Interference effect heat conductance in a Josephson junction and its detection in an rf SQUID

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(Received 4 August 1997)

The energy current through a superconductor-insulator-superconductor Josephson junction consists of a quasiparticle current, an interference current, and a pair current. The quasiparticle part represents the normal dissipative heat current. This part is shown to have a unique temperature dependence. The other two parts depend on the phase drop across the junction θ. When the junction is biased by a fixed temperature drop, the interference current can flow in *either* direction, depending on the sign of $\cos\theta$. This gives rise to an effect in which the total heat current oscillates with the phase drop across the junction. We suggest an experimental setup involving an rf superconducting quantum interference device, which is designed to measure these effects. [S0163-1829(98)03506-1]

The total electrical current through a Josephson junction is usually described by two independent currents:[1,2] one is a normal dissipative current which gives rise to thermoelectric transport—it corresponds to the BCS quasiparticles; the other is an equilibrium supercurrent known as the Josephson current. However, as pointed out by Josephson, there exists a third electrical current that flows through a Josephson junction. This is referred to as the *interference current*, which is understood as a superposition of tunneling of normal electrons and of tunneling of superconducting pairs.[3]

In previous publications[4,5] we studied thermoelectric and thermal transport in superconductor-insulator-superconductor (SIS) Josephson junctions. Usually, thermoelectric effects are attributed to quasiparticle transport. However, we found that the interplay (or interference) between quasiparticles and pairs in the superconducting electrodes comprising the junction gives rise to new thermal and thermoelectric transport phenomena in these systems. In particular, in Ref. 5 we found that the heat current through the junction can be regulated by controlling the superconducting phase difference across the junction. This effect ensues from an anomalous energy transfer term that is analogous to the electrical interference current. In this paper we propose an experiment designed to detect this phase-dependent heat current. We also demonstrate the anomalous temperature dependence of the heat current.

The analytical calculation of the energy current in a Josephson junction, presented in Ref. 5, was based on microscopic theory. In order to calculate the total energy transfer across the SIS junction we employed perturbation theory, similar to the derivation of the electrical current presented by Ambegaokar.[6] The total energy current, flowing from left to right, will be denoted by Q^l_{tot}. The result can be written as a sum of three parts:

$$Q^l_{\text{tot}}=Q^l_{\text{qp}}+Q^l_{\text{qp-pair}}\cos\theta+Q^l_{\text{pair}}\sin\theta, \tag{1}$$

where θ is the superconducting phase difference across the junction. The full expressions are given in Ref. 5.

The form of Eq. (1) is analogous to the expression for the total electric current in a Josephson junction.[3] The first term is just the normal heat current which is carried by the quasiparticles. This is the current that is derived by employing the golden rule.[7] The other two terms in Eq. (1) are related to the occurrence of pair tunneling in the junction, and thus depend on the phase drop across the system. The last term on the right-hand side (RHS) of Eq. (1) is analogous to the Josephson current, whereas the middle term resembles the interference current and will be referred to as the interference energy current. The pair related energy currents produce no dissipation in the system. This issue is explained in detail in Ref. 4.

In this paper we focus on the case where the junction is biased by a temperature drop across the junction δT, but the voltage across it vanishes. We are interested in the behavior of the heat conductance of a Josephson junction. (Note that we do not include the contribution of lattice vibrations to the heat conductance.) It turns out that there is only a contribution from Q_{qp} and $Q_{\text{qp-pair}}$ (henceforth we omit the superscript l in the notation of the heat current). According to Ref. 5 the total heat current is then

$$\begin{aligned}Q_{\text{tot}}&=Q_{\text{qp}}+Q_{\text{qp-pair}}\cos\theta\\&=\frac{8\pi N_l N_r|T_{lr}|^2\delta T}{\hbar T}\int_{\Delta(T)}^{\infty}dw\left(-\frac{df}{dw}\right)\\&\quad\times\frac{w^2[w^2+\Delta^2\cos\theta]}{\sqrt{w^2-\Delta^2(T+\delta T)}\sqrt{w^2-\Delta^2(T)}},\end{aligned} \tag{2}$$

where $f(w)$ is the Fermi distribution function of the quasiparticles. The density of states N_l, N_r and the tunneling matrix element $|T_{lr}|^2$ were taken at the Fermi energy.

