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Size effects in fluctuation spectra of many-valley semiconductors

V. A. Kochelap and V. N. Sokolov
Institute of Semiconductor Physics, National Academy of Sciences of Ukraine, 45 prospect Nauki, 252650 Kiev, Ukraine

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We present the results of theoretical investigation of inhomogeneous fluctuations in submicrometer active layers of many-valley semiconductors with equivalent valleys (Ge, Si type), where the layer dimension, $2d$, is comparable to or less than the intervalley diffusion relaxation length, L_{iv}. The study is based on the Boltzmann-Langevin kinetic equation. Boundary conditions for the fluctuations on the layer surfaces are derived. It is shown that for arbitrary orientations of the valley axes (crystal axes) with respect to the surfaces, the fluctuation spectra depend on the applied small electric field. Some physical phenomena are reported: unlike bulk samples, intravalley fluctuation processes cause the intervalley fluctuations in thin layers; the spectra of fluctuations depend on the layer thickness; with $2d \lesssim L_{\mathrm{iv}}$, a considerable suppression of the fluctuations arises for the fluctuation frequency $\omega \ll \tau_{\mathrm{iv}}^{-1}$, where τ_{iv} is the characteristic intervalley relaxation time. [S0163-1829(98)03123-3]

I. INTRODUCTION

Electron fluctuations in semiconductors have received much attention over several decades because of their importance for both the fundamental and applied physics. For a long time, the low-frequency fluctuations, such as the flicker and generation-recombination noise, have been at the focus of the study. However, since the high frequency region is used in experimental investigations and employed in novel devices, much attention is paid to the nonequilibrium fluctuations in the frequency region where the main type of fluctuations is the noise associated with hot carriers[1–8] and intervalley transitions in many-valley semiconductors.[9–16] These type of fluctuations are studied in weak and strong (heating) electric fields for semiconductors with equivalent (Ge, Si type[9–14]) and nonequivalent (GaAs type[8,15,16]) valleys. All the above papers[1–16] concern bulk semiconductors.

Among important effects inherent in the high-frequency fluctuations, several cases stand out for which the intrinsic mechanisms of limitation or suppression of noise have been discovered recently.[17–26] The suppression of hot-electron noise in bulk samples occurs in compensated semiconductors with strong scattering of electrons by optical phonons (see theory in Ref. 17, experiment in Ref. 18) and in many-valley semiconductors with intensive electron-electron scattering.[14] Micrometer-length-diode structures show the limitation of hot-electron noise.[8,19,20] Thin submicrometer conductive layers, films, etc. demonstrate the suppression of the Nyquist noise,[21] ambipolar drift noise,[14] and hot-electron noise.[22] In nanoscale samples and structures, there also occurs the suppression of shot noise under ballistic and diffusive quantum transport.[23–26]

The results obtained in Refs. 17–26 indicate a fundamental way to control the electron fluctuations and current noise. Most of these works are focused on III-V semiconductors. Meanwhile, silicon remains the basic material of microelectronics. Recently,[27–29] a significant progress has been achieved in the technology of submicrometer Si-based structures and devices for high-speed and low-noise applications. Particularly, in Si-SiGe bipolar transistor the frequency up to about 90 GHz has been realized.[28,29] These materials and structures have multivalley electron energy spectra and show a considerable contribution to the noise from intervalley transitions of carriers.

In this paper, we report on our studies of the intervalley fluctuations in submicrometer active layers for which the boundaries and size effects are important. We show that the size effect allows the intervalley fluctuations to be controlled in such systems. The paper is organized as follows. In Sec. II, we qualitatively analyze the influence of interfaces and boundaries on the intervalley fluctuations and discuss the effect of both the intravalley and intervalley stochastic fluctuation sources. In Sec. III, we describe our model, introduce the basic equations, and derive the boundary conditions for fluctuations in the problem with restricted geometry of a sample. The intervalley fluctuations in the near-equilibrium electron gas are analyzed in Sec. IV. Section V is devoted to a discussion of the results and numerical estimates for the intervalley and generation-recombination fluctuations. Section VI draws the main conclusions of this work.

II. INTERVALLEY FLUCTUATIONS: INFLUENCE OF INTERFACES AND BOUNDARIES

In the theory of electron fluctuations based on the Langevin approach, the central problem is to derive the microscopic stochastic sources of fluctuations and their correlation functions (Ref. 4 and references therein). At least two kinds of sources are essential in many-valley semiconductors. The first is due to the intravalley random scattering of carriers. It leads to the Nyquist noise of the current. The second is caused by the intervalley processes giving rise to the excess noise.[9,10] In this section, we will qualitatively analyze the influence of boundaries on the intervalley fluctuations and discuss the role of both the intervalley and intravalley stochastic sources.

While a separate valley is characterized by its own anisotropy, the total conductivity remains isotropic in the range of nonheating electric fields because of cubic crystal symmetry: $\sigma_{ik} = \Sigma_{\alpha}^{\nu} \sigma_{ik}^{(\alpha)} = \sigma \delta_{ik}$, where $\sigma_{ik}^{(\alpha)}$ is the conductivity tensor of the αth valley, ν is the total number of valleys. Similarly, the correlation function of the Nyquist fluctuations of the current is also isotropic for bulk crystals as a tensor of rank two: $\langle \delta j_i \delta j_k \rangle = \langle (\delta j)^2 \rangle \delta_{ik}$. In the range of electric fields where Ohm's law is valid, the intervalley noise is proportional to the square of the applied electric field $\boldsymbol{E}$. The intensity of fluctuations is described by a tensor of rank four, which cannot be reduced to a scalar quantity due to the crystal symmetry. Thus, in contrast to the Nyquist noise, the excess intervalley noise *is anisotropic*. One can obtain the following comparative estimate for amplitudes of the intervalley (iv) and the Nyquist (i) noise:[13]

$$\frac{(\delta j^2)^{\mathrm{iv}}_{\omega}}{(\delta j^2)^{i}_{\omega}} \approx \frac{v_d^2 \tau_{\mathrm{iv}}}{(1+\omega^2 \tau_{\mathrm{iv}}^2) v^2 \tau_p} \approx \frac{E_x^2}{(1+\omega^2 \tau_{\mathrm{iv}}^2) E_0^2}. \tag{1}$$

Here, τ_{iv} is the intervalley time, τ_p is the electron momentum $\boldsymbol{p}$ relaxation time, v_d and v are the drift and thermal velocities, respectively; the external field is assumed to be along the x direction, $E_0 = k_B T / e L_{\mathrm{iv}}$ is the characteristic field related to the intervalley relaxation length $L_{\mathrm{iv}} = (D \tau_{\mathrm{iv}})^{1/2}$, where D is the diffusion coefficient, T is the temperature, k_B is the Boltzmann constant, e is the electron charge. Expression (1) is given for the actual frequency interval $\omega \tau_p \ll 1$. It is seen that the intervalley noise dominates over the Nyquist one in the frequency region $\omega \tau_{\mathrm{iv}} \lesssim 1$ provided that $E_x \gg E_0$. Since typically the intervalley time τ_{iv} is greater than the electron energy relaxation time τ_{ϵ}, the intervalley scattering of carriers is the main source of the excess noise in the field range

$$E_0 \ll E_x \ll E_{\epsilon}, \quad E_{\epsilon} \equiv k_B T / e L_{\epsilon}, \tag{2}$$

where the hot-electron effect is negligible [$L_{\epsilon} = (D \tau_{\epsilon})^{1/2}$ is the characteristic electron energy relaxation length].

