Hidden non-Abelian gauge symmetries in doped planar antiferromagnets

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We investigate the possibility of hidden non-Abelian local phase symmetries in large-U doped planar Hubbard antiferromagnets, believed to simulate the physics of two-dimensional (magnetic) superconductors. We present a spin-charge separation ansatz, appropriate to incorporate holon spin flip, which allows for such a hidden local gauge symmetry to emerge in the effective action. The group is of the form $SU(2) \otimes U_S(1) \otimes U_{em}(1)$, where $SU(2)$ is a local non-Abelian group associated with the spin degrees of freedom, $U_{em}(1)$ is that of ordinary electromagnetism, associated with the electric charge of the holes, and $U_S(1)$ is a "statistical" Abelian gauge group pertaining to the fractional statistics of holes on the spatial plane. In certain regime of the parameters of the model, namely, strong $U_S(1)$ and weak $SU(2)$, there is the possibility of *dynamical formation* of a holon condensate. This leads to a *dynamical* breaking of $SU(2) \to U(1)$. The resulting Abelian effective theory is closely related to an earlier model proposed as the continuum limit of *large-spin* planar doped antiferromagnets, which lead to an unconventional scenario for two-dimensional parity-invariant superconductivity. [S0163-1829(98)00105-2]

I. INTRODUCTION

The discovery of the quasiplanar high-T_c cuprates[1] prompted considerable theoretical interest in two-dimensional superconductivity of magnetic origin. The strong suppression of the isotope effect was one of the main reasons for looking for alternatives to phonon mechanisms. The main feature of the magnetic superconductivity was believed to be the fractional statistics of the excitations on the planar geometry of the materials. In two spatial dimensions, particles are no longer limited to Bose and Fermi statistics but can acquire an arbitrary interchange phase; such particles with fractional statistics are known as anyons. Laughlin[2] suggested that a gas of anyons may exhibit superconductivity at low temperature. This idea was supported by the results of calculations in the random-phase approximation,[3] which demonstrated that a perfect gas of charged anyons with certain values of the statistics parameter is indeed a superconductor at zero temperature. (For restrictions on the validity of the random-phase approximation in the context of effective field theories of parity-violating anyonic superconductivity see Ref. 4.) This "anyonic superconductivity" is an entirely different phenomenon that has no analog in three-dimensional systems. Motivated by the role of anyonic quasi-particles in the fractional quantum Hall effect, Laughlin went on to suggest that the charge carriers in the copper oxide planes of materials such as La_2CuO_4 and YBa_2CuO_6 might also have fractional statistics and that superconductivity in these materials may be well described by the anyonic model.

From the experimental point of view, however, there seems to be a serious drawback with the anyonic model as a candidate theory of high-T_c superconductivity. A field-theoretic realization of anyonic matter consists of fermions interacting with an Abelian "statistical" gauge field whose dynamics is governed by a Chern-Simons term. As discussed in Ref. 5, this term leads to observable *parity violation* in an anyonic superconductor for which there is, as yet, no conclusive experimental evidence. In Ref. 6 a proposal was made for a simple gauge-theory model that exhibits two-dimensional superconductivity without parity violation. In its most general form, the model consists of two species of massive fermions coupled with opposite signs to an Abelian gauge field representing effective spin interactions among the holon excitations. The two species have equal and opposite masses and hence parity is conserved overall. This theory may be shown[6] to arise as an approximate long-wavelength limit of an idealized model of the dynamics of the charge carriers in doped t-j or Hubbard models. Similar models, but only at a continuum theory level with no attempt to discuss the connection with semimicroscopic condensed-matter systems, have been proposed simultaneously in Refs. 7 and 8.

The treatment in Ref. 6 employed large-spin approximations for the antiferromagnetic model. This resulted in a strong suppression of intrasublattice hopping,[9] which lead to two species of hole excitations for the bipartite lattices used.[9,6] One eventually would like to argue that the same *qualitative* features occur for the realistic value of the spin, $\frac{1}{2}$. It is the purpose of this article to attempt to formulate the above-mentioned effective theory and its physical consequences in a way so as to *avoid* the large-spin S assumption.

To this end, we first review the passage from the statistical large-spin models to the continuum theories and then extend the analysis to spin-$\frac{1}{2}$ models. The local phase symmetries that these models possess play a crucial rôle in this program, and below we study them in some detail. What we shall show here is that the doped large-U Hubbard models possess a local $SU(2) \times U_S(1)$ phase symmetry related to spin interactions. This symmetry will be discovered through

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a *spin-charge separation* ansatz, which allows intersublattice hopping for holons and hence spin flip. The spin-charge separation may be physically interpreted as implying an effective ''substructure'' of the electrons due to the many-body interaction in the medium. This sort of idea, originating from Anderson's resonating valence bond (RVB) theory of spinons and holons,[10] seems to be pursued recently by Laughlin, although from a (formally at least) different perspective than the one discussed here.[11]

The effective long-wavelength model is remarkably similar to a three-dimensional gauge model of particle physics proposed in Ref. 12 as a toy example for chiral symmetry breaking in QCD. In that work, it was argued that *dynamical generation* of a fermion mass gap due to the U(1) group in SU(2)×U(1) breaks the SU(2) group down to a τ_3 U(1) group, where τ_3 is the 2×2 Pauli matrix. From the particle-theory viewpoint this is a Higgs mechanism without an *elementary* Higgs excitation.

The analysis carries over to the present case as well, if one associates the mass gap to the holon condensate. The resulting effective theory of the light degrees of freedom is then similar to the continuum limit of Ref. 6 describing unconventional parity-conserving superconductivity.

From our point of view, the above symmetry-breaking pattern summarizes the effects of doping on the large-U Hubbard model in a dynamical way. In our opinion, the appearance of non-Abelian gauge symmetries, as symmetries of doped antiferromagnets that are broken dynamically by doping, and the analogy of the holon-condensate formation/superconductivity with chiral symmetry breaking in Yang-Mills theories, open up many possibilities for a nonperturbative (exact) treatment of such theories, including the role of nonperturbative effects in the superconductivity mechanism.[13] We also believe that our analysis offers quantitative support to the ideas of Refs. 10 and 11 about effective ''splitting'' of electrons into spinon and holons in the medium in a more general context.

The structure of the article is as follows. In Sec. II we discuss the doped Hubbard (lattice) models from the point of view of the proposed spin-charge separation ansatz and the associated gauge symmetry structure SU(2)×U$_S$(1). In Sec. III we discuss the long-wavelength effective lattice action in the limit of strong U$_S$(1), the dynamical mass generation for the holes, and the connection with (Kosterlitz-Thoules) superconductivity. In Sec. IV we present an analytical derivation of the dynamical breaking of the SU(2)→U(1) on the lattice, in the limit of strong U$_S$(1). Finally, in Sec. V we present our conclusions and the possible predictions of the model.

II. HUBBARD MODELS AND LOCAL PHASE SYMMETRIES

A. Large-spin treatments and their continuum limits

First let us briefly review the large-spin treatments of antiferromagnets.[9,6] In the absence of doping impurities, the quasiplanar materials are antiferromagnetic insulators. The potential importance of antiferromagnetic correlations for high-temperature (cuprate) superconductivity was noted by Anderson,[10] who suggested that the correct model for the dynamics of electrons in the copper oxide layers was the single-band, large-U Hubbard model. The two-dimensional Hubbard Hamiltonian is written in terms of operators $c_{i,\sigma}$ and $c_{i,\sigma}^\dagger$, which annihilate and create electrons in the $d_{x^2-y^2}$ orbital at each copper site,

$$H = -t \sum_{\langle ij \rangle, \sigma} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow}, \qquad (2.1)$$

where t is the electron-hopping matrix element, U is the strong Coulomb repulsion, and $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$ is the occupation number at each site. In the limit $U \to \infty$, a single-occupancy constraint is rigidly imposed. The undoped case is described by the Hubbard model with half-filled band and hence the spins are the only degrees of freedom in this limit. To leading order in large-U perturbation theory, the half-filled Hubbard model is simply equivalent to the two-dimensional Heisenberg antiferromagnet[14]

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j, \qquad (2.2)$$

where $J = 4t^2/U$ and $\mathbf{S}_i$ is the electron spin at site i. Thus we see that in the infinite $U \to \infty$ limit J corresponds to a weak coupling.

The effective long-wavelength degrees of freedom of the antiferromagnet can be described by a ''relativistic'' quantum field theory in (2+1)-dimensional space-time. In particular, the large-S limit of the spin-S Heisenberg antiferromagnet is equivalent, at large length scales, to the quantum nonlinear σ model.[15,16] The relativistic covariance of the effective action arises from the linear dispersion relation for long-wavelength magnons and the spin-wave velocity plays the role of the velocity of light in this formulation.

Doping introduces mobile charges that hop from site to site against the antiferromagnetic background of the spins. The coupled dynamics of holes and spins in the doped system is highly nontrivial. The hopping of holes tends to disorder the spins reducing the antiferromagnetic correlation length and the spins also mediate interactions between the holes. Roughly speaking, there is competition between the influence of the spins that favor a Néel-ordered ground state and that of the holes that tend to form a spin liquid. A general conjecture is that a superconducting pairing of holes arises out of this competition.[17,18] This has been verified in Ref. 6 in an effective large-spin analysis. In that analysis a large-spin $S \to \infty$ has been employed, which leads to two kinds of holes, due to the suppression of intersublattice hopping.[9,6] To be complete, below we review briefly the approach of Ref. 6, with which we shall make contact later on.

To this end, we first note that to describe the dynamics of holes in the model of Ref. 6 one implements a spin-charge separation, which is achieved by representing the electron operators $c_{i,\sigma}$ using a ''slave-fermion'' ansatz[19,20]

$$c_{i,\sigma}^\dagger = \psi_i z_{i,\sigma}^\dagger, \qquad (2.3)$$

where ψ is a Grassmann field representing the absence of a spin at a given site (hole) that carries the electric charge and $z_{i,\sigma}$ is the spin degree of freedom, which can be identified[19] with the magnon field of the CP^1 σ model. The ansatz (2.3) carries information about a local gauge invariance of the model as one can perform simultaneous local phase rotations on ψ_i and $z_{i,\sigma}$ fields with opposite couplings without chang-

ing $c_{i,\sigma}$. It is this symmetry that is responsible for the gauge nature of the interactions between holes. The physical reason for such a symmetry is the restriction of having at most one electron per lattice site. This redundancy of degrees of freedom is the characteristic feature of gauge models.

