q/2\tau\right]g(\boldsymbol{r},\boldsymbol{r}')$$

$$=\delta(\boldsymbol{r}-\boldsymbol{r}').\qquad\text{(B6)}$$

Expressing these functions through the matrices Q and taking into account Eq. (B2), we arrive at the following equation valid in for an arbitrary diffusive system:

$$\begin{aligned}&2\pi^2\left[\frac{\eta(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon)}{\Delta}\delta(\omega)+\frac{\beta(\boldsymbol{r}_1,\boldsymbol{r}_2,\epsilon,\omega)}{\Delta^2}R_2(\omega)\right]\\&\quad=(\pi\nu)^2\mathrm{Re}\{1-\langle Q^{11}_{b1,b1}(\boldsymbol{r}_1)Q^{22}_{b1,b1}(\boldsymbol{r}_2)\rangle_F-2k_d(r)\\&\quad\quad\times\langle Q^{12}_{b1,b1}(\boldsymbol{r}_1)Q^{21}_{b1,b1}(\boldsymbol{r}_1)\rangle_F\},\end{aligned}\qquad\text{(B7)}$$

Here the function k_d is defined in Eq. (5) and $r=|\boldsymbol{r}_1-\boldsymbol{r}_2|$. The separation of the rhs of Eq. (B7) into the singular [proportional to $\delta(\omega)$] and regular parts allows one to obtain the quantities $\eta(\boldsymbol{r}_1,\boldsymbol{r}_2)$ and $\beta(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)$.

For the case of a metallic system in the weak localization regime, the σ-model correlation functions $\langle Q^{11}_{b1,b1}(\mathbf{r}_1)Q^{22}_{b1,b1}(\mathbf{r}_2)\rangle_F$ and $\langle Q^{12}_{b1,b1}(\mathbf{r}_1)Q^{21}_{b1,b1}(\mathbf{r}_2)\rangle_F$ can be calculated for relatively low frequencies $\omega\ll E_c$ with the use of a general method developed in Refs. 37 and 38, which allows one to take into account spatial variations of the field Q. The results are obtained in the form of an expansion in g^{-1}. Up to the terms of order g^{-1}, we obtain

$$\langle Q^{11}_{b1,b1}(\mathbf{r}_1)Q^{22}_{b1,b1}(\mathbf{r}_2)\rangle_F = 1-2\widetilde{R}(\omega)-\frac{4i\Delta}{\pi(\omega+i0)}\Pi_D(\mathbf{r}_1,\mathbf{r}_2)$$

and

$$\langle Q^{12}_{b1,b1}(\mathbf{r}_1)Q^{21}_{b1,b1}(\mathbf{r}_2)\rangle_F = -2\frac{i\Delta}{\pi(\omega+i0)}-2\left(\widetilde{R}(\omega)+\frac{i\Delta}{\pi(\omega+i0)}\right)\Pi_D(\mathbf{r}_1,\mathbf{r}_2).$$

Here the diffusion propagator Π_D is defined by Eq. (3) and we have introduced the function $\widetilde{R}(\omega)=[1+S(\omega)]/2$, where $S(\omega)$ is given by Eq. (16). Note that the two-level correlation function $R_2(\omega)$ is the real part of $\widetilde{R}(\omega)$.

Now, separating regular and singular parts on the rhs of Eq. (B7), we obtain the following result for the autocorrelations of the same eigenfunction:

$$V^2\langle|\psi_k(\mathbf{r}_1)\psi_k(\mathbf{r}_2)|^2\rangle_\epsilon = [1+2k_d(r)][1+2\Pi_D(\mathbf{r}_1,\mathbf{r}_2)], \tag{B8}$$

and for the correlation of amplitudes of two different eigenfunctions ($k\neq l$)

$$V^2\langle|\psi_k(\mathbf{r}_1)\psi_l(\mathbf{r}_2)|^2\rangle_{\epsilon,\omega}-1=2k_d(r)\Pi_D(\mathbf{r}_1,\mathbf{r}_2). \tag{B9}$$

The result (B8) for $\mathbf{r}_1=\mathbf{r}_2$ is the inverse participation ratio previously obtained in Ref. 38, while that for an arbitrary spatial separation was found in the zero-mode approximation ($g=\infty$) in Ref. 39.

Now we turn to the correlation function γ. Similarly to Ref. 29, we obtain a relation

$$\begin{aligned}2\pi^2\left[\frac{\eta(\mathbf{r}_1,\mathbf{r}_2,\epsilon)}{\Delta}\delta(\omega)+\frac{\gamma(\mathbf{r}_1,\mathbf{r}_2,\epsilon,\omega)}{\Delta^2}R_2(\omega)\right]\\ =-(\pi\nu)^2\mathrm{Re}\{\langle Q^{12}_{b1,b1}(\mathbf{r}_1)Q^{21}_{b1,b1}(\mathbf{r}_2)\rangle_F+k_d(r)\\ \times[\langle Q^{11}_{bb}(\mathbf{r}_1)Q^{22}_{bb}(\mathbf{r}_1)\rangle_F\\ +\langle Q^{12}_{b1,b1}(\mathbf{r}_1)Q^{21}_{b1,b1}(\mathbf{r}_2)\rangle_F-1]\}.\end{aligned} \tag{B10}$$

Separating again the rhs into the regular and singular parts, we recover Eq. (B8) and obtain

$$\begin{aligned}V^2\langle\psi^*_k(\mathbf{r}_1)\psi_l(\mathbf{r}_1)\psi_k(\mathbf{r}_2)\psi^*_l(\mathbf{r}_2)\rangle_{\epsilon,\omega}\\ =k_d(r)+[1+k_d(r)]\Pi_D(\mathbf{r}_1,\mathbf{r}_2),\quad k\neq l.\end{aligned} \tag{B11}$$

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