It should be noted that both kinds of the fluctuation sources (intravalley and intervalley) are uncorrelated. As a consequence, for bulk materials the intravalley source does not lead to the intervalley current noise. On the contrary, for thin layers both kinds of the sources give rise to the intervalley fluctuations and current noise. In this work, we investigate the fluctuations in size-restricted samples (layers) of many-valley semiconductors with equivalent valleys. We show that for thin samples of thickness $2d \lesssim L_{\mathrm{iv}}$ the intensity and characteristic frequency of the fluctuations essentially differ from those for bulk crystals. In brief, this difference is as follows. In restricted samples, the fluctuations are inhomogeneous because the surface contributes to the overall relaxation process. The intravalley stochastic source in the αth valley generates the random spatial inhomogeneous flow of electrons, $\tilde{\boldsymbol{I}}^{i}_{\alpha}(\boldsymbol{r},t)$, which, in turn, leads to fluctuations of the local electron density, $\delta n_{\alpha}(\boldsymbol{r},t)$. It can be estimated as $\delta n^{i}_{\alpha} \approx \tau_{\mathrm{iv}}(\mathrm{div}\ \tilde{\boldsymbol{I}}^{i}_{\alpha})$. The fluctuation $\delta n^{\mathrm{iv}}_{\alpha}$ caused by intervalley transitions associated with the appropriate stochastic source, $\tilde{I}^{\mathrm{iv}}_{\alpha}$, must be of the order of $\tau_{\mathrm{iv}} \tilde{I}^{\mathrm{iv}}_{\alpha}$. The relative contribution of both the sources into the spectral density of the fluctuations is given by

$$\gamma = \frac{(\delta n^2_{\alpha})^{i}_{\omega}}{(\delta n^2_{\alpha})^{\mathrm{iv}}_{\omega}}. \tag{3}$$

By using the relations that hold true for correlators of the stochastic sources: $\langle \tilde{I}^{i}_{\alpha k} \tilde{I}^{i}_{\alpha l} \rangle \propto D^{(\alpha)}_{kl}$, $\langle \tilde{I}^{\mathrm{iv}}_{\alpha} \tilde{I}^{\mathrm{iv}}_{\alpha} \rangle \propto \tau_{\mathrm{iv}}^{-1}$, $\langle \tilde{I}^{i}_{\alpha k} \tilde{I}^{\mathrm{iv}}_{\beta} \rangle = 0$, we can estimate the parameter γ taking $\tilde{I}^{i}_{\alpha k} \propto D^{1/2}$, $\tilde{I}^{\mathrm{iv}}_{\alpha} \propto \tau_{\mathrm{iv}}^{-1/2}$, $(\mathrm{div} \tilde{\boldsymbol{I}}^{i}_{\alpha}) \approx (|\tilde{\boldsymbol{I}}^{i}_{\alpha}|/l)$, where l is the characteristic length scale of the fluctuations, $D^{(\alpha)}_{kl}$ represents the diffusion tensor in the αth valley. Then, from Eq. (3) we get $\gamma \approx L^2_{\mathrm{iv}}/l^2$. It is evident for thin layers ($2d \ll L_{\mathrm{iv}}$) we should take $l \approx 2d$. For thick layers ($2d \gg L_{\mathrm{iv}}$), the fluctuation δn^{i}_{α} generated by the random spatial electron flow is characterized by a length scale greater than L_{iv}. Therefore, such a fluctuation is substantially affected by the intervalley relaxation. As a result, the characteristic length scale of the fluctuations proves to be $l = (2 d L_{\mathrm{iv}})^{1/2}$ for thick layers.

These qualitative estimates show that the fluctuations in thick layers are mainly due to the intervalley stochastic sources $\tilde{I}^{\mathrm{iv}}_{\alpha}$: $\gamma \approx L_{\mathrm{iv}}/2d \ll 1$. For thin layers, we have $\gamma \approx L^2_{\mathrm{iv}}/(2d)^2 \gg 1$, which means that the fluctuations are mainly due to the intravalley stochastic sources $\tilde{\boldsymbol{I}}^{i}_{\alpha}$. If $2d \approx L_{\mathrm{iv}}$, both kinds of the stochastic sources are of the same order of magnitude. It is worthwhile to note that the contribution resulting from the intravalley stochastic sources to the intervalley fluctuations has been ignored in previous theoretical works.

In restricted samples, an internal fluctuating electric field arises along the transverse direction with respect to the applied field.[22] This fluctuating field renormalizes the intravalley stochastic sources and, in addition, gives rise to the dependence of the fluctuations on the applied field. Besides the stochastic sources discussed above, there exists an additional source of the fluctuations that originates from intervalley relaxation on the surface. The fluctuations generated on the surface can transfer into the bulk of a layer, while those arisen in the bulk can diffuse to the surface and relax on it. If the intervalley surface scattering rate, S, is large enough ($S \gg D/d$), the fluctuations are redistributed over the spectrum, i.e., the intensity of the fluctuations decreases at low frequencies $\omega \tau_{\mathrm{iv}} \ll 1$ and increases at higher frequencies $\omega \tau_{\mathrm{iv}} \gg 1$ (''blueshift'').

We stress, once again, that the above-mentioned qualitative features of the intervalley fluctuations are completely due to the boundaries and strong surface intervalley relaxation processes. Those features are characteristic of size restricted samples and do not occur in bulk crystals where the surface does not practically affect their electrophysical and fluctuative properties.

III. THE MODEL AND BASIC EQUATIONS

A. The model

The temporal evolution of spatially inhomogeneous quasineutral fluctuations of the carrier density in individual valleys is governed by the set of coupled stochastic continuity equations

$$\frac{\partial}{\partial t} \delta n_{\alpha}(\boldsymbol{r},t) + \mathrm{div}\ \delta \boldsymbol{i}^{\alpha}(\boldsymbol{r},t) = \sum_{\beta \neq \alpha} \left(\frac{\delta n_{\alpha}(\boldsymbol{r},t)}{\tau_{\alpha\beta}} - \frac{\delta n_{\beta}(\boldsymbol{r},t)}{\tau_{\beta\alpha}} \right) + \tilde{I}^{\mathrm{iv}}_{\alpha}(\boldsymbol{r},t), \tag{4}$$

where the intervalley time $\tau_{\alpha\beta}$ corresponds to electron transitions from valley α to valley β ($\alpha,\beta=1\ldots\nu$),

$$\delta \boldsymbol{i}_k^{\alpha}(\boldsymbol{r},t)=-\mu_{kl}^{\alpha}[E_l\,\delta n_\alpha(\boldsymbol{r},t)+n_\alpha\,\delta E_l(\boldsymbol{r},t)]-D_{kl}^{\alpha}\frac{\partial}{\partial x_l}\delta n_\alpha(\boldsymbol{r},t)+\tilde{\boldsymbol{I}}^{i}_{\alpha k}(\boldsymbol{r},t) \tag{5}$$

is a fluctuation of the partial particle flow density (μ_{kl}^{α} is the mobility tensor of the carriers in the αth valley). Equations (4) and (5) contain the Langevin sources of fluctuations due to intervalley and intravalley scattering of electrons:

$$\tilde{I}^{\,\mathrm{iv}}_{\alpha}(\boldsymbol{r},t)=\sum_{\boldsymbol{p}}\chi^{\mathrm{iv}}_{\alpha\boldsymbol{p}}(\boldsymbol{r},t), \tag{6}$$

$$\tilde{\boldsymbol{I}}^{i}_{\alpha}(\boldsymbol{r},t)=\sum_{\boldsymbol{p}}\boldsymbol{v}\tau_p\chi^{i}_{\alpha\boldsymbol{p}}(\boldsymbol{r},t), \tag{7}$$

$\chi^{i,\mathrm{iv}}_{\alpha\boldsymbol{p}}$ are the stochastic microscopic forces with known correlation properties appearing in the Boltzmann-Langevin equation.[4,13] For these forces the following identities are valid:

$$\sum_{\boldsymbol{p}}\chi^{i}_{\alpha\boldsymbol{p}}(\boldsymbol{r},t)=0,\quad \sum_{\alpha=1}^{\nu}\sum_{\boldsymbol{p}}\chi^{\mathrm{iv}}_{\alpha\boldsymbol{p}}(\boldsymbol{r},t)=0, \tag{8}$$

which means the conservation of the partial and total densities of electrons with respect to intravalley and intervalley scattering.

Equations (4)–(7) correspond to low-frequency and long-range fluctuations of hydrodynamic type, which require

$$\omega\tau_p\ll 1,\quad L_p\ll l,L_{\mathrm{iv}}. \tag{9}$$

The conditions of quasineutrality for stationary regime and fluctuations

$$\sum_{\alpha=1}^{\nu}n_\alpha=\nu n_0=N,\quad \sum_{\alpha=1}^{\nu}\delta n_\alpha(\boldsymbol{r},t)=0, \tag{10}$$

complete Eqs. (4) and incorporate complementary requirements for the frequency and spatial-time parameters:

$$\omega\tau_M\ll 1,\quad \tau_M\ll\tau_{\mathrm{iv}},\quad l_D\ll l,\ L_{\mathrm{iv}},\ 2d, \tag{11}$$

τ_M is the Maxwell relaxation time, l_D is the Debye screening length, and N is the overall electron density.