The full partition function of the model is given as a path integral over the Grassmann fields ψ_i and $\psi_i^\dagger$ as well as the CP^1 variables z, $\bar{z}$, and a_μ. The corresponding long-wavelength limit is derived by linearizing the energy spectrum about the chemical potential[9,6] and is given by

$$S_{\text{eff}} = \int_0^\beta d\tau \int d^2x \; \frac{1}{\gamma} \, |(\partial_\mu + i a_\mu)z|^2 + \bar{\Psi}_a \left(i\partial\!\!\!/ - \tau_3 d\!\!\!/ \right.$$

$$\left. + \frac{e}{c} A\!\!\!/ \right) \Psi_a , \tag{2.4}$$

where Ψ are four-component Dirac fermions, representing holes, γ is a constant inversely proportional to the J Heisenberg interaction,[6,19] e is the electric charge, c is the velocity of light in units of the hole Fermi velocity $v_F = 1$, and A is an external electromagnetic field. The Dirac nature of the holes is a result of the flux-phase assumption for each sublattice.[6] The reducible four-component representation of the Dirac algebra in space-time is a result of doubling and follows directly from the local sublattice structure defined by the antiferromagnetic order. The opposite statistical charges of the holes in different sublattices leads to a τ_3 coupling for the gauge field, where $\tau_3 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$ is the 2×2 Pauli σ_3 matrix representation for the generator of the τ_3 U(1) group.[6] (Here and in Ref. 6 we follow the terminology τ_3 U(1) for the Abelian group in spin or sublattice space generated by the 2×2 σ_3 Pauli matrix so as to distinguish it from space-time groups.)

Integrating out the electrically neutral magnon fields and keeping only the leading terms in a derivative expansion,[21,22] one obtains the low-energy effective action of the electrically charged degrees of freedom

$$\mathcal{L} = -\frac{1}{4g^2} f_{\mu\nu} f^{\mu\nu} + \bar{\Psi}_a \left(i\partial\!\!\!/ - S\tau_3 d\!\!\!/ - \frac{e}{c} A\!\!\!/ \right) \Psi_a . \tag{2.5}$$

The dimensionful gauge coupling g^2 is proportional to[22,19]

$$g^2 = (\gamma)^{-1} \sim J\eta , \tag{2.6}$$

where η is the doping concentration in the sample. In the context of the t-j model, which was considered in Ref. 6, this coupling may be taken strong enough so as to generate dynamically a gap in the hole spectrum.

The above analysis essentially postulated the existence of two holon species by suppressing intrasublattice hopping. This was the result of a large-spin analysis. It is the purpose of this article to demonstrate that qualitatively similar long-wavelength results may be obtained for spin-$\frac{1}{2}$ doped antiferromagnets. An important tool in such an analysis is the study of local phase symmetries of the model, which we now turn to. We shall start with a review of (non-Abelian) gauge symmetries that characterize the half-filled (undoped) models and then proceed to a study of the doped case upon constructing an appropriate spin-charge separation ansatz, extending Eq. (2.3) appropriately so as to allow intersublattice

hopping of holons. As we shall show, under the proposed ansatz, the effective Hamiltonian of holon and spinon degrees of freedom is characterized by hidden non-Abelian local phase symmetries. However, the holon condensate breaks the non-Abelian symmetry dynamically down to the Abelian subgroup discussed in Ref. 6 and hence one recovers the above-discussed Abelian model as an effective theory of the light degrees of freedom. Nevertheless, there are remnants of the non-Abelian symmetry structure, which manifests itself in the (mass) spectrum of mesonlike excitations, as we shall discuss in Sec. III. The presence of such excitations constitutes physically testable predictions of the spin-charge separation ansatz proposed in this work.

B. Half-filled spin-$\frac{1}{2}$ antiferromagnets: SU(2) gauge symmetry structure

The large-U (Mott) limit of the half-filled Hubbard model with $j = 4t^2/U$ is the Heisenberg model (2.2). In Ref. 23 it was observed that in this limit there is a local SU(2) symmetry associated with the spin-$\frac{1}{2}$ algebra of the electrons. Indeed, using electron operators c_i^α at a site i, corresponding to spin components up or down, $\alpha = 1,2$, one may represent the Hamiltonian (2.2) as

$$H = J \sum_{\langle ij \rangle} (c^{\dagger,\alpha} \sigma_\alpha^\beta c_\beta)_i (c^{\dagger,\alpha} \sigma_\alpha^\beta c_\beta)_j , \tag{2.7}$$

with *the constraint of one electron per site*

$$c_i^{\dagger,\alpha} c_{i,\alpha} = 1 . \tag{2.8}$$

H is invariant under the usual global SU(2) transformations of the spin-$\frac{1}{2}$ algebra $c_\alpha \rightarrow c_\beta g_\alpha^\beta$, with g_α^β an SU(2) matrix. In Ref. 23 a second SU(2) was constructed out of the doublet of creation operators $(c_2^\dagger, -c_1^\dagger)$. Combining these two doublets in a 2×2 matrix

$$\chi_{\alpha\beta} = \begin{pmatrix} c_1 & c_2 \\ c_2^\dagger & -c_1^\dagger \end{pmatrix} , \tag{2.9}$$

one observes that in addition to the *global* SU(2) transformations $\chi_{\alpha\beta} \rightarrow \chi_{\alpha\gamma} g_\beta^\gamma$, one can[23] define a *local* SU(2) by left multiplication

$$\chi_{\alpha\beta} \rightarrow h_\alpha^\gamma \chi_{\gamma\beta} . \tag{2.10}$$

This local symmetry commutes with the global SU(2) mentioned above. Writing the global SU(2) spin operators **S** appearing in Eq. (2.2) in terms of χ as $\mathbf{S} \propto \text{tr} \, \chi^\dagger \chi \sigma^T$, with T denoting matrix transposition, one can easily see that the Heisenberg interaction (2.7) is invariant under this local SU(2), which is thus the symmetry of the large-U Mott limit of the *half-filled* Hubbard model. It should be stressed of course that this is *not* an exact symmetry of the Hubbard model. As shown in Ref. 23, the very constraint (2.8) of one electron per site, which in terms of χ variables is expressed as

$$\text{Tr} \, \chi^\dagger \sigma^3 \chi = 0 , \tag{2.11}$$

results in a time-dependent local gauge symmetry, when combined with the kinetic term in the Lagrangian

$$L = \frac{1}{2} \sum_i \ \mathrm{tr} \ \chi_i^\dagger \left(i \frac{d}{dt} + A_{0,i} \right) \chi_i - H, \qquad (2.12)$$

where $A_{0,i}$ acts as a Lagrange multiplier implementing the constraint and it may be thought of as the third (temporal) component of the local SU(2) gauge field.[23] Such gauge symmetries appear as a general property of the Gatzwyler projection of one electron per site, due to the fact that such projections are associated with a sort of particle-number conservation. This local gauge symmetry connects various mean-field limits of the half-filled Hubbard model.[24]

To understand the formal meaning of the above symmetry, we return to the CP^1 σ model, which is supposed to describe the low-energy physics of the half-filled Hubbard model in a bosonized framework for the spin excitations. We recall that upon resolving the constraint $\bar{z}z = 1$, with $z = (z_1, z_2)$ a complex SU(2) doublet with boson statistics, the z field can be written as a 2×2 unitary matrix

$$z = \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix} = \exp(i \xi_a \sigma_a), \qquad (2.13)$$

where σ_a, $a = 1,2,3$ are the Pauli generators of SU(2) and the *real* fields ξ_i are dynamical. The gauged σ-model action in this representation reads

$$S_z = \int d^3x \ \gamma_0^{-1} \ \mathrm{tr} |(\partial_\mu - ig B_\mu)z|^2, \qquad (2.14)$$

where γ_0 is a bare coupling constant. In this representation one is free to gauge the full SU(2) local gauge group in the σ-model action, in which case $B_\mu = B_\mu^a \sigma_a$, $a = 1,2,3$, or its Abelian U(1) subgroup $B_\mu = B_3 \sigma_3$. The action (2.14) reads

$$S = \int d^3x \ \gamma_0^{-1} \left[\sum_a (\partial_\mu \xi_a)^2 + g^2 B_\mu^2 + \sum_a B_\mu^a (-2g \partial_\mu \xi_a) \right]. \qquad (2.15)$$

Technically, the above representation separates the Goldstone modes from the rest of the fields relevant at low momenta.[25] The resolution of the constraint implicit in Eq. (2.13) results in a standard mass term for the gauge field B instead of the quartic coupling $\bar{z}B^2 z$.

The possibility of gauging the full SU(2) group in the σ model is equivalent to the local SU(2) symmetry of the Heisenberg action (2.2) found in Ref. 23, given that at half filling only spin excitations (magnons) exist.[10] Of course, the equivalence is understood in terms of bosonization, which in $2+1$ dimensions, unlike $1+1$ dimensions, cannot be expressed in a closed form, but only as an effective derivative expansion.

