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Screening cloud in the k-channel Kondo model: Perturbative and large-k results

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We demonstrate the existence of a large Kondo screening cloud in the k-channel Kondo model using both renormalization-group improved perturbation theory and the large-k limit. We study position r-dependent spin Green's functions in both static and equal-time cases. The equal-time Green's function provides a natural definition of the screening-cloud profile, in which the large scale $\xi_K \equiv v_F/T_K$ appears (v_F is the Fermi velocity; T_K is the Kondo temperature). At large distances it consists of both a slowly varying piece and a piece which oscillates at twice the Fermi wave vector, $2k_F$. This function is calculated at all r in the large-k limit. Static Green's functions (Knight shift or susceptibility) consist only of a term oscillating at $2k_F$, and appear to factorize into a function of r times a function of T for $rT/v_F \ll 1$, in agreement with NMR experiments. Most of the integrated susceptibility comes from the impurity-impurity part with conduction-electron contributions suppressed by powers of the bare Kondo coupling. The single-channel and overscreened multichannel cases are rather similar, although anomalous power laws occur in the latter case at large r and low T due to irrelevant operator corrections. [S0163-1829(97)01446-X]

I. INTRODUCTION

It is well known that the spin-1/2 impurity interacting antiferromagnetically with a Fermi liquid is completely screened at zero temperature.[1] This screening is the essence of the Kondo effect.[2] The question of the screening length is much more subtle. Scaling implies, at least dimensionally, that the low-energy scale of the model, the Kondo temperature $T_K \sim D\exp(-1/\lambda_0)$, should be associated with an exponentially large length scale, $\xi_K = v_F/T_K$. (Here λ_0 is the Kondo coupling times density of states and D is the bandwidth). According to Nozières' Fermi-liquid picture,[3] one could imagine an electron in a region of this size which forms a singlet with the impurity spin. Note that this is a more dynamical type of screening than that which occurs for charge impurities in a Fermi liquid since it involves a linear combination of states where the impurity spin and the screening electron spin are in either an up-down or down-up configuration. In particular, the finiteness of the susceptibility at $T \to 0$ should not be attributed to a static conduction-electron polarization canceling the impurity spin polarization. Rather it results from the tendency of the impurity to form a singlet with the screening electron.

Whether or not this large screening cloud really exists has been a controversial subject in the literature, and has recently attracted some theoretical interest.[4–8] Boyce and Slichter[9] had performed direct Knight-shift measurements of the spin-spin correlator at all temperatures and had concluded that there was no evidence of the so-called screening cloud. Their measurements, however, were limited to very low distances (not more than several lattice spacings), and therefore could not probe directly any possible crossover at the distance scale ξ_K.

To study the screening cloud, we will consider the behavior of spatial spin-spin correlation functions, both zero frequency and equal time. There are two distance scales in the Kondo problem at finite temperature, ξ_K and the thermal scale $\xi_T = v_F/T$. On general scaling grounds, the spatial correlators should depend on the ratio of the distance r to these two scales. Sørensen and Affleck[5] have suggested a scaling form for the r-dependent Knight shift, proportional to the zero-frequency spin susceptibility, which has been justified numerically and perturbatively:[5,6]

$$\chi(r,T) - \rho/2 = \frac{\cos(2k_F r)}{8\pi^2 v_F r^2} f(r/\xi_K, r/\xi_T). \tag{1.1}$$

Here we have subtracted the Pauli contribution $\rho/2$; ρ is the density of states per spin. The g factor for the magnetic impurities is not necessarily equal to that of the conduction electrons. This is especially the case for some rare-earth ions, which have complex multiplet structure. If we take into account this possibility, scaling properties of the local spin susceptibility become not so simple, and we will consider them below. The Knight shift in this case is a sum of two parts, which scale differently.[6]

A possible objection to the naive concept of the screening cloud is based on sum rule arguments. The integral of the local spin susceptibility Eq. (1.1) is proportional to the zero-frequency correlator $\langle S^z_{\rm el} S^z_{\rm tot}\rangle$, where $S^z_{\rm el} = \int d^3\mathbf{r} S^z_{\rm el}(\mathbf{r})$ is the spin of the conduction electrons, $S^z_{\rm tot} = S^z_{\rm el} + S^z_{\rm imp}$. It can be shown that there is no net polarization of the conduction electrons,[10–13] and this correlator should vanish in the scaling limit ($J\rho \to 0$ with T_K held fixed). At $T=0$ this is simply a consequence of the ground state being a singlet. As remarked above, this does not necessarily imply the absence of the screening cloud in the sense of Nozières but only that the screening is a dynamical process.

In order to see the dynamical cloud of conduction electrons let us consider a snapshot of the system, the equal-time correlators. Take $K(r,T)=\langle S^z_{\rm el}(r,0)S^z_{\rm imp}(0)\rangle$ as an example. Note that $\langle S^z_{\rm imp}(0)S^z_{\rm imp}(0)\rangle=1/4$ for a spin-1/2 impurity, while $\langle S^z_{\rm el}(0)S^z_{\rm tot}(0)\rangle=0$ as mentioned above. (For this conserved quantity the equal-time and zero-frequency Green's functions are proportional to each other.) Thus the correlator $\langle S^z_{\rm el}S^z_{\rm imp}\rangle=-1/4$; that is $K(r,T)$ obeys a sum rule. $|K(r,0)|$ is a possible definition of the screening-cloud profile.

The ground-state properties of spatial correlators are determined by the Kondo scale only. In general we expect three different scaling regimes for $\chi(r,T)$ at a given temperature, with the r boundaries defined by the thermal and Kondo length scales. The goal of this paper is to determine scaling behavior of the spin correlators in these regimes.

Exponentially large length scale ξ_K, if present, could have important consequences for the theory of alloys with magnetic impurities. Indeed, typical $T_K\sim 10$ K and $E_F\sim 10$ eV makes $\xi_K\sim 10\,000a$, where a is the lattice spacing, much larger than typical distance between two impurities. Recently this issue was addressed in one dimension (1D) for Luttinger liquids with magnetic impurities,[14] where it was found that a crossover happened for $n_{\rm imp}\sim 1/\xi_K$.

Although perturbative calculations had been done early on,[15] no definite predictions were made regarding the size of the Kondo screening cloud. Chen *et al.*[16] have developed a renormalization-group (RG) approach. They, however, only considered short-range correlations $r\ll\xi_K$. We use the RG-improved perturbative technique, which cannot access lowest temperatures $T<T_K$. In order to gain some insight into what happens at low temperatures, we also consider overscreened $S_{\rm imp}=1/2$ multichannel Kondo effect, where the low-temperature fixed point is accessible perturbatively using $1/k$ expansion, k being the multiplicity of the bands. A very thorough $1/k$ analysis of the multichannel Kondo effect has been performed earlier by Gan,[4] who, however, came to conclusions opposite from ours. We also use the recent conformal field theory approach of one of us and Ludwig[17–19] to calculate the properties of the low-temperature, long distance correlation functions and the crossover at ξ_T. This approach, is valid for all k but only for $r\gg\xi_K$, $T\ll T_K$ and fails to predict the behavior of the spin correlators inside the screening cloud $r\lesssim\xi_K$. The result is nevertheless interesting because, as one could expect, the spin-spin correlators reflect the non-Fermi-liquid nature of the overscreened multichannel fixed point.

The paper is organized as follows. In Sec. II we introduce the model and remind the reader how it is transformed to an equivalent 1D model. We also define notations which we plan to use in the rest of the paper and derive the scaling equations for the spin susceptibility. Section III provides detailed perturbative analysis of the spin-spin correlation functions in the ordinary Kondo model (the Fermi-liquid fixed point). Section IV is devoted to the non-Fermi-liquid overscreened large-k case, where it is possible to obtain results for the spin correlators at all temperatures and distances using the $1/k$ expansion. We discuss our main conclusions in Sec. V. In Appendix A we mention a few details of our perturbative calculations. Appendix B gives the proof of the vanishing of the uniform part of the susceptibility. Appendix C gives results on the overscreened case ($k>1$) at $T\ll T_K$ and $r\gg\xi_K$ obtained from conformal field theory. Some of these results were presented briefly in Ref. 6.

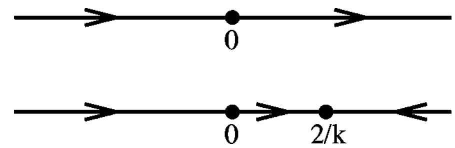

FIG. 1. RG flows for the single-channel and the multichannel Kondo problems.

II. THE MODEL, RENORMALIZATION GROUP, AND SCALING EQUATIONS

In what follows we consider the standard $S_{\rm imp}=1/2$ Kondo model,

$$H=\sum_{\mathbf{k}}\epsilon_k\psi^{\dagger\alpha}_{\mathbf{k}}\psi_{\mathbf{k}\alpha}+J\mathbf{S}_{\rm imp}\cdot\sum_{\mathbf{k},\mathbf{k}'}\psi^{\dagger\alpha}_{\mathbf{k}}\frac{\boldsymbol{\sigma}^\beta_\alpha}{2}\psi_{\mathbf{k}'\beta},\qquad(2.1)$$

and the multichannel $S_{\rm imp}=1/2$ Kondo model. The Hamiltonian for the $S_{\rm imp}=1/2$ k-channel Kondo model also includes summation over different channels j:

$$H=\sum_{\mathbf{k}j}\epsilon_k\psi^{\dagger\alpha j}_{\mathbf{k}}\psi_{\mathbf{k}\alpha j}+J\mathbf{S}_{\rm imp}\cdot\sum_{\mathbf{k},\mathbf{k}',j}\psi^{\dagger\alpha j}_{\mathbf{k}}\frac{\boldsymbol{\sigma}^\beta_\alpha}{2}\psi_{\mathbf{k}'j\beta}.\qquad(2.2)$$

Summation over repeated raised and lowered indiced is implied. The crucial difference between these two models can be seen from the form of the β function:[3]

$$\beta(\lambda)=-\lambda^2+\frac{k\lambda^3}{2}.\qquad(2.3)$$

The flow of the effective coupling is different (Fig. 1) for $k=1$ and $k>1$. The low-temperature fixed point of the multichannel Kondo problem is shown to have a non-Fermi-liquid nature.[17] At large band multiplicity this nontrivial fixed point becomes accessible perturbatively. This difference is not important for the purpose of this section, and we use Eqs. (2.2) and (2.3) for both multichannel and $k=1$ models.

The model is simplified if we assume spherically symmetric Fermi surface. Indeed, linearizing the spectrum and observing that scattering only takes place in the s-wave channel, we can expand the wave functions in spherical harmonics:

$$\psi_{\mathbf{k}\alpha j}=\frac{1}{\sqrt{4\pi}k}\,\psi_{0\alpha j}(k)+(\cdots),$$

$$H_0=\int dk\,\epsilon(k)\psi^{\dagger\alpha j}_0(k)\psi_{0\alpha j}(k)+(\cdots),$$

$$H_{\rm int}=\lambda_0 v_F\int dk\,dk'\,\psi^{\dagger\alpha j}_0(k)\frac{\boldsymbol{\sigma}^\beta_\alpha}{2}\,\psi_{0\beta j}(k')\cdot\mathbf{S}_{\rm imp},\qquad(2.4)$$

where $\epsilon(k)=v_F(k-k_F)$ is the linearized spectrum near the Fermi surface, $(\cdots)$ are higher harmonics, and $\mathbf{k}$ is the $1D$

wave vector. Here $\lambda_0=\rho J$ is the dimensionless coupling constant of the Kondo model, and $\rho=k_F^2/2\pi^2 v_F$ is the density of states per spin.

