Power-law correlations and orientational glass in random-field Heisenberg models

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Monte Carlo simulations have been used to study a discretized Heisenberg ferromagnet (FM) in a random field on simple cubic lattices. The spin variable on each site is chosen from the 12 [110] directions. The random field has infinite strength and a random direction on a fraction x of the sites of the lattice, and is zero on the remaining sites. For $x=0$ there are two phase transitions. At low temperature there is a [110] FM phase, and at intermediate temperature there is a [111] FM phase. For $x>0$ there is an intermediate phase between the paramagnet and the ferromagnet, which is characterized by a $|\mathbf{k}|^{-3}$ decay of two-spin correlations, but no true FM order. The [111] FM phase becomes unstable at a small value of x. At $x=1/8$ the [110] FM has disappeared, but the power-law correlated phase survives. [S0163-1829(98)07001-5]

I. INTRODUCTION

Orientational glasses have been a subject of significant interest for over 15 years. An extended review of the experiments has been given by Höchli, Knorr, and Loidl,[1] and a companion review of the theories has been provided by Binder and Reger.[2] More specialized reviews of electric dipole glasses[3] and the solid hydrogens[4] are also of interest. A primary question, which has remained unresolved to the present time, has been whether the orientational freezing transition which is seen in the experiments represents a true thermodynamic phase transition, with a definite critical temperature T_c. The alternative point of view is that the freezing should be explained as a kinetic effect, which occurs gradually, over a range of temperature. It is surely true that some of the experiments should be explained as kinetic effects. The question is whether a phase transition of this type is possible in a three-dimensional system with realistic interactions.

It was argued by Michel,[5] who performed a mean-field calculation, that the proper model for studying the orientational glass is an $n=3$ Heisenberg model (where n is the number of spin components) with the addition of a cubic symmetry-breaking term due to the single-ion anisotropy and a random-field term due to the alloy disorder. In this work we will use a Hamiltonian of the form

$$H_{\mathrm{RF}}=-J\sum_{\langle ij\rangle}\sum_{\alpha=1}^{3}S_i^{\alpha}S_j^{\alpha}-K\sum_{i}\sum_{\alpha=1}^{3}(S_i^{\alpha})^4 - G\sum_{i'}(\mathbf{S}_{i'}\cdot\mathbf{n}_{i'}-1), \tag{1}$$

where the sites i form a simple cubic lattice and $\langle ij\rangle$ indicates a sum over nearest neighbors. The α are spin indices, each $\mathbf{n}_{i'}$ is an independently chosen random unit vector, and the i' sites are a randomly chosen subset of the lattice containing a fraction x of the sites. Since the defect sites are assumed to be immobile, the random fields do not change with time. As the random fields only occur on a fraction x of the sites, Eq. (1) is a diluted random-field Heisenberg model. There are several alternative choices of a Hamiltonian for an orientational glass,[2] such as using dipolar exchange or quadrupolar exchange instead of the isotropic exchange chosen here. In our present state of knowledge (or ignorance), it may be believed that this choice does not change the qualitative behavior of the orientational freezing transition. We will comment on this point later.

The G term gives the interaction of the orientational order with the point defects. In this work we will study the strong-pinning case of Fukuyama and Lee,[6] in which G is taken to be large and the density of defect sites, x, is taken to be small. In contrast to the one-dimensional case considered by Fukuyama and Lee, the strong-pinning case becomes highly nontrivial in three dimensions when x is small compared to $1-p_c$, where p_c is the site percolation concentration.[7]

It has been accepted for some time that, in the absence of some symmetry-breaking term such as the cubic term, a random field which couples linearly to the order parameter will always destroy the long-range symmetry-breaking order of a three-dimensional system which has a continuous symmetry. This result was first derived by Larkin,[8] and a simpler version of the argument was later presented by Imry and Ma.[9] The absence of long-range order for systems with short-range interactions in the presence of random fields in less than four dimensions has been given a more rigorous basis in the work of Aizenman and Wehr.[10] The simple Imry-Ma argument[9] also makes the stronger claim that a weak random field will cause the ferromagnetic correlation length to be finite at all temperatures in three dimensions. This has recently been questioned for the case of the XY model (i.e., $n=2$) by Gingras and Huse,[11] who argue that these calculations have not taken proper account of the effects of vortex loops. Gingras and Huse suggest that for weak random fields there may be a phase in which there is quasi-long-range order (QLRO), and the two-point correlations have a power-law decay as a function of distance. A recent Monte Carlo calculation by the author[12] has provided strong evidence in favor of this possibility.

The simple Imry-Ma argument[9] assumes that the low-energy direction of the magnetization $\mathbf{M}$ in some domain is determined by the average value of the random field over that domain. While it is surely true that the $\mathbf{k}=0$ Fourier component of the random field is the most important single

component in a model with ferromagnetic exchange, it is not clear that its influence always outweighs the combined effects of all of the other Fourier components. The Imry-Ma argument gives the correct lower critical dimension[10] for the existence of ferromagnetic order in the presence of a random field. However, the situation in three dimensions is very subtle. When we already know that the $\mathbf{k}=0$ component of $\mathbf{M}$ is not dominant over the other small-wave number components, it is no longer safe to assume that the $\mathbf{k}=0$ component of the random field is the only important component of the random field. Competition between different small-wave-number components of the random field may create multiple low-energy minima of the free energy.