The first term in the square brackets on the RHS of Eq. (2) leads to the quasiparticle contribution to the heat current in the superconducting state Q_{qp}. The temperature dependence of Q_{qp} is illustrated in Fig. 1. In order to calculate this contribution we approximated $\Delta\approx 4k_BT_c\sqrt{1-T/T_c}$, where k_B is the Boltzmann coefficient, and where T_c is the superconducting phase transition temperature. [This is a good approximation even at low temperatures due to the exponential dependence on temperature of the integrands in Eq. (2).] For comparison, we also plotted in Fig. 1 the heat current for the

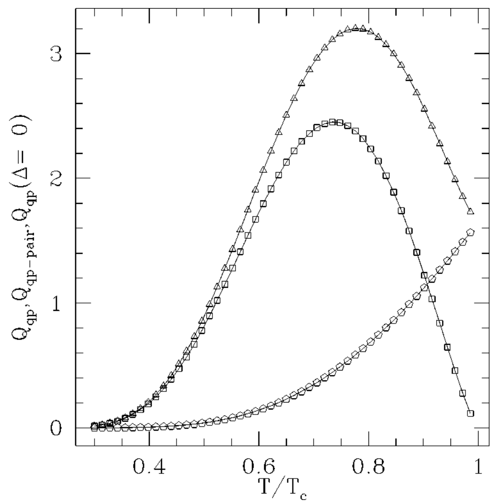

FIG. 1. Heat currents Q_{qp}, $Q_{qp\text{-pair}}$ that flow through the junction as function of temperature, for $T<T_c$. Actually, we only plot the results of the integrals in Eq. (2), divided by k_BT and k_BT_c, which are thus dimensionless and independent of δT and of the tunneling matrix element and densities of states. The triangles represent Q_{qp} which flows in the direction of the temperature drop, and the squares represent the maximum value of the interference energy current $Q_{qp\text{-pair}}$ (i.e., $\cos\theta=1$). This latter term can flow in either direction depending on the magnetic flux. The third curve (pentagons) represents Q_{qp} when the junction is in the normal state, $\Delta=0$. As expected, this coincides with Q_{qp} of the superconducting state at T_c. Note that the heat conductance in the superconducting state exceeds the normal state value, in contrast with the rule for bulk superconductors. The temperature drop across the junction was taken as 0.03 K.

case of a normal metallic junction, i.e., $\Delta=0$. As expected these two plots converge at the transition temperature.

We see that the quasiparticle heat conductance in a Josephson junction shows a behavior unlike the heat conductance in a bulk superconductor. The tunneling heat current is not monotonic in temperature for $T<T_c$ and exceeds its value at T_c. By contrast, in bulks the heat conductance approaches T_c monotonically and is bound from above by the value at T_c.[2] The explanation for this anomalous behavior is the following. When the junction is in the superconducting state there are additional processes that carry charge and energy from side to side.[3,5] These processes involve the breaking and recombination of pairs and quasiparticles, as illustrated in Fig. 2. As a result an effective transport of energy quanta Δ augments the quasiparticle heat conductance (and electrical conductance), as long as the electrodes comprising the junction are superconducting.

The second term in the square brackets in Eq. (2) represents an additional effect, in which the energy current depends on the phase drop across the junction. This latter part represents energy flow due to quasiparticle-pair interference, which can be directed *opposite* to the temperature drop across the junction, depending upon the value of the phase drop. This enables us to control the quasiparticle heat conductance by manipulating the interference energy current via the phase drop. The temperature dependence of this term is plotted in Fig. 1 for the maximum case of $\theta=0$. It is evident that the total heat current $Q_{tot}=Q_{qp}+Q_{qp\text{-pair}}\cos\theta$ is always in the direction of the temperature drop.