We will consider a plate-shaped sample[30,31] of the thickness $2d=L_y$ along the y axis (the smallest size of the sample), with their lateral dimensions $L_{x,z}$ being considerably larger than the thickness: $2d\lesssim L_{\mathrm{iv}}\ll L_x\ll L_z$. Equations (4) can be averaged over the xz plane. After that the problem becomes one dimensional so that all the quantities in Eqs. (4)–(7) are only dependent on the coordinate y, for instance,

$$\delta n_\alpha(y,t)=\frac{2d}{V_0}\int_{-L_x/2}^{L_x/2}dx\int_{-L_z/2}^{L_z/2}dz\,\delta n_\alpha(\boldsymbol{r},t),$$

$V_0=2dL_xL_z$ being the volume of the sample. The internal transverse field arising along the y direction is to be found from Maxwell's equations:

$$\mathrm{rot}\,\boldsymbol{E}=0,\quad \mathrm{div}\,\boldsymbol{j}=0. \tag{12}$$

Supposing the electric circuit in the y direction to be opened on both dc and ac current and taking into account Eqs. (10), we can find the fluctuating field δE_y from the equation

$$\delta i_y(y,t)=0, \tag{13}$$

where $\delta i_y=\Sigma_\alpha\delta i_y^\alpha$. Notice that with the above assumptions we may also take $\delta E_{x,z}=0$.

Further it is convenient to use the Fourier transforms for Eqs. (4):

$$-i\omega\,\delta n_\alpha(y,\omega)+\frac{d}{dy}\delta i_y^{\alpha}(y,\omega)=-\sum_{\beta\neq\alpha}\left(\frac{\delta n_\alpha(y,\omega)}{\tau_{\alpha\beta}}-\frac{\delta n_\beta(y,\omega)}{\tau_{\beta\alpha}}\right)+\tilde{I}^{\,\mathrm{iv}}_{\alpha}(y,\omega). \tag{14}$$

Due to the presence of interfaces, a closed set of equations includes boundary conditions for the fluctuations that in the case under discussion are to be set on the lateral faces at $y=\pm d$. Below, using a simple model, we derive and analyze in detail the boundary conditions to Eqs. (14).

B. Boundary conditions

It is known that there still exists a problem of boundary conditions to the Boltzmann kinetic equation for the one-particle distribution function in the problems where surface scattering of electrons is essential. Finding a solution to this problem is an extraordinarily complex theoretical question, which has not yet been completely examined. In a quantum microscopic approach, the boundary conditions have been derived for the case of surface scattering of the electron momentum in metals.[32] To our knowledge, an analogous treatment for electron fluctuations has not been carried out. Here, the state of lateral faces of the crystal is described by the boundary conditions to the continuity equations (4). Let us consider an extremely thin boundary layer of the thickness δ, where $L_p\ll\delta\ll d$ (a more detailed criterion will be given below). We assume the characteristic intervalley relaxation time within the layer, τ^{s}_{iv}, to be much smaller than that in the bulk, τ_{iv}. Note that in the frame of this model the kinetic parameters may be discontinuous, with integrated (over the layer) singularity. The fluctuative quantities, such as $\delta n_\alpha(y,\omega)$ and $\delta i_y^{\alpha}(y,\omega)$, are continuous functions of the position as being governed by the second-order drift-diffusion equation. Our derivation uses two models with different behavior of the kinetic relaxation parameters within the boundary layer.

For the first model, we assume the $\tau^{s}_{\mathrm{iv}}(y)$ value to be characterized by extremely drastic change on the δ scale. Integrating directly Eqs. (14) over the boundary layer, we get the equations

$$\delta i_y^{\alpha}(y=\pm d,\omega)=\pm\sum_{\beta\neq\alpha}(S^{\pm}_{\alpha\beta}\delta n^{\pm}_{\alpha}-S^{\pm}_{\beta\alpha}\delta n^{\pm}_{\beta})\mp\tilde{u}^{\pm}_{\alpha}(\omega), \tag{15}$$

which are the appropriate boundary conditions to *the bulk* Eqs. (14). Here, we have introduced the surface carrier densities $\delta n^{\pm}_{\alpha}$, the surface rates of intervalley relaxation

$$S^{\pm}_{\alpha\beta}=\pm\int_{\pm(d-\delta)}^{\pm d}\frac{dy}{\tau_{\alpha\beta}(y)},\qquad(16)$$

and the Langevin surface sources

$$\tilde{u}^{\pm}_{\alpha}(\omega)=\pm\int_{\pm(d-\delta)}^{\pm d}\tilde{I}^{\mathrm{iv}}_{\alpha}(y,\omega)dy.\qquad(17)$$

Performing the indicated integration over y, we have taken into account that the fluctuation factor is slowly varying over the layer and can be taken outside the integral. We have also assumed the particle flow density on the crystal boundary with nonconducting surrounding medium to equal zero.

Once the Langevin surface sources are known, the correlation functions (spectral densities) can be immediately calculated. Using the expression (6) and the correlation relations given in Refs. 4 and 13, and taking into account the second of the properties (8), we find

$$(\tilde{u}_1\tilde{u}_2)^{\pm}_{\omega}=-\frac{4dn_0S^{\pm}}{V_0}.\qquad(18)$$

The spectral density (18) is expressed in terms of the surface rate of intervalley relaxation (16).

As is seen from Eq. (15), the fluctuation spectra in restricted samples become dependent on both the sample thickness and the surface intervalley relaxation rate. The most considerable modification of the spectra occurs under the strong surface intervalley scattering:

$$S^{\pm}\gg\frac{D}{d},\qquad(19)$$

that is, when the surface relaxation rate is much larger than the effective diffusion velocity $v_D=D/d$. In the case of Eq. (19), Eqs. (15) can be simplified:

$$\delta n_{\alpha}(y=\pm d,\ \omega)=0.\qquad(20)$$

Really, we can apply an iterative procedure to Eqs. (15), using the assumed large aspect ratio $\Gamma\equiv Sd/D\gg1$ (we put $S^{+}=S^{-}\equiv S$). The first iteration yields

$$\delta n^{\pm}_{\alpha}\approx\Gamma^{-1/2}\propto S^{-1/2}.\qquad(21)$$

A maximal attainable rate of intervalley relaxation on the surface can be estimated as $(1/4)v$.[30] In the drift-diffusion approach this corresponds to $S\rightarrow\infty$. It follows from Eq. (21) that Eqs. (15) reduce to Eqs. (20) in the limit $S\rightarrow\infty$. The boundary conditions of the form (20) have been used in Ref. 33. We also note that the criterion (19) ($S=\infty$) is directly opposite to that of Ref. 30 ($S=0$). Therefore, we may neglect the effect of carrier domain formation that can arise in thin samples.[30]

Obviously, the surface relaxation rate (16) and the spectral density (18) are to depend on the thickness δ. In turn, this dependence is to specify an explicit form of the boundary carrier density fluctuations $\delta n^{\pm}_{\alpha}$ approaching zero when the surface intervalley relaxation time τ^{s}_{iv} tends to zero. To explore such a dependence in more detail, we now consider another model of the boundary layer for which the essential kinetic relaxation parameters are assumed to be constant within the layer. A general solution of equations for the fluctuations is given in the Appendix. We make use of its results to analyze the boundary conditions.

The relation (A10) may be put in a form similar to *boundary* Eqs. (15):

$$\delta i=S\ \delta n(-\delta)-\tilde{u},\qquad(22)$$

with the surface relaxation rate S given by

$$S=\frac{D}{l}\tanh\frac{\delta}{l}.\qquad(23)$$

The spectral density of the correlation function for the Langevin surface source on the right-hand side of Eq. (22) can be readily calculated from Eqs. (A11)–(A15):

$$(\tilde{u}\tilde{u})_{\omega}=\frac{2dn_0S}{V_0}.\qquad(24)$$

By virtue of the continuity property discussed above for the fluctuative variables δn and δi at the boundary, the relation (22) is just the boundary condition in question.

Generally, the expressions (A11)–(A14) and (23)–(24) have been obtained for arbitrary relations between δ and l. It is of interest to analyze these expressions in combination with Eq. (22) in the limiting cases.

