ac conductance of a quantum wire with electron-electron interactions

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The complex ac response of a quasi-one-dimensional electron system in the one-band approximation with an interaction potential of finite range is investigated. It is shown that linear response is exact for this model. The influence of the screening of the electric field is discussed. The complex absorptive conductance is analyzed in terms of resistive, capacitive, and inductive behaviors. [S0163-1829(98)00403-2]

I. INTRODUCTION

Experimental and theoretical investigations of the ac transport in nanostructures are of profound scientific interest, since they provide insight into the behavior of (open) quantum systems in nonequilibrium that are externally controllable within wide ranges of parameters. In addition, possible applications of nanostructures in future electronic devices, which will have to operate at very high frequencies, require detailed knowledge of their frequency- and time-dependent transport behavior.

Electron transport in nanostructures is very strongly influenced by charging effects. Most striking is the Coulomb blockade[1] of the dc current through tiny tunnel junctions when the bias voltage and the temperature are smaller than the ''charging energy,'' $E_C = e^2/2C$ (e is the elementary charge, C the ''capacitance of the tunnel junction''). The use of a capacitance has been justified by observing that its values determined from E_C are consistent with those obtained from the geometry of the junction;[2,3] C was found to be of the order of 10^{-15} F, and much smaller for metallic junctions.[4] Interactions also dominate transport through islands between two tunnel contacts in series in a semiconductor quantum wire. The *linear* conductance shows pronounced peaks[5] if the external chemical potential coincides with the difference between the ground state energies of $N+1$ and N electrons in the island. In the charging model, these energies are again given in terms of a ''capacitance C,'' $E(N) = N^2 e^2/2C$, $C \approx 10^{-15}$ F. One can ask how small a capacitance can be without being influenced by quantum effects. The limitations of the charging model become obvious in the nonlinear transport properties: fine structure in the current-voltage characteristic is related to the quantum properties of the interacting electrons.[6–9]

In recent years, frequency-dependent electrical response of systems with reduced dimensionality became the subject of activities. These techniques are of particular interest, since no current and voltage probes have to be attached to the sample. The current response to microwave and far-infrared radiation on the transport through semiconductor microstructures has been studied.[10–13] It has been found that the absorption of microwaves leads to a characteristic broadening of the conductance peaks of semiconductor quantum dots in the Coulomb-blockade region.[14,15] Infrared absorption[16–18] of quantum dots and wires mainly provided information on the parts of the excitation spectrum that are only weakly influenced by the interaction due to Kohn's theorem. However, Raman scattering from quantum wires and dots showed signatures of the dispersion of the collective excitations.[19–21] The absorption of microwaves in an ensemble of metallic grains has been investigated experimentally and theoretically in many papers.[22,23]

Absorption and scattering of electromagnetic radiation are only one possibility to measure ac-transport properties without applying voltage and current probes that may disturb the system's properties severely. More recently, other highly sophisticated, noninvasive techniques for determining ac conductances have been pioneered.[24] Coupling a system of about 10^5 mesoscopic rings to a highly sensitive superconducting microresonator, the perturbation of the resonance frequencies and quality factors has been used to determine the real and imaginary parts of an ac conductance. Here the fundamental question arises about what the differences between the ''conductances'' as determined by different methods are.[25]

Theoretical approaches have been developed, ranging from semiclassical rate-equation approximations to fully quantum-mechanical attempts. The linear theory of ac-quantum transport has been restricted to noninteracting systems.[26–30] How to define quantum capacitances[28] and inductances[27,29] was addressed. ac transport through mesoscopic structures in the presence of Coulomb interaction was considered by using a self-consistent mean-field method.[31,32] This approach strongly relies on the presence of ''reservoirs,'' ''contacts,'' and ''electrochemical potentials'' which are not necessary ingredients of high-frequency experiments such as the absorption of microwaves.

Photoinduced transport through a tunnel barrier[33] and tunneling through semiconductor double-barrier structures[34] has been considered. Charging effects in small semiconductor quantum dots in the presence of time-varying fields were treated.[30,35] The influence of high-frequency electric fields on the linear and nonlinear transport through a quantum dot with infinitely strong Coulomb repulsion[36] was studied. Photon-assisted tunneling through a double quantum dot has been investigated by using the Keldish technique.[37] The photoinduced transport through a single tunnel barrier in a one-dimensional interacting electron system has been investigated.[38] In most of the latter works, quantum effects of the interaction have been treated only approximatively.

In view of the importance of the interaction for the ac properties of nanostructures, Luttinger systems are of great

interest. Here the interaction can be taken into account exactly. The conductance of a tunnel barrier in a Luttinger liquid with zero-range interaction has been shown[39,40] to scale with the frequency as $\omega^{2/g-2}$ (g is the interaction parameter; see below). At $\omega=0$ repulsive electron-electron interaction suppresses tunneling completely, even for a vanishingly small potential barrier. However, it has been also shown that for $\omega\to0$ there is a *displacement contribution* to the current which can dominate the transport for very strong repulsive interaction and a very high barrier.[41]

The driving voltage has been assumed *ad hoc* in most of these works. Since the driving electric field is determined by the interaction between the electrons,[42,43] the current can be expected to depend on how charges induced by an external electric field are rearranged by the interaction. Even in the limit of dc transport through a Luttinger system this has been argued to be so.[44,45] The dependence of the ac properties of a tunnel barrier in a Luttinger system on the shape of the electric driving field has been investigated.[42] It has been found that the current depends only on the integral over the driving electric field—the external voltage—only in the dc limit, even in the nonlinear regime. One of the side results was to confirm that linear response is exact for the ideal Luttinger system.[42,46,47] The ac properties of the Luttinger liquid with spatially varying interaction strength were also studied.[46–48]

In this paper, we concentrate on the ac-transport properties of electrons described by the Luttinger model with an interaction of finite range. The model is exactly solvable. Since its current response can be determined without approximations, answers to fundamental questions can be found, such as (i) how to identify the specific signatures of the electron-electron interaction in ac transport, (ii) how to understand the influence of the properties of the driving electric field, (iii) how to define conductances, and (iv) how to understand ac-transport in terms of resistive, capacitive, and inductive behaviors. The latter yield quantum analogs of impedances that are common in classical electrodynamics. Such parameters are also often used for describing the transport in nanostructures. Therefore, a quantum approach toward their definition is highly desirable. However, in the quantum regime, they depend not only on the interaction, but also on the frequency and shape of the applied electric field. Together with their microscopic definitions, it is thus important to determine the range of parameters for their validity.

The interaction potential is obtained by using a three-dimensional (3D) screened Coulomb potential (screening length α^{-1}) and projecting to a quasi-1D quantum wire of a finite width d. When $\alpha^{-1}\ll d$, we recover the Luttinger liquid with zero-range interaction. On the other hand, when $\alpha^{-1}\gg d$ we obtain the 1D analog of the Coulomb interaction. The excitation spectrum of this model shows an inflection point at a frequency ω_p that increases monotonically with the interaction strength, and with the inverse of a characteristic length associated with the interaction.[49]

By using linear-response theory, we obtain the complex, frequency-dependent nonlocal conductivity given by the current-current correlation function. It contains the dispersion relation of the elementary excitations, and turns out to be independent of the temperature within the model. There are no nonlinear contributions to the current in this model.