The s-wave operators obey standard one-dimensional anticommutation relations,

$$\{\psi_0^{\dagger\alpha_1 j_1}(k),\psi_{0\alpha_2 j_2}(k')\}=\delta_{\alpha_2}^{\alpha_1}\delta_{j_2}^{j_1}\delta(k-k'). \qquad (2.5)$$

We define left and right movers on a band of width 2Λ around k_F:

$$\psi_{L,R}\equiv\int_{-\Lambda}^{\Lambda}dk e^{\pm ikr}\psi_0(k+k_F). \qquad (2.6)$$

The 3D fermion operators are then written in the form

$$\Psi(r)=\frac{1}{2\sqrt{2}\pi r}\,[e^{-ik_F r}\psi_L(r)-e^{ik_F r}\psi_R(r)]+(\cdots), \qquad (2.7)$$

where $(\cdots)$ are higher harmonics. The left- and right-moving fields defined on $r>0$ obey the boundary condition:

$$\psi_L(0)=\psi_R(0). \qquad (2.8)$$

Flipping the right-moving field to the negative axis, $\psi_L(-r)\equiv\psi_R(r)$, we rewrite the 1D Hamiltonian in terms of the left-moving field only:

$$H=v_F\int_{-\infty}^{\infty}dr\,\psi_L^\dagger(r)(id/dr)\psi_L(r)+2\pi v_F\lambda_0\psi_L^\dagger(0)\frac{\boldsymbol{\sigma}}{2}\psi_L(0)\cdot\mathbf{S}_{\rm imp}. \qquad (2.9)$$

The purpose of this paper is to analyze various spin-spin correlation functions. The most important of them is the distance-dependent Knight shift, which can be measured in NMR experiments. If the impurity spin has a different gyromagnetic ratio from that of the conduction electrons, the uniform magnetic field couples to the spin operator $\mathbf{S}_h=\mathbf{S}_{\rm el}+(g_S/2)\mathbf{S}_{\rm imp}$, where $\mathbf{S}_{\rm imp}$ and $\mathbf{S}_{\rm el}=(1/2)\int d\mathbf{r}\psi^\dagger(\mathbf{r})\boldsymbol{\sigma}\psi(\mathbf{r})$ is the total spin operator of the impurity and conduction electrons, defined with channel sum for the multichannel problem. The expression for the Knight shift then consists of the electron and impurity contributions:

$$\chi(\mathbf{r})\equiv\int_0^\beta d\tau\langle S_{\rm el}^z(r,\tau)S_h^z(0)\rangle=\chi_{\rm el}(r)+\frac{g_s}{2}\chi_{\rm imp}(r). \qquad (2.10)$$

(We set $\mu_B=1$.) We will also consider the equal-time spin correlator $K(\mathbf{r})$, defined by

$$K(\mathbf{r})\equiv\langle S_{\rm el}^z(r,\tau=0)S_{\rm imp}^z(0)\rangle. \qquad (2.11)$$

The above 1D formalism allows to simplify this expression for large $rk_F\gg1$. Substituting Eq. (2.7) in Eq. (2.10), we get

$$\chi_A(r,T)=\frac{\chi_{2k_F,A}(r)}{4\pi^2r^2v_F}\cos(2k_Fr)+\frac{\chi_{{\rm un},A}(r)}{8\pi^2r^2v_F}, \qquad (2.12)$$

where A corresponds to imp or el. For $K(\mathbf{r},T)$ we get a similar expression:

$$K(\mathbf{r},T)=\frac{K_{2k_F}(r)}{4\pi^2r^2v_F}\cos(2k_Fr)+\frac{K_{un}(r)}{8\pi^2r^2v_F} \qquad (2.13)$$

The total electron spin in 1D is

$$\mathbf{S}_{\rm el}=\frac{1}{2\pi}\int_{-\infty}^{\infty}dr\,\psi_L^\dagger(r)\frac{\boldsymbol{\sigma}}{2}\psi_L(r). \qquad (2.14)$$

The uniform and $2k_F$ parts take the form:

$$\chi_{{\rm un},A}(r,T)\equiv v_F\int_0^\beta d\tau\left\langle\left[\psi_L^\dagger(r,\tau)\frac{\sigma^z}{2}\psi_L(r,\tau)+\psi_L^\dagger(-r,\tau)\frac{\sigma^z}{2}\psi_L(-r,\tau)\right]S_{\rm A}^z(0)\right\rangle,$$

$$\chi_{2k_F,A}(r,T)\equiv -v_F\int_0^\beta d\tau\left\langle\psi_L^\dagger(r,\tau)\frac{\sigma^z}{2}\psi_L(-r,\tau)S_{\rm A}^z(0)\right\rangle. \qquad (2.15)$$

Expressions for $K_{\rm un}$ and $K_{\rm imp}$ are analogous to those for $\chi_{\rm un,imp}$ and $\chi_{2k_F,{\rm imp}}$ in Eq. (2.15), although they do not involve integration over τ.

If the spins of the impurity and conduction electron have equal gyromagnetic ratio $(g_s=2)$, the operator S_h^z is the total spin of conduction electrons and impurity, and is conserved. The Knight shift is then given by Eq. (2.15), with $A=$tot. Since the Kondo interaction is local, only boundary $(r=0)$ operators have nonzero anomalous dimensions. Thus the conduction-electron spin operator $S_{\rm el}(r)$ also has zero anomalous dimension, for $r\neq0$. The local spin susceptibility then obeys the following RG equation:

$$\left[D\frac{\partial}{\partial D}+\beta(\lambda)\frac{\partial}{\partial\lambda}\right]\chi(T,\lambda,D,rT/v_F)=0, \qquad (2.16)$$

where D is the ultraviolet cutoff (the bandwidth), and $\beta(\lambda)$ is the β function. Sørensen and Affleck have recently made a conjection,[5] supported by perturbative and numerical results, that in the scaling limit, $rk_F\gg1$, $T\ll E_F$, the spin susceptibility has the following form:

$$\chi\left(\frac{rT}{v_F},\frac{T}{T_K}\right)=\frac{\chi_{2k_F}(rT/v_F,T/T_K)}{4\pi^2r^2v_F}\cos(2k_Fr), \qquad (2.17)$$

where χ_{2k_F} is a universal functions of two scaling variables.[20] This form follows directly from Eqs. (2.15,2.16). In general, one expects that there could be a nonzero phase in Eq. (2.17), and a uniform term. It is easy to see that the phase is zero due to particle-hole symmetry.[5] Indeed, under particle-hole transformation $\psi_L(r)\to\sigma^y\psi_L^\dagger(r)$, so $\mathbf{S}_{\rm tot}\to\mathbf{S}_{\rm tot}$, $\psi_L^\dagger(r)\boldsymbol{\sigma}\psi_L(-r)\to\psi_L^\dagger(-r)\boldsymbol{\sigma}\psi_L(r)$. Particle-hole symmetry of Eq. (2.17) then requires that the phase is zero. This is not so for more realistic Hamiltonians, for which the particle-hole symmetry is broken. For such Hamiltonians there is an additional phase ϕ in Eq. (2.17), but this phase does not renormalize. That is, it is essentially constant in the scaling region $(k_Fr\gg1)$. The fact that the uniform part of the spin susceptibility is zero is less trivial. For the *static* local spin susceptiblity we have proved[6] that all graphs in perturbation theory contain certain integrals that vanish. These properties hold for the electron and impurity parts of the local spin susceptibility Eq. (2.15) separately, for both single-channel and multichannel Kondo effects (see Appendix B). The uniform part and the phase are zero for the Knight shift in case of nontrivial gyromagnetic ratio for the impurity spin ($g_S \neq 1$) as well.

Since we consider the problem perturbatively, it is useful to express the scaling function Eq. (2.17) in terms of some effective coupling constants at an energy scale E,λ_E. This way we eliminate nonuniversal T_K. The energy scales of interest are the temperature T and the distance energy scale v_F/r. We will denote corresponding effective couplings as λ_T and λ_r. Expressions in terms of effective couplings can be easily converted into those in terms of T_K, and vice versa, provided that the β function is known up to the order needed. Indeed,

$$\frac{d\lambda_E}{d\,\ln(E/D)} \equiv \beta(\lambda_E), \tag{2.18}$$

where $D = v_F/\Lambda$ is the bandwidth. Therefore, for the effective coupling at two different energy scales E and E' we have

$$\int_{\lambda_E}^{\lambda_{E'}} \frac{d\lambda}{\beta(\lambda)} = \ln\frac{E'}{E}. \tag{2.19}$$

Since $\lambda_{T_K} \equiv 1$ can well be regarded as one of possible definitions of T_K, we have

$$\int_{\lambda_E}^{1} \frac{d\lambda}{\beta(\lambda)} = \ln\frac{T_K}{E} \tag{2.20}$$

and the arguments of the scaling function in Eq. (2.17) can be replaced by corresponding effective couplings.

The renormalization-group equations for various parts of the local spin susceptibility in Eq. (2.15) are less trivial. Consider first $\chi_{\rm imp}$. Since the Kondo interaction is at the origin, the fermion bilinear operator has zero anomalous dimension, while the operator $S_{\rm imp}$ receives anomalous dimension,[21] $\gamma_{\rm imp} \simeq \lambda^2/2$. Renormalizability implies that the functions $\chi_{B,{\rm imp}}$ ($B = 2k_F$, un) obey equations of the form:

$$\left[D\frac{\partial}{\partial D} + \beta(\lambda)\frac{\partial}{\partial\lambda} + \gamma_{\rm imp}(\lambda)\right]\chi_{B,{\rm imp}}(T,\lambda,D,rT/v_F) = 0, \tag{2.21}$$

where $\gamma_{\rm imp}(\lambda)$ is the anomalous dimension, which in this case is equal to the anomalous dimension of the impurity spin operator. The other correlator $\chi_{B,{\rm el}}$ contains the total conduction-electron spin operator $S_{\rm el}$, in which integration over the electron spin includes a potentially dangerous region near the impurity site. In this region operator mixing occurs between the electron spin and impurity spin. Thus $\chi_{B,{\rm el}}$ obeys the nontrivial mixed RG equation:

$$\left[D\frac{\partial}{\partial D} + \beta(\lambda)\frac{\partial}{\partial\lambda}\right]\chi_{B,{\rm el}}(T,\lambda,D,rT/v_F) = \gamma_{\rm imp}(\lambda)\chi_{B,{\rm imp}}(T,\lambda,D,rT/v_F). \tag{2.22}$$

This equation can be obtained by subtracting Eq. (2.21) from Eq. (2.16). It is more convenient to express the Knight shift for $g_S \neq 1$ in terms of $\chi_{\rm imp}$ and $\chi_{\rm tot}$, which obey ordinary scaling equations Eqs. (2.21),(2.16).

The solution of the scaling equation for $\chi_{B,{\rm imp}}$ Eq. (2.21) has the following form:

$$\chi_{B,{\rm imp}}\left(\lambda_0, \frac{T}{T_K}, \frac{rT}{v_F}\right) = e^{\int_{\lambda_0}^{\lambda_T}[\gamma_{\rm imp}(\lambda)/\beta(\lambda)]d\lambda}\Pi_{B,{\rm imp}}\left(\lambda_T, \frac{rT}{v_F}\right) = \Phi_{B,{\rm imp}}\left(\lambda_T, \frac{rT}{v_F}\right)e^{-\int_0^{\lambda_0}[\gamma_{\rm imp}(\lambda)/\beta(\lambda)]d\lambda}. \tag{2.23}$$

Here $\Phi_{B,{\rm imp}}(\lambda_T, rT/v_F)$, $\Pi_{B,{\rm imp}}(\lambda_T, rT/v_F)$ are some scaling functions to be determined below; $\lambda_0 = \rho J$ is the bare coupling constant. The solution of the scaling equations for $\chi_{\rm imp}$ is a function of T/T_K and rT/v_F, up to some nonuniversal coefficient. We see that the non-universal coefficient $\exp[-\int_0^{\lambda_0}d\lambda(\gamma_{\rm imp}(\lambda)/\beta(\lambda))]$ is equal to unity in the scaling limit of zero bare coupling $\lambda_0 \to 0$, if $\gamma_{\rm imp}(\lambda_0)/\beta(\lambda_0)$ is nonsingular in this limit. This is indeed the case for the Kondo model. The scaling function $\Phi_{B,{\rm imp}}(T/T_K, rT/v_F)$, of course, can differ from $\chi_{2k_F}(T/T_K, rT/v_F)$ in Eq. (2.17). The equal-time correlation functions also obey analogous scaling equations Eq. (2.21) with the anomalous dimension which is a sum of the dimension of the corresponding operators (for $\chi_{\rm imp}$ it is again $\gamma_{\rm imp}$). These equations are also applicable to the uniform part of the correlator, which is now nonzero.

In the rest of this paper we will consider these scaling functions in various regimes, which we now outline. Scaling form is applicable for $r \gg 1/k_F$, and $T \ll D \approx E_F$. For the single-channel Kondo model perturbative treatment is only valid for $T \gg T_K$. From Eq. (1.1) one could expect that there could be two crossovers: one at ξ_T and one at $\xi_K \gg \xi_T = v_F/T$. The latter crossover, however, does *not* happen as a function of r for $T \gg T_K$. The low-temperature correlation functions in the single-channel Kondo effect can be studied using the Fermi-liquid approach.[3] The region of validity for this approach is $r \gg \xi_K$, $T \ll T_K$. It provides important information about the low-temperature long-distance form of the correlation functions, and the crossover at $\xi_T \gg \xi_K$, but is unable to access the most interesting region $r \sim \xi_K$, and answer the question of existence of the screening cloud. For the multichannel Kondo effect the low-temperature long distance correlation functions can be obtained using the conformal field theory approach,[17–19] which is a generalization of Nozières' Fermi-liquid picture. It is also limited to $r \gg \xi_K$, $T \ll T_K$. The interesting low-temperature region with $r \sim \xi_K$ only becomes accessible at large k, when the whole scaling function can be constructed.

III. THE SINGLE-CHANNEL KONDO MODEL

A. The Knight shift

In what follows we consider the Knight shift in the single-channel Kondo model. As mentioned above, the local spin susceptibility only has the oscillating part. We have calculated it up to the third order in perturbation theory.[6] Summing all the relevant diagrams (see Appendix A), we obtain

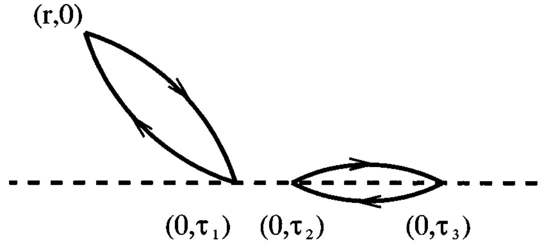

FIG. 2. Singular third-order graph for $\chi(r,T)$.

$$\chi_{2k_F}\left(x=\frac{rT}{v_F},\lambda_0,D\right)$$
$$=\frac{\pi^2}{4\ \sinh(2\pi x)}\{\lambda_0+\lambda_0^2[\ln(D/T)+M(x)+x]$$
$$+\lambda_0^3(\ln^2(D/T)+\ln(D/T)[2M(x)+2x-0.5]$$
$$+[M(x)+x][M(x)+0.5]+\text{const})\},\qquad(3.1)$$

where

$$M(x)=\ln[1-\exp(-4\pi x)].\qquad(3.2)$$

Substituting this expression in Eq. (2.21) we find that scaling is indeed obeyed. At small r, $x\ll 1$, Eq. (3.1) is rewritten as

$$\chi_{2k_F}(r,\lambda_0,D)=\frac{\pi v_F}{8rT}[\lambda_0+\lambda_0^2\ln(\tilde{\Lambda}r)+\lambda_0^3\ln^2(\tilde{\Lambda}r)$$
$$+0.5\lambda_0^3\ln(\tilde{\Lambda}r)-\lambda_0^3\ln(D/T)+\lambda_0^3\text{const}],\qquad(3.3)$$

where $\tilde{\Lambda}=4\pi D/v_F=4\pi\Lambda\sim k_F$. It is clear from Eq. (3.3) that the infrared divergences of the perturbation theory *are not* cut off at low T by going to small r, as was noticed by Gan.[4] In the third order, these divergences are associated with the graph shown in Fig. 2. Due to the nonconservation of momentum by the Kondo interaction, the bubble on the right gives a logarithmic T-dependent factor which is independent of r. Thus, the interior of the screening cloud does *not* exhibit weak-coupling behavior.