For the thin boundary layer ($\delta\ll l$) from Eq. (23), we obtain $S=\delta/\tau^{s}_{\mathrm{iv}}$. It is easy to see that the same expression for S results from Eq. (16), if we take in the integrand the time $\tau_{\alpha\beta}(y)$ to be constant. Consequently, the inequality $\delta\ll l$ is the explicit criterion of the validity of expressions for the surface rate and the Langevin surface sources in the form of Eqs. (16) and (17). Since our primary concern is with large S, then combining the expression for it with the condition (19), we obtain

$$S=\frac{\delta}{\tau^{s}_{\mathrm{iv}}},\quad \frac{l}{d}\ll\frac{\delta}{l}\ll1.\qquad(25)$$

From Eq. (23) it follows that the parameter S increases with increasing δ and tends to saturate. In the opposite limit ($\delta\gg l$), we find $S=l/\tau^{s}_{\mathrm{iv}}$. Similarly to Eq. (25), we can write for its maximal (saturated) value

$$S_{\mathrm{max}}=\frac{l}{\tau^{s}_{\mathrm{iv}}},\quad \frac{d}{l}\gg\frac{\delta}{l}\gg1.\qquad(26)$$

Comparing Eqs. (25) and (26), one can see that *the asymptotic dependences*, $S=S(\delta,\tau^{s}_{\mathrm{iv}})$, for the limiting cases are given by $S\propto(\tau^{s}_{\mathrm{iv}})^{-1/2}$ and $S=S_{\mathrm{max}}\propto(\tau^{s}_{\mathrm{iv}})^{-1/4}$, respectively. The S_{max} value can be approximately estimated from Eq. (26): $S_{\mathrm{max}}=(D\tau^{s}_{\mathrm{iv}})^{1/2}/\tau^{s}_{\mathrm{iv}}\approx v(\tau^{s}_{p}/\tau^{s}_{\mathrm{iv}})^{1/2}$. In particular, with $\tau^{s}_{\mathrm{iv}}\approx\tau^{s}_{p}$ this provides the above-mentioned estimate for the limitary attainable intervalley relaxation rate associated with the intensive surface scattering.[30]

In a similar way, we can analyze the dependence of the carrier density fluctuation, $\delta n(-\delta)$, on the kinetic relaxation parameter, τ^{s}_{iv}. Using the same iterative treatment as above [Eqs. (22) and (24), (25)], we find

$$\delta n(-\delta)\approx S^{-1/2}\propto(\tau^{s}_{\mathrm{iv}})^{1/2}.\qquad(27)$$

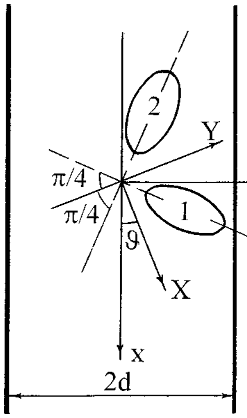

FIG. 1. Orientation of valleys with respect to the lateral faces in the two-valley model.

In the range where $S \approx S_{\max}$ (26), the dependence on τ^s_{iv} becomes more gradual

$$\delta n(-\delta) \approx S^{-1/2} \propto (\tau^s_{\mathrm{iv}})^{1/4}. \tag{28}$$

The expressions (21) and (27) and (28) provide a direct justification of the boundary conditions (20) under the criterion (19). It is worth stressing that our consideration is based on the distinct models and includes the various limiting cases. Nevertheless, the final results demonstrate the same dependence of the boundary density fluctuation on the surface rate of intervalley relaxation [see Eq. (21) and Eqs. (27) and (28)]. Therefore, this approach can be expected to be quite reasonable for the problem of the boundary conditions for the fluctuations in restricted many-valley crystals.

C. Solution of stochastic continuity equations

In order to avoid a confusion of computational detail in the solving of general equations (14), we consider the two-valley model[30] illustrated in Fig. 1. Equations (14) reduce to an equation for the relative fluctuation, $\delta f = \delta n_1/n_0$, which can be written as

$$\hat{\mathcal{L}}[\delta f(\zeta,\omega)] = \tilde{\Phi}(\zeta,\omega). \tag{29}$$

The boundary conditions read

$$\delta f(\zeta = \pm\zeta_0, \omega) = 0. \tag{30}$$

The operator $\hat{\mathcal{L}}$ on the left-hand side and the function $\tilde{\Phi}(\zeta,\omega)$ on the right-hand side of Eq. (29) are given by

$$\hat{\mathcal{L}} \equiv i\omega\tau_{\mathrm{iv}} + \frac{d^2}{d\zeta^2} + a_1(\vartheta)\mathcal{E}\frac{d}{d\zeta} - 1, \tag{31}$$

$$\tilde{\Phi}(\zeta,\omega) \equiv \alpha\left[L_{\mathrm{iv}}^{-1}\frac{d}{d\zeta}\ \tilde{I}^{i}(\zeta,\omega) - \tilde{I}^{\mathrm{iv}}(\zeta,\omega)\right]. \tag{32}$$

The effective stochastic sources of fluctuations are specified by

$$\tilde{I}^{\mathrm{iv}}(\zeta,\omega) \equiv \tilde{I}_1^{\mathrm{iv}}(\zeta,\omega),$$

$$\tilde{I}^{i}(\zeta,\omega) = \tilde{I}_y^{-}(\zeta,\omega) + a\sin 2\vartheta \tilde{I}_y^{+}(\zeta,\omega), \quad \tilde{I}_k^{\pm} = \frac{1}{2}(\tilde{I}^{i}_{1k} \pm \tilde{I}^{i}_{2k}), \tag{33}$$

where the major angular dependence is contained in the anisotropic factor

$$a_1(\vartheta) = \frac{a^2}{2}\sin 4\vartheta, \tag{34}$$

and the rest designations are $\zeta = y/L_{\mathrm{iv}}$, $\zeta_0 = d/L_{\mathrm{iv}}$, $\mathcal{E} = E_x/E_0$, $\alpha = L^2_{\mathrm{iv}}/n_0 D$, $D = D^{(1)}_{xx}(1 - a^2\sin^2 2\vartheta)$, $a = D^{(1)}_{xy}/D^{(1)}_{xx}$, $\tau_{12} = \tau_{21} \equiv 2\tau_{iv}$.

The solution to Eqs. (29) and (30) is found to be

$$\delta f(\zeta,\omega) = \int_{-\zeta_0}^{\zeta_0} \tilde{\Phi}(\zeta,\omega) G_\omega(\zeta,\zeta')\ d\zeta', \tag{35}$$

with $G_\omega(\zeta,\zeta')$ denoting Green's function of the operator (31) with the zero boundary conditions. To find $G_\omega(\zeta,\zeta')$, we consider two different regions, $\zeta' \geqslant \zeta$ and $\zeta' \leqslant \zeta$, for which we introduce

$$G^{>}_\omega(\zeta,\zeta') \equiv G_\omega(\zeta \geqslant \zeta', \zeta'), \quad G^{<}_\omega(\zeta,\zeta) \equiv G_\omega(\zeta \leqslant \zeta', \zeta'). \tag{36}$$

Substituting Eq. (36) into Eq. (35), we get

$$\delta f(\zeta,\omega) = \int_{-\zeta_0}^{\zeta} \tilde{\Phi}(\zeta',\omega) G^{>}_\omega(\zeta,\zeta') d\zeta' + \int_{\zeta}^{\zeta_0} \tilde{\Phi}(\zeta',\omega) G^{<}_\omega(\zeta,\zeta') d\zeta', \tag{37}$$

where Green's functions are given by

$$G^{>}_\omega(\zeta,\zeta') = \frac{\exp[k_{(+)}(\zeta-\zeta')]}{k_{(-)}\sinh[2k_{(-)}\zeta_0]}\sinh[k_{(-)}(\zeta-\zeta_0)] \times \sinh[k_{(-)}(\zeta'+\zeta_0)],$$

$$G^{<}_\omega(\zeta,\zeta') = \frac{\exp[k_{(+)}(\zeta-\zeta')]}{k_{(-)}\sinh[2k_{(-)}\zeta_0]}\sinh[k_{(-)}(\zeta+\zeta_0)] \times \sinh[k_{(-)}(\zeta'-\zeta_0)], \tag{38}$$

with $k_{(\pm)} = (k_1 \pm k_2)/2$ or, alternatively,

$$k_{(+)} = -\frac{1}{2}a_1(\vartheta)\mathcal{E}, \quad k_{(-)} = \sqrt{\frac{a_1^2(\vartheta)}{4}\mathcal{E}^2 + 1 - i\omega\tau_{\mathrm{iv}}}. \tag{39}$$

Finally, in accordance with Eqs. (37)–(39), we can write

$$\delta f(\zeta,\omega) = k^{-1}_{(-)}[\mathcal{F}(\zeta,\zeta_0) + \mathcal{F}(\zeta,-\zeta_0)], \tag{40}$$

where

$$\mathcal{F}(\zeta,\zeta_0) = \frac{\sinh[k_{(-)}(\zeta-\zeta_0)]}{\sinh[2k_{(-)}\zeta_0]}\int_{-\zeta_0}^{\zeta}\tilde{\Phi}(\zeta',\omega) \times \exp[k_{(+)}(\zeta-\zeta')]\sinh[k_{(-)}(\zeta'+\zeta_0)]d\zeta'. \tag{41}$$

