Charge fluctuations in quantum point contacts and chaotic cavities in the presence of transport

M. H. Pedersen
Département de Physique Théorique, Université de Genève, 1211 Genève 4, Switzerland

S. A. van Langen
Instituut-Lorentz, Leiden University, P.O. Box 9506, 2300 RA Leiden, The Netherlands

M. Büttiker
Département de Physique Théorique, Université de Genève, 1211 Genève 4, Switzerland

(Received 9 July 1997)

We analyze the frequency-dependent current fluctuations induced into a gate near a quantum point contact or a quantum chaotic cavity. We use a current- and charge-conserving effective scattering approach in which interactions are treated in the random-phase approximation. The current fluctuations measured at a nearby gate, coupled capacitively to the conductor, are determined by the screened charge fluctuations of the conductor. Both the equilibrium and nonequilibrium current noise at the gate can be expressed with the help of resistances which are related to the charge dynamics on the conductor. We evaluate these resistances for a point contact, and determine their distributions for an ensemble of chaotic cavities. For a quantum point contact these resistances exhibit pronounced oscillations with the opening of channels. For a chaotic cavity coupled to one-channel point contacts, the charge relaxation resistance shows a broad distribution between $\frac{1}{4}$ and $\frac{1}{2}$ of a resistance quantum. The nonequilibrium resistance exhibits a broad distribution between zero and $\frac{1}{4}$ of a resistance quantum. [S0163-1829(98)03103-8]

I. INTRODUCTION

The investigation of fluctuations in mesoscopic conductors is an interesting problem which has found considerable attention both experimentally and theoretically. Two recent reviews provide both an introduction to the subject as well as a discussion of some of the important results.[1,2] In this work we are interested in the frequency-dependent noise spectra of mesoscopic conductors away from the low-frequency white-noise limit. The experimental observation of deviations from the white-noise limit in the current-fluctuation spectra of well conducting samples requires large frequencies.[3] Here we investigate the fluctuations induced into a nearby gate, capacitively coupled to the conductor. These fluctuations are not a correction to an effect that exists already in the zero-frequency limit. We present a discussion which describes the internal potential of the mesoscopic conductor with a single variable. The Coulomb interactions are described with the help of a geometrical capacitance C instead of the full Poisson equation. Furthermore, we will treat the gate as a macroscopic electric conductor. In this case the current fluctuations induced into a nearby gate are determined entirely by the dynamics of the charge fluctuations of the mesoscopic conductor.

Consider a conductor, for instance the quantum point contact,[4–7] shown in Fig. 1. The conductor is described by scattering matrices $s_{\alpha\beta}$ which relate the amplitudes of incoming currents at contact β to the amplitudes of the outgoing currents at α. We find that the charge fluctuations of the mesoscopic conductor can be described with the help of a density-of-states matrix

$$\mathcal{N}_{\delta\gamma}=\frac{1}{2\pi i}\sum_{\alpha} s^{\dagger}_{\alpha\delta}\frac{ds_{\alpha\gamma}}{dE}. \tag{1}$$

The diagonal elements of this matrix determine the density of states of the conductor $N=\Sigma_{\gamma}\mathrm{Tr}(\mathcal{N}_{\gamma\gamma})$; the trace is over all quantum channels. The nondiagonal elements are essential to describe fluctuations. At equilibrium and in the zero-temperature limit, we find that to leading order in frequency the mean-squared current fluctuations at the gate have a spectrum $S_{00}(\omega,V=0)=2\omega^2\hbar|\omega|C_{\mu}^2R_q$. Here $C_{\mu}^{-1}=C^{-1}+(e^2N)^{-1}$, is the *electrochemical* capacitance[8] of the conductor *vis à vis* the gate. The dynamical quantity which determines the fluctuations is the charge relaxation resistance R_q,

$$R_q=\frac{h}{2e^2}\frac{\sum_{\gamma\delta}\mathrm{Tr}(\mathcal{N}_{\gamma\delta}\mathcal{N}^{\dagger}_{\gamma\delta})}{\left[\sum_{\gamma}\mathrm{Tr}(\mathcal{N}_{\gamma\gamma})\right]^2}. \tag{2}$$

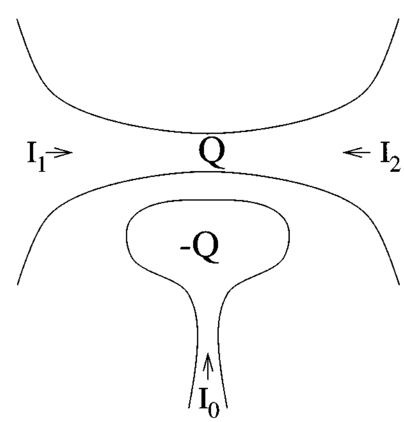

FIG. 1. Geometry of the quantum point contact.

Büttiker, Thomas, and Prêtre[8] showed that the charge relaxation resistance governs the dissipative part of the low-frequency admittance of mesoscopic capacitors. Together with the electrochemical capacitance C_μ, R_q determines the charge relaxation time $R_q C_\mu$ of the mesoscopic conductor. Similarly to the equilibrium noise spectrum, at zero temperature, the nonequilibrium current noise spectrum at the gate, $S_{00}(V,\omega)=2\omega^2 e|V|C_\mu^2 R_v$, is determined by a resistance R_v,

$$R_v=\frac{h}{e^2}\frac{\mathrm{Tr}(\mathcal{N}_{21}\mathcal{N}_{21}^\dagger)}{\left[\sum_\gamma \mathrm{Tr}(\mathcal{N}_{\gamma\gamma})\right]^2}. \tag{3}$$

Whereas the charge relaxation resistance R_q invokes all elements of the density-of-states matrix with equal weight, in the presence of transport the nondiagonal elements of the density-of-states matrix are singled out. Below we present the derivation of these results and evaluate the charge relaxation resistance R_q and the resistance R_v for the quantum point contact and for a chaotic quantum dot.

