Low-temperature electron-phonon interaction in Si MOSFETs

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We investigate electron-phonon coupling in a silicon metal-oxide-semiconductor field-effect transistor by measuring heat flow. Earlier studies of the electron-phonon interaction through similar measurements were complicated by electron diffusion, a competing cooling mechanism. We reduce the diffusion by using an unusually long sample, and account for the remainder by comparing results from two segments of different length. We find that the power dissipation from electron-phonon scattering is proportional to T^5, with a magnitude smaller than theoretically predicted. [S0163-1829(98)04904-2]

In the 1970s and early 1980s, several theoretical and experimental groups worked on the question of low-temperature electron-phonon coupling in two-dimensional electron gases in silicon metal-oxide-semiconductor field-effect transistors (MOSFETs).[1–6] The work was motivated by the difference between the actual and theoretical mobilities of silicon MOSFETs and the possibility of making better devices. However, agreement was never reached on either the magnitude or the temperature dependence of the interaction.

Typical experiments probing the electron-phonon interaction involved applying a known amount of heat to the electron system, and measuring its steady-state temperature, with the assumption that the heat transfer from the electrons was predominantly to the phonon system. As shown below, heat transfer through the competing mechanism of electron out-diffusion may have caused much of the previous disagreement. For example, Payne *et al.*[6] used a short sample with 0.083 mm between the contacts, and took data to below 300 mK. Electron diffusion cooling was not considered, even though it must have influenced the results.

We present data on the electron-phonon coupling, with influences of diffusion minimized through both sample geometry and data analysis. The electron-phonon interaction in Si, although until now unresolved, is an important basic scientific issue. Also, many sources of heating in metallic and semiconducting samples couple directly to the electron system, so understanding how energy is then transferred from the electrons to the rest of the sample is an important practical question for certain applications and for many low-temperature experiments.

Our measurements are made on a Si MOSFET in the form of a Hall bar, with an aluminum gate 0.1 mm wide and 2.5 mm long. Six diffused voltage leads attach to the electron gas at various intervals, as shown in the inset to Fig. 1. The sample was fabricated in the early 1980s, and is analogous to those used by Cham.[3] Sample parameters for the gate voltages V_{GS} used appear in Table I. Connections to the sample are through aluminum wires, wire bonded to gold contact pads. The sample is mounted at the mixing chamber of a dilution refrigerator.

Applying a dc source-drain current I_{SD} along the Hall bar raises the electron temperature through Joule heating. We assume that the phonon temperature remains that of the mixing chamber. In a fixed applied field, the sample resistance varies because of the temperature-dependence of the Shubnikov–de Haas oscillation amplitude.[7] This serves as our electron thermometer. Between 300 and 800 mK, the resistance in a 1.5-T field decreases by about 5%. We measure differential resistance with a small 83-Hz signal added to I_{SD}. We then integrate over the bias current to convert the observed dV_{SD}/dI_{SD} to resistance.[8] To optimize the signal we tune the magnetic field to a Shubnikov–de Haas extremum.

For studying the electron-phonon interaction, the sample geometry should minimize heat loss through the competing mechanism of electron diffusion out the ends of the sample. The power dissipated per electron in steady state, with diffusion as the only mechanism of heat loss, is P

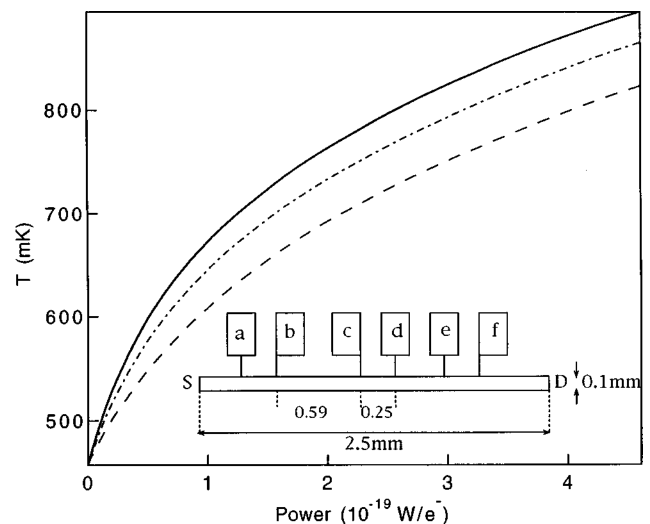

FIG. 1. V_{GS}=3 V, T_{ph}=460 mK. Dashed line: average temperature of the 0.25-mm middle segment of the Hall bar. Dot-dashed line: average temperature of the 0.59-mm segment. Solid line: electron temperature for only electron-phonon cooling. Inset: schematic of the Hall bar.