C. Doped spin-$\frac{1}{2}$ antiferromagnet and non-Abelian gauge symmetry structure

Doping is expected to break the SU(2) symmetry between creation and annihilation pairs of electron operators. Naively speaking, a spatial hopping term of the form $c_{\alpha,i}^\dagger c_{\alpha,j}$ does not seem to be invariant under the local SU(2) (2.10). Away from half filling one would expect that only a local U(1) can survive, which in view of our spin-charge separation ansatz (2.3) seems to be the Abelian subgroup of SU(2) associated with τ_3. This local subgroup is the one gauged in the CP^1 σ

model and also the one associated with the (Berry phase) term describing static holes in the model of Ref. 6. In this article we shall present a dynamical scenario by which the above symmetry breaking is achieved. The scenario will be remarkably similar to a three-dimensional particle-physics toy model for chiral symmetry breaking in QCD.[12]

The key point is to try to uncover the local SU(2) symmetry in the doped case by generalizing the spin-charge separation ansatz (2.3). We seek a representation of the spin-charge separation that will allow spin flip, but would still treat the holons as ''blind'' to the electronic sublattice structure. To this end, we propose to represent the holon degrees of freedom as two-component spinors in a two-dimensional ''color'' space, representing Dirac spin components (ψ_1, ψ_2), while the spin excitations are represented by the CP^1 doublets (z_1, z_2) living in the same color space. However, we amend our construction with a *spin-flip operation*, which, for the z-magnon degrees of freedom, is represented by the conjugate doublet $(-\bar{z}_2, \bar{z}_1)$. Thus the electron annihilation operators can be expressed as

$$c_1 = (\psi_1 \psi_2) \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad c_2 = (\psi_1 \psi_2) \begin{pmatrix} -\bar{z}_2 \\ \bar{z}_1 \end{pmatrix}, \qquad (2.16)$$

while the corresponding creation operators can be obtained by $-c_2^\dagger, c_1^\dagger$, with the dagger denoting Hermitian conjugation. We believe that this ansatz captures the qualitative features behind the RVB idea of Anderson[10] on spinon and holons. Essentially Eq. (2.16) implies that to annihilate an electron with, say, spin up one has to remove all the components of the spin. The spin-charge separation ansatz implies that to some extent the holes should be blind to the spin of the electron (sublattice structure of the antiferromagnet). This is correctly captured in Eq. (2.16) since the hole ''spinors'' in color space are the same for both electron components, while the (magnon) z doublets differ by a spin-flip operation defined above.

Technically, it is convenient to combine the creation and annihilation operators, following the treatment of the half-filled case (2.9). To this end, we propose that for the large-U limit of the doped Hubbard model the following spin-charge separation ansatz occurs at each site i:

$$\chi_{\alpha\beta,i} = \psi_{\alpha\gamma,i} z_{\gamma\beta,i} \equiv \begin{pmatrix} c_1 & c_2 \\ c_2^\dagger & -c_1^\dagger \end{pmatrix}_i$$

$$= \begin{pmatrix} \psi_1 & \psi_2 \\ -\psi_2^\dagger & \psi_1^\dagger \end{pmatrix}_i \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}_i, \qquad (2.17)$$

where the fields $z_{\alpha,i}$ obey canonical *bosonic* commutation relations and are associated with the spin degrees of freedom, while the fields $\psi_{a,i}$, $a = 1,2$ have *fermionic* statistics and are assumed to create holes at the site i with spin index α. They obey the anticommutation relations

$$\{ \psi_{i,\alpha}, \psi_{j,\beta}^\dagger \} = \delta_{ij} \delta_{\alpha\beta}, \quad \{ \psi_{i,\alpha}, \psi_{j,\beta} \} = \{ \psi_{i,\alpha}^\dagger, \psi_{j,\beta}^\dagger \} = 0. \qquad (2.18)$$

The ansatz (2.17) has spin-electric-charge separation since only the fields ψ carry electric charge. From now on, we shall refer to ψ_α as the ''holons'' and to $\bar{z}_\alpha$ as (bosonized)

''spinons.'' The ansatz (2.17) is an obvious generalization of Eq. (2.3) if one allows intersublattice hopping.

It is worth noticing that the anticommutation relations for the electron fields $c_\alpha, c_\beta^\dagger$ do not quite follow from the ansatz (2.17). Indeed, assuming the canonical (anti)commutation relations for z (ψ) fields, one obtains from Eq. (2.17)

$$\{c_{1,i}, c_{2,j}\} \sim 2\psi_{1,i}\psi_{2,i}\delta_{ij},$$
$$\{c_{1,i}^\dagger, c_{2,j}^\dagger\} \sim 2\psi_{2,i}^\dagger\psi_{1,i}^\dagger\delta_{ij},$$
$$\{c_{1,i}, c_{2,j}^\dagger\} \sim \{c_{2,i}, c_{1,j}^\dagger\} \sim 0,$$

$$\{c_{\alpha,i} c_{\alpha,j}^\dagger\} \sim \delta_{ij}\sum_{\beta=1,2}[z_{i,\beta}\bar{z}_{i,\beta} + \psi_{\beta,i}\psi_{\beta,i}^\dagger],$$

$$\alpha=1,2 \quad (\text{no sum over } i).$$

(2.19)

To ensure canonical commutation relations for the c operators therefore we must impose at each lattice site the (slave-fermion) constraints

$$\psi_{1,i}\psi_{2,i} = \psi_{2,i}^\dagger\psi_{1,i}^\dagger = 0,$$

$$\sum_{\beta=1,2}[z_{i,\beta}\bar{z}_{i,\beta} + \psi_{\beta,i}\psi_{\beta,i}^\dagger] = 1.$$

(2.20)

Such relations are understood to be satisfied when the holon and spinon operators act on physical states. Both of these relations are valid in the large-U limit of the Hubbard model and encode the nontrivial physics of constraints behind the spin-charge separation ansatz (2.17). They express the constraint at most one electron or hole per site, which characterizes the large-U Hubbard models we are considering here. From the above analysis, therefore, it becomes clear that the ansatz (2.17) does not characterize a generic Hubbard system, but only the appropriate large-U limit, where the constraint of one electron per site is valid. As we shall discuss in Sec. IV, both of the above constraints (2.20) are consistent with the mass spectrum of the effective long-wavelength theory obtained from dynamical generation of a fermion condensate.

Now let us look at the symmetry structure of the spin separation ansatz (2.17), which in view of the previous analysis coincides with the symmetry structure of the effective large-U Hubbard action. First, it appears to have a trivial local SU(2) symmetry if one defines the transformation properties of the z fields to be given by left multiplication with the SU(2) matrices and those of the $\psi_{\alpha\beta}^\dagger$ matrices by the left multiplication (2.10). In this representation, the gauge group SU(2) is generated by the 2×2 Pauli matrices.

The ansatz (2.17) possesses an additional local $U_S(1)$ statistical phase symmetry, which allows fractional statistics of the spin and charge excitations. This is an exclusive feature of the three-dimensional geometry. This is similar in spirit, although implemented in an admittedly less rigorous way, to the bosonization technique of the spin-charge separation ansatz of Ref. 26, and allows the alternative possibility of representing the holes as slave bosons and the spin excitations as fermions. In addition, as a consequence of the fact that the fermions ψ carry electric charge, one has an extra $U_{em}(1)$ symmetry for the problem.

To recapitulate, the above analysis, based on the spin-charge separation ansatz (2.17) that allows spin flip, leads to the following local-phase (gauge) group structure for the doped large-U Hubbard model:

$$G = SU(2) \times U_S(1) \times U_{em}(1),$$

(2.21)

where the second $U_{em}(1)$ factor refers to electromagnetic symmetry due to the electric charge of the holes. This symmetry appears as a hidden symmetry of the effective holon and spinon degrees of freedom obeying the ansatz (2.17).

The presence of the $U_S(1)$ statistics changing group factor will be crucial in our analysis. As we shall discuss in Sec. IV, in its strong coupling limit it can generate a mass gap[27–29] for the fermionic holon fields ψ, which for each hole component breaks parity, thereby producing a statistics changing dynamical Chern-Simons term. However, due to the even number of fermionic species there is no overall parity violation in the model.[6] Note that, since this statistical gauge field couples also to the z fields, their statistics will be affected as well.

D. Effective Hamiltonian of the doped Hubbard antiferromagnet

Next we focus our attention on showing that the various terms in the action are expressible in terms of the $\chi_{\alpha\beta}$ variables, which would imply that the symmetries of the large-U doped Hubbard model action are the symmetries of the ansatz (2.3). To this end, we first study the hopping term of the dopped Hamiltonian, which broke explicitly the local SU(2) symmetry (2.10) of the electron operators $c_\alpha, c_\beta^\dagger$. Let us rewrite this term in terms of $\chi_{\alpha\beta}$ variables:

$$H_{hop} = -\sum_{\langle ij \rangle} t_{ij}c_{\alpha,i}^\dagger c_{\alpha,j} = -\sum_{\langle ij \rangle} t_{ij}[\chi_{i,\alpha\gamma}^\dagger\chi_{j,\gamma\alpha} + \chi_{i,\alpha\gamma}^\dagger(\sigma_3)_{\gamma\beta}\chi_{j,\beta\alpha}],$$

(2.22)

where σ_3 is a 2×2 Pauli matrix and summation over the spin indices is implied. In terms of the spin and charge excitations, appearing in Eq. (2.17), then, the hopping term may be written as

$$H_{hop} = -\sum_{\langle ij \rangle} t_{ij}[\bar{z}_{i,\beta\kappa}\psi_{i,\kappa\alpha}^\dagger\psi_{j,\alpha\gamma}z_{j,\gamma\beta} + \bar{z}_{i,\beta\kappa}\psi_{i,\kappa\alpha}^\dagger(\sigma_3)_{\alpha\lambda}\psi_{j,\lambda\gamma}z_{j,\gamma\beta}]$$

(2.23)

and is trivially local SU(2) symmetric.

To complete the analysis we should also look at the interaction terms. The Heseinberg term (2.7) can be written in the convenient form[23]

$$H = -\frac{1}{8}J\sum_{\langle ij \rangle} \text{tr}[\chi_i\chi_j^\dagger\chi_j\chi_i^\dagger],$$

(2.24)

which can be linearized in terms of the fermion bilinear functions if one introduces in the path integral a Hubbard-Stratonovich field Δ_{ij} in a standard fashion. The result of the linearization is

$$H = \sum_{\langle ij \rangle} \text{tr}[(8/J)\Delta_{ij}^\dagger\Delta_{ji} + (\chi_i^\dagger\Delta_{ij}\chi_j + \text{H.c.})].$$

(2.25)

We then employ the ansatz (2.17) and perform a Hartree-Fock (mean-field) approximation for the bilinear functions:

$$\langle \overline{z}_i z_j \rangle \equiv |A_1| V_{ij} U_{ij}, \quad (2.26)$$

$$\langle \psi_i^\dagger [-t_{ij}(1+\sigma_3)+\Delta_{ij}]\psi_j \rangle \equiv |A_2| V_{ij} U_{ij},$$

where, according to the previous discussion, we have used the fact that the link variables are $SU(2) \times U_S(1)$ group elements, due to the specific transformation properties of the variables z and ψ. In the above notation V is the $SU(2)$ part and U denotes the Abelian $U_S(1)$ group element. The amplitudes $|A_i|$, $i=1,2$, of the link variables are assumed frozen, as usual. By an appropriate normalization of the z and ψ fields, this amplitude is common for both link variables,

$$|A_1|=|A_2|=K. \quad (2.27)$$

According to the discussion of Ref. 6 the amplitude K is proportional to the Heisenberg exchange interaction $J=4t^2/U$, with t the hopping parameter, and also to the doping concentration in the sample.[19] We shall return to this issue later on.