The conductivity describes the current response of the system with respect to an electric field $E(x',\omega)$. In an experiment, either the current as a function of an *external voltage* (in the dc limit), or the absorption of electromagnetic energy at frequency ω from an external field is determined. In both cases, not too much is known about the internal field. Therefore, it is reasonable to search for quantities that do not depend on the spatial form of the field. One possibility is[50] to define the conductance Γ_1 by using the absorbed power. One finds that, in the dc limit, the result is indeed independent of the shape of the field. However, in ac transport, the shape of the square of the Fourier transform of the electric field appears as a multiplicative factor. The remaining factor is the density of collective excitations of the interacting electrons. It has a resonance at ω_p. For noninteracting electrons $dq/d\omega=v_F^{-1}$, and $\Gamma_1(\omega)$ reveals merely the structure of the electric field.

That the ac conductance depends on the shape of the driving electric field leads to the question of the nature of this field in an interacting system. We show that for the conductivity of an ideal Luttinger liquid, it is the *external electric* field which has to be used, since linear response is exact. Furthermore, we will demonstrate that this is also true for the absorptive conductance.

Having determined the absorptive part of the conductance, $\Gamma_1(\omega)$, the reactive part $\Gamma_2(\omega)$ may be obtained by Kramers-Kronig transformation. The complex conductance $\Gamma(\omega)=\Gamma_1(\omega)+i\Gamma_2(\omega)$ relates the average current with the voltage. The current as a function of time consists of two parts. One, $\propto\Gamma_1$, is in phase with the driving field. The second, $\propto\Gamma_2$, is phase shifted by $\pm\pi/2$.

When ω is small, we can expand

$$\Gamma_1(\omega)=g\frac{e^2}{h}+\gamma_1\omega^2+O(\omega^4). \qquad (1)$$

The first term corresponds to the quantized contact conductance[51] $R_K^{-1}=e^2/h$. It is here renormalized by the interaction parameter[39] g. The term $\propto\omega^2$ indicates whether the current is capacitive ($\gamma_1>0$) or inductive ($\gamma_1<0$). For Γ_2 we find

$$\Gamma_2(\omega)=\omega\gamma_2+O(\omega^3). \qquad (2)$$

This quantity also indicates if the system behaves capacitively and inductively, $\gamma_2<0$ and $\gamma_2>0$, respectively. However, in the latter case, γ_1 can still be positive, indicating capacitive behavior of the real part of the conductance.

For frequencies close to the resonance, the Kramers-Kronig transformation gives

$$\Gamma_2(\omega)\approx\gamma_m(\omega-\omega_m^*), \qquad (3)$$

with $\gamma_m>0$ when $\gamma_2<0$ and $\omega_m^*\approx\omega_m$ [position of maximum of $\Gamma_1(\omega)$]. The quantity γ_m indicates capacitive and/or inductive behavior close to the resonance frequency ($\approx\omega_p$).

The height or, equivalently, the width of the resonance in Γ_1 defines also a resistance, R. In contrast to the contact resistance, it is truly ''dissipative'' and related to the pair excitations of the Luttinger liquid. It is also contained in γ_1, though its numerical value for small ω is different from the one near the resonance. Generally, we find that it is only

possible to define resistances, capacitances, and inductances in certain limited parameter regions.[49]

In order to observe capacitive behavior, the interaction between the electrons should be *sufficiently long range*. This is consistent with the results of a different approach in which Coulomb blockade behavior at a tunnel barrier between two Luttinger liquids has been discussed.[52] Also there, a nonzero range of the interaction is necessary for capacitive behavior.

In Sec. II, we briefly describe the model and the dispersion relations for various interaction potentials. We calculate the ac conductance and study external versus internal driving fields in Sec. III. Section IV contains the identification of quantum impedances. Section V contains the discussion of the results.

II. LUTTINGER LIQUID WITH LONG-RANGE INTERACTION

A. Outline of the model

The Luttinger liquid is a model for the low-energy excitations of a 1D interacting electron gas.[53–56] Its excitation spectrum can be calculated analytically. Also, many of the thermodynamical and transport properties, such as the linear conductivity, can be determined even in the presence of perturbing potentials. The main assumption is the linearization of the free-electron dispersion relation near the Fermi level. The starting point is the Hamiltonian for interacting fermions with, say, periodic boundary conditions,[57]

$$H = \hbar v_F \sum_{k,s=\pm} (sk-k_F)(c_{ks}^\dagger c_{ks} - \langle c_{ks}^\dagger c_{ks}\rangle_0)$$
$$+ \sum_{k_1,s_1\cdots k_4,s_4} V_{k_1,s_1\cdots k_4,s_4} c_{k_1 s_1}^\dagger c_{k_2 s_2}^\dagger c_{k_3 s_3} c_{k_4 s_4}. \quad (4)$$

Here $c_{ks}^\dagger$ and c_{ks} are the creation and annihilation operators for fermions in the states $|ks\rangle$ of wave number $k=2\pi n/L$ ($n=0,\pm 1,\pm 2,\dots$) in the branches $s=\pm$, k_F the Fermi wave number, V the Fourier transform of the electron-electron interaction, and $\langle\cdots\rangle_0$ denotes an average in the ground state.

Formally, the fermion Hamiltonian can be transformed into a bosonized form. For spinless particles with an interaction that depends only on the distance between the particles, $V(|x-x'|)$, and taking into account only forward scattering, one obtains a bilinear form in the boson operators which can be diagonalized by a Bogolubov transformation.[56] The result is

$$H = \sum_q \hbar\omega(q) b_q^\dagger b_q. \quad (5)$$

The spectrum of the pair excitations corresponding to the bosonic creation and annihilation operators $b_q^\dagger$ and b_q is given by the Fourier transform of the interaction potential[58] $V(q)$,

$$\omega(q) = v_F|q|\sqrt{1+\frac{V(q)}{\hbar\pi v_F}}. \quad (6)$$

The particle excitations which change the total electron number are omitted here. The number of particles is assumed to

be constant, $N_0=k_F L/\pi$. The dispersion relation interpolates between the limit of zero-range interaction ($q\to 0$), where $\omega(k)=v|q|$, with the "charge sound velocity" $v\equiv v_F/g$, and the limit of noninteracting particles ($q\to\infty$), $\omega(q) = v_F|q|$. The strength of the interaction is defined as

$$\frac{1}{g^2} \equiv 1 + \frac{V(q=0)}{\hbar\pi v_F}. \quad (7)$$

Noninteracting fermions correspond to $g=1$, repulsive interaction to $g<1$.