The characterization of the current fluctuations in terms of resistances can be motivated as follows. The current fluctuations at the gate contact are directly related to fluctuations of the charge Q on the conductor,

$$S_{00}(\omega,V)=\omega^2 S_{QQ}(\omega,V). \tag{4}$$

In turn, the charge fluctuations are related to the potential fluctuations by the geometrical capacitance C,

$$S_{QQ}(\omega,V)=C^2 S_{UU}(\omega,V). \tag{5}$$

Voltage fluctuations, as is well known, are essentially determined by resistances. However, in contrast to the Nyquist formula for equilibrium voltage fluctuations, we deal here with electrostatic potential fluctuations inside the conductor. The resistances R_q and R_v are related to the charge dynamics rather than the two-terminal dc resistance.

The resistances R_q and R_v probe an aspect of mesoscopic conductors which is not accessible by investigating the dc-conductance or the zero-frequency limit of shot noise. These resistances are not determined by the scattering matrix alone, but also by its energy derivative. According to the fluctuation dissipation theorem, the low-frequency equilibrium current fluctuations of a conductor which permits transmission are determined by the conductance of the system. For a two-terminal conductor the conductance is simply the sum of all transmission eigenvalues T_n. The low-frequency nonequilibrium noise, the shot noise,[9,10] of a two-terminal conductor is determined by the sum of the products $T_n(1-T_n)$, where $T_n=1-R_n$ are again the eigenvalues of the transmission matrix multiplied by its Hermitian conjugate.[11,12] Hence both the equilibrium noise and the shot noise are governed by the transmission behavior of the sample. This is even true for correlations on multiterminal conductors which cannot be expressed in terms of transmission eigenvalues.[11,13,14] In contrast, the dynamic conductance is determined by oscillations of the charge distribution in the conductor.[15] Since charge is a conserved quantity, the oscillatory part of the charge distribution can be represented as a sum of dipoles.[16,17] Similarly, the frequency-dependent fluctuations are governed by the fluctuations of the charge distributions, or more precisely by the fluctuations of dipolar charges.

The charge fluctuations of a noninteracting system can be described with the help of the density-of-states matrix; Eq. (1). However, the charge distribution of a noninteracting system is not dipolar. In fact, without interactions, charge is not conserved, and consequently currents are not conserved. To achieve a dipolar (or higher order multipolar) charge distribution it is necessary to consider interactions. Here we consider the simple approximation in which the charge distribution is effectively represented by a single dipole. We permit the charging of the quantum point contact vis à vis the gate. In Fig. 1 this dipole is indicated by the charges Q and $-Q$. A more realistic treatment of the charge distribution of a quantum point contact includes a dipole across the quantum point contact itself,[16] and in the presence of the gates includes a quadrupolar charge distribution.[17]

The frequency dependence of the noise spectra generated by the fluctuations of the dipolar charges should be distinguished from a purely statistical frequency dependence arising from the Fermi distribution functions:[3,18,19] Even for a conductor with an energy-independent scattering matrix, there exists a frequency dependence due to the Fermi distribution functions of the different reservoirs. For small frequencies the distribution functions are governed by the temperature kT or the applied voltage eV, and a crossover occurs when the frequency $\hbar\omega$ exceeds both kT and eV. We will not emphasize this crossover further, since it is a property of the Fermi distribution alone, and provides no new information on the conductor itself.

Our work is also of interest in view of recent efforts to discuss the dephasing induced by the shot noise of two conductors in close proximity,[20,21] or due to the fluctuating electromagnetic field.[22] Our work shows that what counts are the dipolar charge fluctuations. The discussion presented below cannot be applied to metallic diffusive conductors, for which the potential needs to be treated as a field.[2] Recently Nagaev's[23] classical discussion of shot noise in metallic conductors was extended to investigate the effect of a nearby gate.[24,25] In these works the source of the noise is taken to be frequency independent over the entire range of interest. In contrast for the examples treated here, it is not only the electrodynamic response which is frequency dependent but also the noise itself.

There has been a considerable recent interest in the parametric derivatives of the scattering matrix of chaotic conductors.[26–30] The energy derivative of the scattering matrix determines quantities like the density-of-states matrix; Eq. (1). For electrostatic problems it is the functional derivative of the scattering matrix with respect to the local potential which matters.[31] Only in the limit where we describe the internal electrostatic potential as a single variable (instead of a continuous field), and only if we are satisfied with a WKB-like-description can the energy derivatives of the scattering matrix be used. These two conditions are likely to be fulfilled for a ballistic quantum dot. Then the energy derivative of the scattering matrix and the potential derivative differ just by a sign. For chaotic cavities a theory of the energy derivative of the scattering matrix has permitted a discussion of the distribution of capacitances.[27–29] Fyodorov and Sommers[26,29] used supersymmetric methods to investigate the energy derivatives of the scattering matrix. For a chaotic cavity connected to a reservoir via a single channel lead, Gopar, Mello, and Büttiker[27] found the distribution functions for all universality classes by analyzing directly the statistical properties of the scattering matrix. The single-channel discussion of Gopar, Mello, and Büttiker[27] was generalized by Brouwer, Frahm, and Beenakker,[30] who found the distribution of the scattering matrix and its derivatives for the multichannel problem. This generalization made it possible to investigate the distribution of parametric conductance derivatives like the transconductance dG/dV_0, where G is the conductance and V_0 is the gate voltage.[32] Here we use the result of Ref. 30 to find the distribution of the charge relaxation resistance R_q and the resistance R_v for a chaotic quantum dot coupled to reservoirs via two perfect one-channel leads.

II. CURRENT AND CHARGE FLUCTUATIONS

To find the current fluctuations for the structures of interest, in this section we discuss an approach which includes interaction effects in the random-phase approximation (RPA). This approach was used in Ref. 33 to find the dynamic conductance of mesoscopic structures for the case that the self-consistent potential of the conductor can be taken to be a single variable U. The fluctuations belonging to this approach are discussed in Ref. 8 for the case of a mesoscopic capacitor, and for a more general multiprobe conductor capacitively coupled to a gate in Ref. 34.