The result of the Hartree-Fock approximation, then, for the combined hopping and interaction terms in the Hamiltonian is

$$H_{HF}=\sum_{\langle ij \rangle} \text{tr}\{(8/J)\Delta_{ij}^\dagger \Delta_{ji} + [-t_{ij}(1+\sigma_3)+\Delta_{ij}]$$
$$\times (\psi_j \langle z_j \overline{z}_i \rangle \psi_i^\dagger)\}$$
$$+\sum_{\langle ij \rangle} \text{tr}\{\overline{z}_i \langle \psi_i^\dagger [-t_{ij}(1+\sigma_3)+\Delta_{ij}]\psi_j \rangle z_j\}+\text{H.c.}$$
$$(2.28)$$

and using Eqs. (2.26) and (2.27) one obtains

$$H_{HF}=\sum_{\langle ij \rangle} \text{tr}\{(8/J)\Delta_{ij}^\dagger \Delta_{ji} + K[-t_{ij}(1+\sigma_3)$$
$$+\Delta_{ij}]\psi_j V_{ji} U_{ji}\psi_i^\dagger\}+\sum_{\langle ij \rangle} \text{tr}[K\overline{z}_i V_{ij} U_{ij} z_j]+\text{H.c.}$$
$$(2.29)$$

This is the effective field theory lattice action we propose to describe the dynamics of the large-U Hubbard model. It is understood the constraints (2.20) should be taken into account to complement the description. It is important to note that the fermion fields ψ are 2×2 matrices in the above representation. Notice also that the term $t_{ij}(1+\sigma_3)+\Delta_{ij}$ transforms covariantly under a global U(1) symmetry generated by the Pauli matrix σ_3. This global U(1) symmetry acts on the electron operators χ_i as $\chi_i \rightarrow U\chi_i$, with $U=e^{i\theta}$, θ a global phase. The z-dependent (magnon) terms yield, in the continuum, the CP^1 σ-model Lagrangian (2.14).[19]

In the large-U Hubbard limit we are considering here, one has the order of magnitude estimates

$$J=4t^2/U, \quad t \sim U\eta_{max}, \quad \eta_{max} \ll 1, \quad (2.30)$$

where η_{max} is the maximum doping concentration of the sample, above which superconductivity is destroyed. For underdoped cuprates one may consider the case $\eta_{max} \ll 1$. In this limit, one observes from Eq. (2.29) that the Gaussian fluctuations of the variable Δ_{ij} are of $O(J/|t_{ij}|)$ and hence suppressed compared to the hopping term t_{ij}. This means that one may approximate $t_{ij}(1+\sigma_3)+\Delta_{ij} \simeq t_{ij}(1+\sigma_3)$. Considering the usual case with $t_{ij}=t$ for every i,j, one may absorb such terms into an appropriate rescaling of the fermion fields ψ. This will be understood in what follows. However, we stress once again that in the case of finite-U Hubbard models, one should consider the effects of the Gaussian variable Δ_{ij} in the Lagrangian (2.29). This will be left for future work.

The conventional lattice gauge theory form of the action is derived upon integrating out the magnon fields z in the path integral. As discussed in Refs. 22 and 21 the result of such a path integration of the magnon fluctuations around the mean field yields appropriate Maxwell kinetic terms for the link variable $V_{ij} U_{ij}$, which are the dominant terms in a low-energy derivative expansion. The constraint of at most one electron per lattice site in Eq. (2.20) is crucial in such a derivation since its implementation through a Lagrange multiplier field σ results in a ''mass'' term for the magnon fields z, in the way explained in Ref. 21. The effective Maxwell terms in the continuum are of the generic form

$$S_{kim} \propto \int d^3x \frac{1}{\sqrt{\sigma_0}} (F_{\mu\nu}^2 + \mathcal{G}_{\mu\nu}^2), \quad (2.31)$$

where $F_{\mu\nu}, \mathcal{G}_{\mu\nu}$ denote the $U_S(1)$ and $SU(2)$ field strengths, respectively, and σ_0 is a vacuum expectation value of the Lagrange multiplier field σ. An elementary one-loop renormalization-group analysis yields[21]

$$\sqrt{\sigma_0}=M-4\pi K_R, \quad (2.32)$$

where M is a transmutation mass and K_R is the ''renormalized'' K coefficient of the CP^1 part of the action (2.29). From the analysis of Ref. 19 we may infer that $K_R \propto J\eta$, with J the Heisenberg interaction and η the doping concentration, which for lightly doped cuprates is $\eta \ll 1$. This implies that the order of magnitude of the coefficient of the Maxwell term (2.31), resulting from the z integration in a derivative expansion, is set by the Heisenberg exchange interaction field strength J. The conventional three-dimensional gauge coupling g^2, of dimensions (mass), is related to K_R by the simple relation (2.6)

$$1/g^2 \propto K_R^{-1} \propto J^{-1}\eta^{-1}. \quad (2.33)$$

Thus, from Eq. (2.30) one obtains for the *dimensionless* coupling $g^2 a$, with a the lattice spacing of the antiferromagnetic Hubbard model,

$$\beta_1 \equiv \frac{1}{g^2 a} \propto \frac{1}{\eta \eta_{max}^2 U_a}. \quad (2.34)$$

The magnitude of this coupling depends on the way the limit $U \rightarrow \infty$ is taken. Taking the limit of $U \rightarrow \infty$ such that $Ua\eta\eta_{max}^2 \gg 1$, one obtains a *small* β, i.e., strong coupling for the $U_S(1)$ group. The limit of small β is crucial for the symmetry-breaking patterns of the non-Abelian $SU(2)$ group, as we shall discuss in Sec. III.

We now remark that on the lattice the kinetic (Maxwell) terms (2.31) are given by appropriate plaquette terms of the form

$$\sum_p [\beta_{SU(2)}(1-\text{Tr } V_p)+\beta_{U_S(1)}(1-\text{Tr } U_p)], \quad (2.35)$$

where p denotes sum over plaquettes of the lattice and $\beta_{U_S(1)}\equiv\beta_1$ and $\beta_{SU(2)}\equiv\beta_2=4\beta_1$ are the inverse square couplings of the $U_S(1)$ and SU(2) groups, respectively. The specific relation between the SU(2) and $U_S(1)$ couplings is due to the appropriate normalization of the generators of the groups.

At this point it is worth remarking that for certain Schwinger-Dyson treatments of dynamical symmetry breaking[6,24] a large-N treatment is desirable, in which case one assumes that the spin SU(2) group is replaced by SU(N) with N large enough. In that case the non-Abelian coupling is related to the Abelian one through

$$\beta_{SU(N)}=2N\beta_{U_S(1)}=2N\beta_1. \quad (2.36)$$

This implies that, even in the case of strong $U_S(1)$ coupling, $\beta_1\rightarrow 0$, the large-N (large spin) limit may be implemented in such a way so that $\beta_{SU(N)}$ is *finite*. This is the limit of the analysis of Ref. 12. We shall discuss this case in Sec. III, where we shall make contact with the results of Ref. 6, where such a large-N treatment had been assumed.

Above we did not write explicitly the chemical potential term $\mu\Sigma_{i,\alpha}c_{i\alpha}^{\dagger}c_{i\alpha}$, which determines the doping concentration in the sample. This term is also expressed in terms of the χ variables, and essentially has the form of Eq. (2.22) but for $i=j$, which again may be expressed in a gauge-invariant way upon using the ansatz (2.17). In deriving long-wavelength continuum limits, one linearizes the energy spectrum about the chemical potential.[9,6] For most of our discussion below we shall not write explicitly such terms, as they do not affect the symmetry structure of the theory.

E. Spinor structure for holons and symmetry-breaking patterns

Before closing this section we would like to remark that, as a result of the 2×2 matrix structure of the fermion fields ψ in Eq. (2.29), one may actually change representation of the SU(2) group and instead of working with 2×2 matrices one may use a representation in which the fermionic matrices $\psi_{\alpha\beta}$ are represented as four-component vectors [in color (spin) space]

$$\psi_{\alpha\beta}\rightarrow\mathbf{\Psi}^{\dagger}\equiv(\psi_1 \ -\psi_2^{\dagger} \ \psi_2 \ \psi_1^{\dagger}). \quad (2.37)$$

It is easy to see that in this representation the SU(2) group is generated by the matrices

$$\tau_1=\gamma_3\equiv\begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \quad \tau_2=\gamma_5\equiv i\begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix},$$

$$\tau_3=\Delta\equiv i\gamma_3\gamma_5=\begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \quad (2.38)$$

where the substructures are 2×2 matrices. This is the SU(2) representation used in Ref. 12 in the context of three-dimensional toy models for chiral symmetry breaking. Remarkably, the same type of symmetry arises in our context between creation and annihilation operators of holon pairs in the spin-charge separation ansatz (2.17).