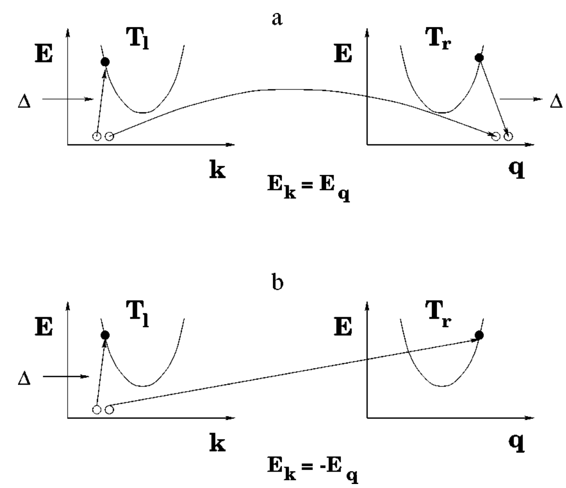

FIG. 2. A schematic description of the additional processes which carry charge and energy across the junction in the superconducting state. On the left and right we have the quasiparticle energy spectrum of the corresponding electrodes. For each of the processes shown we indicate the appropriate conservation law, as dictated by the analytical expressions (Ref. 5).

In order to measure the two effects we suggest the experimental setup illustrated in Fig. 3. We consider a rf superconducting quantum interference device (SQUID) which allows control of the phase drop across a SIS junction by varying the magnetic flux through the ring. A temperature drop across the junction is maintained by connecting one side of the junction to a heat bath T_r and fixing its temperature at a different value than the other side T_l. In order to measure the heat current through the junction, we connect the other side of the junction to a heating device. This can be achieved by using a Peltier circuit, or by attaching a resistor. We will consider the Peltier circuit since the heat current it supplies to the system is proportional to the applied electric current: $Q_{ext}=\Pi I_{ext}$, where Q_{ext} is the heat current generated by the external electrical current I_{ext} (which is controlled by the current source A) and where Π is the Peltier coefficient. This circuit is used to maintain a fixed (and different) temperature

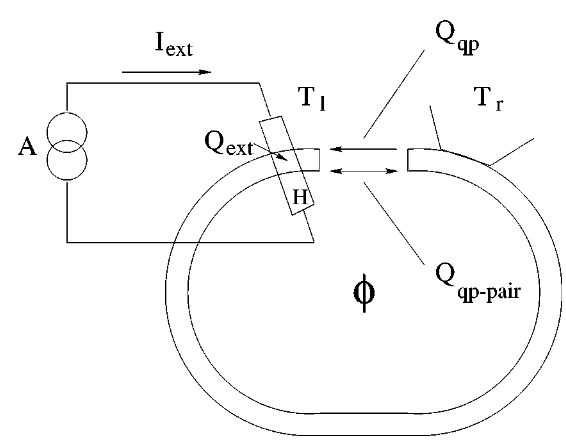

FIG. 3. A schematic description of the suggested experimental setup. The SQUID is threaded with a controllable magnetic flux Φ. Attached to the RHS of the Josephson junction is a heat reservoir at temperature T_r. On the LHS we attach a Peltier device H, which is connected to an external circuit with a feedback mechanism (see text).

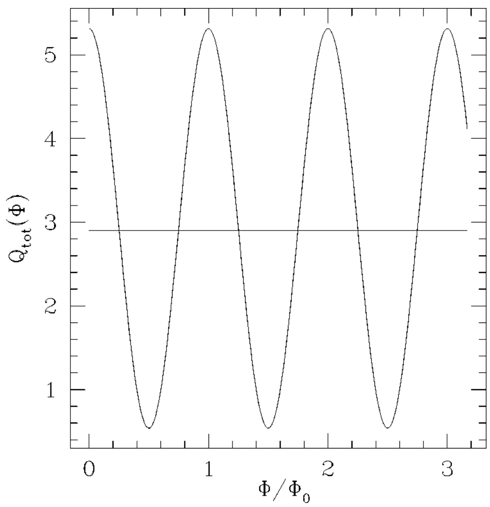

FIG. 4. A numerical solution of the flux dependence of the total heat current through the junction given by Eq. (2). We used the parameters of Fig. 1 and solved for $T/T_c=0.7$. At this temperature $Q_{\rm qp}\sim 2.9$ and the total heat current oscillates around this value. Note that the $Q_{\rm tot}>0$, i.e., heat never flows in the opposite direction of the temperature drop.

on this side of the junction. This can be done for any average temperature, thus enabling a measurement of the temperature dependence of the heat conductance.