Based on renormalization-group calculations, it was suggested by Mukamel and Grinstein[13] that it might be possible to induce a QLRO phase in an $n=3$ random-anisotropy system by adding a cubic crystalline anisotropy. This occurs because a weak cubic anisotropy does not destabilize the $n=3$ Heisenberg critical point in three dimensions, but it removes the Imry-Ma instability. This should apply to the random-field case as well. Even in the presence of a weak random field, we expect a ferromagnetic phase to exist at low temperatures, but there is no longer a stable renormalization-group critical fixed point which describes the transition. Thus, as argued by Mukamel and Grinstein, either the phase transition must become first order, or else there must be some type of intermediate behavior between the ferromagnet and the paramagnet (PM). In the $n=2$ case it was found[12] that both possibilities may occur, at different values of x.

At our current level of understanding we have no reliable analytical method of predicting whether the QLRO which has been found for the $n=2$ case should also occur for larger values of n. There was another suggestion, by Mayer and Cowley,[14] that QLRO should occur for $n=2$, and also for $n=3$ when the cubic term is present. This argument yields a line of fixed points with continuously varying critical exponents, rather than the zero-temperature fixed point of Gingras and Huse.[11] Both of these analyses seem to use the existence of dislocation lines in an essential way. However, there is no direct evidence in the $n=2$ Monte Carlo results[12] that dislocations are crucial. Based on the work of Mézard and Young,[15] one should expect the existence of replica symmetry breaking in the QLRO phase.

We will find that adding a special type of weak cubic anisotropy does allow us to find a QLRO phase, and we will study some of its properties. Because an orientational glass typically exists in the presence of some crystalline lattice potential, this result may be directly applicable to cases of experimental interest. It should be noted that our model does not allow the existence of any lattice defects such as dislocation lines or stacking faults, although these may be needed to explain some of the experimentally observed behavior. We will not conclusively answer here the question of whether the QLRO phase exists also for the isotropic $n=3$ random-field model, but we will present indirect evidence in favor of this. If it does exist even in that case, its properties should be essentially those that we find here.

II. DISCRETIZED HEISENBERG MODELS

Rapaport[16] has discussed the technical advantages of using a discretization of the sphere for Monte Carlo calculations of the Heisenberg model. He also suggested that this might be a useful way of studying the effects of random fields on this model. There is one significant point which was not made in his work and which will be important here. The discretization of the sphere chosen by Rapaport was the set of 30 unit vectors defined by the centers of the edges of an icosahedron, which we will refer to as the $\mathbf{I}_{30}$ model. In the work presented here we will use the set of 12 unit vectors defined by the edge centers of a cube, which in the standard Cartesian notation are called the [110] vectors. We will refer to this as the $\mathbf{O}_{12}$ model (where $\mathbf{O}$ stands for octahedral). In either case, with the simple Heisenberg exchange Hamiltonian

$$H=-J\sum_{\langle ij\rangle}\sum_{\alpha=1}^{3}S_i^{\alpha}S_j^{\alpha}, \tag{2}$$

where each $\mathbf{S}_i$ is now restricted to the discrete set, rather than the entire sphere, there are now two phase transitions. There is the $n=3$ Heisenberg critical point, which occurs at a slightly higher T_c than in the ordinary Heisenberg model, and, in addition, there is a first-order phase transition at a lower T_c, at which the minimum free-energy directions of $\mathbf{M}$ change.

The nature of the low-temperature transition can be understood in a Landau mean-field treatment. We will discuss the $\mathbf{O}_{12}$ case here, and Rapaport's $\mathbf{I}_{30}$ case is essentially similar. Because we now have only a cubic symmetry of the Hamiltonian rather than the full symmetry of the sphere, the Landau free-energy functional $F(T,\mathbf{M})$ should contain terms proportional to $\Sigma_{\alpha}(M^{\alpha})^{2l}$ for all (positive) integer values of l, in addition to the usual isotropic terms. Since we want to find the minimum of the free energy, it is sufficient to consider only the high-symmetry directions [100], [110], and [111].

The coefficient of the term of order M^{2l} is proportional to $8(1/2)^l$ for [100], $2+8(1/4)^l$ for [110], and $6(2/3)^l$ for [111]. The $l=1$ term is, of course, isotropic, and the $l=2$ term favors the [111] directions. Thus, just below the critical point, where the thermal average $\langle|\mathbf{M}|\rangle$ is small, $\langle\mathbf{M}\rangle$ will be aligned along [111].

All of the terms with $l>2$ favor the [110] directions. Therefore, as we go to still lower T and $\langle|\mathbf{M}|\rangle$ grows, we eventually reach a point where it becomes favorable to rotate $\mathbf{M}$ to a [110] direction. Because there are 8 [111] directions and 12 [110] directions, and 12/8 is not an integer, the Landau rules determine that this transition must be first order.