A. Fixed internal potential

We consider a conductor with a fixed internal potential (noninteracting problem), and present the results needed later on to treat the problem with interactions. Consider a conductor described by scattering matrices $s_{\alpha\beta}$ which relate the annihilation operators $\hat{a}_\beta$ in the incoming channels in contact β to the annihilation operators $\hat{b}_\alpha$ of a carrier in the outgoing channel of contact α via[11]

$$\hat{b}_\alpha = \sum_\beta s_{\alpha\beta}\hat{a}_\beta . \tag{6}$$

In a multichannel conductor the s matrix has dimensions $N_\alpha \times N_\beta$ for leads that support N_α and N_β quantum channels. Here α and β run over all contacts of the conductor $\alpha,\beta = 1,2$. (Later, we need indices for the contacts of the conductor and the gate. For this case we will use the labels $\mu,\nu = 0,1,2$). The current at contact α is determined by the difference in the occupation of the incident channels minus the occupation of the outgoing channels

$$\hat{I}_\alpha(\omega) = \frac{e}{\hbar}\int dE[\hat{a}^\dagger_\alpha(E)\hat{a}_\alpha(E+\hbar\omega) - \hat{b}^\dagger_\alpha(E)\hat{b}_\alpha(E+\hbar\omega)]. \tag{7}$$

Using Eq. (6) to eliminate the occupation numbers of the outgoing channels in terms of the incoming channels yields a current operator[11]

$$\hat{I}_\alpha(\omega) = \frac{e}{\hbar}\int dE \sum_{\beta\gamma} \hat{a}^\dagger_\beta(E) A^0_{\beta\gamma}(\alpha,E,E+\hbar\omega)\hat{a}_\gamma(E+\hbar\omega), \tag{8}$$

with a *current matrix*

$$A^0_{\delta\gamma}(\alpha,E,E') = \delta_{\alpha\delta}\delta_{\alpha\gamma}1_\alpha - s^\dagger_{\alpha\delta}(E)s_{\alpha\gamma}(E'). \tag{9}$$

Here the upper index 0 indicates that we deal with noninteracting electrons. The current noise spectra are determined by the quantum expectation value $\langle\cdots\rangle$ of the current operators at contact μ and ν, $\frac{1}{2}\langle\Delta\hat{I}_\mu(\omega)\Delta\hat{I}_\nu(\omega') + \Delta\hat{I}_\nu(\omega')\Delta\hat{I}_\mu(\omega)\rangle \equiv 2\pi S_{\mu\nu}\delta(\omega+\omega')$. The spectral densities in terms of the current matrix are[11]

$$S_{\mu\nu}(\omega) = \frac{e^2}{h}\sum_{\delta\gamma}\int dE\, F_{\gamma\delta}(E,\omega)\mathrm{Tr}[A^0_{\gamma\delta}(\mu,E,E+\hbar\omega) \times (A^0)^\dagger_{\gamma\delta}(\nu,E,E+\hbar\omega)], \tag{10}$$

$$F_{\gamma\delta}(E,\omega) = f_\gamma(E)[1-f_\delta(E+\hbar\omega)] + f_\delta(E+\hbar\omega)[1-f_\gamma(E)]. \tag{11}$$

Here the trace is taken over channels, and f_γ is the Fermi distribution function for contact γ. At equilibrium these fluctuation spectra are related to the ac conductances of the noninteracting problem discussed in Ref. 33. The current operator for the gate has thus far not been defined: that will be achieved only in Sec. II B.

It is natural to decompose the current matrix into two contributions: one at equal energies that determines the dc response of the conductor, and one at differing energies that is associated with the dynamics of the system. Thus we write

$$A^0_{\delta\gamma}(\alpha,E,E') = \delta_{\alpha\delta}\delta_{\alpha\gamma}1_\alpha - s^\dagger_{\alpha\delta}(E)s_{\alpha\gamma}(E) - 2\pi i(E'-E)\mathcal{N}_{\delta\gamma}(\alpha,E,E'), \tag{12}$$

with a *partial density-of-states* matrix

$$\mathcal{N}_{\beta\gamma}(\alpha,E,E') = \frac{i}{2\pi}\frac{s^\dagger_{\alpha\beta}(E)[s_{\alpha\gamma}(E) - s_{\alpha\gamma}(E')]}{E'-E}. \tag{13}$$

This matrix has a simple interpretation: The elements of $\mathcal{N}_{\beta\gamma}(\alpha,E,E')$ are the diagonal and nondiagonal elements of the density-of-states associated with carriers incident from contact β and γ which eventually contribute to the current at contact α. From the continuity equation we find immediately that the total charge fluctuations in the conductor generated by particles incident from contact β and γ irrespective through which contact they leave the conductor are determined by the density of states matrix

$$\mathcal{N}_{\beta\gamma}(E,E') = \sum_\alpha \mathcal{N}_{\beta\gamma}(\alpha,E,E'). \tag{14}$$

Some additional combinations of these matrices have a special meaning.[31,35] We call

$$\overline{\mathcal{N}}_\beta(E,E') = \sum_\alpha \mathcal{N}_{\beta\beta}(\alpha,E,E') \tag{15}$$

the *injectance matrix* of contact β, and call

$$\underline{\mathcal{N}}_{\alpha}(E,E')=\sum_{\beta} \mathcal{N}_{\beta\beta}(\alpha,E,E') \tag{16}$$

the *emittance matrix*. The frequency-dependent injectance is the quantum expectation value of the injectance operator $\overline{N_{\beta}}(\omega)=\langle\Sigma_{\alpha}\hat{\mathcal{N}}_{\beta\beta}(\alpha,E,E+\hbar\omega)\rangle$. Similarly, the frequency dependent emittance is $\underline{N}_{\alpha}(\omega)=\langle\Sigma_{\beta}\hat{\mathcal{N}}_{\beta\beta}(\alpha,E,E+\hbar\omega)\rangle$. Below we will often use only the zero-frequency limit of the density matrix Eq. (14) $(\omega\rightarrow 0)$, which is given by Eq. (1). Similarly we will most often use only the zero-temperature, zero-frequency injectance,

$$\overline{N_{\beta}}=\frac{1}{2\pi i}\sum_{\alpha} \mathrm{Tr}\left(s^{\dagger}_{\alpha\beta}\frac{ds_{\alpha\beta}}{dE}\right), \tag{17}$$

and emittance,

$$\underline{N}_{\alpha}=\frac{1}{2\pi i}\sum_{\beta} \mathrm{Tr}\left(s^{\dagger}_{\alpha\beta}\frac{ds_{\alpha\beta}}{dE}\right). \tag{18}$$

The density matrices introduced above together with the injectances and emittances can now be used to characterize the charge fluctuations of the conductor. The evaluation of the injectances and emittances in the equilibrium state of the conductance limits the theory presented below to linear order in the applied voltage.

