Superfluidity and quantum vortices in systems with pairing of spatially separated electrons and holes in crossed magnetic and electric fields

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The low-temperature behavior of two-dimensional systems with pairing spatially separated electrons and holes is studied. We predict a transition to a superfluid state of a dilute gas of electron-hole pairs in a strong magnetic field normal to conducting layers. In the superfluid phase the crossed electric and magnetic fields are shown to create planar vortices where the pairs rotate in the structure plane. [S0163-1829(98)05420-4]

About 20 years ago it was predicted[1–3] that a rather unusual superconductivity mechanism due to pairing of spatially separated electrons and holes (PSSEH) can exist. In this superconductivity mechanism an electron supercurrent is accompanied by a hole supercurrent, which is equal in value and opposite in direction to the former one. During the last 5–7 years the ideas outlined in Refs. 1–3 have been developed in a number of theoretical papers,[4–9] in which systems with PSSEH have been studied in a strong magnetic field perpendicular to the layers. Experimental papers[10,11] reported the observation of phenomena that are associated[9,11] with the superconductivity mechanism predicted.[1–3] However, the conclusion about superconductivity is drawn on the ground of some indirect results, rather than on measuring, say, the electroresistance. Therefore the experimental observation of a superconducting phase in systems with PSSEH remains doubtful.

In this paper we will consider the behavior of systems with PSSEH below the temperature of a superconducting transition in crossed electric and magnetic fields, and we will show that this behavior resembles the one of ordinary superconductors in a magnetic field. Specifically, the crossed fields can lead to an effect similar to flux quantization in ordinary superconductors, and give rise to vortices in which electron-hole pairs rotate in the structure plane. The observation of these phenomena would be an unequivocal indication that a system with PSSEH experiences a transition to a superconducting state.

Consider a three-layer sandwich consisting of a layer with electron conductivity, a layer with hole conductivity, and a dielectric layer of thickness d between them. Assume a strong magnetic field $\vec{H}$ to be applied perpendicular to the structure plane. We will consider the low-density limit, when the size of a bound electron-hole pair is less than the average distance between them. As is the case in the absence of the magnetic field, the low-density limit seems more favorable for pairing and for a transition of pairs into a superfluid state.

The behavior of a single pair is described by the Schrödinger equation (see, e.g., Ref. 12)

$$\left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\vec{r}^2}+\frac{ie\hbar}{2mc}\gamma\vec{H}\cdot\left(\vec{r}\times\frac{\partial}{\partial\vec{r}}\right)+\frac{e^2}{8mc^2}(\vec{H}\times\vec{r})^2+e\left(\frac{\vec{P}\times\vec{H}}{Mc}+\vec{E}\right)\vec{r}-\frac{e^2}{\varepsilon r}+\frac{P^2}{2M}\right\}\Psi(\vec{r})=\mathcal{E}\Psi(\vec{r}). \tag{1}$$

Here $\vec{r}=\vec{r}_1-\vec{r}_2$ is a three-dimensional vector of the distance between an electron and a hole, $m=m_1m_2/(m_1+m_2)$ is the reduced mass, $M=m_1+m_2$ the total mass of the pair, $\vec{P}$ the momentum of the pair as a whole, $\vec{E}$ the strength of the electric field, and $\gamma=(m_2-m_1)/(m_2+m_1)$. An electric field $\vec{E}$ consists of an external field $\vec{E}_{\mathrm{ext}}$ and the field $\vec{E}'$ created by all remaining pairs. In what follows, we shall disregard the field $\vec{E}'$, whose account in the limit of low pair density yields only an inessential correction to the pair binding energy. I shall confine myself to the case where the external electric field $\vec{E}_{\mathrm{ext}}$ and the pair momentum $\vec{P}$ are perpendicular to the magnetic field $\vec{H}$ directed along the axis z.

Taking into account that in a strong magnetic field the distance between Landau levels $\hbar eH/mc$ considerably exceeds the Coulomb energy $me^4/\varepsilon^2\hbar^2$, at $\vec{P}=0$ and $\vec{E}_{\mathrm{ext}}=0$ Eq. (1) can be solved in the framework of the perturbation theory, taking the value $(me^4/\varepsilon^2\hbar^4)/(\hbar eH/mc)$ as a small parameter. If $\vec{P}\neq 0$ and $\vec{E}_{\mathrm{ext}}\neq 0$, then one needs at first to make a transformation which eliminates from the Hamiltonian the term with the effective electric field $\vec{E}_{\mathrm{eff}}=\vec{E}_{\mathrm{ext}}+\vec{P}\times\vec{H}/Mc$.