Since it is of interest to compare the contributions of the different Langevin sources given in Eqs. (6) and (7) to the noise, we consider their action separately. To this end, we can split the total fluctuation $\delta f(\zeta,\omega)$ into two parts:

$$\delta f(\zeta,\omega)=\delta f^{i}(\zeta,\omega)+\delta f^{\mathrm{iv}}(\zeta,\omega). \qquad (42)$$

From Eqs. (40) and (41), we get for the two terms:

$$\delta f^{i,\mathrm{iv}}(\zeta,\omega)=\frac{\exp[k_{(+)}\zeta]}{k_{(-)}}\left[\int_{\zeta_0}^{\zeta}\tilde{I}^{\,i,\mathrm{iv}}(\zeta',\omega)F^{i,\mathrm{iv}}(-\zeta,\zeta',\omega)d\zeta' + \frac{\sinh[k_{(-)}(\zeta_0-\zeta)]}{\sinh[2k_{(-)}\zeta_0]} \times\int_{-\zeta_0}^{\zeta_0}\tilde{I}^{\,i,\mathrm{iv}}(\zeta',\omega)F^{i,\mathrm{iv}}(\zeta_0,\zeta',\omega)d\zeta'\right], \qquad (43)$$

where straightforward calculations give the expressions

$$F^{\mathrm{iv}}(\zeta,\zeta',\omega)=\exp[-k_{(+)}\zeta']\sinh[k_{(-)}(\zeta+\zeta')], \qquad (44)$$

$$F^{i}(\zeta,\zeta',\omega)=\frac{\partial}{\partial\zeta'}F^{\mathrm{iv}}(\zeta,\zeta',\omega).$$

By virtue of the results obtained in this subsection, it is easy to calculate spectral densities of the correlation functions for fluctuations of the valley carrier density, current density, transverse voltage and to study their field, frequency, and size dependences. This will be done in the next section.

IV. INTERVALLEY NEAR-EQUILIBRIUM FLUCTUATIONS

One of essential features of the electron fluctuations in restricted samples is their spatial inhomogeneity. The macroscopic observable quantities should be averaged over the sample thickness:

$$(\delta j_i\delta j_k)_\omega=\frac{1}{(2d)^2}\int_{-d}^{d}dy_1\int_{-d}^{d}dy_2[\delta j_i(y_1)\delta j_k(y_2)]_\omega . \qquad (45)$$

By using the expressions resulting from Eqs. (5) and (13) for the current density

$$\delta j_x=-2e\left[\tilde{I}_x^{\,+}-an_0D\left(\mathcal{E}\sin(2\vartheta)+\cos(2\vartheta)\frac{d}{d\zeta}\right)\delta f\right] \qquad (46)$$

and the fluctuative transverse electric field

$$\delta\mathcal{E}_y=\frac{L_{iv}}{n_0D}\tilde{I}_y^{\,+}+a\left[\sin(2\vartheta)\frac{d}{d\zeta}-\cos(2\vartheta)\mathcal{E}\right]\delta f, \qquad (47)$$

we find the spectral density of the current density fluctuations:

$$S_j(\omega,E_x)=1+a^2\sin^2(2\vartheta)\mathcal{E}^2S_f(\omega,E_x). \qquad (48)$$

We have introduced the dimensionless quantities

$$S_j(\omega,E_x)\equiv\frac{(\delta j_x\delta j_x)_\omega(E_x)}{(\delta j^2)^{\infty}_{\omega=0}},\quad S_f(\omega,E_x)\equiv\frac{(\delta f\delta f)_\omega(E_x)}{(\delta f^2)^{\infty}_{\omega=0}}, \qquad (49)$$

where $(\delta f^2)^{\infty}_{\omega=0}=\tau_{\mathrm{iv}}/NV_0$, and $(\delta j^2)^{\infty}_{\omega=0}=2e^2ND/V_0$ correspond to the low-frequency equilibrium fluctuations of the valley carrier density and the current density for an infinite crystal, respectively.

In an experiment, the intervalley fluctuations can be studied by measuring besides the excess current noise also fluctuations of the transverse voltage across the sample

$$\delta U=\int_{-d}^{d}\delta E_y(y)\ dy. \qquad (50)$$

By inserting Eq. (47) into Eq. (50), we get

$$S_{\mathcal{U}}(\omega,E_x)=1+a^2\cos^2(2\vartheta)\mathcal{E}^2S_f(\omega,E_x),$$

$$S_{\mathcal{U}}(\omega,E_x)\equiv\frac{(\delta\mathcal{U}\delta\mathcal{U})_\omega(E_x)}{(\delta f^2)^{\infty}_{\omega=0}}, \qquad (51)$$

where $\delta\mathcal{U}\equiv\delta U/2dE_0$ is the dimensionless transverse voltage. As is seen from direct comparison, the expressions (48) and (51) are quite similar and differ in their angular dependences.

An important peculiarity of the excess current noise is the dependence of the spectral density $S_f(\omega,E_x)$ on the applied electric field [the second term in Eq. (48)] in the range of the fields (2). This is due to the fact that for an arbitrary orientation of the valleys relative to the lateral surfaces (see Fig. 1), the electric field enters in Eq. (29). The spectral density $S_f(\omega,E_x)$ has to decrease with increasing the electric field because of the occurrence of drift of the fluctuations to the surface and fast destruction due to intensive surface intervalley relaxation. For symmetrical orientations of the valleys ($\vartheta=0,\pi/4$) the electric field drops from Eq. (29) because the coefficient (34) vanishes for such values of the angle ϑ. In this case, the intervalley fluctuations are identical to those under the thermal equilibrium condition.

Setting in Eq. (31) $E_x=0$, we find the local spectral density $[\delta f(\zeta_1)\delta f(\zeta_2)]_\omega$, which should then be averaged over the thickness similarly to Eq. (45). For the averaged spectral density $S_f(\omega,\zeta_0)\equiv S_f(\omega,E_x=0)$, we obtain

$$S_f(\omega,\zeta_0)=\mathcal{K}^{\mathrm{iv}}(\omega,\zeta_0)+\mathcal{K}^{i}(\omega,\zeta_0). \qquad (52)$$

Here, the functions $\mathcal{K}^{\mathrm{iv},i}(\omega,\zeta_0)$ correspond to the different random sources in Eq. (33) [see also Eqs. (6) and (7)] and provide the size dependence of the spectrum:

$$\mathcal{K}^{\mathrm{iv}}(\omega,\zeta_0)=\frac{1}{1+\omega^2\tau_{\mathrm{iv}}^2}\left[1+\frac{1}{\cosh(\xi_1)+\cos(\xi_2)}\left(\frac{\sinh(\xi_1)}{\xi_1}+\frac{\sin(\xi_2)}{\xi_2}-4\frac{\xi_1\sinh(\xi_1)+\xi_2\sin(\xi_2)}{\xi_1^2+\xi_2^2}\right)\right], \qquad (53)$$

$$\mathcal{K}^{i}(\omega,\zeta_0)=\frac{1}{(1+\omega^2\tau_{\rm iv}^2)^{1/2}}\,\frac{1}{\cosh(\xi_1)+\cos(\xi_2)}\times\left(\frac{\sinh(\xi_1)}{\xi_1}-\frac{\sin(\xi_2)}{\xi_2}\right), \tag{54}$$

where $\xi_1=2\zeta_0\mathrm{Re}(k_{(-)})$, $\xi_2=2\zeta_0\mathrm{Im}(k_{(-)})$ with $\zeta_0=d/L_{\rm iv}$ and $k_{(\pm)}$ given in Eq. (39).

Let us analyze briefly the expressions (52)–(54). The contribution related to the second term in Eq. (52) [see Eq. (54)] is completely determined by the boundaries. It disappears for an infinite crystal, i.e., if $d\to\infty$ (or $\zeta_0\to\infty$) : $\mathcal{K}^{i}\to 0$, $\mathcal{K}^{\rm iv}\to(1+\omega^2\tau_{\rm iv}^2)^{-1}$. For the range of low frequencies $\omega\tau_{\rm iv}\ll 1$ from Eqs. (53) and (54), we find

$$\mathcal{K}^{\rm iv}(\omega=0,\zeta_0)=\frac{3}{2}\left[1-\frac{1}{3}\tanh^2(\zeta_0)-\frac{\tanh(\zeta_0)}{\zeta_0}\right],$$
$$\mathcal{K}^{i}(\omega=0,\zeta_0)=\frac{1}{2}\left[\frac{\tanh(\zeta_0)}{\zeta_0}+\tanh^2(\zeta_0)-1\right]. \tag{55}$$

These expressions result in simple size dependence of the low-frequency spectrum given by

$$S_f(\omega=0,\zeta_0)=1-\frac{\tanh(\zeta_0)}{\zeta_0}. \tag{56}$$

Now consider the limiting cases. For $d\gg L_{\rm iv}$ ($\zeta_0\gg 1$), the main contribution to the spectrum is due to the intervalley scattering. Really, it follows from Eqs. (55) and (56) that $\mathcal{K}^{i}\approx(1/2)(d/L_{\rm iv})^{-1}$, $\mathcal{K}^{\rm iv}\approx 1$, i.e., $\mathcal{K}^{\rm iv}\gg\mathcal{K}^{i}$. On the contrary, for $d\ll L_{\rm iv}$ ($\zeta_0\ll 1$), we obtain $\mathcal{K}^{\rm iv}\ll\mathcal{K}^{i}$. Consequently, in this case the size dependence of the spectrum is primarily determined by the transverse fluctuative electron flow, and it is given by

$$S_f(\omega=0,\zeta_0)=\frac{1}{3}\left(\frac{d}{L_{\rm iv}}\right)^2. \tag{57}$$

Thus, the overall spectral density (52) is dominated for $d\gg L_{\rm iv}$ by the source $\tilde{I}^{\rm iv}$ (6) and for $d\ll L_{\rm iv}$ by the source $\tilde{I}^{i}$ (7). This is in full agreement with the qualitative estimates presented in Sec. II.