In the analysis of Ref. 12, to be discussed in the context of the present model in Sec. III, the statistical group $U_S(1)$ group is responsible for the dynamical generation of a parity conserving mass $\langle\overline{\Psi}\Psi\rangle$. In terms of the dynamical variables describing creation and annihilation of holons $\psi,\psi^{\dagger}$, respectively, the *parity conserving* mass depends on the holon condensate. To see this, it is convenient to split the four-component spinors (2.37) into two-component ones

$$\widetilde{\Psi}_1^{\dagger}=(\psi_1 \ -\psi_2^{\dagger}), \quad \widetilde{\Psi}_2^{\dagger}=(\psi_2 \ \psi_1^{\dagger}). \quad (2.39)$$

In this representation the two-component spinors $\widetilde{\Psi}$ (2.39) will act as Dirac spinors and the γ-matrix (space-time) structure will be spanned by the irreducible 2×2 representation. The Dirac conjugate field $\overline{\widetilde{\Psi}}$ may be identified directly with the Hermitian conjugate fields $\widetilde{\Psi}^{\dagger}$ in terms of holon operators. This is due to the fact that in a path integral over the holon fields, the conjugate fields $\psi^{\dagger}$ can be considered as *independent* degrees of freedom.[9,6,19] In this representation, the local SU(2) gauge group is generated by the familiar 2×2 Pauli matrices σ_a, $a=1,2,3$. The parity transformation is defined as $\widetilde{\Psi}_1\rightarrow\sigma_1\widetilde{\Psi}_2$, $\widetilde{\Psi}_2\rightarrow\sigma_1\widetilde{\Psi}_1$, which in terms of the (microscopic) holon operators ψ_i, $i=1,2$, reads $\psi_1\rightarrow\psi_2^{\dagger}$, $\psi_2\rightarrow-\psi_1^{\dagger}$. With these in mind, it is straightforward to observe that the parity-conserving mass term $\overline{\widetilde{\Psi}}_1\widetilde{\Psi}_1-\overline{\widetilde{\Psi}}_2\widetilde{\Psi}_2$ can be related to the holon condensation

$$\langle\overline{\widetilde{\Psi}}_1\widetilde{\Psi}_1-\overline{\widetilde{\Psi}}_2\widetilde{\Psi}_2\rangle=-2(\psi_1^{\dagger}\psi_1-\psi_2^{\dagger}\psi_2), \quad (2.40)$$

where we took proper account of the anticommutation relations (2.18) among the Grassman $\psi_{i,\alpha}$, $\alpha=1,2$. The terms $\langle\psi_\alpha^{\dagger}\psi_\alpha\rangle$, $\alpha=1,2$, are holon condensates. Notice, in the same context, that the parity-violating mass term $\langle\overline{\widetilde{\Psi}}_1\widetilde{\Psi}_1+\overline{\widetilde{\Psi}}_2\widetilde{\Psi}_2\rangle$ equals an irrelevant constant, which may be subtracted. This result is consistent with the *generic* energetics arguments that disfavor dynamical generation of a parity-violating mass in vector such as theories with *even* flavor number.[30]

The formation of holon condensate due to a statistics changing $U_S(1)$ group is similar in spirit to the approach of Ref. 3 in the context of the anyonic superconductivity. How-

ever, as mentioned above, in our case, due to the four-component structure of the fermions, there is an even number of fermionic species and hence no overall parity violation. Moreover, this mass gap is not a singlet under SU(2), as we shall discuss in Sec. III A, but transforms as a triplet,[12] thereby breaking SU(2) down to its τ_3 U(1) subgroup. This is the τ_3 U(1) symmetry of the ansatz (2.3), leading to the effective action (2.5). This provides a sort of dynamical breaking of the local spin SU(2) group as the result of introducing holes into the system.

The breaking of the SU(2) symmetry down to its Abelian τ_3 subgroup admits the (physical) interpretation of restricting the holon hopping effectively to a single sublattice. In a low-energy effective theory of the massless degrees of freedom this reproduces the results of Refs. 6 and 9. This scenario can be readily seen by using the four-component spinor representation (2.37). Clearly, the two off-diagonal generators of the SU(2) group (2.38) γ_3 and γ_5, corresponding to the gauge bosons acquiring masses dynamically due to the holon condensate, *mix* the two sublattices in the notation of Refs. 9 and 6. Indeed, from Eq. (2.39) it follows immediately that if a holon of spin, say, 1 is created at a site i, these generators would connect it to the destruction of a hole with spin 2 in the neighboring sublattice. On the other hand, the generator Δ of the unbroken τ_3 U(1) is block diagonal, thereby not mixing the sublattices. The intrasublattice hopping in this approach is then suppressed by the mass of the gauge bosons. We are considering here the limit of *infinitely* strong $U_S(1)$.[12] In such a limit the intrasublattice hopping is completely suppressed since the mass (which is proportional to the infinite condensate) is infinite.[12] This situation therefore describes *static* holes. Hole hopping is allowed for strong but finite couplings, in which case the holon condensates and masses are finite.

We shall devote more discussion on the phase diagram of the theory, and its comparison to that of Ref. 6, in the next section. We would like to close this section by noting that, in the context of microscopic models of the form (2.29), dynamical formation of holon condensates, and hence destruction of antiferromagnetic order, would occur above a critical doping concentration.[31] To quantify the above results on symmetry breaking, therefore, one needs proper lattice simulations of these models. This is left for the future.

III. LONG-WAVELENGTH LIMIT OF THE SPIN-$\frac{1}{2}$ DOPED ANTIFERROMAGNET

A. Derivation of the long-wavelength Hamiltonian

We now proceed in the long-wavelength limit of Eqs. (2.29) and (2.35), in the spinor representation for the holon fields, discussed in Sec. II E. To this end, we assume, following the analysis of Ref. 6, a nontrivial *flux phase* for the gauge field $U_S(1)$. This is crucial in yielding a *Dirac* form for the hole effective action.[32,23,6] The long-wavelength continuum limit is then obtained in a similar way as in the Abelian case of Refs. 6 and 19, at low energies, by linearizing about a specific point on the Fermi surface. [In what follows we shall ignore, for simplicity, the shape of the Fermi surface[6] and therefore deal with conventional relativistic lattice models. Of course, this will not be the case in a realistic condensed-matter system, where there are known to be large

Fermi surfaces for holes. The relativistic nature may be accurate in superconducting models with nodes on their Fermi surface, when linearization about a node is performed. However, for our purposes in this work, which are a study of the generic symmetry-breaking patterns of the local group (2.21), their physical implications for superconductivity, and the connection with the results of Ref. 6, such relativistic models will be sufficient.] Due to the assumed flux-phase-π *background* for the gauge field $U_S(1)$ one gets for the hopping (kinetic) terms of the two spinors (2.39) (ignoring interactions for brevity):[6]

$$L_{\text{kin}} \sim \sum_{r,\mu} (-1)^{r_0+\cdots+r_{\mu-1}} \widetilde{\overline{\Psi}}_c(r) \widetilde{\Psi}_c(r+\hat{\mu}) + \text{H.c.},$$

$$(3.1)$$

where $c = 1,2$ is the color index, not to be confused with the space-time (Dirac) index. The factors $(-1)^{r_0+\cdots}$ yield a phase $e^{i\pi} = -1$ per lattice plaquette and this result is produced in our case by the $U_S(1)$ flux-phase background.[6] As discussed in Ref. 32, the form (3.1) corresponds to a Dirac form for the kinetic terms of the fermions $\widetilde{\Psi}$ upon making an (inverse) Kawamoto-Smit transformation[33]

$$\Psi_c(r) = \gamma_0^{r_0} \cdots \gamma_2^{r_2} \widetilde{\Psi}_c(r), \quad \overline{\Psi}_c(r) = \widetilde{\overline{\Psi}}_c(r)(\gamma_2^\dagger)^{r_2} \cdots (\gamma_0^\dagger)^{r_0},$$

$$(3.2)$$

where Ψ are two-component Dirac spinors, carrying color. We stress once again that the color structure is up and above any space-time (Dirac) structure. Notice that in such a picture fermion bilinear functions of the form $\overline{\Psi}_{i,c}\Psi_{i,c'}$ (i is a lattice index), for instance, the condensate (2.40), are just $\widetilde{\overline{\Psi}}_{i,c}\widetilde{\Psi}_{i,c'}$ due to the Clifford algebra $\{\gamma_\mu,\gamma_\nu\} = -2\delta_{\mu\nu}$ and (anti-)Hermiticity properties of the 2×2 γ matrices on the Euclidean lattice. This is useful to have in mind when we study the spectrum of meson states in Sec. IV.

In what follows we shall make use of the above-mentioned (irreducible) 2×2 representation in both the color and space-time indices *on the lattice*. According to the above discussion, then, upon ignoring for the moment the electromagnetic interactions of holes, one obtains the following effective low-energy lattice action for the holon fields, originating from Eqs. (2.29), (2.35), and (3.1):

$$S = \frac{1}{2}K \sum_{i,\mu} \left[\overline{\Psi}_i(-\gamma_\mu)U_{i,\mu}V_{i,\mu}\Psi_{i+\mu} \right.$$

$$+ \overline{\Psi}_{i+\mu}(\gamma_\mu)U_{i,\mu}^\dagger V_{i,\mu}^\dagger \Psi_i \right] + \beta_1 \sum_p (1-\text{tr } U_p)$$

$$+ \beta_2 \sum_p (1-\text{tr } V_p),$$

$$(3.3)$$

where $\mu = 0,1,2$, $U_{i,\mu} = \exp(i\theta_{i,\mu})$ represents the statistical $U_S(1)$ gauge field, $V_{i,\mu} = \exp(i\sigma^a B_a)$ is the SU(2) gauge field, and the plaquette terms are obtained, at low energies, as a result of the z-magnon integration.[22,21] [We would like to mention that, technically, in order to study dynamical formation of fermion condensates on the lattice using Monte Carlo studies as in Ref. 12, one should add to the action (3.3) a bare mass term $m_0\sum_i\overline{\Psi}_i\sigma_3\Psi_i$ and take the limit $m_0\to 0$

only at the very end of the computations. This will be irrelevant for our purposes here.] The fermions Ψ are taken to be two-component spinors, in both Dirac and color spaces. The quantity K is proportional to the holon hopping matrix element, which in turn depends[10,6] on the doping concentration, as stated earlier in Eq. (2.33). According to the discussion following Eq. (2.36), in a large-spin $[\text{SU}(N \to \infty)]$ treatment, as in Ref. 6, the coupling constant $\beta_2 \to \infty$, and hence the non-Abelian gauge group sector of the model is *weakly* coupled in this limit. On the other hand, the coupling of the statistical $U_S(1)$ is considered to be *strongly* coupled in the limit $U \to \infty$. It is known, from either lattice results[28] or a semianalytic Schwinger-Dyson type of analysis,[27,29] that dynamical mass generation in a U(1) theory in three space-time dimensions occurs only for strong coupling, i.e., for values of the gauge coupling that are larger than a given critical value. This mass will break the SU(2) gauge group dynamically. This will be discussed in detail in Sec. IV.