Note that the temperature drop across the junction also induces a normal heat current along the superconducting ring. However, this is a small effect with respect to the tunneling heat current through the junction if the system is built such that the temperature gradient along the ring is small compared to the value of $\delta T/x$, where x is the length of the junction. Moreover, the heat current in the superconducting ring should not oscillate with the applied magnetic flux. This effect is unique to the Josephson junction.

The measurement of the heat current through the junction, as a result of the temperature drop δT, is then reduced to measuring the heat current needed from the Peltier device in order to maintain a fixed temperature drop across the junction. [To this end one would require a feedback device which monitors the temperature on the left-hand side (LHS).] This in turn is equivalent to measuring $I_{\rm ext}$ in the Peltier device. Tuning the magnetic field so that $\theta=\pi/2$, one can measure the anomalous behavior of $Q_{\rm qp}$ as function of temperature. Based on the theoretical prediction in Eq. (2), we expect the temperature dependence of $I_{\rm ext}$ to resemble the top curve in Fig. 1. We also predict that $I_{\rm ext}$ will depend on the magnetic flux Φ as $\cos(2\pi\Phi/\Phi_0)$, where Φ_0 is the quantum unit flux. This measurement should look similar to Fig. 4, where we plotted Eq. (2) as the function of the magnetic flux using the parameters of Fig. 1 and choosing $T/T_c=0.7$. The dependence on magnetic flux is due to the relation between the phase drop across the junction and the magnetic flux threading the SQUID

$$\theta=2\pi\frac{\Phi}{\Phi_0}+\int_{\rm ring} dr\nabla\alpha, \tag{3}$$

where Φ is the total flux threading the ring. $\nabla\alpha$ is the continuous superconducting-phase gradient along the wires of the ring, and the integral is along a counterclockwise path.

In order to estimate the value of the heat conductance, we consider a typical Sn-O-Sn junction ($T_c\approx 2$ K) with a normal-junction resistance $R=[4\pi e^2 N_l N_r|T_{lr}|^2/\hbar]^{-1}$ of the order of 0.1 Ω. This gives us an estimate for the coefficient $8\pi N_l(0)N_r(0)|T_{lr}(0)|^2\sim 10^4$ in Eq. (2). The temperature drop across the junction taken in the numerical solution of Fig. 1 was of the order of 0.01 K. Hence, $\delta T/T\approx 0.01$. Substituting these numbers into Eq. (2), the predicted quasiparticle heat conductance through the junction $\kappa\equiv Q_{\rm qp}/\delta T$ is of the order of 10^{-9} W/K. As illustrated in Fig. 4, this value can vary up to about 50% in either direction by applying a magnetic flux, but will always remain positive.

To conclude, we predict two effects in the heat conductance through a Josephson junction. The phenomena result from the interplay between quasiparticles and pairs, and are unique to Josephson junctions. The theoretical work presented here was restricted to tunnel junctions. However, since the electrical interference current was measured in all types of Josephson junctions (e.g., weak links, etc.),[8] we suspect that the phenomena described here can be found in other types of junctions as well. Experimental verification of the effects in all types of Josephson junctions can be carried out as described above.

Partial support for this work was provided by the Israel Science Foundation, the US-Israel Binational Science Foundation, and a grant from the office of the vice president for research at the Tel-Aviv University. We are grateful to M. Ya. Azbel for enlightening discussions. We would like to thank A. Shnirman for stimulating talks.

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