It is convenient to rewrite this result in terms of effective couplings at the energy scales T and v_F/r. One can easily write down the effective coupling constant[22] at some energy scale ω using the well-known β function Eq. (2.3):

$$\lambda_\omega=\lambda_0+\lambda_0^2\ln(D/\omega)$$
$$+\lambda_0^3[\ln^2(D/\omega)-(1/2)\ln(D/\omega)+\text{const}].\qquad(3.4)$$

We find that the expression for χ_{2k_F} is simplified when we use effective couplings λ_T and λ_E at the energy scales T and $E(x)=T/[1-\exp(-4\pi x)]e$, $x=r/\xi_T$. When $r\ll\xi_T$ the latter becomes the effective coupling at the distance scale r, since $E(x)\propto v_F/r$. Equation (3.1) in terms of these effective couplings takes the form:

$$\chi_{2k_F}\left(x=\frac{rT}{v_F},\lambda_T\right)=\frac{[\lambda_E+(3\pi/2)\lambda_E^2x+\text{const}\lambda_E^3](1-\lambda_T)}{(4/\pi^2)\sinh(2\pi x)}.\qquad(3.5)$$

It is instructive to consider various limiting cases for the scaling function Eq. (3.5). For $r\ll\xi_T$ we find

$$\chi_{2k_F}(x,\lambda_T)=(\pi/8x)(\lambda_r+\text{const}\lambda_r^3)(1-\lambda_T).\qquad(3.6)$$

The factor $(1-\lambda_T)/4T$ in Eq. (3.6) is, to the order under consideration, precisely the total impurity susceptibility, $\chi_{tt}(T)$. This is the total susceptibility less the bulk Pauli term. Thus Eq. (3.6) can be written

$$\chi_{2k_F}(\lambda_T,\lambda_r)\rightarrow\frac{\lambda_r}{2(r/\pi v_F)}\ \chi_{tt}(T).\qquad(3.7)$$

We conjecture that this equation is exact, for arbitrary T/T_K in the short distance limit, $r\ll v_F/T,\xi_K$. According to our conjecture, the infrared divergences which are not cut off by going to small r are simply the ones which produce the function $\chi_{tt}(T)$. since this function has been calculated accurately from the Bethe ansatz[1] [although $\chi(r,T)$ has not], our conjecture, if correct, has considerable predictive power in the experimentally interesting region $r\ll\xi_K$. We show below that this conjecture is consistent with the behavior of the integrated susceptibilities and also with the large-k limit. This conjecture can be understood graphically. In Fig. 2, the bubble at $r=0$ is simply the one-loop, logarithmically divergent, contribution to $\chi_{tt}(T)$. The other loop connecting the origin to the point r makes a factorized contribution. Adding arbitrary additional insertions and loops at $r=0$, while leaving the simple bubble connecting 0 to r, gives the factor of $\chi_{tt}(T)$. We conjecture that all additional insertions where the two lines from r split, etc. are not infrared divergent. As one can see from Eq. (3.5), this factorization breaks down at $r\sim v_F/T$.

We can compare this result with the experiment of Boyce and Slichter,[9] who have measured the Knight shift from Cu nuclei near the doped Fe impurities, at distances up to fifth nearest neighbor. At these very small distances of order of a few lattice spacings, they have found empirically that the Knight shift obeyed a factorized form, $\chi(r,T)\approx f(r)/T+T_K)$, with rapidly oscillating function $f(r)$ for a wide range of T extending from well above to well below the Kondo temperature. Although our condition $r\gg 1/k_F$ is not satisfied in this experiment, this form coincides with Eq. (3.7), since the Bethe ansatz solution for $\chi_{tt}(T)$ may be quite well approximated[1] by $1/(T+T_K)$ at intermediate temperatures $T\sim T_K$.

On the other hand, for $r\gg\xi_T$, the spin susceptibility takes the following (nonfactorized) form:

$$\chi_{2k_F}(x,\lambda_T)=(3\pi^3/4)\lambda_T^2(1-\lambda_T)e^{-2\pi x}.\qquad(3.8)$$

For high temperatures $T\gg T_K$ there is no crossover at $r\sim\xi_K$ in the behavior of the local spin susceptibility.

At low temperatures $T\ll T_K$ and large distances $r\gg\xi_K$ the behavior of $\chi(r)$ is determined by the zero-energy Fermi-liquid fixed point.[5] The Kondo impurity acts as a potential scatterer with a phase shift $\pi/2$ at the Fermi surface.[3] The local susceptibility follows directly from the formula for Friedel's oscillation in the electron density for an s-wave scatterer and $\pi/2$ phase shift,

$$n(r)=n_0-\frac{1}{2\pi^2 r^3}\cos[2k_F r+\pi/2]. \tag{3.9}$$

Since the magnetic field H simply shifts the chemical potential by $\pm g\mu_B H/2$ for spin-up or spin-down electrons,

$$\chi(r,T)=\frac{1}{v_F}\frac{dn}{dk_F}=\frac{\rho}{2}+\frac{1}{4\pi^2 v_F r^2}\cos(2k_F r). \tag{3.10}$$

This implies for the scaling function Eq. (2.21):

$$\chi_{2k_F}=1. \tag{3.11}$$

The finite-temperature properties of $\chi_{2k_F}(r)$, and, in particular, the crossover at $r\sim\xi_T$ can be obtained directly from the Nozières' low-energy Hamiltonian for the Fermi-liquid fixed point:[3,17]

$$H_0=\int_{-\infty}^{+\infty}dr\,\psi_L^\dagger(r)\frac{d}{dr}\psi_L(r)+\frac{\delta(r)}{T_K}\mathbf{S}_{\rm el}^2(r), \tag{3.12}$$

where $\mathbf{S}_{\rm el}(r)\equiv\psi_L^\dagger(r)(\boldsymbol{\sigma}/2)\psi_L(r)$. This definition of T_K differs from one in Eq. (2.20) or $\chi_{\rm tt}\propto 1/(T+T_K)$ by numerical factors $O(1)$ (Wilson ratio). The expression for $\chi_{2k_F}(r)$, Eq. (3.11), is zero order in the leading irrelevant coupling constant $1/T_K$, and the finite-temperature form of $\chi_{2k_F}(r)$ is easily obtained:

$$\chi_{2k_F}(x)=\frac{2\pi x}{\sinh(2\pi x)},\quad x=\frac{rT}{v_F}. \tag{3.13}$$

We can derive corrections to Eq. (3.13) by doing perturbation theory in the leading irrelevant operator. For the first correction we obtain

$$\delta\chi_{2k_F}(x)=\frac{\pi^2 T}{T_K\sinh(2\pi x)}. \tag{3.14}$$

The first correction does not alter the leading-order behavior. At zero temperature the scaling function for $r\gg\xi_K$ takes the following form:

$$\chi_{2k_F}(r/\xi_K)=1+\pi\frac{\xi_K}{2r}. \tag{3.15}$$

This correction gives rise to the first term in the large-distance expansion of our scaling function $\chi_{2k_F}(r/\xi_K)$. The behavior of the scaling function χ_{2k_F} in different regimes is summarized in Fig. 3. $\chi_{2k_F}(r/\xi_K,r/\xi_T)$ exhibits a crossover at low T, when the "screening" cloud is formed. At high temperatures this crossover is absent.

What happens when the g factor of the impurity is anomalous? $\chi_{2k_F}(r,T)$ is a sum of impurity and electron parts, $\chi_{2k_F,\rm imp}$ and $\chi_{2k_F,\rm el}$. As we have discussed in the previous section, the latter obeys a complicated mixed RG equation, Eq. (2.22). It is more convenient to express the spin susceptibility in terms of the correlators $\chi_{2k_F,\rm tot}(r,T)$ and $\chi_{2k_F,\rm imp}(r,T)$, for which RG equations are simple:

$$\chi_{2k_F}(r,T)=(g_S/2)\chi_{2k_F,\rm imp}(r,T)+\chi_{2k_F,\rm el}(r,T),$$

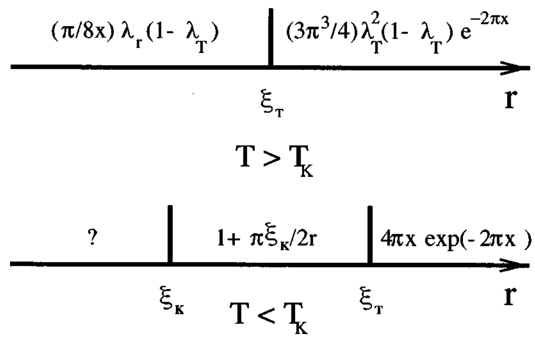

FIG. 3. Scaling regimes for $\chi_{2k_F}(\lambda_T,x=r/\xi_T)$ in the single-channel Kondo effect.

$$\chi_{2k_F,\rm el}=\chi_{2k_F,\rm tot}(r,T)-\chi_{2k_F,\rm imp}(r,T). \tag{3.16}$$

Since we have already determined $\chi_{2k_F,\rm tot}(r,T)$, it is sufficient to consider only $\chi_{2k_F,\rm imp}(r,T)$. From the perturbative analysis (see Appendix A) we obtain

$$\begin{aligned}\chi_{2k_F,\rm imp}&\left(x=\frac{rT}{v_F},\lambda_0,D\right)\\ &=\frac{\pi^2}{4\sinh(2\pi x)}\{\lambda_0+\lambda_0^2[\ln(D/T)+M(x)+0.5]\\ &\quad+\lambda_0^3(\ln^2(D/T)+2\ln(D/T)M(x)\\ &\quad+[M(x)+0.5]^2+\text{const})\},\end{aligned} \tag{3.17}$$

where $M(x)$ is the same as in Eq. (3.2). One can easily check that Eq. (2.21) is obeyed with[22,4]

$$\beta(\lambda)=-\lambda^2+\frac{\lambda^3}{2},\quad \gamma_{\rm imp}(\lambda)=\frac{\lambda^2}{2}. \tag{3.18}$$

We then obtain for the nonuniversal factor in Eq. (2.23):

$$e^{-\int_0^{\lambda_0}[\gamma_{\rm imp}(\lambda)/\beta(\lambda)]d\lambda}\simeq 1+\frac{\lambda_0}{2}, \tag{3.19}$$

and the local impurity spin susceptibility takes the following form:

$$\chi_{2k_F,\rm imp}\simeq\left(1+\frac{\lambda_0}{2}\right)\chi_{2k_F}^{(1)}(\lambda_T,x), \tag{3.20}$$

where the scaling function

$$\chi_{2k_F}^{(1)}(\lambda_T,x)=\frac{(\lambda_E+\text{const}\lambda_E^3)(1-\lambda_T)}{(4/\pi^2)\sinh(2\pi x)} \tag{3.21}$$

differs from that for the conserved local susceptibility Eq. (3.5). For the electron part we obtain

$$\chi_{2k_F,\rm el}\approx-\frac{\lambda_0}{2}\chi_{2k_F}^{(1)}(\lambda_T,x)+\frac{(3\pi/2)\lambda_E^2 x(1-\lambda_T)}{(4/\pi^2)\sinh(2\pi x)}. \tag{3.22}$$

The second contribution does not vanish in the scaling limit $\lambda_0\to 0$. However, it only becomes substantial at large distances $r\sim\xi_T$, where there is no additional smallness associated with the factor $x=r/\xi_T$. We conclude that two different scaling functions are present in the experimentally measured Knight shift, and their share depends upon the gyromagnetic ratios for the impurity and the conduction electrons.