V. DISCUSSION

In this work, the intervalley fluctuations in the electron gas of restricted semiconductors are studied under conditions when the thickness of a sample, or an active layer, $2d$, is comparable to or less than the intervalley diffusion relaxation length, $L_{\rm iv}$. Both kinds of stochastic sources, the bulk and surface ones, are taken into account. Reduction in the layer thickness results in that the fluctuation spectra depend on the thickness and surface intervalley relaxation processes [Eqs. (15) and (40) and (48)]. This allows one to state the occurrence of considerable size effects in the fluctuation spectra of restricted many-valley semiconductors. The above analysis of fluctuation processes makes clear the set of parameters controlling the surface noise sources and relaxation.

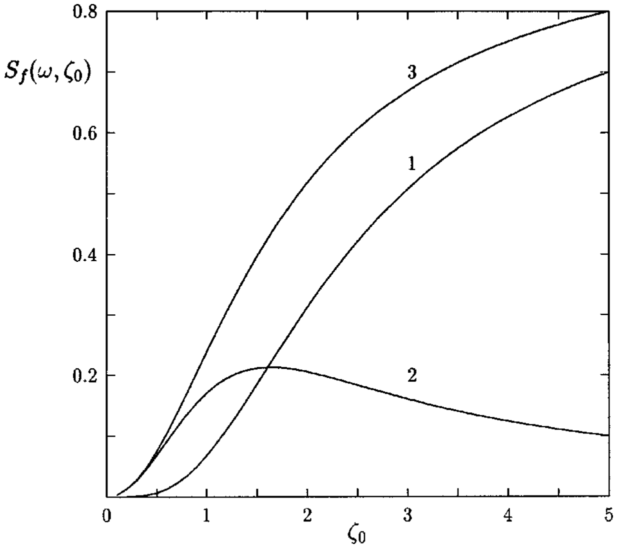

FIG. 2. Various contributions (55) to the low-frequency ($\omega\tau_{\rm iv}\ll 1$) spectral density (56) compared with the total contribution as a function of dimensionless thickness $\zeta_0=d/L_{\rm iv}$: 1, $\mathcal{K}^{\rm iv}(\omega,\zeta_0)$; 2, $\mathcal{K}^{i}(\omega,\zeta_0)$; 3, $S_f(\omega,\zeta_0)=\mathcal{K}^{\rm iv}(\omega,\zeta_0)+\mathcal{K}^{i}(\omega,\zeta_0)$.

The spectra of the intervalley fluctuations in the near-equilibrium electron gas with strong intervalley surface scattering of the particles [see criteria (2) and (19)] essentially differ from the spectra for bulk samples where they have the ordinary Lorentz form. Unlike the fluctuation behavior, the stationary transport undergoes no qualitative modifications and remains unchanged for the above both cases. Hence, investigation of the spectra of the intervalley fluctuations in small electric fields must provide information of microscopic processes in the electron gas, particularly, about the intensity of surface scattering.

An important feature of the fluctuation spectra considered is that the low-frequency spectral densities for thin samples prove to be much smaller than that for an infinite crystal. This follows directly from the expressions (48), (51), and (57). Such *a suppression of the low-frequency fluctuations* is the most pronounced in the case of strong surface intervalley scattering [in the sense of the criterion (19)]. *The suppression effect* clearly manifests itself in fluctuations of the valley carrier density, the current density, and the transverse voltage [see Eqs. (48) and (51) and Eqs. (52)–(54) as well as Figs. 2–4]. Detailed analysis of the parameters governing the surface processes (Sec. III and Appendix) points the way to control the current noise. Properly chosen impurity doping of very thin layer(s) at the surface(s) can suppress the noise while the dc current does not suffer any changes.

Another interesting feature concerns the noise anisotropy. Note that the noise anisotropy is significant in bulk n-Si. For instance, providing the dc current is along [100] or [110] directions this is determined by the factor $[(\delta j_x\delta j_x)^{\rm iv}_{\omega[100]}/(\delta j_x\delta j_x)^{\rm iv}_{\omega[110]}]=4$. From our results it is clear that under the size effect the anisotropy decreases and completely vanishes for an extremely thin layer ($d\ll L_{\rm iv}$).

It is furthermore important to note a change of the role of intravalley and intervalley Langevin sources for bulk and restricted semiconductors. In restricted samples ($d\lesssim L_{\rm iv}$), the characteristic diffusion time $t_d=d^2/D$ (i.e., the time needed for diffusion of a fluctuation to the surface) becomes compa-

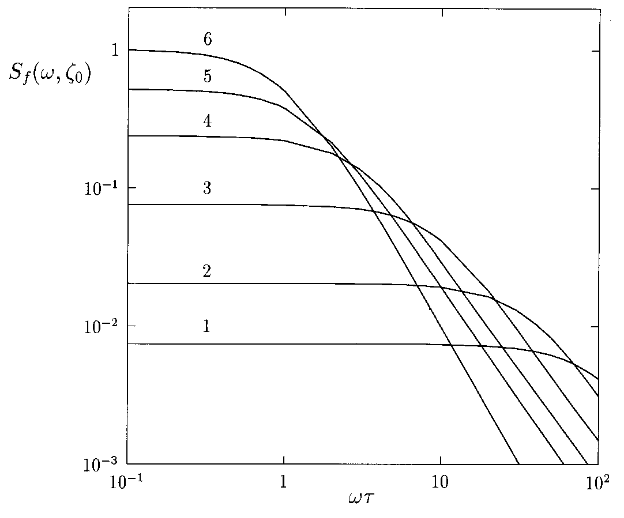

FIG. 3. Frequency dependences of the spectral density $S_f(\omega,\zeta_0)$ (52) for different values of dimensionless thickness $\zeta_0=d/L_{iv}$:1, 0.5; 2, 1.0; 3, 2.0; 4, 3.0; 5, 5.0; 6, ∞.

rable to the intervalley time τ_{iv}. As a consequence, the fluctuating inhomogeneous transverse flow of the carriers gives the same order of magnitude contribution to the frequency spectrum as that due to the direct intervalley scattering. In thin samples ($d\ll L_{iv}$), *the diffusion intravalley term* (54) completely dominates the noise (Fig. 2).