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TABLE I. Parameter values for our data and that of Ref. 16, along with the theoretical prediction of Ref. 1.

	n (m^{-2})	μ (m^2/V s)	R_{sq} (Ω)	$Pn^{3/2}/(T_e^5-T_{ph}^5)$ (W/K^5 m^3)
V_{GS}=3 V	1.42×10^{16}	0.92	480	1.4×10^6
V_{GS}=4 V	1.93×10^{16}	0.81	400	1.3×10^6
V_{GS}=5 V	2.42×10^{16}	0.74	351	2.0×10^6
Ref. 15	0.88×10^{16}	1.31	542	3.4×10^6
Ref. 1				2.2×10^7

$=(4\mathcal{L}/R_{sq}L^2n)(T_c^2-T_{ph}^2)$. Here R_{sq} is the resistance per square of the sample, L is the length, n the electron number density, and $\mathcal{L}$ the Lorenz constant $\frac{1}{3}(\pi k_B/e)^2=2.44\times10^{-8}$ WΩ/K^2. T_c is the electron temperature reached at the center of the sample. We assume the Ohmic contacts at the sample ends remain at the mixing chamber temperature. Alternatively, for the electron-phonon interaction acting alone, a calculation for deformation potential scattering[9–11] gives

$$P=\frac{3\zeta(5)\Xi_u^2k_B^5(2m^*)^{1/2}}{\pi\rho\hbar^4 s_l{}^4\varepsilon_F{}^{3/2}}\left[D^2+D+\frac{3}{8}+\frac{1}{8}\left(\frac{s_l}{s_t}\right)^4\right]\times(T_{max}^5-T_{ph}^5),$$

with $\zeta(x)$ the Rieman zeta function, m^* the electron effective mass, Ξ_u and D the deformation potential constants, ρ the density, s_l and s_t the longitudinal and transverse sound velocities, and ε_F the Fermi energy. The electron-phonon interaction is independent of position along the sample, resulting in a constant electron temperature T_{max}. Calculations assuming different temperature regimes or electron-phonon scattering mechanisms yield other expressions of the form $P\alpha(T_{max}^n-T_{ph}^n)$, with the exponent n greater than two for all low-temperature predictions. The stronger temperature dependence of electron-phonon dissipation implies that diffusion cooling will dominate at sufficiently low temperatures. In addition, the phonon cooling is independent of the sample length, while the power dissipated via diffusion cooling falls off as $1/L^2$. Thus phonon cooling is more important for longer samples.

In our device, both cooling mechanisms are significant. This leads to a temperature profile along the sample which cannot be found analytically, but can easily be calculated numerically. At the ends of the sample the electron temperature T_e equals the phonon temperature. T_e rises to a maximum in the center. For sufficiently long samples, phonon cooling dominates in the center, and the temperature near the middle of the sample is nearly constant at T_{max}. Near the ends, regardless of sample length, the temperature changes quickly as a function of position, and diffusion is important.

We measure resistance simultaneously on two segments of our Hall bar: the middle section, of length 0.25 mm, and an adjacent section, the longest on the bar, of length 0.59 mm. Typical data for steady-state electron temperature as a function of power dissipation are shown in Fig. 1. The two lower curves are the average electron temperatures of the two sections. The upper curve is T_{max}, the electron temperature that would be achieved in the absence of diffusion, derived from the other two curves as described below. Note that for a given current through the Hall bar, the power per electron is equal in the two sections, because the resistivity is nearly temperature independent. Comparing data from the two segments reveals additional information about the relationship between diffusion and phonon cooling. First, we find that for a given current, the temperature of the middle section (cd) is lower than that of the longer section (bc). This means that electron diffusion cooling occurs mainly through the voltage leads, rather than along the Hall bar. Diffusion solely along the length of the bar would leave the center section warmer. If diffusion cooling through the voltage leads is the dominant diffusion path, we in essence measure the average temperatures of two fairly short bars, rather than the temperature at the center of a long bar. This explains why the temperature difference between the two segments persists even at higher temperatures where most of the heat is transfered to the phonons: the bar ends, with $T_e<T_{max}$, have relatively more importance in the shorter bar, leading to the lower measured temperature.