B. Integrated susceptibilities

It is instructive to consider the integral of $\chi(\mathbf{r},T)$ over all space. This quantity determines the polarization of the screening cloud in external magnetic field. We immediately see that the contribution from large distances vanishes because of the oscillatory behavior of $\chi(\mathbf{r},T)$ at large r. Nevertheless the integral can be finite due to the contributions at small distances $r\sim 1/k_F$. We will specify three different spin correlators:

$$\chi_{\rm tt}(T)\equiv\frac{\langle S^z_{\rm tot}S^z_{\rm tot}\rangle}{T},\quad \chi_{\rm ti}(T)\equiv\frac{\langle S^z_{\rm imp}S^z_{\rm tot}\rangle}{T},$$

$$\chi_{\rm ii}(T)\equiv\int_0^\beta\langle S^z_{\rm imp}(\tau)S^z_{\rm imp}(0)\rangle d\tau. \tag{3.23}$$

For this choice of correlators the RG equations are simplified, and have the form Eq. (2.21). It seems more natural to define correlators of the impurity spin and the total conduction-electron spin $S_{\rm el}$ instead of $S_{\rm tot}$:

$$\chi_{\rm ee}(T)\equiv\int_0^\beta\langle S^z_{\rm el}(\tau)S^z_{\rm el}(0)\rangle d\tau-\chi_0,$$

$$\chi_{\rm ei}(T)\equiv\int_0^\beta\langle S^z_{\rm el}(\tau)S^z_{\rm imp}(0)\rangle d\tau, \tag{3.24}$$

where χ_0 is the free-electron susceptibility, proportional to the volume of the system. However, for this set of spin correlators the RG equations are mixed.

Two of the three spin-correlation functions can be measured. The first one is the bulk susceptibility, $\chi_{\rm tt}(T)$. The electron-spin polarization in the presence of an impurity is determined by the spatial integral of $\chi(r)$ measured in the Knight-shift experiment Eq. (2.10), or, equivalently, by $\chi_{\rm tt}(T)-\chi_{\rm ti}(T)$. If the gyromagnetic ratio for the impurity is different from 2, the experimentally measured magnetic susceptibility is $(g_s^2/4)\chi_{\rm ii}+g_s\chi_{\rm ie}+\chi_{\rm ee}$, while the integrated electron susceptibility is given by $(g_s/2)\chi_{\rm ie}+\chi_{\rm ee}$.

Since $S^z_{\rm tot}$ is conserved, the spin susceptibilities obey the RG equation, Eq. (2.21), with anomalous dimensions determined by the dimension $\gamma_{\rm imp}(\lambda)$ of the operator $S^z_{\rm imp}$. For the three different susceptibilities: $\gamma_{\rm tt}=0$, $\gamma_{\rm ti}=\gamma_{\rm imp}$, and $\gamma_{\rm ii}=2\gamma_{\rm imp}$. The solutions of these equations take the form Eq. (2.23),

$$4T\chi_j(T)=\Phi_j(\lambda_T)e^{-\int_0^\lambda[\gamma_j(\lambda)/\beta(\lambda)]d\lambda}, \tag{3.25}$$

where j labels tt, ti or ii. From our third-order perturbative analysis using Wilson's result[23] for $\chi_{\rm tt}(T)$ we have obtained that the functions $\Phi_j(\lambda_T)\simeq 1-\lambda_T$ coincide for all three susceptibilities up to and including terms of order λ_T^2. If this is indeed the case in the Kondo model, we then obtain from Eq. (3.25) that in the scaling limit $\lambda_0\to 0$ both $\chi_{\rm ee}(T)$ and $\chi_{\rm ie}(T)$ vanish. At finite bare couplings these susceptibilities also become finite, with nonuniversal amplitudes. We then obtain from Eq. (3.25) for the impurity-electron and electron-electron pieces of the spin susceptibility:

$$\chi_{\rm ie}\simeq-\frac{\lambda_0}{2}\chi_{\rm tt}(T),$$
$$\chi_{\rm ee}\simeq\frac{\lambda_0^2}{4}\chi_{\rm tt}(T). \tag{3.26}$$

Thus, the integrated distance-dependent Knight shift obeys

$$\int\chi(r,T)d\mathbf{r}=\chi_{\rm ee}(T)+\chi_{\rm ie}(T)\approx-\frac{\lambda_0}{2}\chi_{\rm tt}(T). \tag{3.27}$$

The major contribution to Eq. (3.27) comes from the electron-impurity correlator. It should be emphasized that the result is nonzero at finite bare coupling λ_0. [A typical experimental value of λ_0 might be $1/\ln(E_F/T_K)\approx 0.15$.] It is easy to see that the integral in Eq. (3.27) is dominated by $r\sim 1/k_F$. Thus most of the small net polarization of the electrons in a magnetic field (with the free-electron value subtracted) comes from very short distances. However, this should not be interpreted as meaning that the screening cloud is small as can be clearly seen from the equal-time correlation function discussed in the next subsection.

If the equality of the scaling functions $\Phi_j(\lambda_T)$ defined in Eq. (3.25) holds at all T, the integrated electron-spin susceptibility vanishes in the scaling limit of zero bare coupling at all T. The fact that $\chi_{\rm ie}$ and $\chi_{\rm ee}$ are suppressed in the scaling limit has been known or conjectured from a variety of different approaches over the years. The earliest result of this sort that we are aware of, in the context of the Anderson model, predates the discovery of the Kondo effect and is referred to as the Anderson-Clogston compensation theorem.[10] It was later established at $T=0$ from the Bethe ansatz solution.[12] A very simple and general proof[13] of this result follows from the Abelian bosonization approach.[24] Beginning with left-moving relativistic fermions on the entire real line, as in Eq. (2.9), we may bosonize to obtain left-moving spin and charge bosons. The charge boson decouples and the Hamiltonian for the Kondo Hamiltonian can be written in terms of the left-moving spin boson, ϕ_L which obeys the canonical commutation relation:

$$[\phi_L(r'),\partial_r\phi_L(r)]=(i/2)\delta(r-r'). \tag{3.28}$$

The Hamiltonian becomes

$$H=\int_{-\infty}^{\infty}dr\left[v_F(\partial_r\varphi_L(r))^2-\frac{h_e}{\sqrt{2\pi}}\partial_r\varphi_L(r)\right]+H_K-h_iS^z,$$

$$H_K=2\pi v_F\lambda_0\left[\frac{S^z\partial_r\varphi_L(0)}{\sqrt{2\pi}}+\text{const}\cdot(S^+e^{i\sqrt{8\pi}\varphi_L(0)}+\text{H.c.})\right]. \tag{3.29}$$

Here h_i and h_e are the magnetic fields acting on the impurity and the conduction electrons, correspondingly. These fields may differ by the ratio of corresponding g factors. We can get rid of the $\int dr\,\partial_r\varphi_L(r)$ term by shifting the bosonic field:

$$\varphi_L(r)=\tilde{\varphi}_L(r)+h_er/v_F\sqrt{8\pi}. \tag{3.30}$$

The Hamiltonian in terms of the new bosons takes the form

$$H=\int_{-\infty}^{\infty}drv_F(\partial_r\tilde{\varphi}_L(r))^2-\frac{h_e^2L}{4\pi v_F}+\sqrt{2\pi}v_F\lambda_0S^z\partial_r\tilde{\varphi}_L(0)$$
$$+\text{const}\cdot 2\pi v_F\lambda_0(S^+e^{i\sqrt{8\pi}\tilde{\varphi}_L(0)}+\text{H.c.})-\left(h_i-\frac{h_e\lambda_0}{2}\right)S^z. \qquad (3.31)$$

Thus, our original Hamiltonian with nonzero field h_e acting on the conduction electrons is exactly equivalent to the one with no field acting on conduction electrons and modified impurity field. The same argument was given in Ref. 25 except that the field shift by $h_e\lambda_0/2$ was not obtained because another, noncommuting, canoncial transformation was performed first to eliminate the z component of the Kondo interaction.

In terms of the free energy, this is written as

$$F(h_i,h_e)=-\frac{h_e^2L}{4\pi v_F}+F\left(0,h_i-\frac{h_e\lambda_0}{2}\right). \qquad (3.32)$$

Taking magnetic-field derivatives, we easily find

$$\chi_{\text{ii}}=-\frac{\partial^2F}{\partial h_i^2},$$
$$\chi_{\text{ie}}=-\frac{\partial^2F}{\partial h_i\partial h_e}=-\frac{\lambda_0}{2}\chi_{\text{ii}}, \qquad (3.33)$$
$$\chi_{\text{ee}}=-\frac{\partial^2F}{\partial h_e^2}-\chi_0=\left(\frac{\lambda_0}{2}\right)^2\chi_{\text{ii}},$$

where $\chi_0=L/2\pi v_F$ is the Pauli term. It is easy to see that this is valid for the anisotropic Kondo model as well, with λ_0 being the z component of the Kondo interaction.

C. Equal-time spin-spin correlator

The equal-time spin correlators provide a snapshot of the Kondo system. The quantity of interest is

$$K(\mathbf{r},T)=\langle S^z_{\text{el}}(\mathbf{r},0)S^z_{\text{imp}}(0)\rangle. \qquad (3.34)$$

As we have shown in the previous sections, it satisfies a nonzero sum rule:

$$\int d\mathbf{r}K(\mathbf{r},T)=-1/4. \qquad (3.35)$$

The proof that $\chi_{\text{un}}(r)=0$ is based on the fact that the time integral for the Feynman diagrams is zero in all orders in perturbation theory (see Appendix B). For the equal-time correlator we do not integrate over the time variable, so the uniform part does not have to vanish. $K(\mathbf{r},T)$ can be rewritten in 1D in terms of the uniform and $2k_F$ parts, Eqs. (2.17,2.15). For the same reason as for the impurity part of the Knight shift, the equal-time correlator obeys the scaling equation (2.21), with solutions of the form Eq. (2.23). Since the decomposition of $K(r,T)$ into the uniform and $2k_F$ parts is only valid in the scaling region $k_Fr\gg1$, the sum rule Eq. (3.35) does not necessarily extend to $K_{\text{un}}(r,T)$. The region $r\sim 1/k_F$ could produce a large contribution to the sum rule Eq. (3.35).

Consider now the equal-time correlators $K_{\text{un}}(r,T)$ and $K_{2k_F}(r,T)$ perturbatively. In the third order we obtain (see Appendix A):

$$K_{\text{un}}(r,T)=\frac{\pi^2\{-\lambda_0^2+(1/2)\lambda_0^3-2\lambda_0^3\ln[D/T]\}T}{\exp(2\pi r/\xi_T)-1}+\pi^2T\lambda_0^3G_1(r/\xi_T),$$
$$K_{2k_F}(r,T)-T\chi_{2k_F}(r,T)=\pi^2\lambda_0^3T\left[G_2(r/\xi_T)-\frac{\ln(1-e^{-2\pi r/\xi_T})}{4\sinh(2\pi r/\xi_T)}\right]. \qquad (3.36)$$

$G_{1,2}(x)$ are some functions which can be represented as integrals:

$$G_1(x)=\int_0^1\frac{2ds}{1-s}\left[\frac{1}{e^{2\pi x}-1}+\frac{s}{1-s}\ln\left(\frac{1-e^{-2\pi x}}{1-se^{-2\pi x}}\right)\right],$$
$$G_2(x)=\int_0^1 ds\frac{e^{-2\pi x}s}{(1-s)(1-se^{-4\pi x})}\ln\left[\frac{1-se^{-2\pi x}}{1-e^{-2\pi x}}\right]. \qquad (3.37)$$

It is easy to check that Eq. (2.21) is satisfied for both uniform and $2k_F$ parts. The solutions are found in the form Eq. (2.17) with the nonuniversal factor Eq. (3.19). The scaling functions are easily obtained from Eq. (3.36). The final expressions are simplified in the most interesting limiting cases. For $r\ll\xi_T$ we obtain

$$K_{\text{un}}(\lambda_r,r/\xi_T,\lambda_0)=-\frac{\pi v_F\lambda_r^2(1+\lambda_0/2)}{2r},$$
$$K_{2k_F}(\lambda_r,r/\xi_T,\lambda_0)=\frac{\pi v_F\lambda_r(1+\lambda_0/2)}{8r}. \qquad (3.38)$$

In case of $r\gg\xi_T$, these functions take the form:

$$K_{\text{un}}(\lambda_r,r/\xi_T,\lambda_0)=-\pi^2T\lambda_T^2\left(1+\frac{\lambda_0}{2}\right)e^{-2\pi r/\xi_T},$$
$$K_{2k_F}(\lambda_r,r/\xi_T,\lambda_0)=\frac{\pi^2T\lambda_0}{2}\left(1+\frac{\lambda_0}{2}\right)(1-\lambda_T)e^{-2\pi r/\xi_T}. \qquad (3.39)$$

Note that K_{2k_F} is suppressed in this limit by the small value of the bare coupling. Like the local spin susceptibility, the equal-time correlator does not have crossover at $r\sim\xi_K$ at high temperatures. Instead, the corresponding scaling function for $r\gg\xi_T$ has a factorized form, $K(r/\xi_T,T/T_K)\propto f_1(r/\xi_T)f_2(T/T_K)$.

The behavior of the equal-time correlation function at $T\ll T_K$ and $r\gg\xi_K$ can be calculated using Noziéres Fermi-liquid approach. Indeed, the impurity spin at the infrared Kondo fixed point should be replaced by the local spin density $\mathbf{J}(0)$ for $r\approx0$, up to a constant multiplicative factor[17,18]

$$\mathbf{S}_{\text{imp}}\propto\frac{v_F\mathbf{J}_L(0)}{T_K}. \qquad (3.40)$$

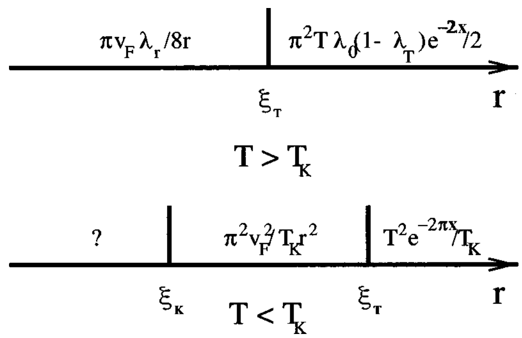

FIG. 4. Scaling regimes for the oscillating part of the equal-time spin-spin correlator $K_{2k_F}(\lambda_T, x = r/\xi_T)$ in the single-channel Kondo effect.