The decrease of the intensity of fluctuations in the range of low frequencies, $\omega\tau_{iv}\ll 1$, is attended by its increasing for higher frequencies, $\omega(d^2/D)\gtrsim 1$ (''blueshift''). Thus, a redistribution of the intensity over the spectrum occurs on account of the boundaries with strong intervalley relaxation (Fig. 3). The surface relaxation substantially affects not only the intensity, but also the frequency characteristics of the intervalley fluctuations. Using Eqs. (52)–(54), one can obtain different high-frequency asymptotics for the spectral density (52) [see also Eqs. (48) and (51)] depending on the thickness $2d$. The case of thick samples [$d/L_{iv}\equiv\zeta_0\gg 1$] implies that $\tau_{iv}^{-1}\gg D/d^2$. Then the $\mathcal{K}^i(\omega,\zeta_0)$ in Eq. (52) is negligible and the corresponding spectral density for the frequency $\omega\gg\tau_{iv}^{-1}$ is given by

$$S_f(\omega,\zeta_0\gg 1)\approx\mathcal{K}^{iv}(\omega,\zeta_0\gg 1)\approx(\omega\tau_{iv})^{-2}.$$

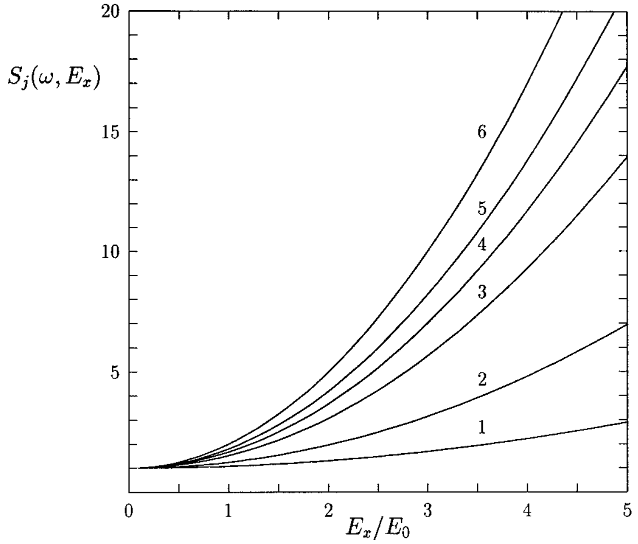

FIG. 4. Field dependences of the low-frequency ($\omega\tau_{iv}\ll 1$) spectral density $S_j(\omega,\zeta_0)$ (48) corresponding to $\vartheta=\pi/4$ (see Fig. 1) for different values of dimensionless thickness $\zeta_0=d/L_{iv}$:1, 0.5; 2, 1.0; 3, 2.0; 4, 3.0; 5, 5.0; 6, ∞.

On the opposite, for thin samples ($\zeta_0\ll 1$) and for the frequency range $\omega\gg D/d^2$ [$D/d^2\gg\tau_{iv}^{-1}$], we get $\mathcal{K}^{iv}(\omega,\zeta_0\ll 1)\approx(\omega\tau_{iv})^{-2}$, $\mathcal{K}^i(\omega,\zeta_0\ll 1)\approx\zeta_0^{-1}(\omega\tau_{iv})^{-3/2}$. The large value of the ratio $[\mathcal{K}^i(\omega,\zeta_0\ll 1)/\mathcal{K}^{iv}(\omega,\zeta_0\ll 1)]\approx\zeta_0^{-2}\gg 1$ provides the predominance of the second term in Eq. (52). In turn, this leads to the asymptotic dependence

$$S_f(\omega,\zeta_0\ll 1)\approx\zeta_0^{-1}(\omega\tau_{iv})^{-3/2},$$

which is characteristic of diffusion limited noise.[34]

Under criterion (2) the intervalley noise is quasiequilibrium.Accordingly, in bulk samples the intensity of intervalley current fluctuations is proportional to E_x^2, and a deviation from this law occurs in strong electric fields due to the heating effect. In restricted samples, the E_x^2 dependence is realized only for symmetrical orientations of the valleys. Otherwise, it becomes more complicated through the appearance of the transverse fluctuating electric field, which results in the spectral density $S_f(\omega,E_x)$ dependence on the field E_x [see Eq. (48)].

The real physical situation corresponding to the two-valley model discussed in the text is a thin plate of n-Ge or n-Si with such orientation of the lateral faces relative to principal crystal axes as it behaves like a two-valley semiconductor. For instance, in the case of n-Ge the axes X,Y,Z must be oriented along crystal axes of the fourfold symmetry, with a plate cut out in accordance with Fig. 1. In the case of n-Si, two situations close to the two-valley model are possible. (i) The x,y axes are in the (110) plane, with the x axes making an angle ϑ with the [110] crystal axis. Thereby, one set of valleys consists of the [100] and [010] valleys while another does of the [001] valley. (ii) The x,y axes are in the (001) plane, with the x axis making an angle ϑ with the [110] crystal axis. Then, there are two valleys along [100] and [010] to be added by the third *isotropic valley* along the [001] axis.

In particular, we consider the semiconductor with parameters of n-Si (assuming dc current along the [100] direction): $N=10^{14}$ cm^{-3}, $T=77$ K, $\mu_{77}=10^4$ cm^2/V s, $\epsilon_0=11.7$, $m_\perp=0.19m_0$, $m_\parallel=0.92m_0$, $v=10^7$ cm/s, where ϵ_0 is the static dielectric constant, $m_\perp$ and $m_\parallel$ are the transverse and longitudinal masses of X valleys, respectively; m_0 is the free-electron mass. The ideology of the drift-diffusion approach and quasineutrality condition require the validity of inequalities (9) and (11). The characteristic lengths are the Debye length $l_D=(\epsilon_0 k_B T/4\pi e^2 N)^{1/2}=2.1\times 10^{-5}$ cm, the mean free path $L_p=(3/v)(k_B T/e)\mu_{77}=2\times 10^{-5}$ cm, the intervalley length $L_{iv}=(D\tau_{iv})^{1/2}=2\times 10^{-4}$ cm [for the above parameters the diffusion coefficient is $D_{77}=66.4$ cm^2/s], the characteristic diffusion field is $E_0=33.2$ V/cm. The corresponding characteristic times are the Maxwellian time τ_M

$=\epsilon_0/4\pi eN\mu_{77}=6.5\times10^{-12}$ s, the electron momentum relaxation time $\tau_p=L_p/v=2\times10^{-12}$ s, the electron energy and intervalley relaxation times: $\tau_\epsilon=7\times10^{-11}$ s, $\tau_{\rm iv}=6\times10^{-10}$ s.[35]

Let us briefly discuss the intensity of surface relaxation. The criterion (19) reduces to

$$\frac{\sqrt{3}}{4}\left(\frac{\tau_{\rm iv}}{\tau_p}\right)^{1/2}\gg1,$$

which contains in explicit form the microscopic relaxation parameters. This results from the order-of-magnitude estimates of the characteristic quantities in Eq. (19): $D\approx v^2\tau_p/3$, $d\approx L_{\rm iv}=(D\tau_{\rm iv})^{1/2}$, and we use the ultimate value of intervalley relaxation rate on the surface $(1/4)v$.[30] Numerical estimates of the relationships (25), (26) show that selective impurity doping of a thin surface layer provides a relevant value of the parameter S. Large values of S ($S=3\times10^5-2\times10^6$ cm/s) have been observed for the silicon surfaces.[35] These estimates show that the above parameters enable all required criteria of the theory to be satisfied.

For the chosen parameters, a contribution of generation-recombination (GR) fluctuations to the total noise may prove to be considerable. Now we find the conditions imposed on the frequency ω and semiconductor parameters under which their influence is negligible. The characteristic time $\tau_{\rm GR}$ associated with impurity recombination and ionization processes[36] is given by

$$\tau_{\rm GR}=\frac{1-u}{B_TN_Du(2-u)},$$

where

$$u\equiv N/N_D=2[1+4g(N_D/N_c)\exp(\epsilon/k_BT)+1]^{-1/2}$$

is the fraction of ionized impurities, B_T is the recombination coefficient, N_c is the effective density of states in the conduction band, g is the impurity ground-state (of the energy ϵ) degeneracy factor. For instance, using $B_T=10^{-6}$ cm^3/s,[36] $\epsilon=0.049$ eV, $N_D=10^{14}$ cm^{-3}, $T=77$ K , we obtain $u=0.72$, $\tau_{\rm GR}=3\times10^{-9}$ s. For the accepted parameters we may assume the condition $\tau_{\rm GR}\gg\tau_{\rm iv}$ to be valid. In the frequency range $\tau_{\rm GR}^{-1}\ll\omega\ll\tau_{\rm iv}^{-1}$, the GR noise undergoes a strong spectral dispersion $[(\tau_{\rm GR}\omega)^2\gg1]$ whereas the dispersion of the intervalley noise still may be ignored $[(\tau_{\rm iv}\omega)^2\ll1]$. For the Lorentz-type spectra, the ratio of noise intensities is given by

$$\frac{(\delta j_x\delta j_x)_\omega^{\rm iv}}{(\delta j_x\delta j_x)_\omega^{\rm GR}}\approx\omega^2\tau_{\rm iv}\tau_{\rm GR},$$

where a numerical factor of the order of unity which depends on the number of valleys and their orientation has been omitted. It is seen that the intervalley noise is dominant in the frequency range

$$\tau_{\rm GR}^{-1}\ll(\tau_{\rm GR}\tau_{\rm iv})^{-1/2}\ll\omega\ll\tau_{\rm iv}^{-1}.$$

We conclude by noting that currently considerable study is given to electron fluctuations and current noise and their suppression in conducting microstructures of small intercontact distance, $d_\parallel$: the ballistic conductor in the regime of classical or quantum transport[23] ($d_\parallel$ is smaller than the electron elastic mean free path); the diffusive conductor in the regime of quasiclassical transport[24–26] ($d_\parallel$ is much greater than the mean free path, but much smaller than an inelastic scattering length). In terms of the ideology of this paper all mentioned cases can be classified as longitudinal size effects (see also Ref. 19). In contrast, the effects analyzed in this paper may refer to a different class of size effects when the transverse to the current dimension, $d_\perp$, of a sample is the smallest one. For such cases the longitudinal dimension considerably exceeds all characteristic lengths ($d_\parallel\gg L_{\rm iv}\gg L_\epsilon\gg L_p$). The size effects for such cases of sample geometry can be classified as transverse size effects in electron fluctuations and noise.[21,22] Thus, the excess noise suppression can be observable for different types of noise (shot noise, hot electron noise, intervalley noise) and for both longitudinal and transverse size effects, and also in certain exotic cases of bulk materials.[17,18]

VI. CONCLUSION

To summarize, we have shown that the fluctuation features of the electron gas in submicrometer structures of many-valley semiconductors considerably differ from those in bulk crystals. This difference is due to the strong influence of the structure's boundaries characterized by intensive intervalley (surface) scattering of the carriers. In particular, the spectra of valley carrier density fluctuations depend on the applied small (under Ohm's law) electric field.