It is natural to expect that the behavior of the $\mathbf{I}_{30}$ model would be closer to the isotropic limit than that of the $\mathbf{O}_{12}$ discretization used here. This is surely true at most T. For example, it is likely that the reason Rapaport did not report the existence of the low-temperature transition is that in the $\mathbf{I}_{30}$ model it occurs at a T_c which is smaller than the minimum used in his Monte Carlo simulations. However, because the 12 [110] states are not a subset of the 30 icosahedral edge-center states, results for both the $\mathbf{O}_{12}$ model and the $\mathbf{I}_{30}$ model cannot be combined in a simple way to obtain an extrapolation to the isotropic limit.

One might think at first that a similar intermediate-temperature FM phase would occur for the discretization based on the edge centers of a tetrahedron. These six states,

however, are equivalent to the six [100] vectors of a cube. This model has only two phases, with a first-order phase transition between the FM and PM phases.[17]

Our discretization of the spin variables using the 12 [110] vectors automatically builds a cubic anisotropy into the free energy. Thus, since we are only trying to understand the qualitative aspects of orientational ordering and not attempting a quantitative model of a particular experiment, we do not need to keep the K term in the Hamiltonian. If we now take the limit $G\rightarrow\infty$ with each of the $\mathbf{n}_{i'}$ chosen from the set of [110] unit vectors, to simplify the calculation, Eq. (1) reduces in form to Eq. (2), with the dynamical restriction that a fraction x of the spins (the ones on the i' sites) point in fixed random directions, parallel to the local random fields on the i' sites.

It is interesting to note that the stability of the [111] phase is of purely entropic origin. The energy always favors the [110] alignment. A significant consequence of this is that the walls between different [111] domains must be broad, rather than sharply defined. As a result, the [111] FM phase is easily destabilized by a random field.

III. MONTE CARLO CALCULATION

Because all of the $\mathbf{S}_i$ are chosen from the [110] set, Eq. (2) has the helpful property that the energy of every state is an integral multiple of 1/2. Thus it becomes easy to write a Monte Carlo program to study Eq. (2) which uses integer arithmetic to calculate energies. This, plus the fact that each spin has only 12 possible states, gives substantial improvements in performance over working with the general form of Eq. (1), for both memory size and speed. It is also be possible to use integer arithmetic if G is chosen to be an integer. When G is infinite, the random field is implemented by assigning a fraction x of the sites to be the i' sites. The spins on these sites are given random [110] directions and then left fixed for the remainder of the calculation.

The Monte Carlo program used two linear congruential pseudorandom number generators. In order to avoid unwanted correlations, the random number generator used to select which sites would be assigned the random fields was different from the one used to assign the initial $\mathbf{S}_i$. A heat bath method was used for flipping the spins, which at each step reassigned the value of a spin to one of the 12 allowed states, weighted according to their Boltzmann factors and independent of the prior state of the spin.

$L\times L\times L$ simple cubic lattices with periodic boundary conditions were used throughout. The values of L used ranged from 16 to 64. Away from any T_c the samples were run for 10 240 Monte Carlo steps per spin (MCS) at each T, with sampling after each 10 MCS. Near a T_c they were run several times longer. The initial part of each data set was discarded for equilibration. Typically, two different random field configurations with a given L were studied for a given $x>0$. This gives a rather crude estimate of the finite-size dependence of the various thermodynamic properties. Unfortunately, however, in the presence of the random field high-precision finite-size scaling is not a very effective tool, because the sample-to-sample variations for a given size are large and not well behaved.[12,18]

Both random and ordered initial conditions were used for the unpinned spins. For large samples, typically two or three random initial states were tested, and brief tests were made of all of the 12 ordered states. For $x=1/16$ (or less), in a ground state essentially all of the unpinned spins are aligned along one of the [110] directions. Thus, in these cases it is easy to equilibrate the system at low temperatures, by starting from an ordered state. This is not true for $x=1/8$, however.

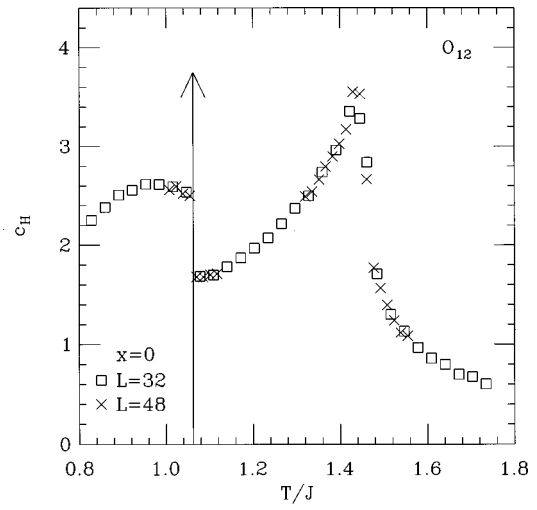

FIG. 1. Specific heat vs temperature for the pure $\mathbf{O}_{12}$ model on $L\times L\times L$ simple cubic lattices. The large vertical arrow indicates a first-order phase transition.