Substituting this in the definition of $K_{\rm un}$ and K_{2k_F}, we obtain at finite T

$$K_{2k_F}(r/\xi_T) = -(1/2)K_{\rm un}(r/\xi_T) = \frac{{\rm const}T^2}{T_K \sinh^2(\pi T r/v_F)}. \tag{3.41}$$

Thus, at $T\rightarrow 0$ the equal-time correlator K decays as $\sin^2 k_F r/r^4$ [see Eq. (2.13)]. This result was obtained by Ishii[26] in the context of the Anderson model. The behavior of the equal-time correlators $K_{2k_F}(r)$ and $K_{\rm un}(r)$ in different regimes is summarized in Figs. 4 and 5.

IV. LARGE-k MULTICHANNEL KONDO MODEL

The information that one gets for the single-channel Kondo model using perturbative RG is very limited, and further numerical analysis is required. To justify the presence of the Kondo length scale more, we analyze the multichannel model with large band multiplicity. The generalization of the above perturbative analysis to the multichannel case is quite straightforward. The Hamiltonian for the $S_{\rm imp} = 1/2$ k-channel Kondo model is given by Eq. (2.2). Further analysis of Sec. II applies to the multichannel case as well. Some of the relevant perturbative $1/k$ calculations for the multichannel Kondo effect were done by Gan.[4] His scaling equations and conclusions about the screening cloud are, however, different from ours. We refer to some of his results below.

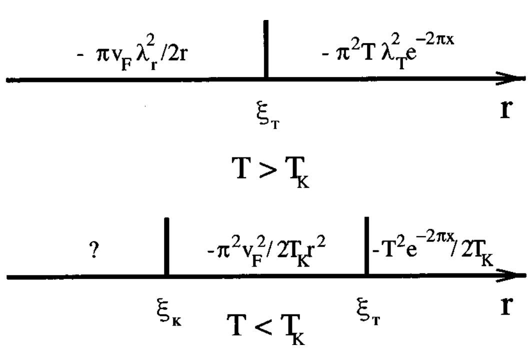

FIG. 5. Scaling regimes for the uniform part of the equal-time correlator $K_{\rm un}(\lambda_T, x = r/\xi_T)$ in the single-channel Kondo effect.

A. The local spin susceptibility

Spin susceptibilities of the multichannel Kondo problem also satisfy RG equation Eq. (2.21). However, the diagrams which contribute to the same order in $1/k$ are different from the single-channel case. Since the low-temperature fixed point for coupling constant is $\sim 1/k$, each vertex produces a $1/k$ factor. [Here we assume that the bare coupling, λ_0, is also $O(1/k)$.] Each loop, on the other hand, gives a large factor of k. Combination of these factors determines the diagrams that one needs to calculate to a given order in $1/k$. The number of diagrams is finite (see Appendix A for details). We shall calculate the spin correlators of interest up to the first nonzero order in $1/k$.

The solution of the scaling equations for the coupling constant up to subleading order in $1/k$ were obtained by Gan.[4] From the calculation of the conduction electron self-energy he found that the β function is given by

$$\beta(\lambda_E) = -\lambda_E^2 + \frac{1}{2}k\lambda_E^3 + \frac{1}{2}ka\lambda_E^4 - \frac{1}{4}k^2\lambda_E^5, \tag{4.1}$$

where a is some nonuniversal number, which depends on the cutoff procedure. The flow for the overscreened Kondo model is shown in Fig. 1. The low-temperature physics is determined by the intermediate-coupling stable fixed point λ^* given by $\beta(\lambda^*) = 0$:

$$\lambda^* = \frac{2}{k}\left(1 + \frac{2-4a}{k}\right). \tag{4.2}$$

The position of the fixed point is not universal. On the other hand, the slope of the β function at this fixed point, $\Delta \equiv \beta'(\lambda^*)$ is the dimension of the leading irrelevant operator,[17] and should be universal:

$$\Delta = \beta'(\lambda^*) = \frac{2}{k+2}. \tag{4.3}$$

This fact is readily checked from Eq. (4.1).

It is sufficient for our purposes to consider the β function in the leading order in $1/k$, Eq. (2.3). At this order $\lambda^* = \Delta = 2/k$. Solving Eq. (2.18), we obtain

$$\frac{k}{2}(\phi_E + \ln|\phi_E|) = \ln\frac{E}{T_K}, \tag{4.4}$$

where

$$T_K = D\ \exp\left[\frac{k}{2} - \frac{1}{\lambda_0}\right]\left(\frac{k\lambda_0/2}{1-k\lambda_0/2}\right)^{k/2},$$

$$\phi_E \equiv \frac{2}{k\lambda_E} - 1. \tag{4.5}$$

We assume that the bare coupling λ_0 is sufficiently weak on the $1/k$ scale, $\lambda_0 < 2/k$. Then the solution for the running coupling constant is rewritten as

$$\lambda_E = \frac{\lambda^*}{F^{(-1)}[(E/T_K)^\Delta] + 1}. \tag{4.6}$$

Here $F^{(-1)}(y)$ is the function inverse to $F(x) = x\ \exp(x)$. The asymptotic form of this solution at $E \ll T_K$ is also useful:

$$\lambda_E=\lambda^*-\left(\frac{E}{T_K}\right)^{\Delta}. \qquad (4.7)$$

The analysis of the local spin susceptibility is parallel to the single-channel case (Sec. III). It is easy to see that the uniform part of the local spin susceptibility should vanish in the multichannel model as well (see Appendix B). Therefore, for the most general magnetic impurity (i.e., with gyromagnetic ratio not necessarily 2) we are left with electron and impurity parts of the oscillating local spin susceptibility $\chi_{2k_F,\mathrm{el}}(r,T),\chi_{2k_F,\mathrm{imp}}(r,T)$. The RG equations that these quantities satisfy were considered in Sec. II. The only difference with the single-channel case is that the β function and anomalous dimension are different, with $\gamma_{\mathrm{imp}}(\lambda)$ now given by

$$\gamma_{\mathrm{imp}}(\lambda)=\frac{k\lambda^2}{2}. \qquad (4.8)$$

The nonuniversal scale factor for the solution of the RG equations Eq. (2.17) then is

$$\exp\left[-\int_0^{\lambda_0}\frac{\gamma_{\mathrm{imp}}(\lambda)}{\beta(\lambda)}d\lambda\right]\simeq\frac{1}{1-k\lambda_0/2}. \qquad (4.9)$$

The scaling functions in the large-k limit are determined from the perturbative analysis (see Appendix A). We find again that the scaling equations Eq. (2.21) are obeyed, and the solutions are given by

$$\chi_{2k_F,\mathrm{imp}}(\lambda_T,x)=\frac{1}{1-k\lambda_0/2}\,\frac{\chi^{(1)}_{2k_F}(\lambda_T,x)}{\sinh(2\pi x)},$$

$$\chi_{2k_F,\mathrm{tot}}(\lambda_T,x)=\frac{(3\pi^3/8)k\lambda_E^2x}{\sinh(2\pi x)}+\frac{\chi^{(1)}_{2k_F}(\lambda_T,x)}{\sinh(2\pi x)}, \qquad (4.10)$$

where

$$\chi^{(1)}_{2k_F}(\lambda_T,x)=(\pi^2/4)(1-k\lambda_T/2)^2\frac{k\lambda_E}{1-k\lambda_E/2}, \qquad (4.11)$$

and λ_E is the coupling at the energy scale $E(x)=T/[1-\exp(-4\pi x)]e$, just like in the single-channel case. λ_E, λ_T are functions of E/T_K or T/T_K given by Eq. (4.6). Using Eq. (4.10) together with Eq. (4.6) we determine the scaling functions $\chi_{2k_F,\mathrm{imp}}$ and $\chi_{2k_F,\mathrm{el}}$ up to the leading order in $1/k$ in the scaling limit $\lambda_0\to 0$, $r\gg 1/k_F$ at all temperatures:

$$\chi_{2k_F,\mathrm{imp}}(T/T_K,x)=\frac{\pi^2}{2\,\sinh(2\pi x)}\left(\frac{F^{(-1)}[(T/T_K)^{\Delta}]}{F^{(-1)}[(T/T_K)^{\Delta}]+1}\right)^2\times\frac{1}{F^{(-1)}[(E/T_K)^{\Delta}]},$$

$$\chi_{2k_F,\mathrm{el}}(T/T_K,x)=\frac{3\pi^3x}{2k\,\sinh(2\pi x)}\,\frac{1}{\{F^{(-1)}[(E/T_K)^{\Delta}]+1\}^2}. \qquad (4.12)$$

It is interesting to note that Eq. (4.11) has a factorized form, where the T dependence is once again that of the spin

$$\chi^{(1)}_{2k_F}(\lambda_T,x)=\frac{2\pi^2T\chi_{\mathrm{tt}}(T)}{F^{(-1)}[(E/T_K)^{\Delta}]}. \qquad (4.13)$$

Consider now Eq. (4.10) in various limits. Obviously, $\lambda_E=\lambda_r$ for $r\ll\xi_T$, and $\lambda_E=\lambda_T$ for $r\gg\xi_T$. At high temperatures $\xi_T\ll\xi_K$, and the crossover at $r\sim\xi_K$ does not happen—just like we have seen in the single-channel case. For $r\gg\xi_T$ the correlation functions decay exponentially, just as we have seen in the single-channel case. The most interesting is the low-temperature limit $T\ll T_K$, $r\ll\xi_T$. In this limit we find

$$\chi_{2k_F,\mathrm{imp}}\left(\frac{T}{T_K},\frac{r}{\xi_T}\right)=\frac{\pi\xi_T}{4r}\,\frac{(T/T_K)^{2\Delta}}{F^{(-1)}[(\xi_K/[4\pi r])^{\Delta}]}\,\frac{1}{1-k\lambda_0/2},$$

$$\chi_{2k_F,\mathrm{el}}\left(\frac{T}{T_K},\frac{r}{\xi_T}\right)=\frac{3\pi^2}{4k}\,\frac{1}{\{F^{(-1)}[(\xi_K/[4\pi r])^{\Delta}]+1\}^2}+(k\lambda_0/2)\chi_{2k_F,\mathrm{imp}}\left(\frac{T}{T_K},\frac{r}{\xi_T}\right). \qquad (4.14)$$

The scaling function for the electron piece in the limit $\lambda_0\to 0$ appears in the subleading order in $1/k$. For nonzero bare coupling there is also a piece in the leading order, which is proportional to the impurity scaling function in Eq. (4.14) and the anomalous factor.

As in the single-channel case, the weak-coupling behavior is *not* recovered inside the screening cloud. Outside the screening cloud, for $T\ll T_K$ and $\xi_K\ll r\ll\xi_T$, the local spin susceptibility takes the form

$$\chi_{2k_F,\mathrm{tot}}\left(\frac{T}{T_K},\frac{rT}{v_F}\right)=\frac{\pi v_F}{4rT}\left(\frac{4\pi rT^2}{\xi_KT_K^2}\right)^{\Delta}+\frac{3\pi^2}{4k}. \qquad (4.15)$$

The T divergence is not removed at low temperatures, and Eq. (4.15) does not have the Fermi-liquid form, as one could expect for a non-Fermi-liquid low-temperature fixed point. The distance-dependent Knight shift for overscreened Kondo fixed point can also be understood at $r>\xi_K$ using the generalization of the Nozières' Fermi-liquid approach developed by Affleck and Ludwig.[17]

The spin susceptibility is obtained as the leading term at the low-temperature fixed point plus corrections in the leading irrelevant operator. For the overscreened Kondo fixed point the leading irrelevant operator contribution corresponds to the second term in Eq. (4.15). It is surprising that the dominant divergent term [the first term in Eq. (4.15)] in the limit $T\to 0$, $r\ll\xi_T$, or in the limit $k\to\infty$ appears in the first order in the leading irrelevant coupling. An interested reader can find the details on this technical point in Appendix C.