The particle density $\rho(x)$ can be written in terms of the phase variable of the Luttinger model

$$\vartheta(x) = i \sum_{q\neq 0} \text{sgn}(q)\sqrt{\frac{1}{2L|q|}} e^{\varphi(q)-iqx}(b_q^\dagger + b_{-q}) \quad (8)$$

where

$$e^{2\varphi(q)} = \frac{v_F|q|}{\omega(q)}. \quad (9)$$

With the mean particle density $\rho_0=N_0/L$, we write

$$\rho(x) \equiv \rho_0 + \frac{1}{\sqrt{\pi}}\partial_x\vartheta(x). \quad (10)$$

For later use in the linear-response theory, we need the coupling to the driving voltage $U(x,t)$ and the current density.[42] The former is given by

$$H_U = e\int_{-\infty}^\infty dx\ \rho(x)U(x,t). \quad (11)$$

The electric field is $E(x,t)=-\partial_x U(x,t)$. The external voltage is assumed to be given by $\int_{-\infty}^\infty dx\ E(x,t)\equiv U(t)$. The current operator is defined by using the 1D continuity equation for the Heisenberg representation of the operators,

$$J(x,t) \equiv -\frac{e}{\sqrt{\pi}}\dot{\vartheta}(x,t). \quad (12)$$

B. Interaction potential

In order to obtain the dispersion relation explicitly, we need a specific model for the interaction. Since we eventually want to draw some conclusions on quantum wires, we start from a 3D screened Coulomb interaction

$$V(r) = V_0\frac{e^{-\alpha r}}{r}, \quad (13)$$

with $V_0=e^2/4\pi\varepsilon\varepsilon_0$, and project onto the quasi-1D states of the quantum wire.

For the latter, we assume a parabolic confining potential in the y and z directions independent of x. The corresponding states are ($\zeta=\sqrt{y^2+z^2}$)

$$\psi_k(x,\zeta) = \frac{e^{ikx}}{\sqrt{L}}\varphi(\zeta). \quad (14)$$

In the following, we assume, for the confining wave function

$$\varphi(\zeta) = \sqrt{\frac{2}{\pi d^2}} e^{-\zeta^2/d^2}, \qquad (15)$$

where d represents the ''diameter'' of the wire.

We obtain the effective interaction potential for the motion in the x direction from the matrix elements of Eq. (13) in state (15) by performing the integrations with respect to y and z,

$$V(x) = -\frac{2V_0}{\alpha d^2} \int_0^\infty d\zeta \, e^{-\zeta^2/\alpha^2 d^2} \frac{d}{d\zeta} [e^{-\sqrt{\alpha^2 x^2 + \zeta^2}}]. \quad (16)$$

Its Fourier transform is

$$V(q) = V_0 e^{(d^2/4)(q^2+\alpha^2)} E_1 \left(\frac{d^2}{4}(q^2+\alpha^2) \right). \qquad (17)$$

The function E_1 is the exponential integral.[59]

Two limiting cases are of particular interest. When $z \gg 1$, $E_1(z) \approx \exp(-z)/z$. Thus, for $\alpha d \gg 1$,

$$V(q) = \frac{4V_0}{d^2} \frac{1}{q^2 + \alpha^2}. \qquad (18)$$

This is the Fourier transform of

$$V(x) = \tfrac{1}{2} V_L \alpha e^{-\alpha|x|}. \qquad (19)$$

For $\alpha \to \infty$, but with $V_L \equiv 4V_0/\alpha^2 d^2 = $ const, this is $V_L \delta(x)$, the zero-range interaction, with the strength V_L of the conventional Luttinger liquid.

When $\alpha d \ll 1$ we still obtain the above result (18) as long as $qd \gg 1$ which implies that in x space $V(x \to 0)$ behaves as Eq. (19), but with $V(x=0) = \sqrt{\pi} V_0/d$. For $z \to 0$, $E_1(z) \approx -\ln z$ so that, for $qd \ll 1$,

$$V(q) \approx -V_0 \ln[(\alpha^2 + q^2)d^2]. \qquad (20)$$

This is the same behavior as obtained by starting from the 1D equivalent of the Coulomb interaction,[49,58,60] implying the interaction falls off as x^{-1} in space.

In many of the quantum wire experiments metallic gates are present in some distance, say D, from the wire (diameter $d \ll D$). In order to discuss the changes in the interaction induced by the presence of the gates, we can consider an infinite metal plate parallel to the wire. This changes the interaction potential according to

$$V_D(r) = V(r) - V(|\vec{r} + \vec{D}|), \qquad (21)$$

due to the presence of the mirror charge. It is clear that this influences the results only when $\alpha^{-1} \gtrsim D$. Assume then that $\alpha = 0$. The cutoff of the Coulomb tail of the potential is in this limit given by D instead of α^{-1}. The results to be discussed below for $\alpha d \ll 1$ also apply for this limit, with α replaced by D^{-1}.

C. Results for the dispersion law

Results for the dispersion are shown in Fig. 1 for interaction parameter $g_0 = 0.1$ $[V_0 \equiv \hbar \pi v_F(g_0^{-2} - 1)]$ and various αd. There is a crossover between the interacting and noninteracting limits for $q \to 0$ and $q \to \infty$, respectively, at the in-
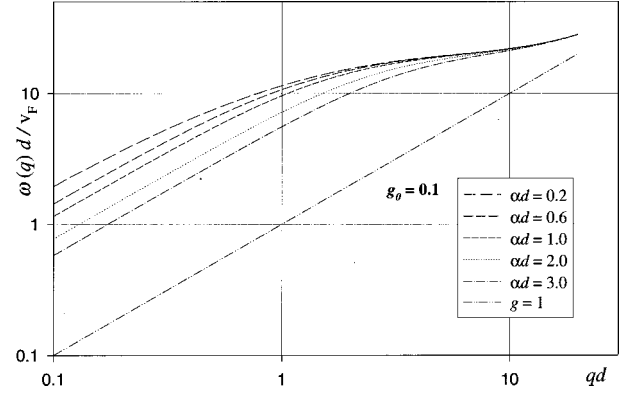

FIG. 1. Double logarithmic plot of the dispersion relation $\omega(q)$ of the Luttinger model with $g_0 = 0.1$ and different ranges αd (α is the inverse potential range in position space, d is the diameter of quantum wire).

termediate wave number q_p corresponding to the characteristic frequency ω_p. It is related to the finite range of the interaction in the wave-number space. For zero-range interaction the dispersion becomes linear, and $\omega(q) = v_F q/g$. Figure 2 shows the excitation density $dq/d\omega$ for various g_0 and $\alpha d = 1$.

The frequency ω_p and the corresponding wave number q_p as a function of $(g_0^{-2} - 1)^{-1}$ are shown in Fig. 3 for various αd. For a broad range of g_0 the frequency ω_p and q_p decay as g_0^{-1} and $g_0^{-1/2}$, respectively. The data for ω_p obey the scaling law ($\beta_0 \equiv g_0^{-2} - 1$)

$$\omega_p(\beta_0; \alpha d) = \omega_p[\beta_0 h^{-2}(\alpha d); 0] h(\alpha d). \qquad (22)$$

The scaling function $h(\alpha d)$ is shown in the bottom left inset of Fig. 3, and it is proportional to the limit of ω_p for infinitesimally small interaction.

