Electron superconductivity in coupled electron-hole layers

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We investigate superconducting pairing of electrons when coupled to an adjacent layer of holes in a GaAs semiconductor heterostructure. When the density of the electrons is much higher than the density of holes, the repulsive electron-electron interaction is reduced by dynamical coupling to the strongly correlated holes. For the effective-mass ratio of electrons to holes in GaAs we find that the effective electron-electron interaction becomes attractive when the layer separation is smaller than a critical value. With equal effective masses this effect does not occur. We estimate the attraction and repulsion parameters for a superconducting state in the electron layer. A typical value of the superconducting transition temperature for experimentally accessible carrier densities and layer separations is $T_c = 100$ mK. [S0163-1829(98)01603-8]

The current interest in coupled layers of charge carriers in semiconductor heterostructures is partly motivated by the possibility of such systems forming exotic ground states. These are generated by the Coulomb interactions between charges in the opposite layers and so would not occur for a single layer of charges. An interesting possibility is the formation of a superconducting ground state. This was first proposed by Lozovik and Yudson[1] who suggested for coupled electron-hole layers that superconducting pairs could form across the layers. Malozovsky *et al.* and Longe and Bose[2] considered superconducting pairing of electrons with the attraction coming from plasmon exchange. This is in analogy with conventional BCS phonon exchange. Thakur *et al.*[3] and Platzman *et al.*[4] have discussed superconducting pairing of electrons in one layer in the presence of a Wigner crystal of holes in the second layer. Canright and Vignale[5] have discussed a superconducting transition in a related system consisting of two types of electrons of different effective masses coexisting in the same two-dimensional layer with the pairing mechanism coming from the exchange of electron-hole excitations.

In this paper we propose a quite different mechanism for superconducting pairing of electrons in one layer due to the presence of relatively strongly correlated holes in the other layer. The attractive interaction between the electrons is generated by Coulomb correlation effects and does not depend on exchange of plasmons or phonons. The attraction is not dependent on the holes being in a solid state.

We discuss the superconducting pairing using a general effective potential for two indistinguishable electrons. This takes into account the vertex and ladder corrections to the polarization. The potential is constructed assuming both the species are in the liquid phase. It can be used even when the correlations are very strong and it remains applicable if one of the species is driven to a localized state by the strong correlations.

When calculating the two-body potential with one species localized, it is conventional to assume that the properties of the localized species are independent of the other species. Also, the excitation spectra of the localized modes are usually calculated within the harmonic approximation. While for localized ions, which are much more massive than holes, these approximations are generally adequate, they cannot be expected to hold when the localized species are holes. Specifically, (i) Wigner phonons are known to be strongly anharmonic because of their large vibrationally amplitudes,[6] and (ii) the correlations within each layer of a coupled layer system are affected by the correlations in the other layer.[7] This has a significant effect on the Wigner phonon frequencies. In our approach we avoid these problems. Our derivation of the potential treats the two species on an interdependent and equal footing. The dependence of the eigenmodes of the system on correlations both within the layers and between layers is taken into account without resorting to the harmonic approximation.

The effective potential[8] between two electrons in an electron layer coupled to a hole layer is given by

$$V_{\rm eff}(\boldsymbol{q},\omega) = V_{ee}(\boldsymbol{q}) + \sum_{i,j=e,h} V_{ei}(\boldsymbol{q})[1-G_{ei}(\boldsymbol{q})] \times \chi_{ij}(\boldsymbol{q},\omega) V_{je}(\boldsymbol{q})[1-G_{je}(\boldsymbol{q})]. \quad (1)$$

The $V_{ij}(\boldsymbol{q})$ are the bare Coulomb interactions between carriers $i,j=e,h$ for a layer separation d and semiconductor substrate dielectric constant ϵ. The $G_{ij}(\boldsymbol{q})$ are static local fields.

In our approach we extract the $G_{ii}(\boldsymbol{q})$ from exact numerical simulation data for the ground state[9] of a single layer.