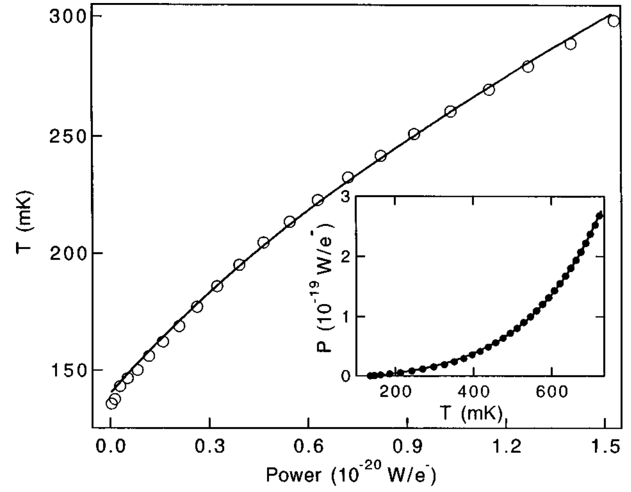

FIG. 2. Open circles: measured electron temperature of segment cd at V_{GS}=4 V and T_{ph}=140 mK. Line: fit of diffusion-cooling form, $T_e=(P/B+T_{ph}^2)^{1/2}$. Inset: dots are data; for clarity only every third point is shown. The line is a fit to $P=A(T_e^5-T_{ph}^5)+B(T_e^2-T_{ph}^2)$.

We expect electron diffusion to be most relevant at low phonon temperatures and low heating levels, and for the shorter segment. Figure 2 shows typical low-temperature data. The best fit to $T_e=(P/B+T_{ph}^n)^{1/n}$ gives $n=2.06$; a single-parameter fit with $n=2$ gives $B=2.15\times10^{-19}$ W/K^2. The calculated coefficient $4\mathcal{L}/R_{sq}L^2n$ is 2.37×10^{-19} W/K^2 for V_{GS}=3 V.[12] Our data show the relevance of electron diffusion, and raise serious questions about the early experiments on Si MOSFETs. From the information given about the sample geometries in those previous studies, one expects diffusion cooling to be comparable to electron-phonon cooling. We believe this led to identifications of exponents less than 5, as well as to large measured power dissipation. Diffusion was almost certainly a problem in the 0.05- and 0.08-mm samples of Refs. 5 and 6, and probably also in those of Ref. 4. In the last case the Hall bars were 0.4 mm long, but the location and size of the voltage leads was unspecified.

At higher temperatures, we find a crossover from electron

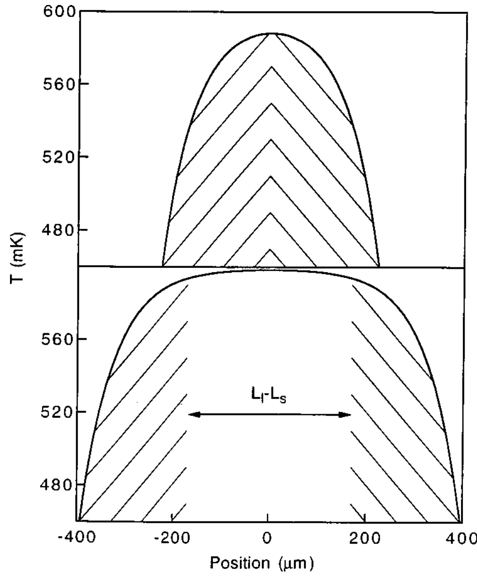

FIG. 3. Use of two Hall bar segments to determine T_{max}, the ideal electron temperature from electron-phonon cooling in the absence of electron diffusion. The temperature profiles are calculated with coefficients for diffusion and electron-phonon cooling derived from fits to our data, and are nearly identical in the shaded regions of the two segments.

diffusion to electron-phonon scattering. The inset of Fig. 2 shows a two-parameter fit, $P=A(T_e^5-T_{ph}^5)+B(T_e^2-T_{ph}^2)$, over the entire temperature range. This functional form is correct far from the crossover temperature and a reasonable approximation even in the crossover region.[13] Fixing $B=2.15\times10^{-19}$ W/K^2 and varying the exponent for the electron-phonon contribution gives a best fit at $n=4.97$.