B. Effective current matrix

Our goal is to derive a current matrix which includes the effect of screening[2] and replaces the current matrix, Eq. (9) of the noninteracting problem. To this extent we next determine the operator $\hat{U}$ for the internal potential. The charge on the conductor is determined by the Coulomb interaction. Here we describe the interaction with the help of a single geometrical capacitance. Hence the charge on the conductor is $\hat{Q}=C\hat{U}$. Here we have assumed that the gate is macroscopic, and has no dynamics of its own. We can also determine the charge $\hat{Q}$ as the sum of the bare charge fluctuations $e\hat{\mathcal{N}}$ and the induced charges generated by the fluctuating induced electrical potential. In the RPA the induced charges are proportional to the average frequency-dependent density of states $N(\omega)$ times the fluctuating potential. Thus the net charge is determined by

$$\hat{Q}=C\hat{U}=e\hat{\mathcal{N}}-e^{2}N\hat{U}. \tag{19}$$

Solving this equation gives us, for the operator of the potential fluctuations,

$$\hat{U}=Ge\hat{\mathcal{N}}, \tag{20}$$

with

$$G(\omega)=[C+e^{2}N(\omega)]^{-1}. \tag{21}$$

Here $G(\omega)$ takes into account the effective interaction potential.

The total current at probe α is determined by the particle current, and in addition by a current due to the fluctuating potential. The fluctuation of the internal potential creates additional currents at all the contacts. The current fluctuations generated by the induced potential fluctuations at contact α are determined by $i\omega e^{2}\underline{N}_{\alpha}(\omega)\hat{U}(\omega)$. Here the response to the internal potential is determined by the emittance[31,33] of the conductor into contact α. Thus the total current at contact α of the conductor is

$$\hat{I}_{\alpha}(\omega)=\hat{I}^{0}_{\alpha}(\omega)-i\omega e^{2}\underline{N}_{\alpha}(\omega)\hat{U}(\omega), \tag{22}$$

where $\hat{I}^{0}_{\alpha}$ is the current operator for fixed internal potential. The current induced into the gate is given by the time derivative of the total charge, and hence by

$$\hat{I}_{g}(\omega)=i\omega C\hat{U}(\omega). \tag{23}$$

Expressing $\hat{U}$ in terms of the density of states matrix gives for the current operators Eqs. (22) and (23) an expression which is of the same form as Eq. (8), but with the current matrix Eq. (9) replaced by an effective current matrix

$$A_{\delta\gamma}(\alpha,E,E+\hbar\omega)=A^{0}_{\delta\gamma}(\alpha,E,E+\hbar\omega) + i\omega e^{2}\underline{N}_{\alpha}G\mathcal{N}_{\delta\gamma}(E,E+\hbar\omega). \tag{24}$$

Equation (24) determines the current at the contacts of the conductor. The current induced into the gate contact is determined by a current matrix

$$A_{\delta\gamma}(0,E,E+\hbar\omega)=-i\omega CG\mathcal{N}_{\delta\gamma}(E,E+\hbar\omega). \tag{25}$$

The sum of all currents at the contacts of the sample and the current at the gate is conserved. Indeed, labeling the index which runs over all contacts by ν $(\nu=0,1,2)$, we find

$$\sum_{\nu} A_{\delta\gamma}(\nu,E,E+\hbar\omega)=0. \tag{26}$$

Equation (26) follows from the relation between the bare current matrix and the density-of-states matrix, Eqs. (12)–(14) and the fact that $1-e^{2}NG=CG$. Before continuing we notice that for these effective current matrices $A_{\delta\gamma}(\nu)$, the index ν runs over all contacts but the indices δ and γ run only over the contacts of the sample. This ''asymmetry'' is a consequence of our macroscopic treatment of the gate.

C. Charge fluctuation spectra

With the help of the effective current matrices, Eqs. (24) and (25), we can find the current fluctuation spectra $S_{\mu\nu}(\omega,V)$ as in the noninteracting case: In Eq. (10) we have to replace the bare current matrix $A^{0}_{\delta\gamma}(\alpha)$ by the effective current matrix $A_{\delta\gamma}(\nu)$. This determines a matrix $S_{\mu\nu}(\omega)$ of fluctuation spectra for the mean-square current fluctuations at the contacts of the conductor and the gate and for the correlations between any two currents. As a consequence of current conservation, $\Sigma_{\mu}S_{\mu\nu}(\omega)=\Sigma_{\nu}S_{\mu\nu}(\omega)=0$. At equilibrium the fluctuation spectra which we find with the help of the effective current matrix are related via the fluctuation dissipation theorem to the frequency-dependent conductances of the interacting system given in Ref. 33. The spectra also agree with the expression given in Ref. 34. Here we are interested in the current fluctuations at the gate determined by the spectrum $S_{00}(\omega,V)$. This spectrum is entirely determined by the charge fluctuations of the conductor [see Eq.

(4)]. Defining the frequency-dependent capacitance of the conductor to the gate $C_\mu(\omega)\equiv e^2N(\omega)CG(\omega)$, and using Eq. (25), we find

$$S_{QQ}(\omega)=C_\mu^2(\omega)N^{-2}(\omega)\sum_{\delta\gamma}\int dE\ F_{\gamma\delta}(E,\omega)\times\mathrm{Tr}[\mathcal{N}_{\gamma\delta}(E,E+\hbar\omega)\mathcal{N}^\dagger_{\gamma\delta}(E,E+\hbar\omega)]. \tag{27}$$

Two limits are of special interest. At equilibrium, at zero temperature, we find, for the charge fluctuation spectrum in the low-frequency limit, $S_{QQ}(\omega)=2C_\mu^2R_q\hbar|\omega|$, where the electrochemical capacitance is given by its zero-frequency value, and where the charge relaxation resistance is determined by Eq. (2).