We express the density-density response function matrix elements $\chi_{ee}(\boldsymbol{q},\omega)$, $\chi_{hh}(\boldsymbol{q},\omega)$, and $\chi_{eh}(\boldsymbol{q},\omega)=\chi_{he}(q,\omega)$ in terms of Lindhard functions for noninteracting electrons $\chi_e^0(\boldsymbol{q},\omega)$, and holes $\chi_h^0(\boldsymbol{q},\omega)$,

$$\chi_{ii}(\boldsymbol{q},\omega)=\frac{\chi_i^0(\boldsymbol{q},\omega)}{1+\widetilde{\mathcal{V}}_{ii}(\boldsymbol{q},\omega)\chi_i^0(\boldsymbol{q},\omega)},\quad i=e,h, \tag{2}$$

$$\chi_{eh}(\boldsymbol{q},\omega)=\frac{\chi_e(\boldsymbol{q},\omega)V_{eh}(\boldsymbol{q})[1-G_{eh}(\boldsymbol{q})]\chi_h(\boldsymbol{q},\omega)}{1+\{V_{eh}(\boldsymbol{q})[1-G_{eh}(\boldsymbol{q})]\}^2\chi_e(\boldsymbol{q},\omega)\chi_h(\boldsymbol{q},\omega)}, \tag{3}$$

where

$$\chi_i(\boldsymbol{q},\omega)=\frac{\chi_i^0(\boldsymbol{q},\omega)}{1+V_{ii}(\boldsymbol{q})[1-G_{ii}(\boldsymbol{q})]\chi_i^0(\boldsymbol{q},\omega)}, \tag{4}$$

$$\widetilde{\mathcal{V}}_{ii}(\boldsymbol{q},\omega)=V_{ii}(\boldsymbol{q})[1-G_{ii}(\boldsymbol{q})]-V_{ij}(\boldsymbol{q})[1-G_{ij}(\boldsymbol{q})]\times\chi_j(\boldsymbol{q},\omega)V_{ji}(\boldsymbol{q})[1-G_{ji}(\boldsymbol{q})] \tag{5}$$

for $i=e,h$ and $j\neq i$.

Equation (1) can be written as

$$V_{\rm eff}(\boldsymbol{q},\omega)=U_{ee}(\boldsymbol{q},\omega)+U_{ehe}(\boldsymbol{q},\omega), \tag{6}$$

where $U_{ee}(\boldsymbol{q},\omega)$ is the contribution to $V_{\rm eff}(\boldsymbol{q},\omega)$ from the electron layer only,

$$U_{ee}(\boldsymbol{q},\omega)=V_{ee}(\boldsymbol{q})-V_{ee}^2(\boldsymbol{q})[1-G_{ee}(\boldsymbol{q})]^2\frac{\chi_e^0(\boldsymbol{q},\omega)}{1+V_{ee}(\boldsymbol{q})[1-G_{ee}(\boldsymbol{q})]\chi_e^0(\boldsymbol{q},\omega)}, \tag{7}$$

and $U_{ehe}(\boldsymbol{q},\omega)$ contains the effects of electron-hole interactions,

$$U_{ehe}(\boldsymbol{q},\omega)=-\left[\frac{V_{eh}(\boldsymbol{q})[1-G_{eh}(\boldsymbol{q})]}{1+V_{ee}(\boldsymbol{q})[1-G_{ee}(\boldsymbol{q})]\chi_e^0(\boldsymbol{q},\omega)}\right]^2\frac{\chi_h^0(\boldsymbol{q},\omega)}{1+M(\boldsymbol{q},\omega)\chi_h^0(\boldsymbol{q},\omega)}, \tag{8}$$

where

$$M(\boldsymbol{q},\omega)=V_{hh}(\boldsymbol{q})[1-G_{hh}(\boldsymbol{q})]-\{V_{eh}(\boldsymbol{q})[1-G_{eh}(\boldsymbol{q})]\}^2\frac{\chi_e^0(\boldsymbol{q},\omega)}{1+V_{ee}(\boldsymbol{q})[1-G_{ee}(\boldsymbol{q})]\chi_e^0(\boldsymbol{q},\omega)}. \tag{9}$$

$U_{ee}(\boldsymbol{q},\omega)$ in Eq. (7) is analogous to the direct screened electron-electron repulsion term in the BCS Hamiltonian. In Eq. (8) the factor $\chi_h^0(\boldsymbol{q},\omega)/[1+M(\boldsymbol{q},\omega)\chi_h^0(\boldsymbol{q},\omega)]$ is the polarizability of the holes in the presence of the electrons, while the direct electron-hole interaction $V_{eh}(\boldsymbol{q})[1-G_{eh}(\boldsymbol{q})]$ is screened by the electrons.

In a metallic solid the electron-phonon contribution to the effective two-body electron potential has a similar structure to Eq. (8);[10] i.e., it contains the square of the electron-phonon matrix element and the phonon propagator. The phonon frequencies are often calculated directly from the ion interaction potential. In contrast in the present system the eigenfrequencies of the plasmons in the two layers are strongly affected by correlations. The direct hole-hole potential $V_{hh}(\boldsymbol{q})[1-G_{hh}(\boldsymbol{q})]$ in Eq. (9) is modified by the screened electron-hole interaction. By treating both species on an equal basis, our expression for $V_{\rm eff}(\boldsymbol{q},\omega)$ remains valid even when the holes are strongly correlated.