To extract the strength of the electron-phonon coupling, one would ideally measure the temperature at the center of an extremely long sample. Lacking such a sample, we combine the results from the two different Hall bar segments to determine T_{max}. We assume only (1) that the voltage leads are identical, and (2) that the temperature T_{max} is achieved at the center of the shorter segment. The temperature profile of the ends of the long segment is identical to that of the short segment. Since at the center of the short segment the temperature is T_{max}, the entire extra length in the middle of the long segment is at T_{max}. Then $T_l=[T_sL_s+T_{max}(L_l-L_s)]/L_l$, where L_l, L_s, T_l, and T_s are the lengths and average temperatures of the long and short segments. This relation is illustrated in Fig. 3. We can then substitute the known lengths and solve for T_{max}.

From two-wire resistance measurements, we believe condition (1) above holds to within 5%. Condition (2) holds at sufficiently high electron temperature. The temperature profiles in Fig. 3 are calculated with the electron-phonon interaction strength derived from our T_{max} data. For $T_{ph}=460$ mK and $T_{max}=598$ mK, the temperature in the center of the short segment reaches 588 mK, which would lead to an error of 1 mK in T_{max}.[14]

Previous work on separating the contributions to power dissipation focused on determining electron diffusion at relatively high temperatures, above 2.5 K.[15] The technique involved applying heat locally and measuring temperature at nearby points. Our lower-temperature measurements have

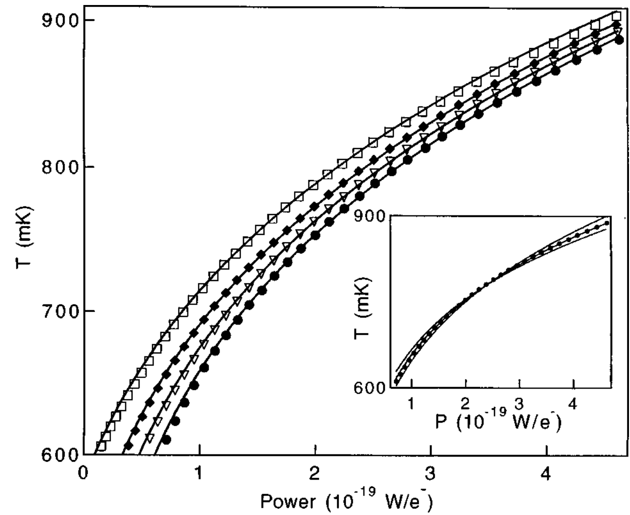

FIG. 4. T_{max} vs power for $V_{GS}=3$ V and $T_{ph}=340$ mK (●), 460 mK (▽), 520 mK (♦), and 580 mK (□). Lines are $T_{max}=(P/A+T_{ph}^5)^{1/5}$, with $A=8.37\times10^{-19}$ W/K^5. Inset: $T_{ph}=340$-mK data with best fits $T_{max}=(P/A+T_{ph}^n)^{1/n}$, varying A, for $n=4.5$, 5, and 5.5.

the *opposite* goal: we introduce a method for extracting the strength of the electron-phonon interaction in the presence of substantial diffusion cooling, by applying heat uniformly and comparing average temperatures of fairly long segments. The different temperature ranges of the experiments reflect the dominance of electron-phonon interaction at high temperatures and electron diffusion at sufficiently low temperatures.

Our main result is the temperature dependence of the electron-phonon interaction. We perform two-parameter fits, $T_{max}=(P/A+T_{ph}^n)^{1/n}$, on 26 curves T_{max} such as that in Fig. 1, with phonon temperatures from 40 to 580 mK. Based on the information just derived for diffusion cooling and T_{max}, we restrict our attention to the range $T_{max}>600$ mK to avoid effects of diffusion cooling. The best-fit exponent n ranges from 4.80–5.15, strongly suggesting $n=5$. In Fig. 4 we plot four of our data sets. A collective fit to these curves, with $n=5$ and the prefactor A as a single adjustable parameter, yields $A=8.37\times10^{-19}$ W/K^5. The resulting fit to the four curves is shown. To indicate the sensitivity to the exponent, we plot in the inset one-parameter fits for a single T_{ph} with n fixed at 4.5, 5.0, and 5.5.