The second limit we wish to consider is the zero-temperature, low-frequency limit of the charge fluctuations to leading order in the applied voltage V. Evaluation of Eq. (27) gives $S_{QQ}(\omega)=2C_\mu^2R_v|eV|$, with a resistance R_v given by Eq. (3). Thus the nonequilibrium noise is determined by a nondiagonal element of the density-of-states matrix. If both the frequency and the voltage are nonvanishing, we obtain, to leading order in $\hbar\omega$ and V, $S_{QQ}(\omega)=2C_\mu^2R(\omega,V)\hbar|\omega|$ with a resistance

$$R(\omega,V)\hbar|\omega|=\begin{cases}R_q\hbar|\omega|, & \hbar|\omega|\geqslant e|V|\\ R_q\hbar|\omega|+R_V(e|V|-\hbar|\omega|), & \hbar|\omega|\leqslant e|V|,\end{cases} \tag{28}$$

which is a frequency- and voltage-dependent series combination of the resistances R_q and R_v. For the experimentally relevant cases the variation of C_μ is very small, and the measured noise is directly proportional to the resistance defined above. Below, we discuss the resistances R_q and R_v in detail for two examples: a quantum point contact and a chaotic cavity.

III. QUANTUM POINT CONTACT

Quantum point contacts (QPC's) are formed with the help of gates. It is therefore interesting to ask what the fluctuations are which would be measured at one of these gates. For simplicity, we consider a symmetric contact: We assume that the electrostatic potential is symmetric for electrons approaching the contact from the left or from the right. Furthermore, we combine the capacitances of the conduction channel to the two gates, and consider a single gate, as shown schematically in Fig. 1. If only a few channels are open, the potential has in the center of the conduction channel the form of a saddle:[36]

$$V(x,y)=V_0+\tfrac{1}{2}m\omega_y^2y^2-\tfrac{1}{2}m\omega_x^2x^2, \tag{29}$$

where V_0 is the electrostatic potential at the saddle and the curvatures of the potential are parametrized by ω_x and ω_y. For this model the scattering matrix is diagonal, i.e., for each quantum channel [energy $\hbar\omega_y(n+1/2)$ for transverse motion] it can be represented as a 2×2 matrix. For a symmetric scattering potential and without a magnetic field the scattering matrix is of the form

$$s_n(E)=\begin{pmatrix}-i\sqrt{R_n}\exp(i\phi_n) & \sqrt{T_n}\exp(i\phi_n)\\ \sqrt{T_n}\exp(i\phi_n) & -i\sqrt{R_n}\exp(i\phi_n)\end{pmatrix}, \tag{30}$$

where T_n and $R_n=1-T_n$ are the transmission and reflection probabilities of the nth quantum channel, and ϕ_n is the phase accumulated by a carrier in the nth channel during transmission through the QPC. The probabilities for transmission through the saddle point are[36]

$$T_n(E)=\frac{1}{1+e^{-\pi\epsilon_n(E)}}, \tag{31}$$

$$\epsilon_n(E)=2[E-\hbar\omega_y(n+\tfrac{1}{2})-V_0]/(\hbar\omega_x). \tag{32}$$

The transmission probabilities determine the conductance $G=(e^2/h)\Sigma_nT_n$ and the zero-frequency shot noise[10,11] $S(\omega=0,V)=(e^2/h)(\Sigma_nT_nR_n)e|V|$. As a function of energy (gate voltage) the conductance rises steplike.[4,5] The shot noise is a periodic function of energy. The oscillations in the shot noise associated with the opening of a quantum channel have recently been demonstrated experimentally by Reznikov *et al.*[6] and Kumar *et al.*[7]

To obtain the density of states we use the relation between density and phase $N_n=(1/\pi)\phi_n$, and evaluate it semiclassically. The spatial region of interest for which we have to find the density of states is the region over which the electron density in the contact is not screened completely. We denote this length by λ. The density of states is then found from $N_n=1/h\int_{-\lambda}^{\lambda}(dp_n/dE)\,dx$, where p_n is the classically allowed momentum. A simple calculation gives a density of states

$$N_n(E)=\frac{4}{h\omega_x}\ \mathrm{arsinh}\left(\sqrt{\frac{1}{2}\frac{m\omega_x^2}{E-E_n}}\lambda\right) \tag{33}$$

for energies E exceeding the channel threshold E_n, and

$$N_n(E)=\frac{4}{h\omega_x}\ \mathrm{arcosh}\left(\sqrt{\frac{1}{2}\frac{m\omega_x^2}{E_n-E}}\lambda\right) \tag{34}$$

for energies in the interval $E_n-(1/2)m\omega_x^2\lambda^2\leqslant E<E_n$ below the channel threshold. Electrons with energies less than $E_n-\frac{1}{2}m\omega_x^2\lambda^2$ are reflected before reaching the region of interest, and thus do not contribute to the density of states. The resulting density of states has a logarithmic singularity at the threshold $E_n=\hbar\omega_y(n+\frac{1}{2})+V_0$ of the nth quantum channel. (We expect that a fully quantum-mechanical calculation gives a density of states which also exhibits a peak at the threshold but which is not singular). The total density of states as function of energy (gate voltage) is shown in Fig. 2 for $\omega_y/\omega_x=3$, $V_0=0$ and $m\omega_x\lambda^2/\hbar=18$. Each peak in the density of states of Fig. 2 marks the opening of a new channel. With the help of the density of states we also obtain the capacitance $C_\mu^{-1}=C^{-1}+(e^2N)^{-1}$. For the experimentally most relevant case $(e^2/C)\gg N^{-1}$ the variations in the capacitance are small and the noise spectra are dominated by the energy dependence of R_q and R_v, which we will now discuss.

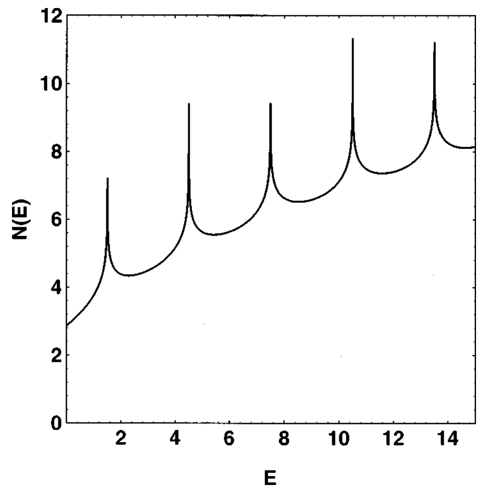

FIG. 2. Density of states in units of $4/(h\omega_x)$ for a saddle-point constriction as function of energy, $E/(\hbar\omega_x)$. $\omega_y/\omega_x=3$.