III. LINEAR RESPONSE

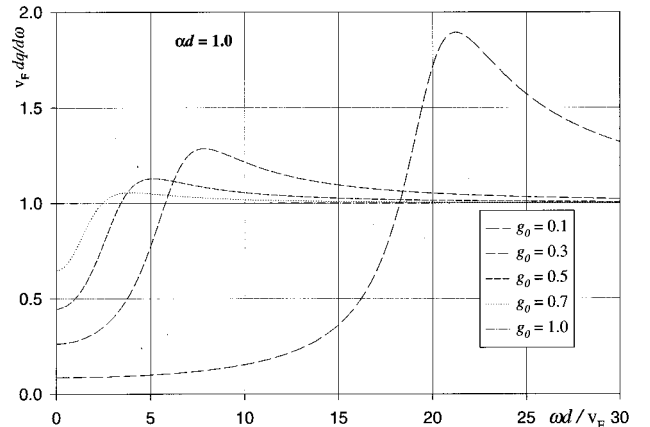In this section we outline the calculation of the conductance with linear-response theory.

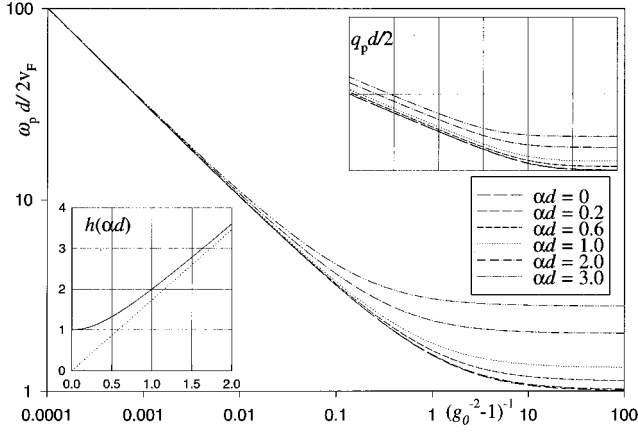FIG. 2. The density of pair excitations, $dq/d\omega$ for $\alpha d = 1$, different g_0.

FIG. 3. The resonance frequency ω_p and the corresponding wave number, q_p (inset, top right) as a function of the strength of the interaction $(g_0^{-2}-1)^{-1}$ of the Luttinger liquid with interaction of finite range as indicated. Inset, bottom left: scaling function $h(\alpha d)$.

A. Conductivity

Using the above current, Eq. (12), one obtains the complex conductivity, $\sigma(x-x';t-t')$ which relates the current at a given point x at time t with the driving electric field $E_{\text{ext}}(x',t')$,

$$J(x,t)=\int_{-\infty}^{\infty}dx'\int_{-\infty}^{t}dt'\,\sigma(x-x';t-t')E_{\text{ext}}(x',t').$$
(23)

While the nonlocality of the conductivity is unimportant in the dc limit, it is crucial for time-dependent transport. By assuming that the electric field is concentrated only near a given point, say $x'=x_0$, we obtain

$$J(x,t)=\int_{-\infty}^{t}dt'\,\sigma(x-x_0;t-t')U_{\text{ext}}(t')$$
(24)

with the voltage drop $U_{\text{ext}}(t')=\int dx' E_{\text{ext}}(x',t')$ dropping only near x_0. We see that $\sigma(x-x_0,t-t')$ then plays the role of a ''conductance'' that relates the current at some point x in the system with a voltage dropping at some other point. By using near-field microscopy, one could possibly perform such a nonlocal experiment. However, it seems to us that it is very hard to measure the current locally in a quantum wire, especially in the region of high frequency.

By Fourier transformation, Eq. (23) is equivalent to

$$J(q,\omega)=\sigma(q,\omega)E_{\text{ext}}(q,\omega).$$
(25)

The conductivity kernel can be expressed either by the current-current correlation function or, by using the continuity equation, as

$$\sigma(q,\omega)\equiv\sigma_1(q,\omega)+i\sigma_2(q,\omega)=\frac{-i\omega e^2}{q^2}R(q,\omega),\quad(26)$$

with the charge-charge correlation function

$$R(x,t)=-\frac{i}{\hbar}\Theta(t)\langle[\rho(x,t),\rho(0,0)]\rangle.$$
(27)

Here $\langle\cdots\rangle\equiv\text{Tr}[\exp(-\beta H)\cdots]/\text{Tr}\exp(-\beta H)$ is the usual thermal average (temperature $\propto\beta^{-1}$). By using expression (10), we find the exact result

$$R(q,\omega)=\frac{v_F}{\hbar\pi}\frac{q^2}{\omega^2(q)-(\omega+i0^+)^2}.$$
(28)

B. Analogy with a Brownian particle

A most remarkable feature of the above result becomes transparent by applying the imaginary-time path-integral approach. With this, the time-dependent nonlinear response to an electric field of arbitrary spatial shape of a Luttinger system was calculated recently.[42] It was found that the average of the phase field (8) obeys the equation of motion of a Brownian particle with mass $M\to0$,

$$M\ddot{\vartheta}(x,t)+\int_{-\infty}^{t}dt'\,\gamma(t-t')\dot{\vartheta}(x,t')=\mathcal{L}(x,t),\quad(29)$$

subject to an effective external force with the Fourier transform

$$\mathcal{L}(x,\omega)=-\frac{e}{\sqrt{\pi}}\frac{1}{\sigma(x=0,\omega)}\int_{-\infty}^{\infty}dx'E(x',\omega)\sigma(x-x',\omega),$$
(30)

and a damping term $\gamma(t)$ which is given by the nonlocal conductivity. The Fourier transform of $\gamma(t)$ is

$$\gamma(\omega)=\frac{e^2}{\pi}\frac{1}{\sigma(x=0,\omega)}.$$
(31)

From the solution of the equation of motion the current is found,

$$J(x,t)=\int_{-\infty}^{\infty}dx'\int_{-\infty}^{t}dt'\,\sigma(x-x',t-t')E(x',t').\quad(32)$$

The linear response is exact for the Luttinger liquid.

C. Driving electric field

In this section, we investigate the influence of screening on the response to an external electric field. In particular, we discuss the dc limit, and show that the two limits $\omega\to0$ and $q\to0$ cannot be interchanged. It will turn out that for the Luttinger model, where linear response is exact, one can use the external field for the calculation of the current. The results will be used to derive absorptive and reactive conductances.

The dielectric response function, which describes the dynamical screening of a charge, is defined as

$$\varepsilon(q,\omega)=\frac{U_{\text{ext}}(q,\omega)}{U_{\text{tot}}(q,\omega)},$$
(33)

where

$$U_{\text{tot}}(q,\omega)=U_{\text{ext}}(q,\omega)+U_{\text{sc}}(q,\omega)$$
(34)

is the total potential, and U_{ext} and U_{sc} are the external and screening potentials, respectively. Within the linear screen-

ing model, the dielectric response function can be written in terms of the charge-charge correlation function as

$$\frac{1}{\varepsilon(q,\omega)} = 1 - V(q)R(q,\omega). \tag{35}$$

By using result (28) for $R(q,\omega)$, we find the explicit expression

$$\varepsilon(q,\omega) = \frac{\omega^2(q) - \omega^2}{\omega_0^2(q) - \omega^2}, \tag{36}$$

with the dispersion of the noninteracting electrons $\omega_0(q)$.