Equation (1) is equally valid whether or not the hole layer is at a density where the holes are in a Wigner crystal state. However, experimentally it has not yet proved possible to fabricate a layer at a density low enough for the carriers to form a Wigner crystal. For the present calculation we took electron densities from $r_s^e=1$ to 2. The hole density we fixed at $r_s^h=5$. At this density the holes are in the liquid state. These electron and hole densities are accessible to experiments. The effective masses $m_e^\star=0.03m_e$, $m_h^\star=0.21m_e$, and dielectric constant $\epsilon=13$ were all set at the values for GaAs.

For the local field factors we used numerical simulation data[9] of the static structure factor $S_i(\boldsymbol{q})$ for an isolated electron layer to determine $G_{ee}(\boldsymbol{q})$ and $G_{hh}(\boldsymbol{q})$ following the procedure described in Ref. 7. We neglected $G_{eh}(\boldsymbol{q})$.

Figure 1 shows the dependence of the static $V_{\rm eff}(\boldsymbol{q},0)$ on the separation between the layers d [see Eq. (6)]. At a critical

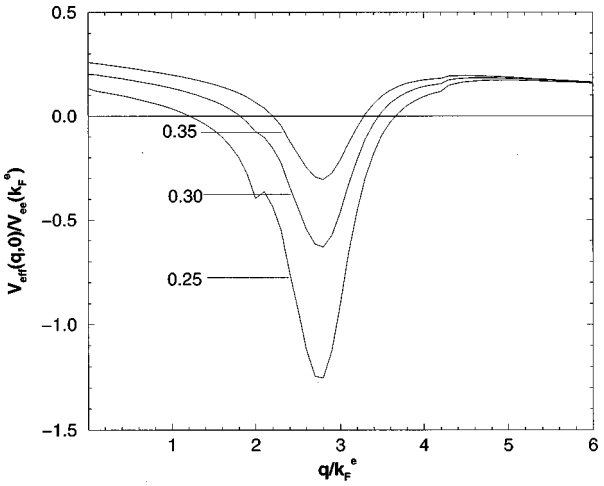

FIG. 1. Effective electron-electron interaction $V_{\rm eff}(\boldsymbol{q},0)$ for different layer separations. The electron and hole densities are $r_s^e=1.5$ and $r_s^h=5$. k_F^e is the electron Fermi momentum. The labels give the layer separations d in units of $a_B^\star=\hbar^2\epsilon/m_e^\star e^2=9.8$ nm, the electron effective Bohr radius in GaAs.

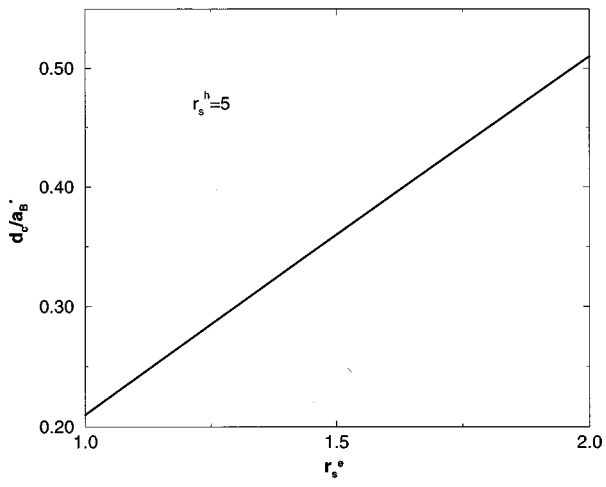

FIG. 2. Critical layer separation $d_c/a_B^\star$ as a function of the electron layer density r_s^e. The hole layer density $r_s^h=5$ is fixed.

separation $d=d_c$ the total potential $V_{\rm eff}(\boldsymbol{q},0)$ becomes attractive for a small range of $\boldsymbol{q}$. Further decreasing d extends the range of $\boldsymbol{q}$ over which $V_{\rm eff}(\boldsymbol{q},0)$ is attractive. This effect is caused by the behavior of $U_{ehe}(\boldsymbol{q},0)$ [Eq. (8)], which increases strongly with decreasing d. The contribution $U_{ee}(\boldsymbol{q},0)$ [Eq. (7)] depends only weakly on d. Figure 2 shows the dependence of the critical d_c on r_s^e for fixed $r_s^h=5$. The critical separation increases with r_s^e as the electron correlations get stronger. For the carrier densities and layer separations with which we work there is no tendency in the system towards a charge density wave instability.