In the absence of any external field, random or uniform, the rotation of $\mathbf{M}$ between different [110] directions is a slow process. Because all of these 12 directions are equivalent, however, there is no need for the Monte Carlo program to average over the minima in this case. In the presence of the random field the different [110] ferromagnetic Gibbs states have different energies. If the system is started in a high-energy [110] direction, it will eventually jump to a more favorable direction (unless T is so low that this does not happen in the time available). In this, as in many other respects, the model behaves like the random-field Ising model.[18]

IV. NUMERICAL RESULTS FOR $x=0$

Specific heat data for the pure ($x=0$) $\mathbf{O}_{12}$ model with $L=32$ and $L=48$, obtained by numerically differentiating the energy, are displayed in Fig. 1. The Heisenberg critical point occurs at $T_{c1}/J=1.453\pm0.001$, which is less than 1% above the value found for the standard isotropic Heisenberg model,[19,20] which is about 1.4430 ± 0.0001. The T_c for the isotropic model is known to greater precision because far more computing resources have been used to calculate it. As argued by Rapaport,[16] given equal resources one should be able to get more than equal precision for the discretized model. Within the accuracy calculated here, the most effective way of estimating T_c is to assume that the energy $E(T_c)$ is the same for both models. Our estimate of T_{c1} for the $\mathbf{O}_{12}$ model is based on $E(T_{c1})=0.994$, as found in the isotropic model.[19,20] It is, however, necessary to consider the L dependence of E. This method also works for the $n=2$ case,[12] and reflects the fact that the two-spin correlation function at the

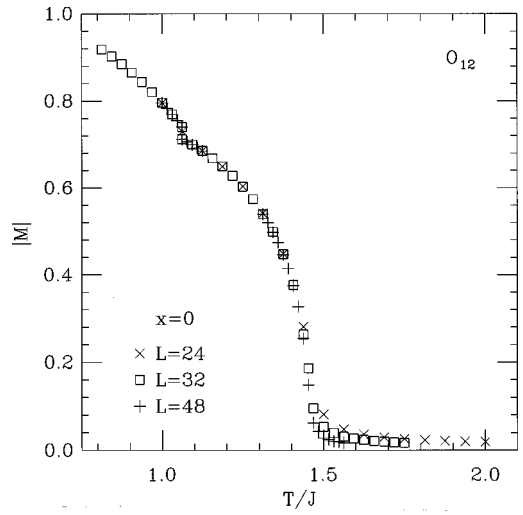

FIG. 2. Magnetization vs temperature for the pure $\mathbf{O}_{12}$ model on $L\times L\times L$ simple cubic lattices.

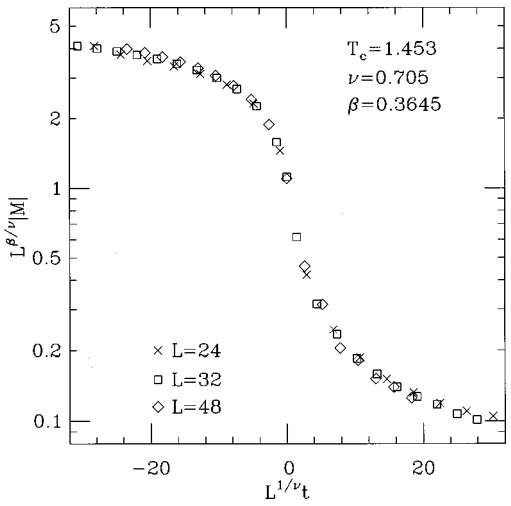

FIG. 3. Finite-size scaling of the magnetization near T_c for the pure $\mathbf{O}_{12}$ model on $L\times L\times L$ simple cubic lattices. The y axis is scaled logarithmically.

Heisenberg critical point is virtually identical at all distances in the discretized model and the isotropic model, even though strict universality only demands that they be identical at large distances. The short-distance part of the two-spin correlation function, and thus $E(T_c)$, does depend on the lattice structure, even in a Bethe-Peierls mean-field treatment.

The first-order transition from the [111] FM phase to the [110] FM phase is indicated in Fig. 2 by the vertical arrow. This transition occurs at $T_{c2}/J=1.0625\pm0.0075$, where the error bar indicates the approximate width of the region of metastability, rather than a statistical error. Measured at $T/J=1.0625$, the latent heat at this transition is measured to be $\Delta Q=0.0561\ J$ for $L=32$, and $\Delta Q=0.0571\ J$ for $L=48$. The observed increase of ΔQ as L increases confirms that the transition is indeed first order. Because the specific heat is substantially larger in the [110] phase than in the [111] phase near T_{c2}, the largest contribution to the uncertainty in ΔQ comes from the uncertainty in T_{c2}.