In the structures with small thickness of the active region ($d\ll L_{\rm iv}$), strong surface intervalley relaxation results in suppression of the low-frequency ($\omega\tau_{iv}\ll1$) intervalley noise without affecting the Nyquist component of noise and the dc current. The anisotropy of the intervalley fluctuations is also suppressed under these conditions. It is shown that a diffusion-type dependence of the noise intensity ($\sim\omega^{-3/2}$) is characteristic of the high-frequency ($\omega\gg D/d^2\gg\tau_{\rm iv}^{-1}$) behavior of the noise in contrast with the ordinary Lorentz spectra ($\sim\omega^{-2}$) for bulk crystals. Using our consideration, one can point out such a range of the parameters for n-Si, n-Ge where the intervalley and GR noise separate by their characteristic frequencies.

As a concluding remark we stress that by varying the properties of the surfaces and interfaces, it is possible to control the intervalley noise in semiconductor structures of submicrometer sizes.

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APPENDIX

Let us consider a transitional near-surface layer $(-\delta,0)$ of thickness δ with the position $y=0$ corresponding to the real crystal boundary. The boundary conditions (15), in their structure, provide a relation of the particle flow density fluctuation, δi, to the carrier density fluctuation, δn, at a certain physical boundary[30] determined by the position $y=-\delta$. To put down an analogous relation for the model layer involved, one must solve the set of equations

$$-i\omega\,\delta n(y,\omega)+\frac{d}{dy}\,\delta i(y,\omega)=-\frac{\delta n(y,\omega)}{\tau^{s}_{\text{iv}}}+\tilde{I}^{\text{iv}}(y,\omega), \tag{A1}$$

$$\delta i(y,\omega)=-D\frac{d}{dy}\,\delta n(y,\omega)+\tilde{I}^{i}(y,\omega), \tag{A2}$$

with the boundary conditions

$$\delta i(y=0,\omega)=0, \quad \delta i(y=-\delta,\omega)=\delta i. \tag{A3}$$

We assume two equivalent valleys oriented symmetrically relative to the boundary planes (see Sec. III C and Fig. 1). Physically, the first of Eqs. (A3) means the absence of the fluctuating particle flow density at $y=0$ while the second corresponds to the given value of it at $y=-\delta$. The procedure we use here is as follows. Having written down the general solution to Eqs. (A1)–(A3), we get the relation of the fluctuation $\delta n(-\delta)$ to the fluctuation δi. Then, we can find the inverse relation, having expressed the fluctuating flow density δi at the boundary in terms of fluctuation of the carrier density $\delta n(-\delta)$. The latter must provide an explicit form of the surface relaxation rate and the Langevin surface sources.

The general solution to Eqs. (A1) and (A2) with the conditions (A3) can be written as

$$\delta n(y)=\delta n_{0}(y)+\int_{-\delta}^{0}G(y,y')\tilde{f}(y)\,dy'. \tag{A4}$$

Here, $\delta n_0(y)$ is the general solution to the homogeneous equation for δn, being obtained by inserting Eq. (A2) into Eq. (A1), with the boundary conditions:

$$D\frac{d}{dy}\,\delta n_{0}(y)\bigg|_{y=0}=\tilde{I}^{i}(0),$$

$$D\frac{d}{dy}\,\delta n_{0}(y)\bigg|_{y=-\delta}=-\delta i+\tilde{I}^{i}(-\delta), \tag{A5}$$

$G(y,y')$ is Green's function of the operator $\hat{\Lambda}(y)=(d/dy)-l^{-2}$ with zero boundary conditions, $l^{2}=D\tau^{s}_{\text{iv}}/(1-i\omega\tau^{s}_{\text{iv}})$. The effective Langevin source in the integrand (A4) is given by

$$\tilde{f}(y)=D^{-1}\left(-\tilde{I}^{iv}+\frac{d}{dy}\tilde{I}^{i}\right). \tag{A6}$$

For Green's function we readily obtain the expressions:

$$G(y,y')=l\,\frac{\cosh[(y+\delta)/l]/\cosh[(y'+\delta)/l]}{\tanh(y'/l)-\tanh[(y'+\delta)/l]}, \quad y\leqslant y'; \tag{A7}$$

$$G(y,y')=l\,\frac{\cosh(y/l)/\cosh(y'/l)}{\tanh(y'/l)-\tanh[(y'+\delta)/l]}, \quad y\geqslant y'.$$

In order to determine $\delta n(-\delta)$, we need the expression for Green's function evaluated at $y=-\delta$:

$$G(-\delta,y')=-l\;\frac{\cosh(y'/l)}{\sinh(\delta/l)}. \tag{A8}$$

Finally, setting in Eq. (A4) $y=-\delta$ and taking into account Eqs. (A6) and (A8), and the expression for $\delta n_0(-\delta)$ given by

$$\delta n_{0}(-\delta)=\frac{l}{D\,\tanh(\delta/l)}\{\delta i+\tilde{I}^{i}(0)\cosh^{-1}(\delta/l)-\tilde{I}^{i}(-\delta)\}, \tag{A9}$$

we get the required relation in the form

$$\delta n(-\delta)=\frac{l}{D\,\tanh(\delta/l)}(\delta i+\tilde{u}). \tag{A10}$$

Here $\tilde{u}=\tilde{u}^{\text{iv}}+\tilde{u}^{i}$ with

$$\tilde{u}^{\text{iv}}=\frac{1}{\cosh(\delta/l)}\int_{-\delta}^{0}\tilde{I}^{\text{iv}}(y')\cosh(y'/l)\,dy', \tag{A11}$$

$$\tilde{u}^{i}=\frac{1}{l\,\cosh(\delta/l)}\int_{-\delta}^{0}\tilde{I}^{i}(y')\sinh(y'/l)\,dy'. \tag{A12}$$

To obtain $\tilde{u}^{i}$ in Eq. (A12), we have used integration by parts in the integral on the right-hand side of Eq. (A4) containing the position derivative with respect to y [Eq. (A6)].

The expressions (A11), (A12) define the Langevin surface sources of the fluctuations in the model under consideration. Substituting further the Langevin sources $\tilde{I}^{\text{iv}}_{\alpha}$, $\tilde{\boldsymbol{I}}^{i}_{\alpha}$ from Eqs. (6) and (7) into Eqs. (A11) and (A12) and using their correlation relations,[4,13] we can easily calculate the spectral densities of the correlation functions for the Langevin surface sources:

$$(\tilde{u}^{\text{iv}}\tilde{u}^{\text{iv}})_{\omega}=\frac{2dn_{0}}{V_{0}}(\tau^{s}_{iv})^{-1}\frac{\delta}{2\,\cosh^{2}(\delta/l)}\left\{\frac{\sinh(2\delta/l)}{2\delta/l}+1\right\}, \tag{A13}$$

$$(\tilde{u}^{i}\tilde{u}^{i})_{\omega}=\frac{2dn_{0}}{V_{0}}(D/l^{2})\frac{\delta}{2\,\cosh^{2}(\delta/l)}\left\{\frac{\sinh(2\delta/l)}{2\delta/l}-1\right\}, \tag{A14}$$

$$(\tilde{u}^{\text{iv}}\tilde{u}^{i})_{\omega}=0. \tag{A15}$$

The last property (A15) is clearly a consequence of the fact that the Langevin sources $\tilde{I}^{\text{iv}}$ and $\tilde{I}^{i}$ are uncorrelated.

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