B. Integrated susceptibilities

As we have seen for the single-channel case, the static spin susceptibility is mostly given by the impurity-impurity correlation function, $\chi_{\mathrm{ii}}(T)$, while other pieces contribute only a small fraction which is proportional to the bare coupling constant, or bare coupling constant squared. It is easy to see that this is also the case for the multichannel model. Indeed, according to Gan,[4]

$$\chi_{\rm ii}(T)=\frac{1}{4T}\left(1-k\lambda_0^2\ln\frac{D}{T}\right),$$
$$\chi_{\rm ie}(T)=-\frac{\lambda_0 k}{8T}\left(1-k\lambda_0^2\ln\frac{D}{T}\right), \tag{4.16}$$
$$\chi_{\rm ee}(T)=\frac{\lambda_0^2k^2}{16T}\left(1-k\lambda_0^2\ln\frac{D}{T}\right).$$

Thus, from the scaling equations Eq. (2.21) we obtain

$$\chi_{\rm ii}=\frac{\chi_{\rm tt}}{(1-k\lambda_0/2)^2}, \quad \chi_{\rm it}=\frac{\chi_{\rm tt}}{1-k\lambda_0/2}, \tag{4.17}$$

where the scaling function for the total spin susceptibility is given by

$$\chi_{\rm tt}=\frac{1}{4T}\left(1-\frac{k\lambda_T}{2}\right)^2. \tag{4.18}$$

Using Eq. (4.6), we can rewrite Eq. (4.18) in the form

$$\chi_{\rm tt}(T)=\frac{1}{4T}\left(\frac{F^{(-1)}[(T/T_K)^\Delta]}{F^{(-1)}[(T/T_K)^\Delta]+1}\right)^2. \tag{4.19}$$

The electron-impurity and impurity-impurity correlators contain smallness associated with the bare coupling:

$$\chi_{\rm ie}\simeq-\frac{k\lambda_0}{2}\chi_{\rm tt}(T), \quad \chi_{\rm ee}\simeq\frac{k^2\lambda_0^2}{4}\chi_{\rm tt}(T). \tag{4.20}$$

Thus, the spin susceptibility is given mostly by the impurity-impurity spin correlator, and for a system with impurity g factor $g\neq 2$ there are corrections to the bulk susceptibility proportional to the bare coupling. For $T<T_K$ the scaling function for the total spin susceptibility takes the form:

$$\chi_{\rm tt}\simeq\frac{1}{4T}\left(\frac{T}{T_K}\right)^{2\Delta}. \tag{4.21}$$

As in Sec. III B, this fact is easily understood in the bosonic language using canonical transformation. The bosonized Kondo Hamiltonian for the k-channel model has the form

$$H=\int_{-\infty}^{\infty}dr\left[(\partial_r\varphi_L(r))^2-h_e\frac{\sqrt{k}}{\sqrt{2\pi}}\partial_r\varphi_L(r)\right]$$
$$+H_0^{\rm para}+\sqrt{2\pi k}\lambda_0S^z\partial_r\varphi_L(0)+{\rm const}\cdot\lambda_0$$
$$\times(S^+e^{i\sqrt{8\pi/k}\varphi_L(0)}O^{\rm para}+{\rm H.c.})-h_iS^z. \tag{4.22}$$

Here φ is the canonically normalized total spin boson, i.e., the sum of the spin bosons for each channel divided by $\sqrt{k}$. The additional, independent degrees of freedom which couple to the impurity correspond to the $SU(2)_k$ Wess-Zumino-Witten model with one free boson factored out. This is the Z_k parafermion model.[27] For the $k=2$ case it corresponds to an extra Ising degree of freedom, or equivalently a Majorana fermion. These extra degrees of freedom play no role in the canonical transformation.

Changing the bosonic field $\varphi_L(r)=\tilde{\varphi}_L(r)+\sqrt{k}h_er/\sqrt{8\pi}$, the Hamiltonian takes the form

$$H=\int_{-\infty}^{\infty}dr(\partial_r\tilde{\varphi}_L(r))^2+H_0^{\rm para}-\frac{k}{8\pi}h_e^2(2L)$$
$$+\sqrt{2\pi k}\lambda_0S^z\partial_r\tilde{\varphi}_L(0)$$
$$+{\rm const}\cdot\lambda_0(S^+e^{i\sqrt{8\pi/k}\tilde{\varphi}_L(0)}O^{\rm para}+{\rm H.c.})-\left(h_ik\lambda_0\frac{h_e}{2}\right)S^z. \tag{4.23}$$

Thus, for the free energy, we have

$$F(h_i,h_e)=-\frac{Lk}{4\pi v_F}h_e^2+F\left(0,h_i-\frac{h_ek\lambda_0}{2}\right). \tag{4.24}$$

For the susceptibilities we then obtain

$$\chi_{\rm ie}=-\frac{k\lambda_0}{2}\chi_{\rm ii}, \quad \chi_{\rm ee}=\left(\frac{k\lambda_0}{2}\right)^2\chi_{\rm ii}, \tag{4.25}$$

with $\chi_0=(k/2\pi v_F)L$, the Pauli term. This agrees with the large-k results.

Let us now return to the issue of screening. The electron-total piece of the spin susceptibility, $\chi_{\rm et}=\chi_{\rm ee}+\chi_{\rm ei}$, is given by the integral of the local spin susceptibility $\chi(r,T)$. As in the single-channel case, since $\chi(r,T)$ only has the oscillating piece, this integral is determined by the short-distance contribution, $r\sim1/k_F$. The form of $\chi(r,T)$ at $r\sim 1/k_F$ is cutoff dependent. However this dependence disappears in the integral, which describes conduction-electron spin polarization. In case of a 3D Fermi gas the cutoff procedure is well defined. The fact that the net conduction-electron spin polarization due to impurity comes mainly from $r\sim1/k_F$ is indeed justified to the orders we worked in perturbation theory. From Eq. (4.10), with $\lambda_E\simeq\lambda_r\simeq\lambda_0\lesssim1/k$ we can write for the local spin susceptibility:

$$\chi(\mathbf{r},T)=\frac{k\lambda_0\chi_{\rm tt}(T)}{1-k\lambda_0/2}\left(\frac{\cos2k_Fr}{8\pi r^3}-\frac{\sin2k_Fr}{16\pi k_Fr^4}\right). \tag{4.26}$$

We have checked this conjecture to the leading order in $1/k$. Integration of this expression over r gives the correct result for $\chi_{\rm et}$,

$$\chi_{\rm et}(T)=\frac{-k\lambda_0/2}{1-k\lambda_0/2}\chi_{\rm tt}(T). \tag{4.27}$$

Obviously, the major contribution to the integral

$$\int_0^\infty d^3\mathbf{r}\chi(\mathbf{r},T)\propto\int_0^\infty dr\frac{d}{dr}\left(\frac{\sin2k_Fr}{2k_Fr}\right) \tag{4.28}$$

comes from $r\sim1/k_F$.

C. Equal-time correlation function

As we have seen above, the zero-frequency spin correlator vanishes as $T^{2\Delta}$ when $T\to0$. It also obeys a zero-sum rule. As in the single-channel case, the equal-time spin correlator $K(r,T)=\langle S_{\rm el}^z(r,0)S_{\rm imp}^z(0)\rangle$ has a nonvanishing sum rule, since $\langle S_{\rm el}^zS_{\rm imp}^z\rangle=-1/4$. The uniform part of the equal-time spin correlator is nonzero.

Consider the equal-time correlators $K_{\rm un}(r,T)$ and $K_{2k_F}(r,T)$ using the $1/k$ expansion. $K(r,T)$ satisfy scaling

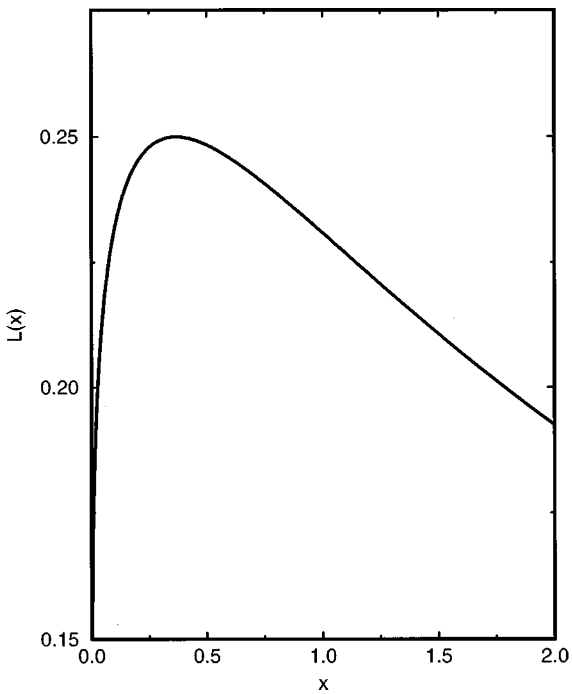

FIG. 6. Scaling function $L(x)$.

equations Eq. (2.21). As in the single-channel case, for $r>\xi_T$ the spin correlators decay exponentially. The behavior is most interesting for $r\ll\xi_T$, where our expressions are considerably simplified. Expressing our results in terms of effective coupling at scale r,λ_r, we get

$$K_{\rm un}(\lambda_r,r/\xi_T)=-\frac{1}{1-k\lambda_0/2}\frac{\pi v_F k\lambda_r^2}{2r}(1-k\lambda_r/2)^2, \tag{4.29}$$

$$K_{2k_F}(\lambda_r,r/\xi_T)=\frac{1}{1-k\lambda_0/2}\frac{\pi v_F}{8r}k\lambda_r(1-k\lambda_r/2).$$

We can rewrite these expressions using Eq. (4.6) in terms of $T/T_K,r/\xi_T$ variables. Suppressing the anomalous factor $1/(1-k\lambda_0/2)$, we obtain

$$K_{\rm un}\left(\frac{T}{T_K},\frac{r}{\xi_T}\right)=-\frac{2\pi T\xi_T}{kr}L^2[(4\pi r/\xi_K)^\Delta],$$

$$K_{2k_F}\left(\frac{T}{T_K},\frac{r}{\xi_T}\right)=\frac{\pi T\xi_T}{4r}L[(4\pi r/\xi_K)^\Delta], \tag{4.30}$$

where $L(x)$ is the function defined by

$$L(x)\equiv\frac{F^{(-1)}(1/x)}{[F^{(-1)}(1/x)+1]^2}. \tag{4.31}$$

A plot of this function is shown in Fig. 6. As we have discussed in Sec. III, the integral of $K(r,T)$ should not vanish. It is given by Eq. (3.35), as in the single-channel case. The integral over long distances $r\sim\xi_K$ can be calculated explicitly from Eq. (4.29) by changing variable $r\to\lambda_r$. Using Eq. (2.18),

$$\int_0^\infty\frac{drK_{\rm un}(\lambda_r,r)}{2\pi v_F}=\int_{\lambda_0}^{\lambda^*}\frac{k\lambda_r-2}{2-k\lambda_0}\frac{d\lambda_r}{2\lambda^*}=\frac{k\lambda_0-2}{8}. \tag{4.32}$$

Thus, in this case the screening length $\sim\xi_K$ is explicitly present. The dependence on the bare coupling constant λ_0 is surprising, since it should not be there according to the sum rule Eq. (3.35). The missing part of the sum rule comes from the short distances. To provide the most transparent demonstration of this, we write the second equation in Eq. (4.29) for a 3D Fermi gas, so that $\cos(2k_Fr)$ is replaced by $\cos(2k_Fr)-[\sin(2k_Fr)/2k_Fr]$, as in Eq. (4.26). The short-distance integral, which is analogous to Eq. (4.28), gives precisely the compensating term $-k\lambda_0/8$ needed for the sum rule Eq. (3.35) to be obeyed.

The low-temperature decay of the equal-time correlator at $r\gg\xi_K$ in the overscreened multichannel Kondo model can be obtained using conformal field theory approach (see Appendix C). Indeed, at the low-temperature fixed point we have[17,18]

$$S_{\rm imp}\to{\rm const}\;\boldsymbol{\phi}(0,0)T_K^{-\Delta}, \tag{4.33}$$

where $\boldsymbol{\phi}$ is the $s=1$ primary of dimension $\Delta=2/(2+k)$, const is a nonuniversal constant. We then obtain for K_{2k_F} from conformal invariance:

$$K_{2k_F}(r)\propto\left\langle\frac{(\boldsymbol{\phi}(0,0)\cdot\boldsymbol{\sigma})}{2T_K^\Delta}\psi_L^\dagger(0,r)\psi_L(0,-r)\right\rangle\propto\frac{1}{T_K^\Delta r^{1+\Delta}}, \tag{4.34}$$

in agreement with the large-k result Eq. (4.30). The same leading-order calculation gives zero for $K_{\rm un}$, since $\langle\boldsymbol{\phi}(0,0)\cdot\mathbf{J}(0,r)\rangle=0$. The first-order term in the leading irrelevant operator gives

$$K_{\rm un}(r)\propto\frac{1}{T_K^{2\Delta}r^{1+2\Delta}}, \tag{4.35}$$

which also agrees with Eq. (4.30). It is interesting to note that, unlike the single-channel Kondo model, the long-distance decay of the uniform and $2k_F$ correlators is different.

V. CONCLUSION

Although the techniques employed in this paper—renormalization-group improved perturbation theory and the large-k limit—are of limited validity, they have led to one exact result (all orders in perturbation theory) and suggested a certain conjecture which, if true, leads to a rather complete picture of the Kondo screening cloud. We first summarize the exact result and the conjecture, pointing out a consistency check between them and then state the resulting conclusions.

(i) The uniform part of the r-dependent susceptibility, vanishes to all orders in perturbation theory. On the other hand, the equal-time correlation function has a nonzero uniform part, varying on the scale ξ_K at $T=0$.

(ii) The $2k_F$ part of the r-dependent susceptibility has a factorized form at $r\ll v_F/T,\xi k$:

$$\chi_{2k_F}(r,T) \to \frac{k\pi v_F}{2r}\lambda_r\chi_{\rm tt}(T), \tag{5.1}$$

where $\chi_{\rm tt}(T)$ is the total susceptibility (less the free-electron Pauli part) and where λ_r is the effective coupling at scale r. This was verified to third order in perturbation theory and in the large-k limit [including the $O(1/k)$ correction].