We have taken data below 1 K for three gate voltages. In addition, at 4-V gate voltage we extended the measurements to higher temperature on a pumped helium probe. The high-temperature data uses the temperature dependence of the zero-field resistance as the electron thermometer. Further evidence in favor of $n=5$ is the matching of data with T_{ph} of 540 mK and 1.3 K, shown in Fig. 5. We plot $T_{max}^5-T_{ph}^5$ against power; the line, a fit to both data sets, has slope $1/A$.

Although a T^5 form describes well the temperature dependence of the electron-phonon cooling, the observed interaction strength A is smaller than the theoretical calculation by a factor of about 15, depending on the precise treatment of the deformation potential constants. This discrepancy has not been pointed out previously, to our knowledge. Early work in Si found too strong an interaction because of the influence

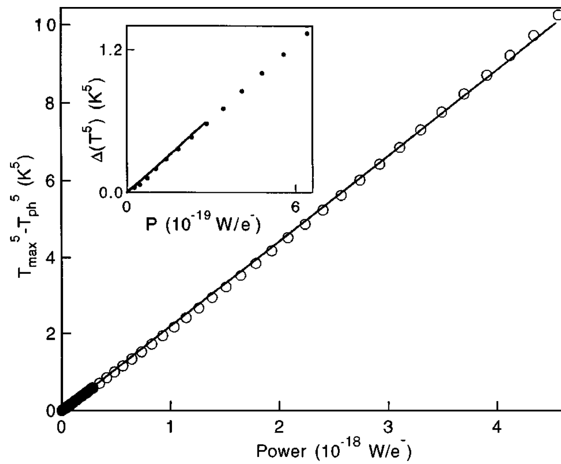

FIG. 5. V_{GS}=5 V, T_{ph}=540 mK, and 1.3 K. The dense points in the lower left have T_{ph}=540 mK. The solid line is a one-parameter fit to all the data. Inset: expansion of low-power region.

of electron diffusion. Recent measurements in SiGe agree roughly with our data,[16] but these authors apparently neglected the error in Ref. 1.

Theoretical studies[10,11] do predict a crossover to a strong screening regime at low temperatures. The screening weakens the coupling. In addition, the temperature dependence of the screening interactions changes the temperature dependence of the power dissipated by phonon emission to T^7. A T^7 power law combined with electron diffusion could mimic a T^5 behavior over a small temperature range, but is inconsistent with the matching between our low- and high-temperature data, and also with the two-exponent fits (Fig. 2) which rule out T^7. Another possibility is that impurities change the q dependence, and hence the temperature dependence, of the electron-phonon scattering. For our sample at V_{GS}=3 V and T=0.5 K, ql=1.4 for longitudinal phonons, so we could be near a crossover to hydrodynamic behavior.[17]

We have shown that power dissipation from electron-phonon scattering is proportional to T^5 at low temperatures. We have also demonstrated a method for measuring electron-phonon coupling in the presence of significant electron diffusion. Further experiments using longer samples could explore the electron-phonon interaction at lower temperatures, and might elucidate the role of screening in the coupling. Other interesting questions include the effect of mobility on the electron-phonon interaction, and how the metal gate may influence interactions in the semiconductor.

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[9] References 10 and 11 give the calculation for GaAs, which is identical except for the term for anisotropy of the deformation potential. Reference 10 approximates $\zeta(5)$ as 1, and also loses a factor of 0.5 while evaluating integrals. Reference 1 does the calculation for Si, but its equation 2.4 improperly switches the order of a limit and integration: the resulting expression for dissipation is too small by a factor of 5 in the low-temperature limit [e.g., Eq. (A5) of Ref. 1]. With these corrections, the results from Refs. 1, 10, and 11 agree.

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[12] The calculated value is for B at the center of the segment. A more accurate model would average B over the segment (which reduces B), and also take into account the length of the voltage leads (which increases B). Our results for B from a variety of simple models are all within a factor of 2 of the fit value.

[13] We have checked through computer simulations that our nonlinear $R(T)$ does not significantly affect the fit exponents.

[14] For this calculation, we add 0.2 mm to the length of each segment, to approximate each voltage lead as equivalent to one square of the Hall bar.

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