Equations (33) and (34) imply that

$$E_{\text{tot}}(q,\omega) = E_{\text{ext}}(q,\omega)[1 - V(q)R(q,\omega)]$$

$$\equiv E_{\text{ext}}(q,\omega)F(q,\omega). \tag{37}$$

Using conductivity (26) with Eq. (28) and comparing with Eq. (36), we see that

$$\sigma(q,\omega) = \frac{\sigma_0(q,\omega)}{\varepsilon(q,\omega)}. \tag{38}$$

Here σ_0 is the conductivity of the noninteracting electrons. This implies that

$$E_{\text{tot}}(q,\omega)\sigma_0(q,\omega) = E_{\text{ext}}(q,\omega)\sigma(q,\omega). \tag{39}$$

If we calculated the conductivity from the response to the total field, the conductivity would turn out to be that of non-interacting electrons.

This result is true as long as one can use linear screening. It also implies that the voltage drop at frequency ω is the same for both fields,

$$U_{\text{tot}}(\omega) = \int_{-\infty}^{\infty} dx \, E_{\text{tot}}(x,\omega) = U_{\text{ext}}(\omega), \tag{40}$$

since for any finite nonzero frequency $\varepsilon(q \rightarrow 0,\omega) = 1$.

For a monochromatic external field,

$$E_{\text{ext}}(q,t) = E_{\text{ext}}(q)\cos\omega t, \tag{41}$$

one finds the result

$$E_{\text{tot}}(q,t) = E_{\text{ext}}(q)[\text{Re}F(q,\omega)\cos\omega t + \text{Im}F(q,\omega)\sin\omega t], \tag{42}$$

which means that there is a phase shift between the total and the external field.

A final remark concerns the static limit. While we have for any nonzero frequency $\varepsilon(q \rightarrow 0,\omega) = 1$, we find, for $\omega = 0$,

$$\lim_{q \rightarrow 0} \varepsilon(q,0) = \lim_{q \rightarrow 0} \frac{\omega^2(q)}{\omega_0^2(q)} = \frac{1}{g}. \tag{43}$$

By inserting the dispersion relation of the Luttinger model into Eq. (28), one obtains

$$E_{\text{tot}}(q) = E_{\text{ext}}(q) \frac{1}{1 + V(q)/\hbar v_F \pi}. \tag{44}$$

The limits $\omega \rightarrow 0$ and $q \rightarrow 0$ cannot be interchanged. The latter result has been used recently, in order to explain that in quantum wires the dc conductance is not renormalized by the interaction[44,45] [see also Eq. (44)]. Here we are discussing frequency-dependent properties. Thus for small frequencies we always obtain a conductance that is renormalized by the interaction, since we consider the limit of infinite system length.

D. Absorptive and reactive conductances

Since it is very difficult to detect the nonlocal conductivity, experimentally some average has to be performed. One possibility is to use the absorbed power $P(t)$ in order to define the conductance. This appears to be a natural choice if one wants to describe infrared or microwave experiments,

$$P(t) = \int_{-\infty}^{\infty} dx \, J(x,t)E_{\text{tot}}(x,t)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} dq \, J(q,t)E_{\text{tot}}(-q,t). \tag{45}$$

We define the average

$$\bar{P} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \, P(t). \tag{46}$$

Using the Laplace transform

$$P(s) = \int_0^{\infty} dt \, e^{-st}P(t), \tag{47}$$

we obtain $\bar{P} = \lim_{s \rightarrow 0} sP(s)$. The *absorptive conductance* can then be defined by

$$\Gamma_1 = \frac{\bar{P}}{U_{\text{ext}}^2}, \tag{48}$$

with $U_{\text{ext}}(t) = \int dx \, E_{\text{ext}}(x,t)$. It is independent of the amplitude of the external field, but depends on its shape in space and time. Physically, it is the absorption constant for electromagnetic radiation. Using Eq. (42), one can show that the absorbed power for a monochromatic time dependence is the same for both fields due to the time average. We obtain

$$\bar{P}_\omega = \frac{1}{4\pi} \int_{-\infty}^{\infty} dq \, \text{Re}\sigma(q,\omega)|E_{\text{ext}}(q)|^2. \tag{49}$$

With (28) we find, for the real part of the conductivity,

$$\text{Re}\sigma(q,\omega) = \frac{v_F e^2}{2\hbar}\{\delta[\omega - \omega(q)] + \delta[\omega + \omega(q)]\}. \tag{50}$$

This gives the expression

$$\Gamma_1(\omega) = v_F \frac{e^2}{h} L[q(\omega)]\frac{dq}{d\omega}, \tag{51}$$

with the Fourier transformed autocorrelation function of the external electric field
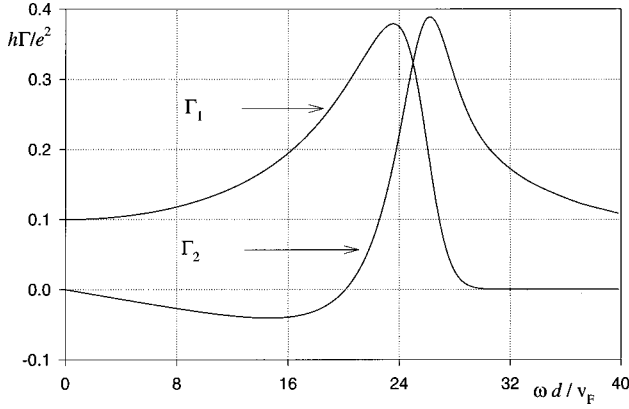
FIG. 4. Real and imaginary parts, $\Gamma_1(\omega)$ and $\Gamma_2(\omega)$, of the ac conductance of a Luttinger wire with finite range interaction; $\alpha d = 1$, $g = 0.1$, range of the electric field $\ell/d = 1/4$.

$$L(q) \equiv \frac{1}{2\overline{U}^2_{\text{ext}}} \left| \int_{-\infty}^{\infty} dx\ e^{iqx} E_{\text{ext}}(x) \right|^2. \qquad (52)$$

By a Kramers-Kronig transformation, we can also define a reactive conductance

$$\Gamma_2(\omega) = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} d\omega' \frac{\Gamma_1(\omega')}{\omega - \omega'}$$

$$= \frac{1}{4\pi\overline{U}^2_{\text{ext}}} \int_{-\infty}^{\infty} dq\ \text{Im}\sigma(q,\omega)|E_{\text{ext}}(q)|^2; \qquad (53)$$

it contains information about phase shifts.

For a zero-range interaction the conductance becomes

$$\Gamma_1(\omega) = \frac{ge^2}{h} L\left(\frac{g\omega}{v_F}\right), \qquad (54)$$

the same as without interaction,[26] except for the renormalization of the prefactor and the Fermi velocity with the interaction strength g and g^{-1}, respectively. In the general case of an interaction potential of finite range, we obtain asymptotically ($\omega\to\infty$),

$$\Gamma_1(\omega) \approx \frac{e^2}{h} L\left(\frac{\omega}{v_F}\right). \qquad (55)$$

This reflects that for large q the dispersion is not influenced by an interaction of a finite range.