There is an important consistency check relating this result and conjecture and the result $\chi_{\rm ie} = -(\lambda_0/2)\chi_{\rm ii}$, following from the formula:

$$\chi_{\rm et} = \int d^3r\chi(r). \tag{5.2}$$

Since $\chi(r)$ is an RG invariant, it has no explicit dependence on the bare coupling. If the uniform part had been nonzero, its integral would have given a contribution to $\chi_{\rm et}$ which would be unsuppressed by any powers of the bare coupling. The integral involving $\chi_{2k_F}(r)$ gives 0 for $r \gg 1/k_F$ due to the $\cos(2k_Fr)$ factor and hence is determined by the value of χ_{2k_F} at short distances of $O(1/k_F)$. In this limit $\chi_{2k_F}(r) \propto \lambda_r \approx \lambda_0$ and integrating Eq. (5.1) gives $\chi_{\rm et} \simeq -(\lambda_0 k/2)\chi_{\rm tt}$.

Strictly speaking this consistency check requires yet another conjecture:

$$\chi(r) \approx \frac{\chi_{2k_F}}{4\pi^2 r^2 v_F}\left[\cos(2k_Fr) - \frac{\sin 2k_Fr}{2k_Fr}\right]$$
$$\approx \frac{k\lambda_r\chi_{\rm tt}(T)}{8\pi r^2}\left[\cos(2k_Fr) - \frac{\sin 2k_Fr}{2k_Fr}\right], \tag{5.3}$$

for $r \ll \xi_K,\ v_F/T$. This last conjecture involves corrections of $O(1/k_F O(1/k_F r)$ which we have not calculated systematically and go beyond the scope of the one-dimensional model. We did check the result in lowest order in $1/k$.

Despite the limitations of our calculational approach, we are thus led to a fairly complete understanding of the Kondo screening cloud. The heuristic picture of Nozières and others of the Kondo ground state is seen to be correct. The impurity essentially forms a singlet with an electron which is in a wave function spread out over a distance of $O(\xi_K)$. This is seen from our calculation of the $T=0$ equal-time correlation function which varies over the scale ξ_K.

On the other hand, the behavior of static susceptibilities is considerably more subtle. A naive picture that an infinitesimal magnetic field fully polarizes the impurity but induces a compensating polarization of the electrons is certainly wrong. Rather the impurity polarization is proportional to the weak magnetic field and the *integrated* polarization of the electrons (with the free-electron value subtracted) is much smaller (proportional to λ_0). The finiteness of the $T=0$ impurity susceptibility results from its tendency to form a singlet with the electrons.

If we now examine the r dependence of the electron polarization, we find that it is small at short distances $[O(\lambda_0)]$. However, it exhibits a universal oscillating form at long distances which is not suppressed by any powers of λ_0 but only by a dimensional factors of $1/r^2$. The fact that it is purely oscillating ensures that the contribution to the integrated polarization is negligible. The envelope of this oscillating susceptibility, consists of the dimensional factor of $1/r^2$ times an interesting and universal scaling function of r/ξ_K and T/T_K. This scaling function factorizes into $\chi_{\rm tt}(T)f(r/\xi_K)$ for $v_F/T \gg r$.

Our work leaves various open questions for further study. It seems plausible that our conjecture could be proven to all orders in perturbation theory, thus putting this work on a more solid foundation. There are three interesting universal scaling functions which we have introduced, one for the $2k_F$ susceptibility and two for the uniform and $2k_F$ equal-time correlation functions. A general calculation of these functions could perhaps be accomplished by quantum Monte Carlo or exact integrability methods. Results on the $T=0$ limit of the susceptibility scaling function were given in Ref. 5. An obvious generalization of our calculations is to general frequency-dependent Green's functions.

Most importantly, experimental results on the Kondo screening cloud are very limited. The NMR experiments of Boyce and Slichter only probe extremely short distances, $r \approx 1/k_F$. Our work shows that these results are entirely consistent with a large screening cloud. However, these experiments do not directly probe the scale ξ_K. NMR is probably not a feasible technique for doing this since it is difficult to study distances of more than a few lattice constants. One possibility might be neutron scattering, which could, in principle, measure $\chi(q,\omega)$ for $q \approx 2k_F$. An alternative is to study small samples with dimensions of $O(\xi_K)$.[28]

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APPENDIX A: PERTURBATIVE RESULTS

The diagram technique for interactions involving spin operators is complicated due to their nontrivial commutation relations. It is possible to express these operators in terms of pseudofermion operators:[22,29]

$$\mathbf{S}_{\rm imp} = \frac{1}{2}\sum_{\alpha,\beta=1,2} f^{\dagger\alpha}\boldsymbol{\sigma}_\alpha^\beta f_\beta. \tag{A1}$$

The problem in using the fermion substitution Eq. (A1) is that the $\boldsymbol{\sigma}$ matrices have dimensionality 2, while the fermion space is four dimensional. Thus, only the states with

$$N = \sum_\alpha f^{\dagger\alpha} f_\alpha = 1 \tag{A2}$$

are physical. This constraint is imposed by choosing appropriate chemical potential.[22] For example, Popov's technique[29] adds an imaginary chemical potential, $i\pi T/2$, to the pseudofermions. Then the contribution of the nonphysical states to the partition function is zero. The diagram technique then becomes the standard fermion technique with the one-dimensional conduction electron (left-movers) propagator $(i\omega_n+v_F k)^{-1}$, the pseudofermion propagator $(i\omega_n-i[\pi T/2])^{-1}$, and the interaction Hamiltonian

$$H_{\rm int}=v_F\lambda_0\frac{\boldsymbol{\sigma}_\alpha^\beta\boldsymbol{\sigma}_\gamma^\delta}{4}\psi_L^{\dagger\alpha}(0)f^{\dagger\gamma}f_\delta\psi_{L\beta}(0). \qquad (A3)$$

For our purpose of computation of spatial correlators it is convenient to work in the coordinate r,τ space, where the propagator for the left movers takes the form

$$G_0(z)=\frac{\pi T}{\sin[\pi T z]},\quad z=v_F\tau+ix. \qquad (A4)$$

For the lowest-order diagrams it may be more convenient to calculate time-ordered impurity-spin averages directly. Such spin operator Green's-function approach was applied successfully, for example, in case of long-range Heisenberg ferromagnets.[30] Consider

$$\langle S^i S^j\cdots S^k\rangle, \qquad (A5)$$

where $i,j,\ldots,k=\{z,+,-\}$. Obviously, this average is zero when the total number of S^+ operators is not equal to the total number of S^-. Consider first averages containing only S^z operators. For odd number of spin operators it vanishes. In our simple $S=1/2$ case $\mathrm{Tr}([S^z]^{2n})=1/4^n$. One can use spin commutation relations and the relations $S^+S^-=(1/2)+S^z$, $S^-S^+=(1/2)-S^z$ to calculate the average Eq. (A5).

All diagrams for the spin susceptibility $\chi(r,T)$ up to third order are shown in Fig. 7. Graphs (a)–(d) represent the electron-impurity part, while graphs (e)–(i) the electron-electron part. We only show the electron Green functions on these diagrams. The dashed line represents the boundary. For the electron-impurity spin-correlation function the external electron-spin operator $S_{\rm el}(r)$ takes the propogator away from the boundary. In the case of the electron-electron part of the Knight shift there are two such operators. We have to integrate over the position of one of these operators.

Straightforward calculations lead to the final results stated in Eqs. (3.1), (3.17) of Sec. III A. To calculate the equal-time correlator $\langle S_{\rm el}(r,0)S_{\rm imp}(0)\rangle$, we need to evaluate the graphs (b)–(d) of Fig. 7 once again. The first graph (a) is frequency independent, i.e., it is the same as for the electron-impurity part of the local spin susceptibility. Both uniform and $2k_F$ parts are now nonzero. The result of this calculation is given by Eq. (3.36) of Sec. III C.

For the discussion of static susceptibilities in Sec. III B we need to calculate the impurity-impurity part, in addition to space integrals of $\chi_{\rm ie}(r,T)$ and $\chi_{\rm ee}(r,T)$. The second- and third-order graphs for $\chi_{\rm ii}(T)$ are shown in Fig. 8. The leading order is, of course, $1/4T$. We find that

$$4T\chi_{\rm ii}=1-\lambda_0^2\left(\ln\frac{D}{T}+A_1\right)-\lambda_0^3\left(\ln^2\frac{D}{T}+A_2\ln\frac{D}{T}+\text{const}\right),$$

$$4T\chi_{\rm ie}=-\frac{\lambda_0}{2}+B_1\lambda_0^2+\frac{\lambda_0^3}{2}\ln\frac{D}{T}+\text{const}\lambda_0^3, \qquad (A6)$$

$$4T\chi_{\rm ee}=\frac{\lambda_0^2}{4}+\text{const}\lambda_0^3.$$

In general, the constants A_1, A_2, and B_1 in Eq. (A6) depend on the cutoff procedure. However, these three constants are connected, $A_2+4B_1-2A_1=0$, as follows directly from the results of Wilson[23] on the scaling properties of the total spin susceptibility. Using this connection and Eq. (3.25), the fact that all three scaling functions for the spin susceptibility are equal up to the terms $\sim\lambda_T^2$ is easily demonstrated.

Consider now the multichannel case. As we have mentioned in the text, the graph selection in this case is different, since each vertex λ is $\sim 1/k$. To the order $1/k$ we need to calculate all the graphs in Fig. 7, except (c) and (i), which are of the order $1/k^2$. In addition, we need to calculate the fourth-order graph shown in Fig. 9. The result of this calculation is given by Eq. (4.10) in the text.

Calculations of the equal-time correlator are somewhat more involved. While $K_{2k_F}(r,T)$ in Eq. (4.29) is also nonzero up to this order, $K_{\rm un}(r,T)$ vanishes. We need to go to the next order in $1/k$ to find the answer. For the terms of the order $1/k^2$, we need to calculate graph (c) in Fig. 7, and additional fourth- and fifth-order graphs shown in Fig. 10.

The bulk susceptibility results are found again by calculating $\chi_{\rm ii}(T)$ and r integrating the Knight shift. In the leading order we only need to consider second-order graph in Fig. 8.

APPENDIX B: PROOF THAT THE UNIFORM PART OF THE LOCAL SUSCEPTIBILITY VANISHES

As clarified in the text, the local spin susceptibility can be written as a sum of impurity and electron parts [see Eq. (2.10)]. We will consider these two parts separately for the purpose of this proof.

Consider first the impurity part, $\chi_{\rm un,imp}(r)$. Using Eq. (2.15), one can write

$$\chi_{\rm un,imp}(r,T)=v_F\left\langle T\left[\int_0^\beta d\tau\psi_L^\dagger(r,\tau)\frac{\sigma^z}{2}\psi_L(r,\tau)S_{\rm imp}^z(0)\times\exp\left\{-\int_0^\beta d\tau' H_{\rm int}(\tau')\right\}\right]\right\rangle+(r\leftrightarrow -r), \qquad (B1)$$

where $H_{\rm int}$ is given by Eq. (2.9). The fact that this contribution vanishes is very easily seen when we perform the τ integration. Indeed, in every order in perturbation theory $\chi_{\rm un,imp}(r,T)$ can be written as

$$\chi_{\rm un,imp}(r,T)=\int_0^\beta\int_0^\beta d\tau_1 d\tau_2 I(\tau_1,\tau_2,r)\times F(\tau_1,\tau_2), \qquad (B2)$$

where

$$I(\tau_1,\tau_2,r)=\int_0^\beta d\tau G(r,\tau-\tau_1)G(-r,\tau_2-\tau), \qquad (B3)$$

or, equivalently,

$$I=\int_0^\beta\frac{d\tau(\pi T)^2}{\sin[\pi T(v_F\tau-v_F\tau_1+ir)]\sin[\pi T(v_F\tau_2-v_F\tau-ir)]}. \qquad (B4)$$

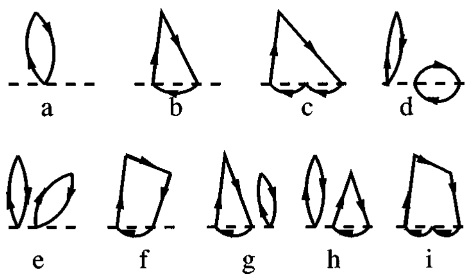

FIG. 7. Perturbative diagrams for $\chi(r,T)$ up to third order.

After the change of integration variable, $\tau \rightarrow \exp(i2\pi T v_F \tau)$, one encounters contour integration with two poles on one side (see Fig. 11), and $I=0$.

Consider now the electron part, $\chi_{\mathrm{un,el}}(r)$. Here the cancellation of $\chi_{\mathrm{un,el}}(r)$ is less trivial since there are other graphs in addition to those with the integration Eq. (B4) (see Fig. 12):

$$G(r',-\tau_1)G(r-r',\tau)G(-r,\tau_2-\tau)\,\eta(\tau_2-\tau_1),$$
$$G(r,\tau-\tau_1)G(r'-r,-\tau)G(-r',\tau_2)\,\eta(\tau_2-\tau_1), \tag{B5}$$

where $\eta(\tau_2-\tau_1)$ is determined by the full perturbative series. We now introduce the complex notation, $z \equiv \pi T(v_F\tau + ir)$, and remember that $G(z)=\pi T/\mathrm{sin} z$. Then the sum of the graphs in Fig. 12 gives

$$\frac{\eta(z_2-z_1)}{\sin(z-z')}\left[\frac{1}{\sin(z_2-z)\sin(z'-z_1)} - \frac{1}{\sin(z_2-z')\sin(z-z_1)}\right] = \frac{\eta(z_2-z_1)\sin(z_1-z_2)}{\sin(z-z_1)\sin(z_2-z)\sin(z'-z_1)\sin(z_2-z')}, \tag{B6}$$

which is graphically presented in Fig. 12. Integration over τ Eq. (B4) yields zero in this case as well. Generalization of this proof to the multiple number of channels is quite trivial. Indeed, the graphs that cancel have the same channel-dependent factor. As we have seen above, the crucial step of the proof is that the integral Eq. (B4) is zero. Thus, the $q=0$ part of the correlator is absent only for zero-frequency spin-spin correlators, not for equal-time correlators.