The above limit has been studied in Ref. 12, where the model (3.3) has been used as a toy model for studying chiral symmetry-breaking patterns of QCD. Remarkably, as we have described above, this model can also be used to describe the physics of the spin-charge separation of strongly correlated electrons in a doped Hubbard model in its large-U limit. In this analogy the holon fields $\psi_{\alpha\beta}$ behave like the ''quarks'' of QCD, which are thus viewed as substructures of the physical electron $\chi_{\alpha\beta}$. It seems to us that this point of view is similar in spirit to that pursued in the context of anyonic models by Laughlin.[11] However, we should stress that from our point of view this ''splitting'' is viewed as a many-body effect for the holon dynamics in such systems and hence we do not ascribe to it any further significance.

B. Symmetry structure in the continuum

It will be instructive to study first the symmetry structure of the model (3.3) in the continuum, following the analysis of Ref. 12. This will help the reader understand better the interplay between the irreducible (2×2) and the reducible (4×4) representations of the Dirac and color (gauged chiral symmetry) groups. To this end, we first note that the continuum limit of the model (3.3) is described by the Lagrangian[12]

$$\mathcal{L} = -\tfrac{1}{4}(F_{\mu\nu})^2 - \tfrac{1}{4}(\mathcal{G}_{\mu\nu})^2 + \bar{\Psi} D_\mu \gamma_\mu \Psi - m_0 \bar{\Psi}\Psi, \quad (3.4)$$

with $D_\mu = \partial_\mu - i g_1 a_\mu^S - i g_2 \sigma^a B_{a,\mu}$ and $F_{\mu\nu}, \mathcal{G}_{\mu\nu}$ the corresponding field strengths for the Abelian (statistical) gauge field a_μ^S and the spin SU(2) gauge field B_μ^a. The parity-conserving bare mass m_0 term has been added by hand, as mentioned above, to facilitate Monte Carlo studies of dynamically generated fermion masses as a result of the formation of fermion condensates $\langle \bar{\Psi}\Psi \rangle$ by the strong $U_S(1)$ coupling. The $m_0 = 0$ limit should be taken at the end.

To understand better the nature of this SU(2) gauge symmetry, it is instructive to look first at the global SU(2) group, whose gauging produces the action (3.4). To this end we observe that the γ_μ, $\mu = 0,1,2$, matrices, which span the reducible 4×4 representation of the Dirac algebra in three dimensions in a fermionic theory with an even number of fermion flavors, assume the form[27]

$$\gamma_0 = \begin{pmatrix} \sigma_3 & \mathbf{0} \\ \mathbf{0} & \sigma_3 \end{pmatrix},$$

$$\gamma^1 = \begin{pmatrix} i\sigma_1 & \mathbf{0} \\ \mathbf{0} & -i\sigma_1 \end{pmatrix}, \quad (3.5)$$

$$\gamma^2 = \begin{pmatrix} i\sigma_2 & \mathbf{0} \\ \mathbf{0} & -i\sigma_2 \end{pmatrix},$$

where σ are 2×2 Pauli matrices and the (continuum) space-time is taken to have a Minkowskian signature. As is well known,[27] there exist two 4×4 matrices that anticommute with γ_μ, $\mu = 0,1,2$:

$$\gamma_3 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \quad \gamma_5 = i\begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}, \quad (3.6)$$

where the substructures are 2×2 matrices. These are the generators of the chiral symmetry for the massless-fermion theory

$$\Psi \to \exp(i\theta\gamma_3)\Psi, \quad \Psi \to \exp(i\omega\gamma_5)\Psi. \quad (3.7)$$

Note that these transformations do not exist in the fundamental two-component representation of the three-dimensional Dirac algebra and therefore the above symmetry is valid for theories with even fermion flavors only.

The set of generators $\{\mathbf{1}, \gamma_3, \gamma_5, i\gamma_3\gamma_5 \equiv \Delta\}$ form[12] a global SU(2)×U(1) symmetry. The identity matrix $\mathbf{1}$ generates the U(1) subgroup, while the other three form the SU(2) part of the group. The currents corresponding to the above transformations are[12]

$$J_\mu^\Gamma = \bar{\Psi}\gamma_\mu \Gamma \Psi, \quad \Gamma = \gamma_3, \gamma_5, i\gamma_3\gamma_5 \quad (3.8)$$

and are conserved in the absence of a fermionic mass term. It can be readily verified that the corresponding charges $Q_\Gamma \equiv \int d^2x\, \Psi^\dagger \Gamma \Psi$ lead to an SU(2) algebra[12]

$$[Q_3, Q_5] = 2iQ_\Delta, \quad [Q_5, Q_\Delta] = 2iQ_3, \quad [Q_\Delta, Q_3] = 2iQ_5 \quad (3.9)$$

If a mass term is present then there is an anomaly

$$\partial^\mu J_\mu^\Gamma = 2m\bar{\Psi}\Gamma\psi, \quad (3.10)$$

while the current corresponding to the generator $\mathbf{1}$ is *always* conserved, even in the presence of a fermion mass.[12]

The bilinear functions

$$\mathcal{A}_1 \equiv \bar{\Psi}\gamma_3\Psi, \quad \mathcal{A}_2 \equiv \bar{\Psi}\gamma_5\Psi, \quad \mathcal{A}_3 \equiv \bar{\Psi}\Psi \quad (3.11)$$

$$B_{1\mu} \equiv \bar{\Psi}\gamma_\mu\gamma_3\Psi, \quad B_{2\mu} \equiv \bar{\Psi}\gamma_\mu\gamma_5\Psi, \quad B_{3\mu} \equiv \bar{\Psi}\gamma_\mu\Delta\Psi,$$

$$\mu = 0,1,2$$

transform as *triplets* under SU(2). The SU(2) singlets are

$$\mathcal{A}_4 \equiv \bar{\Psi}\Delta\Psi, \quad B_{4,\mu} \equiv \bar{\Psi}\gamma_\mu\Psi, \quad (3.12)$$

i.e., the singlets are the parity-violating mass term and the four-component fermion number.

In two-component notation for the spinors Ψ, the above bilinear functions read[12]

$$\mathcal{A}_1 \equiv -i[\bar{\Psi}_1\Psi_2 - \bar{\Psi}_2\Psi_1], \quad \mathcal{A}_2 \equiv \bar{\Psi}_1\Psi_2 + \bar{\Psi}_2\Psi_1,$$

$$\mathcal{A}_3 \equiv \bar{\Psi}_1\Psi_1 - \bar{\Psi}_2\Psi_2,$$

$$B_{1\mu} \equiv \bar{\Psi}_1\sigma_\mu\Psi_2 + \bar{\Psi}_2\sigma_\mu\Psi_1,$$

$$B_{2\mu} \equiv i[\bar{\Psi}_1\sigma_\mu\Psi_2 - \bar{\Psi}_2\sigma_\mu\Psi_1],$$

$$B_{3\mu} \equiv \bar{\Psi}_1\sigma_\mu\Psi_1 - \bar{\Psi}_2\sigma_\mu\Psi_2, \quad (3.13)$$

$$\mathcal{A}_4 \equiv \bar{\Psi}_1\Psi_1 + \bar{\Psi}_2\Psi_2, \quad B_{4,\mu} \equiv \bar{\Psi}_1\sigma_\mu\Psi_1 + \bar{\Psi}_2\sigma_\mu\Psi_2,$$

$$\mu = 0,1,2,$$

with Ψ_i denoting two-component Dirac spinors. For later convenience we have passed onto a three-dimensional Euclidean lattice formalism, in which $\tilde{\Psi}$ is identified with $\Psi^\dagger$ [cf. Eq. (2.39)]. In this convention the bilinear functions (3.13) are Hermitian quantities. It is this Euclidean formalism that we shall use for our lattice treatment in section 4. (On the continuum, of course, $\tilde{\Psi} = \Psi^\dagger\gamma_0$, with γ_0 a 2×2 Dirac matrix, and the Hermiticity properties of the bilinear functions depend on the representation of the Clifford algerba chosen.[12])

One may gauge the above group SU(2) and arrive at the continuum action (3.4), which as we discussed above describes the low-energy continuum field theory limit of the large-U Hubbard model (2.29) and (2.35). In this way, as we shall discuss below, one can generate the fermion condensate $\mathcal{A}_3$ dynamically. In this context, energetics prohibits the generation of a parity-violating gauge invariant SU(2) term[30] and so a parity-conserving mass term necessarily breaks[12] the SU(2) group down to a τ_3 U(1) sector,[6] generated by the σ_3 Pauli matrix in two-component notation.

C. Connection with superconductivity

We now compare the model presented in this article and that of Ref. 6, which is known to exhibit unconventional parity-invariant superconductivity, upon coupling the system to external electromagnetic potentials A_μ. First we note that there is an important physical difference between the two models, concerning the mechanism for mass generation. In our model in this article the gauge group that generates dynamically the fermion mass term is the strongly coupled statistical $U_S(1)$, while the τ_3 U(1) remnant of the weakly coupled SU(2) group is weakly coupled and as such incapable of inducing mass generation. On the other hand, in Ref. 6 the fermion gap that led to superconductivity was due to the τ_3 U(1) gauge boson. This may lead to important differences between the finite-temperature phase diagrams of the two models. Such studies are left for future investigations.