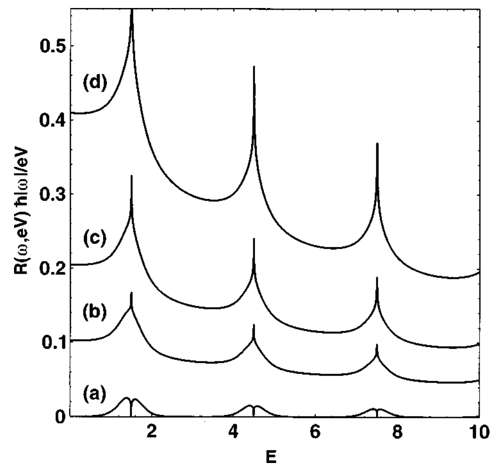

FIG. 3. Effective resistance, in units of h/e^2, as function of energy, $E/(\hbar\omega_x)$, for the cases (curve a)$\hbar\omega/(eV)=0$, (curve b)$\hbar\omega/(eV)=0.25$, (curve c)$\hbar\omega/(eV)=0.5$, and (curve d)$\hbar\omega/(eV)=1$, where V is bias voltage. $\omega_y/\omega_x=3$.

It is instructive to evaluate the resistances R_q and R_v explicitly in terms of the parameters which determine the scattering matrix. We find for the density of states matrix of the nth quantum channel, we find

$$\mathcal{N}_{11}=\mathcal{N}_{22}=\frac{1}{2\pi}\frac{d\phi_n}{dE}, \tag{35}$$

$$\mathcal{N}_{12}=\mathcal{N}_{21}=\frac{1}{4\pi}\frac{1}{\sqrt{R_nT_n}}\frac{dT_n}{dE}. \tag{36}$$

Inserting these results into Eq. (2) gives, for the charge relaxation resistance,[33]

$$R_q=\frac{h}{e^2}\frac{\sum_n (d\phi_n/dE)^2}{\left[\sum_n (d\phi_n/dE)\right]^2}. \tag{37}$$

It is determined by the derivatives of the phases (densities) evaluated at the Fermi energy. The resistance R_v is given by

$$R_v=\frac{h}{e^2}\frac{\sum_n \frac{1}{4R_nT_n}\left(\frac{dT_n}{dE}\right)^2}{\left[\sum_n (d\phi_n/dE)\right]^2}. \tag{38}$$

It is sensitive to the variation with energy of the transmission probability. Note that the transmission probability has the form of a Fermi function. Consequently, the derivative of the transmission probability is also proportional to T_nR_n. The numerator of Eq. (38) is thus also maximal at the onset of a channel, and vanishes on a conductance plateau.

In Fig. 3 the effective resistance $R(\omega,V)$ is shown for four frequencies $\hbar\omega/(eV)=0$, 0.25, 0.5, and 1, where V is the applied voltage. At the highest frequency $\hbar\omega/(eV)=1$ the resistance $R(\omega,V)$ is completely dominated by the equilibrium charge relaxation resistance R_q. The uppermost curve (d) of Fig. 3 is nothing but R_q and determines the noise due the zero-point equilibrium fluctuations. The fluctuations reach a maximum at the onset of a channel, since R_q takes its maximum value $R_q=h/e^2$. At the lowest frequency $\hbar\omega=0$ the resistance $R(\omega,V)$ is determined by R_v. The lowermost curve (a) of Fig. 3 is the nonequilibrium resistance R_v. It is seen that the nonequilibrium resistance R_v is very much smaller than R_q. We will also encounter such a large difference between these two resistances for the chaotic cavity. Furthermore R_v exhibits a double-peak structure: The large peak in the density of states at the threshold of a quantum channel nearly suppresses the nonequilibrium noise at the channel threshold completely. Two additional curves [b and c for $\hbar\omega/(eV)=0.25$ and $\hbar\omega/(eV)=0.5$] describe the crossover from R_v to R_q.

IV. QUANTUM CHAOTIC CAVITY

The general theory is now applied to a chaotic quantum dot[37–39] with two ideal single-channel leads and capacitive coupling to a macroscopic gate, as shown schematically in Fig. 4. For such samples, averages lose their meaning and below we give the distribution functions of the resistances which characterize the noise induced into the gate contact. We compute the statistical distribution of the charge relaxation resistance R_q and the resistance R_v from random matrix theory,[40] assuming that the classical dynamics of the cavity is fully chaotic. We will again consider the case

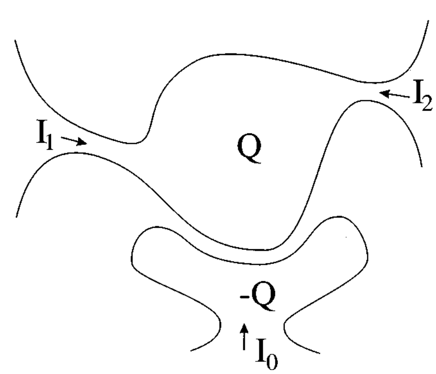

FIG. 4. Quantum dot coupled to two open leads, and coupled to a gate.

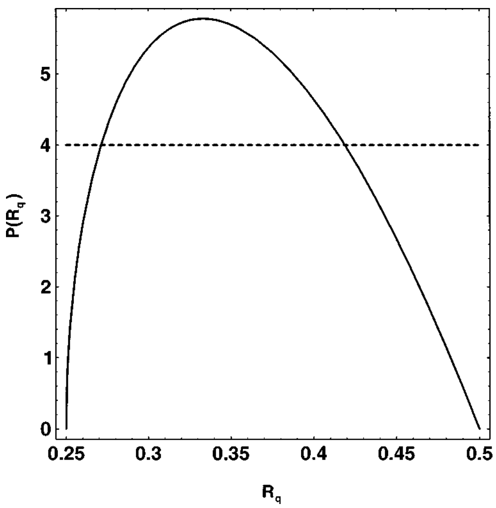

FIG. 5. Distribution of the charge relaxation resistance of a chaotic quantum dot for the orthogonal ensemble (dashed) and the unitary ensemble (solid line) line.