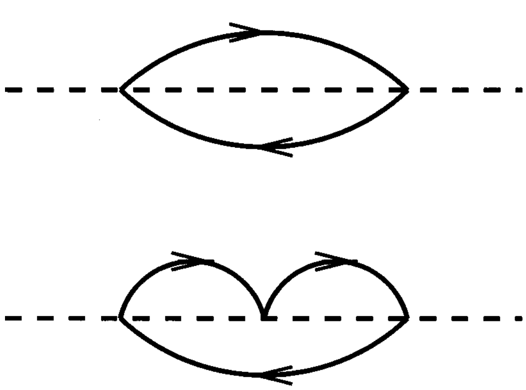

FIG. 8. Second- and third-order graphs for the impurity-impurity part of the spin susceptibility, χ_{ii}.

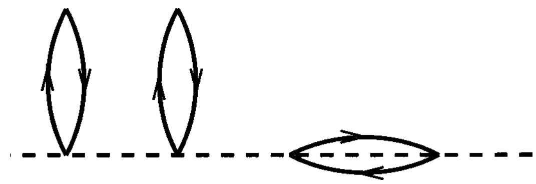

FIG. 9. Fourth-order graph of the order $1/k$.

Note that the absence of the uniform part in the distance-dependent Knight shift becomes trivial in the bosonic language (see Sec. III B). Indeed, since

$$S^z_{\mathrm{el}}(v_F\tau+ir)=\frac{1}{\sqrt{2\pi}}\partial_r\varphi_L(v_F\tau+ir), \tag{B7}$$

we find

$$\chi_{\mathrm{un}}(r) \propto \int_0^\beta d\tau \left\langle \frac{1}{\sqrt{2\pi}}\partial_r\tilde{\varphi}_L(v_F\tau+ir)S^z\right\rangle = -\frac{i}{\sqrt{2\pi}}\left(\langle\tilde{\varphi}_L(ir+v_F\beta)S^z\rangle - \langle\tilde{\varphi}_L(ir)S^z\rangle\right)=0, \tag{B8}$$

because $\tilde{\varphi}_L(z)$ is periodic in the imaginary time variable. Note that we do not need to worry about a potential short-distance singularity because the total spin has been replaced by the impurity spin in the expression for χ_{un} using the above argument. A similar argument for $\chi_{\mathrm{un}}=0$ was given in Ref. 13.

APPENDIX C: LOW-TEMPERATURE LONG DISTANCE LOCAL SUSCEPTIBILITY IN THE MULTICHANNEL KONDO MODEL

$\chi_{2k_F}(r,T)$ is determined by the infrared stable fixed point for $r \gg \xi_K$, $T \ll T_K$ and any value of the ratio rT/v_F. For $k>1$ (and $S_{\mathrm{imp}}=1/2$) this fixed point is of the non-Fermi-liquid type. The low-temperature non-Fermi-liquid multichannel Kondo fixed point was analyzed by Ludwig and Affleck[17–19] using conformal field theory. We refer the reader to these works and a recent review[31] for details. In the

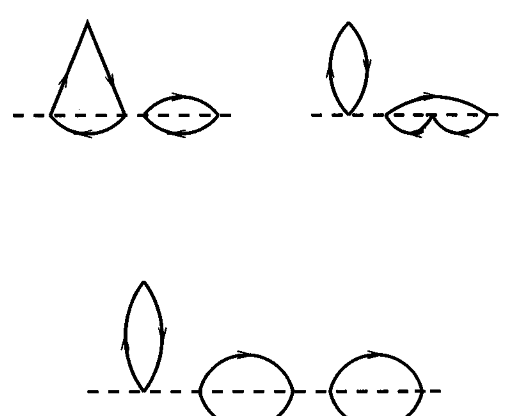

FIG. 10. Fourth- and fifth-order graphs for $\chi(r,T)$ that contribute to the order $1/k^2$ in the $1/k$ expansion.

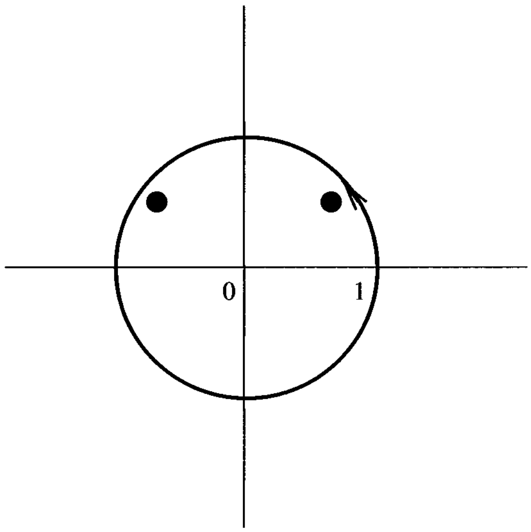

FIG. 11. Contour of integration for I.

bosonized form spin, charge, and flavor sectors of the free fermion Hamiltonian are separate. Only the spin sector is interesting in the Kondo problem, since the impurity spin couples to the spin current. The effect of the strong-coupling fixed point[17] is such that the low-temperature Hamiltonian density is written in terms of new spin currents,

$$H_s=\frac{1}{2\pi(k+2)}\mathbf{J}^2(x), \tag{C1}$$

where

$$\mathbf{J}(x)=\sum_j \psi_{Lj}^\dagger(x)\frac{\boldsymbol{\sigma}}{2}\psi_{Lj}(x)+2\pi\mathbf{S}\delta(x). \tag{C2}$$

The Fourier modes of the spin currents for a system with Hamiltonian density Eq. (C1) defined on a large circle of circumference $2l$,

$$\mathbf{J}_n=\frac{1}{2\pi}\int_{-l}^{l}dx e^{in\pi x/l}\mathbf{J}(x), \tag{C3}$$

satisfy the usual Kac-Moody (KM) commutation relations,

$$[J_n^a,J_m^b]=i\epsilon^{abc}J_{n+m}^c+\frac{1}{2}kn\delta^{ab}\delta_{n+m,0}. \tag{C4}$$

Here ϵ^{abc} is the antisymmetric tensor and k is the Kac-Moody level. To the leading order, the Knight shift is given by

$$\chi_{2k_F}(r,T)=-\frac{v_F}{2\pi}\int_0^\beta\int_{-\infty}^{+\infty}d\tau dy \times\left\langle \psi_L^\dagger(0,r)\frac{\sigma^z}{2}\psi_L(0,-r)J^z(\tau,y)\right\rangle. \tag{C5}$$

Using operator product expansion (OPE)

$$\frac{\mathbf{J}(\zeta)\boldsymbol{\sigma}}{2}\psi_L(z)=-\frac{3/4}{\zeta-z}\psi_L(z)+\mathrm{Reg}(\zeta-z), \tag{C6}$$

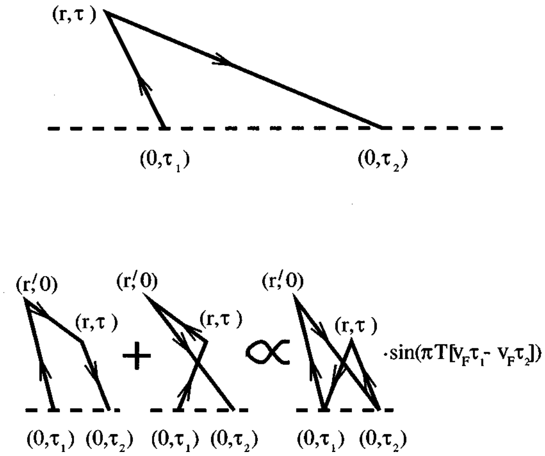

FIG. 12. Cancellation of the uniform part of the local spin susceptibility.

$$\frac{\mathbf{J}(\zeta)\boldsymbol{\sigma}}{2}\psi_L^\dagger(z)=\frac{3/4}{\zeta-z}\psi_L^\dagger(z)+\mathrm{Reg}(\zeta-z), \tag{C7}$$

where $\mathrm{Reg}(\zeta-z)$ denotes a function which is regular at $\zeta\to z$, we rewrite $\chi_{2k_F}(r,T=0)$ as

$$\chi_{2k_F}(r)=-\frac{v_F}{8\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}d\tau dy\left[-\frac{1}{\tau+iy-ir}+\frac{1}{\tau+iy+ir}\right] \times\langle\psi_L(0,-r)\psi_L^\dagger(0,r)\rangle. \tag{C8}$$

The Green's function for two points on the opposite sides of the boundary takes the form

$$\langle\psi_L^\dagger(z_1)\psi_L(\overline{z_2})\rangle=\frac{S_{(1)}}{z_1-\overline{z}_2}, \tag{C9}$$

where

$$S_{(1)}=\frac{\cos[2\pi/(2+k)]}{\cos[\pi/(2+k)]} \tag{C10}$$

is the S-(scattering) matrix, calculated in Ref. 19. This is a universal complex number, which depends on the universality class of the boundary conditions. In the one-channel Kondo effect $S_{(1)}=-1$, corresponding to a $\pi/2$ phase shift. At the overscreened Kondo fixed points $|S_{(1)}|<1$, which means multiparticle scattering. Substracting the free-electron contribution and performing the integrals, we find

$$\chi_{2k_F}(r)=k\frac{1-S_{(1)}}{2}. \tag{C11}$$

In the limit $k\to\infty$ this gives $\chi_{2k_F}(r)\simeq3\pi^2/4k$, in agreement with the large-k result of Sec. IV. For $k=1$ it agrees with the Fermi-liquid result Eq. (3.15). Note that no anomalous power laws occur in the leading order in irrelevant coupling constants. Only the normalization reflects the non-Fermi-liquid behavior. As in the single-channel case, finite-temperature calculations multiply this expression by the factor $2\pi x/\sin(2\pi x)$, where $x=rT/v_F$.

Consider now corrections to this expression. The leading irrelevant operator which appears[17] in the effective Lagrangian at the overscreened Kondo fixed point is $\mathbf{J}_{-1}\cdot\boldsymbol{\phi}$, where

$\boldsymbol{\phi}$ is the $s=1$ SU(2) KM primary field with the dimension $\Delta=2/(2+k)$. The dimension of this singlet operator is $1+\Delta$. We can again write this additional piece as

$$H_{\rm int}\sim\frac{1}{T_K^{\Delta}}(\mathbf{J}_{-1}\cdot\boldsymbol{\phi}(0)). \tag{C12}$$

Thus the correction is given by

$$\delta\chi_{2k_F}(r,T)=-\frac{v_F}{2\pi T_K^{\Delta}}\int_0^{\beta}\int_0^{\beta}\int_{-\infty}^{+\infty}d\tau d\tau_1 dy \times\left\langle \psi_L^{\dagger}(0,r)\frac{\sigma^z}{2}\psi_L(0,-r)J^z(\tau,y)\times[\mathbf{J}_{-1}\cdot\boldsymbol{\phi}(\tau_1,0)]\right\rangle. \tag{C13}$$

To find the most singular part of this expression as $r\to 0$, we use the boundary OPE

$$\psi_L^{\dagger}(0+ir)\frac{\boldsymbol{\sigma}}{2}\psi_L(0-ir)\to\frac{C\boldsymbol{\phi}(0,0)}{r^{1-\Delta}}. \tag{C14}$$

From conformal invariance, this zero-temperature correlator

$$\langle(\boldsymbol{\phi}(0)\cdot\mathbf{J}(z_1))(\mathbf{J}_{-1}\cdot\boldsymbol{\phi}(z_2))\rangle=\frac{C'}{|z_1|^{2\Delta}|z_1-z_2|^2|z_2|^{2\Delta}}. \tag{C15}$$

The finite-temperature correlation function which appears under the integral in Eq. (C13) can be obtained using conformal mapping, a conformal transformation which maps the finite-temperature geometry (half-cylinder) onto the zero-temperature half-plane.

$$z=\tan(\pi T w). \tag{C16}$$

Here $w=\tau+ir$ in the finite-temperature geometry. A Virasoro primary operator $A(z)$ of left-scaling dimension Δ_A transforms as

$$A(w)=\left(\frac{dw}{dz}\right)^{-\Delta_A}A(z), \tag{C17}$$

under conformal transformation. Using $dw(z)/dz=1/\pi T(1+z^2)$, we express the finite-temperature correlators in terms of the zero-temperature ones. The net effect is such that the factors $1/(z_1-z_2)$ for the half-plane get replaced by $\pi T/\sin(\pi T[w_1-w_2])$ on the half-cylinder. Doing the integral in Eq. (C13) and dropping the constants, we obtain

$$\delta\chi_{2k_F}(r,T)\propto\frac{1}{r^{1-\Delta}T^{1-2\Delta}T_K^{\Delta}}, \tag{C18}$$

in agreement with the large-k result of Sec. IV. This term is subdominant, for $r\ll v_F/T$, compared to the leading term in Eq. (C11). On the other hand, it becomes larger than the "leading" term if we take $T\to 0$ with $r\gg\xi_K$ held fixed. Anomalous powers appear from irrelevant operator corrections.

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