Nevertheless, as far as the mechanism of superconductivity is concerned, the two models appear to be qualitatively similar and it is in this sense that the large-spin treatment of Ref. 6 is justified by the results of the present work. Indeed, the global $U_{em}(1)$ symmetry, which is a subgroup of the
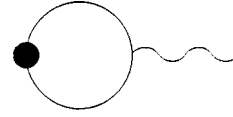

FIG. 1. Anomalous one-loop Feynman matrix element, leading to a Kosterlitz-Thouless-like breaking of the electromagnetic $U_{em}(1)$ symmetry, and thus superconductivity, once a fermion mass gap opens up. The wavy line represents the SU(2) gauge boson B_μ^3, which remains massless, while the filled circle denotes an insertion of the fermion-number current $J_\mu = \bar{\Psi}\gamma_\mu\Psi$. Continuous lines represent fermions.

local symmetry of the ansatz (2.17), corresponds to the *electromagnetic symmetry* in the statistical model. This symmetry can be gauged by coupling the action (3.3) to an external electromagnetic field on the spatial plane as in Ref. 6.

As discussed there, then, *superconductivity* is obtained upon the opening of the gap in the fermion (hole) spectrum due to the one-loop anomalous effect corresponding to the following Feynman matrix element, depicted in Fig. 1:

$$\mathcal{S}^a = \langle B_\mu^a | J_\nu | 0 \rangle, \quad a = 1,2,3, \quad J_\mu = \bar{\Psi}\gamma_\mu\Psi, \quad (3.14)$$

with Ψ four-component spinors, which correspond to the continuum limit of Eq. (2.37). It should be stressed that as a result of the color group structure only the massless B_μ^3 gauge boson of the SU(2) group, corresponding to the σ_3 generator in two-component notation, contributes to the graph. The result is[7,6]

$$\mathcal{S} = \langle B_\mu^3 | J_\nu | 0 \rangle = (\text{sgn } M)\epsilon_{\mu\nu\rho}\frac{p_\rho}{\sqrt{p_0}}, \quad (3.15)$$

where M is the parity-conserving fermion mass (or the holon condensate in the context of the doped antiferromagnet). This observation is consistent with the symmetry-breaking patterns of the $U_{em}(1)$ group since the B_μ^3 color component remains massless and therefore plays the role of the Goldstone boson.[6] As discussed in Refs. 7 and 6, this unconventional symmetry breaking, however, does not have a local order parameter and thereby resembles, but is not identical to, the Kosterlitz-Thouless mode of symmetry breaking.[34] The massless gauge boson B_μ^3 of the unbroken U(1) subgroup of SU(2) is responsible for the appearance of a massless pole in the electric current-current correlator,[6] which is the characteristic feature of any superconducting theory. In this sense, in Ref. 6 the field B_μ^3, or rather its *dual* ϕ defined by $\partial_\mu\phi \equiv \epsilon_{\mu\nu\rho}\partial_\nu B_\rho^3$, was identified with the Goldstone boson of the broken $U_{em}(1)$ (electromagnetic) symmetry. In the non-Abelian context there are also Goldstone bosons associated with the breaking of the SU(2) symmetry.[12] These will be discussed in the next subsection.

IV. DYNAMICAL GAUGE SYMMETRY BREAKING ON THE LATTICE

In this section we derive the symmetry-breaking patterns and discuss in detail the excitation spectrum of the theory obtained from the effective long-wavelength *lattice* action (3.3). We are interested in the effective action of the holon degrees of freedom, after integrating out the fractional-

statistics $U_S(1)$ field. From the above discussion it becomes obvious that this field plays an auxiliary role in the spin separation ansatz and as such it should be integrated out in the effective action of the physical degrees of freedom.

We shall concentrate on the $\beta_1 = 0$ strong-coupling limit for the $U_S(1)$, which from the point of view of the doped Hubbard model corresponds to an infinite-U limit. In this limit the $U_S(1)$ gauge field may be easily integrated out in the path integral with the result[12]

$$\int dV \, d\bar{\Psi} \, d\Psi \, \exp(-S_{\text{eff}}), \qquad (4.1)$$

where

$$S_{\text{eff}} = \beta_2 \sum_p \, (1 - \text{tr} \, V_p) + \sum_{i,\mu} \, \ln I_0(\sqrt{y_{i,\mu}}),$$

$$y_{i\mu} \equiv K^2 \bar{\Psi}_i(-\gamma_\mu) V_{i\mu} \Psi_{i+\mu} \bar{\Psi}_{i+\mu}(\gamma_\mu) V_{i\mu}^\dagger \Psi_i, \qquad (4.2)$$

and I_0 is the zeroth-order Bessel function. The quantity $y_{i\mu}$ may be written in terms of the bilinear functions

$$M_{ab,\alpha\beta}^{(i)} \equiv \Psi_{i,b,\beta} \bar{\Psi}_{i,a,\alpha}, \qquad (4.3)$$

where a, b denote the color groups, α, β denote the Dirac group, and i is the lattice site. The result is

$$y_{i\mu} = -K^2 \, \text{tr}[M^{(i)}(-\gamma_\mu) V_{i\mu} M^{(i+\mu)}(\gamma_\mu) V_{i\mu}^\dagger]. \quad (4.4)$$

In the analogous language of particle physics,[12] the quantities $M^{(i)}$ would represent physical *meson* states. In the context of our spin-charge separation ansatz the mesons would be composite states of holons. We have already seen that the physical electrons are composites of magnon and holons. In the theory (3.3) the magnon degrees of freedom have been integrated out. In this context, the low-energy (long-wavelength) effective action is written as a path integral in terms of gauge field and meson states[12]

$$Z = \int [dV \, dM] \exp\left(-S_{\text{eff}} + \sum_i \, \text{tr} \ln M^{(i)}\right), \quad (4.5)$$

where the meson-dependent term comes from the Jacobian in passing from fermion integrals to meson ones.[33]

In Ref. 12 a method was presented for identifying the symmetry-breaking patterns of the gauge theory (3.3) by studying the dynamically generated mass spectrum. The method consists of first expanding $\Sigma_{i,\mu} \ln I_0(\sqrt{y_{i,\mu}})$ in powers of $y_{i\mu}$ and concentrating on the lowest orders, which will yield the gauge boson masses, while higher orders describe interactions. Keeping only the linear term in the expansion yields[12]

$$\ln I_0(\sqrt{y_{i\mu}}) \simeq \tfrac{1}{4} y_{i\mu} = -\tfrac{1}{4} K^2 \, \text{tr}[M^{(i)}(-\gamma_\mu) V_{i\mu} M^{(i+\mu)}$$

$$\times (\gamma_\mu) V_{i\mu}^\dagger]. \qquad (4.6)$$

It is evident that symmetry-breaking patterns for SU(2) will emerge out of a nonzero vacuum expectation value (VEV) for the meson matrices $M^{(i)}$.

Lattice simulations of the model (3.3), with only a global SU(2) symmetry, in the strong $U_S(1)$ coupling limit $\beta_1 = 0$ and in the quenched approximation for fermions have

shown[35] that the states generated by the bilinear functions $\mathcal{A}_1$ and $\mathcal{A}_2$ [cf. Eq. (3.11)] are massless and therefore correspond to Goldstone bosons, while the state generated by the bilinear function $\mathcal{A}_3$ is massive. In the context of our statistical model [cf. Eq. (2.39)] these meson states may be expressed in terms of the holon operators as

$$\mathcal{A}_{1,i} = -i(\bar{\Psi}_1 \Psi_2 - \bar{\Psi}_2 \Psi_1)_i = -2i(\psi_1^\dagger \psi_2 - \psi_2^\dagger \psi_1)_i,$$

$$\mathcal{A}_{2,i} = (\bar{\Psi}_1 \Psi_2 + \bar{\Psi}_2 \Psi_1)_i = -2(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1)_i \qquad (4.7)$$

and the bilinear function $\mathcal{A}_3$ is given by [see Eq. (2.40)]

$$\mathcal{A}_{3,i} = (\bar{\Psi}_1 \Psi_1 - \bar{\Psi}_2 \Psi_2)_i = -2(\psi_1^\dagger \psi_1 - \psi_2^\dagger \psi_2)_i. \quad (4.8)$$

The fact that members of the triplet SU(2) representation acquire different masses is already evidence for symmetry breaking. We shall confirm this explicitly later on. For the moment we note that lattice analyses[35,36] show that in the strong-coupling limit $\beta_1 = 0$ the condensate $u \equiv \langle \mathcal{A}_3 \rangle$ and the mass of $\mathcal{A}_3$ are infinite. Of course the masses and the condensate are finite for finite β_1, which is the case of finite-U Hubbard models [cf. Eq. (2.34)]. In addition, in this approximation this is the only meson state that develops a nonzero VEV. This therefore constitutes a prediction for the infinite-U Hubbard model and the spin-separation ansatz (2.17). The fact that the VEV of the Goldstone boson states $\mathcal{A}_{1,2}$ vanish implies the absence of a spin flip (on average) at a site $\langle \psi_{1,i}^\dagger \psi_{2,i} \rangle = \langle \psi_{2,i}^\dagger \psi_{1,i} \rangle = 0$, which is also consistent with the slave-fermion constraints (2.20). This is also comforting from the point of view of the equivalence of the above $U \rightarrow \infty$ Hubbard model with that of Ref. 6, whose symmetry-breaking dynamical patterns are characterized by the absence of a local order parameter. (The absence of VEV's for the Goldstone bosons $\mathcal{A}_{1,2}$ eliminates a potentially dangerous source of a possible appearance of a local order parameter in the model. Notice that the dynamical breaking of the electromagnetic U_{em} symmetry as a result of the holon condensate occurs without a local order parameter.[6])

One has the following expansion for the meson states in terms of the SU(2) bilinear functions (3.13):[12]

$$M^{(i)} = \mathcal{A}_3(i)\sigma_3 + \mathcal{A}_1(i)\sigma_1 + \mathcal{A}_2(i)\sigma_2 + \mathcal{A}_4(i)\mathbf{1} + i[B_{4,\mu}\gamma^\mu$$

$$+ B_{1,\mu}(i)\gamma^\mu \sigma_1 + B_{2,\mu}\gamma^\mu \sigma_2 + B_{3,\mu}\gamma^\mu \sigma_3], \qquad (4.9)$$

with $\mu = 0,1,2$, γ_μ (anti-Hermitian) Dirac (space-time) 2×2 matrices, and σ_i, $i = 1,2,3$, the (Hermitian) 2×2 SU(2) color. Pauli matrices. Note that the VEV of the matrix $\langle M^{(i)} \rangle = u\sigma_3$ is proportional to the chiral condensate. Upon substituting Eq. (4.9) into Eq. (4.6), taking into account that the SU(2) link variables may be expressed as

$$V_{i\mu} = \cos(|\mathbf{B}_{i\mu}|) + i \, \boldsymbol{\sigma} \cdot \mathbf{B}_{i\mu} \, \sin(|\mathbf{B}_{i\mu}|) / |\mathbf{B}_{i\mu}|, \quad (4.10)$$

and performing a naive perturbative expansion over the fields $\mathbf{B}$, one finds

$$\ln I_0(\sqrt{y_{i\mu}}) \propto K^2 u^2 [(B_{i\mu}^1)^2 + (B_{i\mu}^2)^2] + \text{interaction terms}.$$

$$(4.11)$$

From this it follows that two of the SU(2) gauge bosons, namely, the B^1, B^2, become massive, with masses proportional to the chiral condensate u:

$$B^{1,2} \text{ boson masses} \propto K^2 u^2, \qquad (4.12)$$

while the gauge boson B^3 remains massless.