The magnetization $\langle|\mathbf{M}(T)|\rangle$ is shown in Fig. 2, for $L=24$, 32, and 48. We see that the size dependence rapidly becomes small below $T/J=1.40$. The jump in $|\mathbf{M}|$ at the first-order transition, again using data taken at $T=1.0625$, is 0.0277 for $L=32$ and 0.0286 for $L=48$. The finite-size scaling function for $|\mathbf{M}(T)|$ at the Heisenberg critical point is displayed in Fig. 3. The values of the critical exponents used for this figure are the usual field theory estimates[21] for $n=3$. Thus, if we determine T_c by the energy condition discussed above, there are no free fitting parameters. The width of the scaling region is slightly larger than that of another $n=3$ model[22] for which the cubic anisotropy is probably somewhat stronger.

V. NUMERICAL RESULTS FOR $x>0$

In addition to $x=0$, Monte Carlo data were also obtained at $x=1/32$, 1/16, and 1/8. A semiquantitative picture of the phase diagram obtained from these results is shown in Fig. 4. The limit of stability of the [110] FM ground state is slightly less than $x=1/8$. For the nonzero values of x used in these calculations, the [110] FM has a transition into the QLRO phase. The [111] FM phase, which is stable at $x=0$, should also extend to small positive values of x. As noted above, however, the domain walls in this phase are broad and have a low cost in free energy. Thus, the [111] FM is easily destabilized by the random field. It is difficult to obtain meaningful numerical results for very dilute random fields, due to crossover effects.[23] Therefore, the [111] FM-QLRO phase boundary was not observed directly, and its existence is shown in Fig. 4 as a dotted line.

The QLRO-to-PM transition is second order for small x and first order for larger x, with a tricritical point somewhere between $x=1/16$ and $x=1/8$. The shift in T_c for the QLRO-to-PM transition is linear in x for small x, with a slope of

$$\frac{d}{dx}\left(\frac{T_c(x)}{T_c(0)}\right)=-3.75\pm0.20. \tag{3}$$

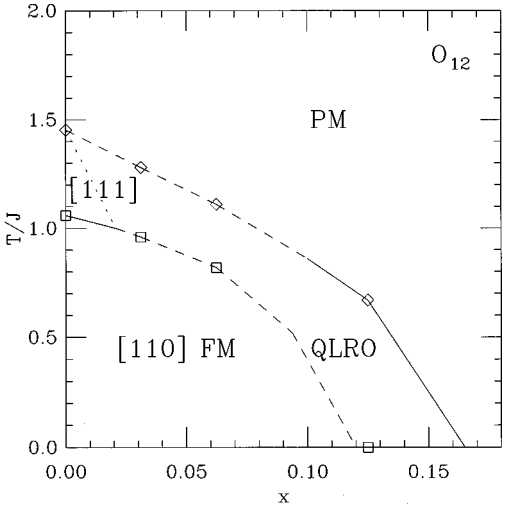

FIG. 4. Phase diagram of the dilute random-field $\mathbf{O}_{12}$ model on simple cubic lattices, showing the paramagnetic (PM), ferromagnetic (FM), and quasi-long-range order (QLRO) phases. The plotting symbols show estimates obtained from the Monte Carlo data. The solid lines indicate first-order transitions, the dashed lines indicate second-order transitions, and there were no data taken on the dotted line.

This should be compared to the corresponding quantity for the $n=2$ case, which is[12] -3.42 ± 0.14. ($x=1/16$ is in the linear regime for $n=2$ also.) Having the shift in $T_c(x)$ increase by about 10% as one goes from the $n=2$ case to the $n=3$ case is about what should be expected if the QLRO phase survives for isotropic spin variables. If the existence of the QLRO was actually dependent on the discretization, the author would expect this shift to be substantially larger for the $\mathbf{O}_{12}$ model than we have found it to be.

It is interesting to note that these numbers are about 3 times as large as the shifts in $T_c(x)$ which one obtains from a simple quenched site dilution,[24] although a naive mean-field approximation predicts that the shift should be the same in the two cases. It is also true in the site dilution case that the size of the shift increases as n increases.

Our results indicate that the QLRO phase still occurs for $x=1/8$ when $n=3$, while for $n=2$ it was found that the limit of stability of the QLRO phase was[12] less than $x=1/8$. This is an indication that the stability of the QLRO phase increases with n. On this basis, we might expect it to occur for all n, and thus its existence would be independent of any topological singularities. This encourages us to hope that it might be possible to study the QLRO using some analytical technique which uses $1/n$ as an expansion parameter. The existence of such an expansion must, for now, be considered mere speculation. It could certainly still be true that the QLRO phase really is of topological origin, in which case it should not exist for $n>3$.

It is difficult to study transitions which occur at low T using Monte Carlo calculations. The $T=0$ end points of both the [110] FM-QLRO boundary and the QLRO-PM boundary in Fig. 4 are illustrated in schematic fashion. The author does not mean to imply that $T_c(x)$ is actually linear in x near the $T=0$ end point in either case.

The evolution of the specific heat as x is increased is shown in Fig. 5. The data displayed were obtained by numerically differentiating the calculated values of the energy with respect to T. The specific heat was also computed by calculating the fluctuations in the energy at fixed temperature, yielding similar but noisier results. We see that the data for different samples with the same value of x agree fairly well, although some differences are visible near the phase transitions.