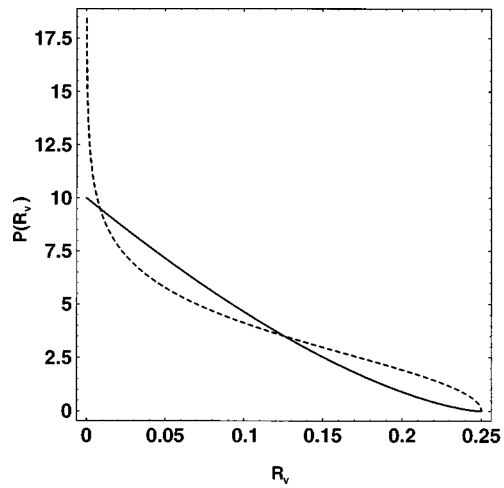

FIG. 6. Distribution of the resistance R_v of a chaotic quantum dot for the orthogonal ensemble (dashed) and the unitary ensemble (solid line) line.

$e^2/C \gg N^{-1}$ for which the distribution function[41] of the electrochemical capacitance becomes very sharp.

The distribution of the two-terminal, zero-frequency shot noise $S = T(1-T)$ in units of $S_0 = 2e(e^2/h)|V|$, follows from the distribution $P(T) \propto T^{-1+\beta/2}$ of the dimensionless conductance T. The symmetry index β equals 1 or 2 depending on whether time-reversal symmetry is present or broken. The latter is a result of the uniform distribution of the scattering matrix on the set of unitary ($\beta=2$) or unitary symmetric ($\beta=1$) 2×2 matrices.[42] Thus one finds for the distribution of $S \in [0,1/4]$

$$P(S) = \begin{cases} \dfrac{\sqrt{1+\sqrt{1-4S}}+\sqrt{1-\sqrt{1-4S}}}{\sqrt{16S(1-4S)}}, & \beta=1 \\ (1/4-S)^{-1/2}, & \beta=2. \end{cases} \tag{39}$$

For the orthogonal ensemble the distribution of the shot noise is bimodal, and has square-root singularities at $S=0$ and $\frac{1}{4}$. In the unitary ensemble the distribution remains finite at zero shot noise, and has a square-root singularity only at $S=1/4$.

In contrast, for the low-frequency spectrum, Eq. (27), of the charge fluctuations, one needs the matrices $\mathcal{N}_{\alpha\beta}$, which are just blocks of the well-known Wigner-Smith delay-time matrix $(1/2\pi i)\, s^\dagger (ds/dE)$[43]. The distribution of this matrix has recently been found.[30] To compute $P(R_q)$ it is sufficient to know the joint distribution of the eigenvalues $P(\{q_i\})$, whereas for $P(R_v)$ we also need that the eigenvectors are distributed uniformly, and independently from the eigenvalues. As in Ref. 32, we integrate over the eigenvalues with an extra weight factor $\Sigma_i q_i$, which is the fluctuating density of states. For instance, the distribution of R_q (in units of h/e^2) follows from

$$P(R_q) = \int dq_1 dq_2 P(q_1,q_2)(q_1+q_2)\,\delta\!\left(R_q - \frac{q_1^2+q_2^2}{2(q_1+q_2)^2}\right). \tag{40}$$

The weight factor appears because, in the limit $e^2/C \gg N^{-1}$, the ensemble is generated either by *uniformly varying* the total charge (rather than E_F) in case the gate voltage is swept, or at *constant* charge (rather than E_F) if an other parameter like the magnetic field is swept. In both cases the average can be replaced by a random matrix average, provided the density of states is used as a Jacobian.[32] Thus we find the distribution of the charge relaxation resistance $R_q \in [\frac{1}{4},\frac{1}{2}]$,

$$P(R_q) = \begin{cases} 4, & \beta=1, \\ 30(1-2R_q)\sqrt{4R_q-1}, & \beta=2. \end{cases} \tag{41}$$

It is shown in Fig. 5.

For the resistance R_v (also in units of h/e^2) the distribution is shown in Fig. 6. It is limited to the range $R_v \in [0,\frac{1}{4}]$ and given by

$$P(R_v) = \begin{cases} 2\ln\left[\dfrac{1-2R_v+\sqrt{1-4R_v}}{2R_v}\right], & \beta=1, \\ 10(1-4R_v)^{3/2}, & \beta=2. \end{cases} \tag{42}$$

For the orthogonal ensemble the distribution is singular at $R_v=0$. Both distribution functions tend to zero at $R_v=\frac{1}{4}$.

We see that, as for the quantum point contact, the resistance R_v is always smaller than the charge relaxation resistance R_q. The distributions shown in Figs. 5 and 6 demonstrate that interesting information can be obtained from the measurement of frequency-dependent shot noise on chaotic quantum dots.

V. DISCUSSION

We investigated the spectrum of the current noise induced into the gate of a quantum point contact and of a chaotic cavity. The current noise spectrum is a direct measure of the charge or potential fluctuations of the conductor. For this calculation, we assumed that the external circuit exhibits zero impedance for the fluctuations. If the impedance of the external circuit is not zero, it is also necessary to investigate the effect of fluctuating reservoir voltages. The fluctuation spectra of a mesoscopic sample embedded in a circuit with nonvanishing impedance will then also depend on the properties of the external circuit.

We treated interactions in the random-phase approximation. Since exchange effects[13,14] play a role, a treatment of interactions on the Hartree-Fock level is very desirable. The discussion presented above also includes of course exchange effects but treats the self-consistent potential in random phase approximation. The importance to go beyond the single-parameter potential approximation, and to treat a continuous potential distribution, has already been emphasized. A theory already exists for the low-frequency fluctuations of a mesoscopic capacitor.[2]

We have found it useful to express the noise spectra with the help of resistances R_q and R_v. The charge relaxation resistance has a clear physical meaning since it also determines the dissipative, low-frequency admittance of a mesoscopic conductor.[8] The charge relaxation resistance differs from the two-terminal resistance, which one might naively want to use to characterize charge relaxation. Whether the resistance R_v introduced here will be useful beyond the discussion of noise properties is not presently apparent.