A most important feature of result (51) is the factorization into a part that depends only on the internal properties of the interacting electron system, $dq/d\omega$, and a part that contains the information about the shape of the electric field. Only in the limit of vanishing frequency is the shape of the latter unimportant.[26,46] In general, the ac response depends on the spatial properties of the electric field,[42] which is certainly determined by the interactions.

Most remarkably, the temperature does not enter the result, although $T = 0$ was assumed in the derivation. This is due to the linearization of the spectrum. As long as this assumption is justified, the response of the Luttinger liquid is independent of temperature.

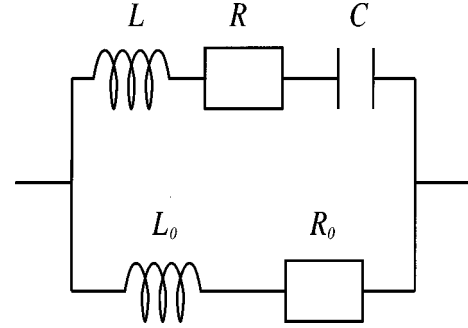A typical result for $\Gamma(\omega)$ is shown in Fig. 4. The Fourier transform of the electric field has been assumed to be a Gaussian $E(x) = E_0\exp(-2x^2/\ell^2)$. If the range of the electric field is zero, the zero of $\Gamma_2(\omega)$, ω_m^*, and the position of the maximum of $\Gamma_1(\omega)$, ω_m, do not agree. However, as soon as ℓ is finite, $\omega_m \approx \omega_m^*$ for a wide region of parameters. In the Coulomb case, $\alpha = 0$, $\Gamma_1 \propto (\ln\omega)^{-1/2}$ for $\omega\to 0$, due to the logarithmic singularity of the dispersion for $q\to 0$.

FIG. 5. Classical circuit for simulating the frequency behavior of the complex conductance.

IV. "QUANTUM IMPEDANCES"

In this section, we analyze the results for the complex conductance presented above with respect to "resistive," "capacitive," and "inductive" behavior. We compare the Luttinger system with an equivalent classical circuit.

A. Impedance network

Our system of interacting electrons shows a resonance in the ac conductance. It can be useful to consider a circuit of capacitances, inductances, and resistances, in order to simulate the frequency behavior. The circuit shown in Fig. 5 contains the minimum set of elements that are necessary for reproducing both the low-frequency behavior and near the resonance. Its complex impedance $Z(\omega)$ is given by

$$Z^{-1}(\omega) = \frac{i\omega C}{1 + i\omega RC - \omega^2 LC} + \frac{1}{R_0 + i\omega L_0}. \qquad (56)$$

The resistance R_0 is fixed to be the resistance at zero frequency, h/ge^2. The circuit shows a resonance near $\omega_0 = (LC)^{-1/2}$, with a width depending on R.

At low frequency, the real and imaginary parts of $Z^{-1}(-\omega)$, $\Gamma_1(\omega)$ and $\Gamma_2(\omega)$, respectively, behave as

$$\Gamma_1(\omega) = R_0^{-1} + \gamma_1\omega^2, \quad \Gamma_2(\omega) = \gamma_2\omega, \qquad (57)$$

with

$$\gamma_1 = RC^2 - \frac{L_0^2}{R_0^3}, \quad \gamma_2 = -C + \frac{L_0}{R_0^2}. \qquad (58)$$

The circuit is defined to behave "capacitively" if $\gamma_1 > 0$ and $\gamma_2 < 0$ simultaneously. If $\gamma_1 < 0$ and $\gamma_2 > 0$, simultaneously, the circuit is clearly "inductive." If $\gamma_2 = 0$, i.e., there is no phase shift between current and voltage, γ_1 indicates capacitive or inductive behavior depending on whether R/R_0 is larger or smaller than 1, respectively. Note that until $O(\omega^2)$ the inductance L does not play any role. Whether the circuit
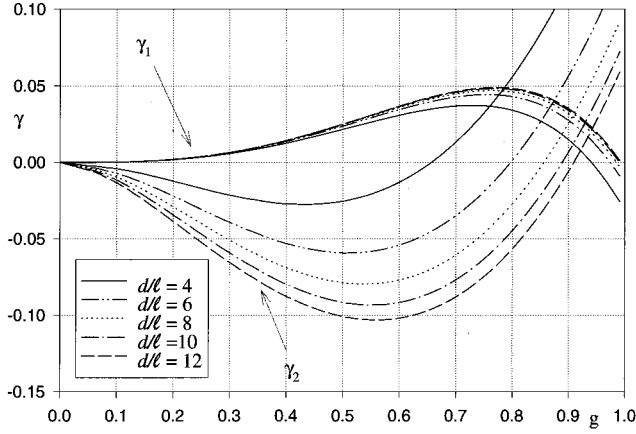
FIG. 6. The parameters γ_1 and γ_2 (in units of $e^2 d^2/h v_F^2$ and $e^2 d/h v_F$, respectively) which characterize the low-frequency behavior of the ac conductance of the Luttinger liquid as a function of the interaction parameter g for different ranges of the electric field, ℓ.

behaves capacitively or inductively near the resonance is therefore to a certain extent independent of its behavior at small frequency.

B. Low frequency

We have extracted the parameters γ_1 and γ_2 [cf. Eqs. (1) and (2)] which characterize the low-frequency behavior of the conductance from the ac-conductance of the Luttinger liquid. By assuming, as above, the electric field to be Gaussian distributed, we obtain the explicit results [in units of e^2/h with $V(0) = V(q=0)$]

$$\gamma_1 = \frac{g^3 d^2}{4 v_F^2} \left[\frac{3(1-g^2)}{2} \left(\frac{4 V_0}{\alpha^2 d^2 V(0)} - 1 \right) - \frac{\ell^2}{d^2} \right],$$

$$\gamma_2 = -\frac{2 v_F}{\pi} \int_0^\infty dq\, e^{-q^2 \ell^2/4} \left[\frac{1}{\omega^2(q)} - \frac{1}{\omega_g^2(q)} \right] + \frac{g^2 \ell}{v_F \sqrt{\pi}},$$
(59)

where $\omega_g(q) = v_F |q|/g$.

Figure 6 show γ_1 and γ_2 as functions of g. Depending on the range of the field, ℓ, the behavior changes from capacitive (small ℓ, $\gamma_1 > 0$, $\gamma_2 < 0$) to inductive (large ℓ, $\gamma_1 < 0$, $\gamma_2 > 0$). As functions of g, γ_1 and γ_2 also change signs. Always, this change of sign happens at smaller value of g for γ_2. The ''phase separation'' lines defined by $\gamma_1(\ell, g_1) = \gamma_2(\ell, g_2) = 0$ are shown in Fig. 7 for $\alpha d = 1$. By decreasing αd, i.e., increasing the range of the interaction, the end points of the two trajectories at $g = 0$ are shifted to higher values of ℓ/d. The region of capacitivelike behavior increases at the expense of the inductive region for increasing interaction strength.