The sharp peaks which occur for $x=0$ (in Fig. 1) have become rounded at $x=32$, and they have moved to lower T. The QLRO-to-PM transition actually occurs slightly below the T of the upper specific heat peak. The [110] FM-to-QLRO transition appears to be continuous, rather than first order, and the specific heat increases as we approach this transition from either direction. Going on to $x=1/16$, all of these trends are enhanced. At $x=1/8$, the FM phase has disappeared entirely. Due to the long relaxation times, for $x=1/8$ it was not practical to equilibrate an $L=64$ lattice at $T/J<0.65$. The small specific heat peak near $T=0.67$ marks the QLRO-to-PM transition, which is first order at this value of x. For $x=1/32$ and $x=1/16$, where this transition is second order, there is no clear signature of the transition observable in the specific heat.

Looking at the dependence of $\langle|\mathbf{M}|\rangle$ on x and L provides additional insight. The data for $x=1/32$, 1/16, and 1/8 are shown in Fig. 6. In the [110] FM phase, $\langle|\mathbf{M}|\rangle$ is almost

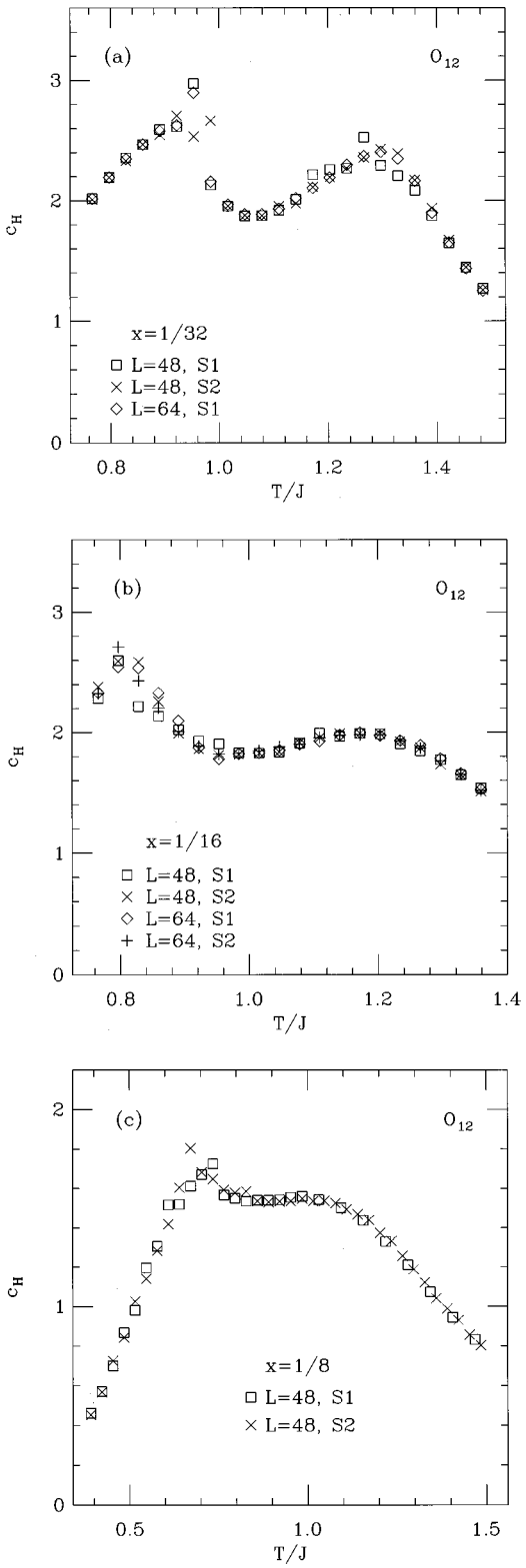

FIG. 5. Specific heat vs temperature for the dilute random-field $\mathbf{O}_{12}$ model on $L\times L\times L$ simple cubic lattices. (a) $x=1/32$, (b) $x=1/16$, and (c) $x=1/8$.

independent of L, except very close to T_c. In the QLRO phase, $\langle|\mathbf{M}|\rangle$ decreases slowly as L increases, probably decaying as $1/\ln(L)$. In the PM phase, $\langle|\mathbf{M}|\rangle$ decreases as $(L/\xi)^{-3/2}$, where ξ is the ferromagnetic correlation length. In Fig. 6(c) we see that the data for large L and T between 0.70 and 1.45 seem to form a set of parallel lines on this semilog