The current fluctuations induced into the gate are proportional to the square of the electrochemical capacitance of the conductor to the gate. The noise will thus be the smaller the more effectively the charge on the conductor is screened. The strong dependence on interaction of the properties discussed in this work are another illustration of the importance of screening in the discussion of dynamical effects in mesoscopic samples.

The frequency-dependent noise induced into a nearby gate is a first-order effect: It is not a small correction to an effect that exists already in the zero-frequency limit. This lets us hope that experimental detection of this noise is possible. From our work it is clear that such experiments would greatly contribute to our understanding of the dynamics of mesoscopic conductors and the role of interactions.

ACKNOWLEDGMENTS

The work of M.H.P. and M.B. was supported by the Swiss National Science Foundation. S.A.v.L. was supported by the Stichting voor Fundamenteel Onderzoek der Materie (FOM) and by the TMR network Dynamics of Nanostructures.

[1] M. J. M. de Jong and C. W. J. Beenakker, in *Mesoscopic Electron Transport*, edited by L. L. Sohn, L. P. Kouwenhoven, and G. Schön, NATO ASI Series E: Applied Sciences, Vol. 345 (Kluwer, Dordrecht, 1997), p. 225.

[2] M. Büttiker, J. Math. Phys. **37**, 4793 (1996).

[3] R. J. Schoelkopf, P. J. Burke, A. A. Kozhevnikov, and D. E. Prober, Phys. Rev. Lett. **78**, 3370 (1997).

[4] B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. **60**, 848 (1988).

[5] D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, J. Phys. C **21**, L209 (1988).

[6] M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, Phys. Rev. Lett. **75**, 3340 (1995).

[7] A. Kumar, L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **76**, 2778 (1996).

[8] M. Büttiker, H. Thomas, and A. Prêtre, Phys. Lett. A **180**, 364 (1993).

[9] V. A. Khlus, Zh. Eksp. Teor. Fiz. **93**, 843 (1987) [Sov. Phys. JETP **66**, 1243 (1987)].

[10] G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 513 (1989) [JETP Lett. **49**, 513 (1989)].

[11] M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990); Phys. Rev. B **46**, 12 485 (1992).

[12] Th. Martin and R. Landauer, Phys. Rev. B **45**, 1742 (1992).

[13] Ya. M. Blanter and M. Büttiker, Phys. Rev. B **56**, 2127 (1997).

[14] S. A. van Langen and M. Büttiker, Phys. Rev. B **56**, R1680 (1997).

[15] M. Büttiker, Phys. Rev. B **45**, 3807 (1992).

[16] T. Christen and M. Büttiker, Phys. Rev. Lett. **77**, 143 (1996).

[17] M. Büttiker and T. Christen, in *Mesoscopic Electron Transport* (Ref. 1), p. 259.

[18] S.-R. Eric Yang, Solid State Commun. **81**, 375 (1992).

[19] M. Ueda and A. Shimizu, J. Phys. Soc. Jpn. **62**, 2994 (1993).

[20] I. L. Aleiner, N. S. Wingreen, and Y. Meir, Phys. Rev. Lett. **79**, 3740 (1997).

[21] Y. Levinson, Europhys. Lett. **39**, 299 (1997).

[22] P. Mohanty and R. A. Webb, Phys. Rev. C **55**, R13 452 (1997).

[23] K. E. Nagaev, Phys. Lett. A **169**, 103 (1992).

[24] K. E. Nagaev, LANL preprint cond-mat/9706024.

[25] Y. Naveh, D. Averin, and K. K. Likharev, Phys. Rev. Lett. **79**, 3482 (1997).

[26] Y. V. Fyodorov and H. J. Sommers, Phys. Rev. Lett. **76**, 4709 (1996).

[27] V. A. Gopar, P. A. Mello, and M. Büttiker, Phys. Rev. Lett. **77**, 3005 (1996).

[28] P. W. Brouwer and M. Büttiker, Europhys. Lett. **37**, 441 (1997).

[29] Y. V. Fyodorov and H. J. Sommers, J. Math. Phys. **38**, 1918 (1997).

[30] P. W. Brouwer, K. M. Frahm, and C. W. J. Beenakker, Phys. Rev. Lett. **78**, 4737 (1997).

[31] M. Büttiker, J. Phys.: Condens. Matter **5**, 9361 (1993).

[32] P. W. Brouwer, S. A. van Langen, K. M. Frahm, M. Büttiker, and C. W. J. Beenakker, Phys. Rev. Lett. **79**, 913 (1997).

[33] M. Büttiker, A. Prêtre, and H. Thomas, Phys. Rev. Lett. **70**, 4114 (1993); A. Prêtre, H. Thomas, and M. Büttiker, Phys. Rev. B **54**, 8130 (1996).

[34] M. Büttiker, in *Noise in Physical Systems and 1/f Fluctuations*, edited by P. H. Handel and A. L. Chung, AIP Conf. Proc. No. 285 (AIP, New York, 1993), p. 3.

[35] V. Gasparian, T. Christen, and M. Büttiker, Phys. Rev. A **54**, 4022 (1997).

[36] M. Büttiker, Phys. Rev. B **41**, 7906 (1990).

[37] C. M. Marcus, A. J. Rimberg, R. M. Westervelt, P. F. Hopkins,

and A. C. Gossard, Phys. Rev. Lett. **69**, 506 (1992).

[38] A. M. Chang, H. U. Baranger, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **73**, 2111 (1994).

[39] I. H. Chan, R. M. Clarke, C. M. Marcus, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **74**, 3876 (1995).

[40] C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).

[41] For the case of a quantum dot coupled to a single perfect channel, see Ref. 27; I. L. Aleiner and L. I. Glazman, LANL preprint cond-mat/9710195.

[42] H. U. Baranger and P. A. Mello, Phys. Rev. Lett. **73**, 142 (1994); R. A. Jalabert, J.-L. Pichard, and C. W. J. Beenakker, Europhys. Lett. **27**, 255 (1994).

[43] E. P. Wigner, Phys. Rev. **98**, 145 (1955); F. T. Smith, *ibid.* **118**, 349 (1960).