From the behaviors deep in the capacitive and inductive regions, one can deduct formulas for equivalent inductances and capacitances. However, these are not always unique, although their scaling properties with the parameters of the system are. For instance, in the region denoted by L in Fig. 7, γ_2 is given by the second term in Eq. (59) only. This leads, by comparing with Eq. (58), to defining L_0
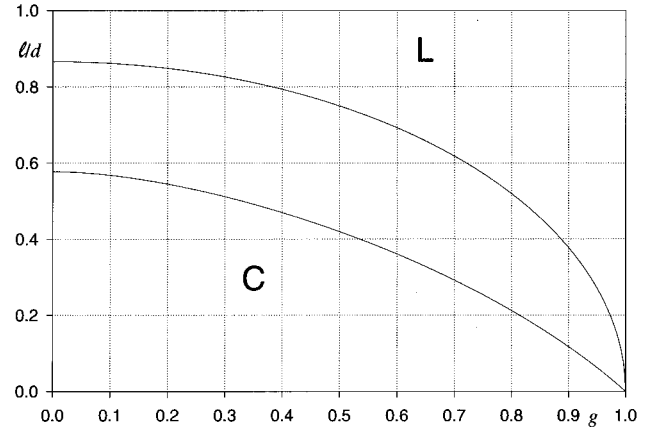

FIG. 7. ''Phase trajectories'' in the (ℓ/d) g plane separating capacitive from inductive behaviors of the Luttinger liquid for $\alpha d = 1$.

$= h \ell / \sqrt{\pi} e^2 v_F$ independently of the interaction parameter. Qualitatively the same result is obtained when using the expression for γ_1. Apart from a different numerical prefactor, the scaling of the inductance with ℓ is the same. This is related to the fact that the behavior of Γ_2 at low frequency is determined via the Kramers-Kronig transformation to the behavior of Γ_1, also at high frequency. The latter depends strongly on the shape of the field. Only if this is assumed in order to reproduce the ω^{-2} behavior of the classical circuit, can one expect γ_1 and γ_2 to yield the same L_0.

In the capacitive region, where $g \ll 1$, the second terms on the right-hand sides of Eq. (59) can be neglected. The first terms can be used to define an equivalent capacitance C and a dissipative resistance R by comparing with Eq. (58). For $\alpha d \ll 1$, in the limit of Coulomb interaction, we obtain, from γ_2,

$$C \approx \frac{e^2}{h} \frac{4 \beta_0}{\alpha v_F} \frac{1}{(1 - 2 \beta_0 \ln \alpha d)^2},$$
(60)

which is independent of the electric field and diverges for infinite interaction range, $\alpha \to 0$. By comparing with the first term of γ_1 we obtain the dissipative resistance

$$R \approx \frac{h}{e^2} \frac{3}{32} \frac{1}{\beta_0} (1 - 2 \beta_0 \ln \alpha d)^{3/2}.$$
(61)

For $\alpha d > 1$, we find

$$C \approx \frac{e^2}{h} \frac{d}{v_F} \frac{\beta_0}{(z^2 + \beta_0^2)^{3/2}} \exp\left[\frac{\beta_0 + z^2}{8} \frac{\ell^2}{d^2} \right]$$

$$\times \left[1 - \Phi\left(\frac{\ell}{2d} \sqrt{\frac{\beta_0 + z^2}{2}} \right) \right],$$
(62)

with $z = \alpha d/2$, and Φ the error function. Here the capacitance depends on the electric field, for instance,

$$C = \frac{e^2}{h} C_1 \frac{g(1-g^2)}{v_F \alpha}$$
(63)

with $C_1 = 2$ for $g/\ell \alpha \gg 1$ and $C_1 = (8\sqrt{2/\pi})(g/\alpha \ell)$ for $g/\ell \alpha \ll 1$. The corresponding dissipative resistances are

$$R = \frac{h}{e^2} R_1 \frac{g}{1-g^2}, \tag{64}$$

with $R_1 = \frac{3}{8}$ and $R_1 = (3\pi/256)(\alpha \ell /g)^2$ for $g/\ell \alpha \gg 1$ and $g/\ell \alpha \ll 1$, respectively.

C. Near resonance

As seen in Fig. 4, the absorptive conductance shows a resonance near the frequency ω_p. For this, the interaction has to be very strong, i.e., $g \ll 1$. Furthermore, the range of the electric field should not be too large (see below). Due to the scaling [cf. Eq. (22)], the results in the Coulomb and Luttinger limits are closely related. Indeed, in contrast to the limit $\omega \to 0$, in the Coulomb limit $\alpha d \ll 1$ the conductance does not show any singular behavior near the resonance frequency ω_m. Since near the resonance the zero-frequency resistance does not play any role, R_0 and L_0 are neglected when fitting C, L, and R to the ac conductance. The parameters of the circuit can be obtained by fitting to the resonance frequency, and the width and the height of the resonance.

For small range of the electric field, $\ell \ll q_p^{-1}$, we find

$$C \propto \frac{e^2}{h v_F} \beta_1^{3/2} \lambda, \tag{65}$$

$$L \propto \frac{h}{v_F e^2} \beta_1^{1/2} \lambda, \tag{66}$$

$$R \propto \frac{h}{e^2} \beta_1^{1/2} \tag{67}$$

with $\beta_1 = g$, $\lambda = \alpha^{-1}$ and $\beta_1 = g_0$, $\lambda = d$ for $\alpha d \gg 1$ and $\alpha d \ll 1$, respectively.

If, on the other hand, the range of the electric field is large, $\ell > q_p^{-1}$, the resonance becomes smaller and very broad. The parameters of the circuit depend here again on the field range. We find

$$C \propto \frac{e^2}{h v_F} \frac{\lambda^2}{\ell} \beta_1^2, \tag{68}$$

$$L \propto \frac{h}{v_F e^2} \ell, \tag{69}$$

$$R \propto \frac{h}{e^2} \frac{1}{\beta_1} \frac{\ell^3}{\lambda^3}. \tag{70}$$

Remarkably, the capacitance obtained here scales in the same way with the system parameters as the one obtained in the limit of low frequency, cf. Eq. (63). Also, the inductance scales as for $\omega \to 0$, although $L \ne L_0$. The dissipative resistance, however, scales differently, and depends much more strongly on ℓ than on L and C. This reflects that the dissipative resistance is much more sensitive to the width and height of the resonance than L and C. The product of the latter is fixed by the resonance frequency, which is quite

insensitive to the range of the electric field in a broad region of field ranges. If we assume $L = L_0$ the scaling behavior of C near the resonance follows directly from $\omega_m = (LC)^{-1/2}$. Thus the somewhat astonishing result of this is that one can identify a region of parameters for the 1D electron system, in which the scaling properties of L and C are independent of the frequency, though the numerical prefactors can be different.