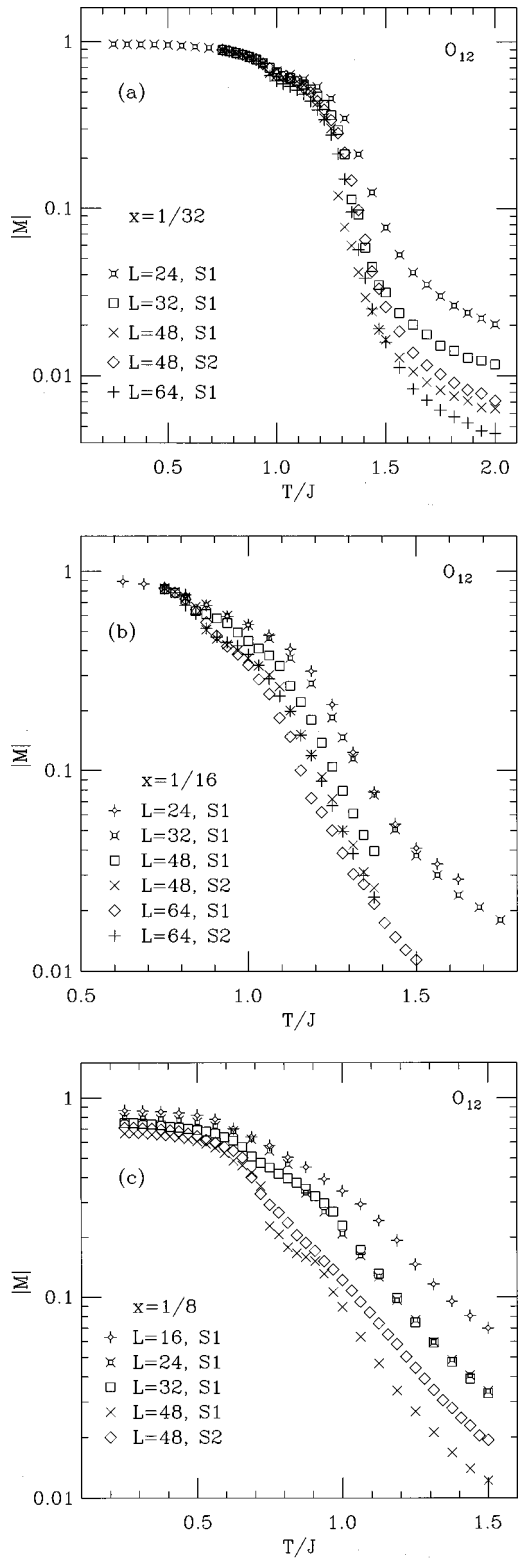

FIG. 6. Magnetization vs temperature for the dilute random-field $\mathbf{O}_{12}$ model on $L\times L\times L$ simple cubic lattices. The y axis is scaled logarithmically. (a) $x=1/32$, (b) $x=1/16$, and (c) $x=1/8$.

plot. This implies that ξ remains constant over this range of temperature for $x=1/8$. The saturation of ξ in the PM phase indicates that the QLRO-to-PM transition is first order at this value of x.

We can get valuable information by looking at the magnetic structure factor of samples of size $L=64$. The structure

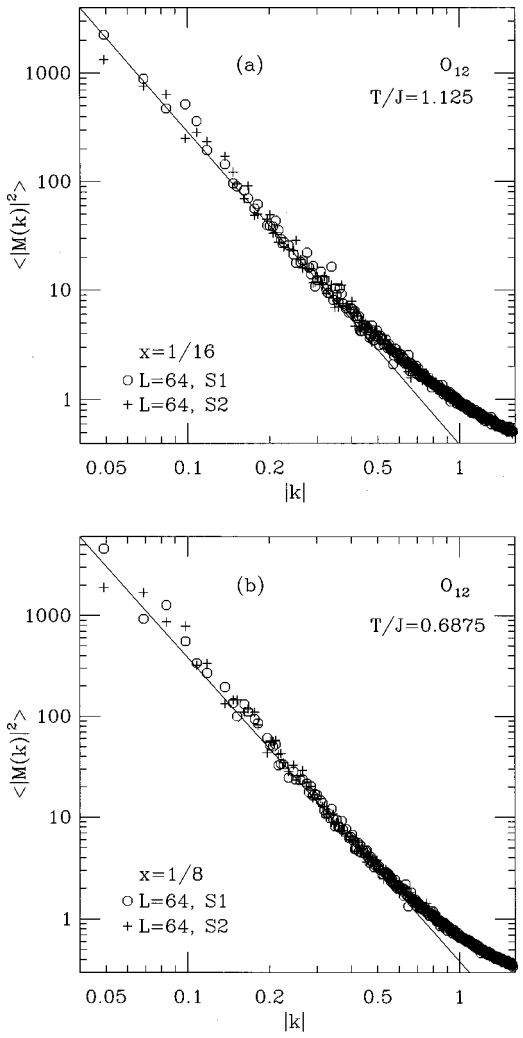

FIG. 7. Angle-averaged magnetic structure factor for the dilute random-field $\mathbf{O}_{12}$ model on $64\times64\times64$ simple cubic lattices, log-log plot. Each data set shows averaged data from 2 states sampled at 10 240 MCS intervals. (a) $x=1/16$, $T=1.125$, the line has a slope of -2.87; (b) $x=1/8$, $T=0.6875$, the line has a slope of -3.00. Note that the vertical scales differ in (a) and (b).

factor is the spatial Fourier transform of $\langle\mathbf{M}^2\rangle$, and it can be measured by x-ray- and neutron-scattering experiments. Near a critical point the small-wave number behavior of the structure factor of a random-field model is expected to have the form

$$|\langle\mathbf{M}(\mathbf{k})\rangle|^2\approx(1/\xi^2+|\mathbf{k}|^2)^{-(4-\overline{\eta})/2}. \tag{4}$$

The correlation length ξ is infinite in the QLRO phase (essentially by definition).