This mass term breaks SU(2) to a U(1) subgroup, and in view of the above analysis one recovers the effective action for the massless modes occurring in the large-spin treatment of Ref. 6 and reviewed in Sec. II. It is understood that a full analysis for finite values of β_1 is necessary before definite conclusions are reached in connection with the exact properties and physical implications of the ansatz (2.17) for finite-U doped Hubbard, or t-j, models. We hope to return to these issues in the future.

We would like now to draw the reader's attention to the similarity of the above mechanism for symmetry breaking with the situation in the adjoint gauge-Higgs model.[37] There the SU(2) symmetry is also broken down to a U(1) whenever the constant multiplying the Higgs-gauge interaction is larger than a critical value. In our case the role of this constant is played by K^2, as can be seen by the formal analogy between the adjoint-Higgs-gauge interaction terms and Eq. (4.6). Of course, in our approach symmetry breaking was achieved due to the infinitely strong $U_S(1)$ coupling. In view of the above analogy with the adjoint-Higgs model,[37] however, one may speculate that interesting phase diagrams for the symmetry breaking of SU(2) could also emerge due to the K^2 coupling, in a way independent of the $U_S(1)$ coupling. In this respect, we would like to stress once again that in the context of our statistical models[19] the amplitude K is proportional to the doping concentration in the sample, $K \propto J\eta$. Since the adjoint-Higgs-like symmetry breaking requires strong enough coupling, then the above analysis, if true in this context, may be seen to suggest a natural and simple explanation, in the context of a gauge theory of the fact that in planar antiferromagnetic models of finite-U Hubbard or t-j type, antiferromagnetic order is destroyed, in favor of superconductivity, *above* a critical doping concentration. As mentioned at the end of Sec. II, this point of view seems to be supported by preliminary results of lattice simulations.[31] More detailed investigations along this line of thought are left for future work.

V. CONCLUSIONS AND OUTLOOK

In this article we have discussed lattice models for *planar* spin-$\frac{1}{2}$ Heisenberg antiferromagnets away from half filling (doped). We have worked in the infinite $U \to \infty$ limit of the Hubbard model, which is characterized by the Gatzwyler projection, namely, a constraint of *no more than one electron per lattice site*. Upon implementing a spin-charge separation ansatz (2.17) in a way consistent with holon spin flip, we have argued that the *doped* model is still characterized by a local SU(2)$\times$U$_S$(1)$\times$U$_{em}$(1) symmetry upon coupling to external electromagnetic fields. Of these, the U$_S$(1) is an auxiliary ''statistical'' gauge symmetry, associated with the fractional statistics of the spin and charge excitations in the ansatz (2.17). This possibility arises because of the planar spatial structure of the lattice model.

We have argued that for strong enough U$_S$(1) couplings, dynamical generation of a holon condensate can occur, with the result of breaking the SU(2) group to τ_3 U(1). This is the same local phase symmetry as the one characterizing super-

conducting effective theories of doped antiferromagnets in large-spin $S \to \infty$ treatments,[9,6] although the mechanisms for mass generation are different. Nevertheless, the superconductivity scenarios appear qualitatively similar. In this way we have explained two things in a dynamical way: (i) the *breaking*, as a result of *doping*, of the local SU(2) spin symmetry that characterizes half-filled large-U Hubbard models and (ii) the qualitative justification of large-spin treatments and in particular the suppression of intrasublattice hopping of holes. Indeed, the latter is associated with massive SU(2) gauge boson states, which acquire their masses through holon condensation. There are many features of the models that still have to be worked out. Finite-U treatments and extension of these ideas to t-j models are worth pursuing. Given the dependence of the coupling constants of such models on the doping concentration in the sample, then a renormalization-group study of the respective phase diagrams could provide useful quantitative information on the order of magnitude of the maximum doping concentration for superconductivity and, in general, shed more light on the physics of the spin-charge separation in the models. We hope to arrive at a more systematic study of such issues in the future.

Further consistency checks of our approach may also come from a study of the renormalization-group structure of the *normal phase* of the model in the infrared. By normal phase we mean the phase where there is no dynamical opening of a gap. In this respect we mention that in three space-time dimensions the natural coupling constant appearing in the Lagrangian of a U(1) gauge theory with fermions is a parameter with dimensions of $\sqrt{\text{mass}}$. In analytic Schwinger-Dyson treatments one can define a dimensionless coupling, which is essentially the ratio of the coupling constant over a characteristic mass scale of the theory, playing the role of the ultraviolet cutoff.[27] In a recent series of papers,[38] it was argued that this dimensionless coupling decreases slowly with the momentum scale. Its growth towards the infrared regime, however, is cut off by the appearance of a *nontrivial infrared fixed point*. The latter phenomenon is responsible for deviations from Fermi-liquid behavior[39,38] and, if the infrared fixed-point value of the coupling is strong enough,[27] also for mass generation. These features are expected to persist in the present model. However, in the present case, the full non-Abelian SU(2)$\times$U$_S$(1) symmetry will be present in the normal phase. A full analysis along the lines of Ref. 38 remains to be done.

Above we have dealt with relativistic low-energy limits, obtained by linearizing about specific points on the Fermi surface for the holons. As argued in Ref. 38, this may still capture certain qualitative features of realistic nonrelativistic holon models. Eventually, one would like to be able to extend quantitatively the above results to nonrelativistic case as well. We mention, however, that our relativistic limits may be related to condensed-matter systems with Fermi surfaces that have nodes. Such systems are known to exist in nature and in particular they are antiferromagnetic planar systems with a strong spin-chain anisotropy as far as Heisenberg interactions are concerned. Upon doping and linearization around holon Fermi-surface nodes, one might then obtain the effective relativistic models discussed in this work and in Ref. 6.

An important issue we would like to raise as a result of the present work is the fact that non-Abelian local gauge symmetries, arising in the strong-U Hubbard antiferromagnets, imply the possibility of existence of nonperturbative effects (monopole instantons in the form of hedgehog configurations, etc.). Their precise role in the superconductivity mechanism associated with these models needs to be investigated in detail.[6,13] This becomes particularly important in view of the claimed association of this scenario for superconductivity with Kosterlitz-Thouless–like phase transitions.[6] There are important similarities between the two scenarios: since both are characterized by the absence of local order parameters for the Goldstone bosons associated with the symmetry breaking. It is known that in Kosterlitz-Thouless transitions the symmetry breaking occurs when nonperturbative degrees of freedom are liberated. A preliminary analysis[6,13] in the effective theory model of Ref. 6, which, as a result of the present work, may be viewed as an effective theory of the massless degrees of freedom of the non-Abelian case, has shown that nonperturbative effects appear to be bound in pairs in the superconducting phase. This issue, however, deserves further investigations that require going beyond perturbation theory.

In this latter respect, we mention that the treatment of nonperturbative effects requires *exact* results. Of course the superconductivity mechanism advocated in Ref. 6 occurs through an anomaly, which is an exact one-loop result. However, this is not sufficient for an exact quantitative treatment of the low-energy effective action. However, it is known that exact results in effective action treatments in higher than one spatial dimension can be derived in certain *supersymmetric non-Abelian* gauge theories, as a result of special nonrenormalization theorems and strong-coupling–weak-coupling duality symmetries.[40] In such theories, one invokes a duality symmetry to map a strongly coupled problem to a weakly coupled dual model that can be solved exactly.

We now remark that t-j models are known, under certain restrictions among their parameters, namely, $t=j$, to exhibit *hidden supersymmetries* in space-time.[41] There are graded algebras among the three possible states on a lattice site of the t-j model:[41] $|a\rangle = \{|0\rangle, |1\rangle, |2\rangle\}$, corresponding to the empty, spin-up, and spin-down states, respectively. The model is supersymmetric up to a shift in the chemical potential, in the sense that there exist two supercharge operators Q_σ^+, $\sigma = 1,2$ [SU(2) spin index], connecting Fermi and Bose sectors and leaving the action invariant. So far this supersymmetry structure was not given any dynamical significance. This is because this supersymmetry refers to electron operators. Our ansatz (2.17), however, which implies electron substructure, when and if extended to this case, might imply hidden supersymmetries among holon and spinons. These might have nontrivial consequences on the dynamics, following the spirit of Ref. 40, provided one could extend it to this case. In such a context, the superconductivity model of Ref. 6 could be viewed as an *effective* theory of the light degrees of freedom, arising in the gauge symmetry-broken phase of a supersymmetric SU(2)$\times$U(1)$\times$U$_{em}$(1) field-theory model of a doped antiferromagnet with $t=j$.

At present, we lack any microscopic dynamics underlying Eq. (2.17) that would allow us to check on its generalization to the $t=j$ case and on the existence of the above-conjectured supersymmetric structure. At any rate, we believe that our work of associating holon condensation with a dynamical breaking of a Yang-Mills gauge theory in doped antiferromagnetic planar systems is an interesting observation, which deserves further serious investigations. We hope to return to a study of some of the above-mentioned issues in due course.

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