V. DISCUSSION

We considered the ac-transport properties of quantum wire with finite range interaction. The linear-response theory was found to be exact, consistent with earlier work, as a result of the linearization of the dispersion relation. The dependence of the current on the electric field is given by the microscopic nonlocal conductivity. However, the latter is not very useful when aiming at a description of experiments. What is needed is a description in terms of externally accessible macroscopic quantities, as for instance given by Ohm's law. Such a relation can also be found in the present, nonlocal quantum region. By assuming the electric field to be non-zero near a given point x_j, and the current to be detected by a probe at a point x_i one finds $\Gamma(\omega) \equiv \Gamma_{ij}(\omega) = \sigma(x_i - x_j, \omega)$. By generalizing to several probe positions $x_1 \dots x_p$, one obtains

$$J_i(\omega) = \sum_j \Gamma_{ij}(\omega) U_j(\omega). \tag{71}$$

Such an approach has been used recently[32] in order to generalize the Landauer dc approach to finite frequency. In the present work, the nonlocal conductances $\Gamma_{ij}(\omega)$ are natural results of the response theory when suitable assumptions are made for the shape of the electric field. However, it seems to us that in the ac regime this approach is not necessarily appropriate, since it may be very difficult to apply experimentally ac fields locally.

We find it more suitable to define a global average conductance via the time average of the absorbed power. If we consider a monochromatic electric driving field, this absorptive ac conductance can be considered as the real part of a complex conductance. It provides information about the magnitude of the current in phase with the electric field. The imaginary part of the ac conductance provides information about the phase shift between current and voltage. It was obtained by a Kramers-Kronig transformation from the real part.

We found that the absorptive ac conductance factorizes into a product of the density of excitations and the Fourier-transformed autocorrelation function of the external electric field. A question related to this is whether the electric field to be used is the external one or whether one has to use the internal electric field that contains screening contributions. We found that one can use the external electric field in order to obtain the ac conductance of the interacting system. If one uses the total field, the same result is obtained for the conductance. In any case, the result for $\omega \ne 0$ depends on the spatial shape of the field.

Only in the dc limit does the conductance become independent of the shape of the applied electric field, and it de-

pends only on the applied external voltage. It is renormalized by the interaction parameter $\Gamma_1 = ge^2/h$. This does not contradict other recent results, which indicate that for a Luttinger system of a finite length connected to Fermi liquid leads the conductance is not renormalized by the interaction. We can argue that the interaction influences transport for system lengths L above $q^{-1}(\omega)$, the wave length of the charge density wave. When $L < q^{-1}(\omega)$, the effect of the interaction can be neglected.[61,62] Since we are considering the thermodynamic limit, this region is outside the range of the validity of our model.

There is a resonance in the absorptive conductance at a frequency which corresponds to the inverse of a characteristic length scale of the interaction. In the limit of a 1D Coulomb interaction the latter is given by the cutoff length d. For the Luttinger limit, it is the interaction range α^{-1}. Its height and width are influenced by the electric field, but its position is not. However, when keeping in mind a ''realistic'' situation, a word of caution is in order here. If one wants to use the Luttinger model as a model for a quantum wire, the parameters should be such that (i) linearization of the free-electron dispersion is a good approximation, i.e. $q < k_F$; and (ii) interband transitions are not important for the absorptive conductance, $m^*\omega < hd^{-2}$ (m^* is the effective mass). We found above that $q_p \propto d^{-1}$ in the Coulomb limit. The Fermi energy should be smaller than the interband energy distance. Therefore, the Fermi wave number is restricted to $k_F < d^{-1}$. Then, $q_p \approx k_F$, and corrections to linearization should be taken into account. Since furthermore $\omega_m \propto v_F/d \propto h/m^*d^2$, interband transitions could also become important near the resonance. For the Luttinger limit of the interaction the situation does not improve. One also needs to take into account interband mixing induced by the interaction for a proper description of the frequency region near the resonance.

However, at low frequency, and if the Fermi level is well below the onset of the second lowest subband, interaction-induced mixing of the bands can be neglected, and interband transitions are unimportant. Here the real and imaginary part of the conductance depend quadratically and linearly on ω, respectively. The signs of the prefactors of these terms indicated capacitivelike or inductivelike behavior of the system. By comparing the frequency behavior of the microscopic model with that of a ''minimal'' classical circuit of capacitances, inductances, and resistances, we find microscopic expressions for these quantities. They are, however, not unique. Their validity is restricted to certain parameter regions. Only the scaling properties are the same. Astonishingly, one can identify a parameter region in which the scaling properties with the parameters of the system are the same for low frequency and near the resonance. This fact gives us some confidence that, although the frequency region near the resonance is somewhat out of the range of validity of the model, certain general features of the results remain valid.

In particular, the impedances strongly depend in general on the shape of the applied electric field. There is a competition between the range of the field and the range of the interaction, which determines whether the system behaves as a capacitor ($\ell\alpha \ll 1$) or as an inductor ($\ell\alpha \gg 1$). In the former case, depending on the ratio between the interaction parameter g and $\ell\alpha$, the capacitance and the dissipative resistance may ($g/\ell\alpha \ll 1$) or may not ($g/\ell\alpha \gg 1$) depend on

the field range. The infinite range of the interaction removes the dependence on the range of the field, such that one can interpret the impedances as genuine properties of the system that are only determined by the parameters of the microscopic model.

It is particularly interesting to note that the model allows us to identify two conceptually different ''resistances.'' In the dc limit, we have the equivalent of the ''contact resistance'' h/ge^2, which is, however, renormalized by the interaction parameter. Near the resonance frequency, one can define a ''dissipative resistance,'' approximately given by the inverse of the height of the resonance. The former depends on if and how the system is connected with the ''outside world'' via contacts. It changes to h/e^2 if the system length is smaller than the wavelength of the charge-density wave. In contrast, we expect that the latter is not changed as long as $L > q_p^{-1} \propto d$. By comparing with the classical circuit one also notes that the dissipative resistance is also present at low frequency, though it scales differently with the system parameters.

VI. CONCLUSION

How to calculate the frequency-dependent low-temperature electronic quantum transport properties of nanostructures when interaction effects are important is presently a subject of strong debate. Since experiment can access this regime by using modern fabrication technology, developing theoretical concepts for the treatment of transport processes, taking into account interactions, is important. In addition, future information technology will require high-frequency signal transportation in nanoscale systems. A proper theoretical understanding of the underlying microscopic processes will be essential for developing this technology in the far future.

In order to obtain insight into some of the peculiar microscopic features of this transport region, we investigated the ac response of a simple 1D model of an interacting electron system. We found that in spite of the simplicity of the model the definition of transport parameters already provides nontrivial questions to be solved. The transport parameters depend on the purpose for which they are supposed to be used. For instance, when asking for the current in some probe as a response to an electric field applied to another probe, it is the nonlocal conductivity which has to be used as the ''conductance.'' On the other hand, when interested in the absorption of electromagnetic radiation, an average of the conductivity provides an ''absorptive conductance.''

By starting from the absorptive conductance, we made an attempt to define for our system the quantum counterparts of the impedance parameters used in the transport theory of classical circuits. We found that they cannot be defined uniquely in terms of the parameters of the microscopic model, in accordance with the above general statement. However, certain scaling properties remain valid in astonishingly large regions of parameters. They even resemble the scaling properties of the corresponding classical definitions. This encourages us to find similar quantities for more complicated systems such as tunnel barriers and multiple tunnel barriers in the presence of interactions.

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