$V_{\rm eff}(\boldsymbol{q},0)$ is sensitive to the effective mass ratio $m_h^\star/m_e^\star$. Setting the two effective masses equal leads to a $V_{\rm eff}(\boldsymbol{q},0)$ that remains repulsive for all layer separations. Correlations are also crucial for the attraction. Within the RPA $V_{\rm eff}(\boldsymbol{q},0)$ remains repulsive for even very small d. (We recall the RPA neglects correlations.)

We now turn to the parameters that determine superconducting properties like the transition temperature T_c. The attraction parameter λ is defined as the Fermi surface average of the static interactions,[8]

$$\lambda=\rho_e\int_0^{2k_F^e}d\boldsymbol{q}\frac{1}{2k_F^e}\left[\frac{V_{eh}(\boldsymbol{q})[1-G_{eh}(\boldsymbol{q})]}{1+V_{ee}(\boldsymbol{q})[1-G_{ee}(\boldsymbol{q})]\chi_e^0(\boldsymbol{q},0)}\right]^2\times|\chi_{hh}(\boldsymbol{q},0)|. \tag{10}$$

ρ_e is the density of states for electrons at the Fermi surface and

$$|\chi_{hh}(\boldsymbol{q},0)|=\frac{2}{\pi}\int_0^\infty\frac{d\omega}{\omega}|\mathrm{Im}\ \chi_{hh}(\boldsymbol{q},\omega+i\delta)|. \tag{11}$$

The repulsion parameter μ is defined as the s-wave Fermi surface average of the direct electron-electron static interactions,

$$\mu=\rho_e\int_0^{2k_F^e}d\boldsymbol{q}\frac{1}{2k_F^e}U_{ee}(\boldsymbol{q},0). \tag{12}$$

When the layer separation is much less than the critical d_c, the effective interaction $V_{\rm eff}(\boldsymbol{q},0)$ is attractive for a wide range of $\boldsymbol{q}$ and we can use the McMillan formula[11] to estimate T_c,

$$T_c=\omega_0\exp\left(-\frac{1}{\lambda^\star-\mu^\star}\right), \tag{13}$$

where $\lambda^\star=\lambda/(1+\lambda)$ and $\mu^\star=\mu/[1+\mu\ln(m_h^\star/m_e^\star)]$ are the renormalized parameters. We approximate the characteristic frequency ω_0 by the hole plasmon frequency as determined by the potential $M(\boldsymbol{q},\omega=0)$ at wave vector $|\boldsymbol{q}|=k_F^e$.

Using appropriate Fermi surface averaging we calculated λ and μ. Both λ and ω_0 are strongly dependent functions of d. We find that T_c increases rapidly with decreasing layer separation and decreasing electron density. For $d/a_B^\star=0.25$ and $r_s^e=1.5$ we obtain a value of $T_c=120$ mK. We can estimate the coherence length using the expression $\xi_0=(hv_F^e)/(2\pi\Delta)$, where according to BCS the zero-temperature energy gap is given by $\Delta=1.75k_BT_c$. v_F^e is the electron Fermi velocity. We find ξ_0 is much larger than the average interelectron spacing.

The value of T_c is very sensitive to the values of λ and μ and it also strongly depends on ω_0. Within the standard BCS approach Platzman *et al.*[4] discussed Wigner-phonon-induced superconductivity for coupled electron-hole layers. They calculated the frequency spectrum of Wigner phonons using the harmonic approximation. As we have pointed out, the harmonic approximation is inapplicable for Wigner phonons because of their large vibrational amplitudes.[6] They also assume that the Wigner phonon frequencies are unaffected by electron correlations, which is contradicted by results showing that correlations within a layer are affected by correlations in the other layer.[7] This has a significant effect on the Wigner phonon frequency. Platzman *et al.* used an approximate method to calculate λ and μ. They found for $r_s^h=3.24$ and $r_s^e=0.56$ that the electron layer becomes superconducting for layer separation $d=75$ Å. In our calculation for these values of the parameters the $V_{\rm eff}(\boldsymbol{q},0)$ is everywhere repulsive and there is no superconducting instability.

In this paper we have discussed an effective electron-electron interaction $V_{\rm eff}(\boldsymbol{q},0)$, which takes into account the exchange and correlation effects between the electrons and between the electrons and the medium. These features are particularly important for this type of superconductivity where the mass of the holes is comparable to the mass of the superconducting electrons. Using a generalized random-phase approximation for the density response functions we incorporated the exchange correlation effects through local-field factors $G_{ij}(\boldsymbol{q})$. Using the resulting effective potential we find with experimentally reasonable electron and hole densities that the critical layer separations and the transition temperatures for the superconducting transition lie in a range that should be experimentally detectable using current techniques.

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