In three dimensions, Eq. (4) requires that $\overline{\eta}\geq1$. To estimate $\overline{\eta}$, we measure the slope of the structure factor on a log-log plot. This is shown, averaged over angles, for two $L=64$ lattices with $x=1/16$ at $T=1.125$, in Fig. 7(a). The slope of the best fit to the data at small $|\mathbf{k}|$ is -2.87 ± 0.05, and so we find

$$\overline{\eta}=1.13\pm0.05. \tag{5}$$

This is essentially the same as the result found for this quantity in the $n=2$ case.[12] It is, well known that the value of an η exponent is often insensitive to the value of n, and so one should not believe on these grounds that the values of $\overline{\eta}$ for $n=2$ and $n=3$ are identical. The data for $x=1/32$ (not shown) have a smaller value of the slope at T_c. This, however, should not be interpreted as measuring the value of $\overline{\eta}$, because these data are taken in the crossover region[23] from pure system behavior, which has a slope of -1.97 for the structure factor at T_c.

Repeating the above procedure for two $L=64$ with $x=1/8$ lattices at $T=0.6875$, using cold start initial conditions, we find the results shown in Fig. 7(b). This value of T is slightly above the best estimate of T_c at this value of x, and the measured values of $\langle|\mathbf{M}|\rangle$ for these data are 0.2518 for $S1$ and 0.2880 for $S2$. Similar states, but with slightly higher energy and lower magnetization, were achieved using random initial states. We see that the structure factor again shows a power-law behavior at small $|\mathbf{k}|$, but that the slope at T_c has now assumed the minimum allowed value of -3. This is the value inside the QLRO phase and yields

$$\overline{\eta_0}=1. \tag{6}$$

This is an independent confirmation of the first-order nature of the transition at this value of x. The latent heat is too small to measure accurately, being less than 0.01 J. The small latent heat may be an indication that we are close to the tricritical point.

Although no first-order behavior was seen along the QLRO-to-PM transition line[12] for $n=2$, it probably exists for x near the end point of the QLRO phase in that case also. The tricritical point on the QLRO-to-PM transition line is likely to have the same origin as the analogous tricritical point which occurs for the random-field Ising model.[25]

VI. DISCUSSION

What we have found is that, despite the weak destruction of the ferromagnetic long-range order caused by the Imry-Ma instability, there is actually a good correspondence in three dimensions between the behavior of the QLRO-to-PM transition in $n=2$ and $n=3$ random-field models and the FM-to-PM transition in the random-field Ising model. The author sees no reason why the QLRO phase should become completely destabilized by a weak uniaxial anisotropy. Therefore, it should be possible for an $n=3$ system with a weak uniaxial anisotropy and a random field to exhibit a QLRO phase between its PM and FM phases. This is, in fact, a good description of the diluted antiferromagnet in a magnetic field systems which serve as the prototypical experimental random-field Ising models. Therefore, the ''metastability'' which is seen in these experimental systems[26] is probably caused by the presence of a QLRO phase, at least those cases where the uniaxial anisotropy is weak.

Since the QLRO phase exists for both $n=2$ and $n=3$ in three dimensions, and possibly for larger values of n also, there cannot be any simple correspondence between the random-field models and the pure n-vector models in one lower spatial dimension. We do not yet know if the QLRO phase exists for $n\geqslant 4$, and so we cannot say whether the QLRO is properly attributed to topological excitations, as the Kosterlitz-Thouless phase in the pure model is.

As mentioned in the Introduction, many orientational glass systems have quadrupolar interactions, rather than the vector interactions studied here. In cases where the parallel alignment of the quadrupoles is the low-energy one, such as the diluted alkali-cyanides,[1] our model should apply, at least to the extent that lattice defects can be neglected. In those cases where the low-energy alignment is the T configuration, however, such as the solid hydrogens,[4] things are more complicated. Another interesting experimental system with a quadrupolar order parameter, for which our results should be relevant, is the isotropic-nematic transition of a liquid crystal in silica gel.[27,28]

VII. CONCLUSION

In this work we have used Monte Carlo simulations to study the $\mathbf{O}_{12}$ version of the diluted random-field ferromagnet in three dimensions. We have found that there are two types of ordered phases, just as in the $n=2$ case. In addition to the anisotropy-stabilized ferromagnet, we find an intermediate phase displaying a $|\mathbf{k}|^{-3}$ decay of two-spin correlations. There is a tricritical point on the QLRO-to-PM transition line. When this transition is second order, the critical exponent $\overline{\eta}$, which characterizes the magnetic structure factor on the critical line, has a value which is indistinguishable from its value in the $n=2$ case. The exponent $\overline{\eta_0}$, which is observed within the QLRO phase, is also the same as in the $n=2$ case. The results should be applicable to a variety of experimental systems.

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