We seek $\Psi(\vec{r})$ in the form (cf. Ref. 13)

$$\Psi(\vec{\rho},z)=\Phi(\vec{\rho}-\vec{\rho}_0,z)\exp\left(-i\frac{\gamma\vec{\rho}\cdot\vec{P}'}{2\hbar}\right), \tag{2}$$

where $\vec{\rho}$ is a two-dimensional radius vector in the xy plane,

$$\vec{P}'=\vec{P}-M\vec{u},\quad \vec{u}=\frac{c}{H^2}\vec{E}_{\mathrm{ext}}\times\vec{H},\quad \vec{\rho}_0=\frac{c}{eH^2}\vec{H}\times\vec{P}'. \tag{3}$$

Equation (1) is reduced to the form

$$\left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\vec{r}^2}+\frac{ie\hbar}{2mc}\gamma\vec{H}\cdot\left(\vec{\rho}\times\frac{\partial}{\partial\vec{\rho}}\right)+\frac{e^2}{8mc^2}H^2\rho^2-\frac{e^2}{\varepsilon[(\vec{\rho}+\vec{\rho}_0)^2+z^2]^{1/2}}+\frac{P^2-P'^2}{2M}\right\}\Phi(\vec{r})=\mathcal{E}\Phi(\vec{r}). \tag{4}$$

The new Hamiltonian does not contain the term with the field $\vec{E}_{\mathrm{eff}}$. For this, one has to pay with a shift of $\vec{\rho}$ by $\vec{\rho}_0$ in the potential energy, which seems to be a smaller nuisance, since the potential energy is accounted for in the framework of the perturbation theory.

As Elliot and Loudon have shown,[14] in the case when an electron and a hole belong to the lowest Landau level, the wave function $\Phi(\vec{r})$ from Eq. (4) can be written in the first approximation as

$$\Phi(\vec{r})=\frac{1}{\sqrt{2\pi}\ell_H}\exp\left(-\frac{\rho^2}{4\ell_H^2}\right)\psi(z)\equiv\varphi(\vec{\rho})\psi(z), \qquad (5)$$

where $\ell_H=\sqrt{c\hbar/eH}$ is the magnetic length. The wave function $\psi(z)$ must obey the equation obtained by averaging Eq. (4) with the help of $\varphi(\vec{\rho})$.

We consider two cases separately. First, we will assume the thickness of conducting layers to be slightly above the Bohr radius, $a_0=\varepsilon\hbar^2/me^2$. Because of the relation $\hbar eH/mc \gg me^4/\varepsilon^2\hbar^2$, the inequality $a_0\gg\ell_H$ holds. If the dielectric thickness is small ($d\ll\ell_H$), then one proves easily that the function $\psi(z)$ obeys the equation

$$\left\{-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}+U(z)\right\}\psi=\left(\mathcal{E}-\frac{P^2-P'^2}{2M}-\frac{\hbar eH}{mc}\right)\psi, \qquad (6)$$

where

$$U(z)=-\frac{e^2}{2\pi\varepsilon\ell_H^2}\int\frac{e^{-\rho^2/2\ell_H^2}}{[(\vec{\rho}+\vec{\rho}_0)^2+z^2]^{1/2}}d^2\rho. \qquad (7)$$

The consideration of Eq. (6) with the potential $U(z)$ from Eq. (7) shows that the binding energy of an electron-hole pair at rest is equal to $(2\hbar^2/ma_0^2)\ln^2(a_0/\ell_H)$, whereas the part of the pair energy depending on the momentum $\vec{P}$ and drift velocity $\vec{u}$, has the form[13]

$$\delta\mathcal{E}=\frac{P^2}{2M_H}+\left(1-\frac{M}{M_H}\right)\vec{u}\cdot\vec{P}-\frac{1}{2}\left(1-\frac{M}{M_H}\right)Mu^2, \qquad (8)$$

where the mass M_H is equal to

$$M_H=m\frac{a_0^2}{\ell_H^2}2\ \ln\left(\frac{a_0}{\sqrt{2}\ell_H}\right). \qquad (9)$$

If the thicknesses of the conducting layers are much less than ℓ_H and d, one can regard the layers as purely two dimensional. This permits one to omit the derivative over z in Eq. (6), and to substitute the coordinate z in expression (7) with the dielectric thickness d. As a result, the correction to the energy of an electron-hole pair due to the Coulomb interaction between an electron and a hole will amount to

$$\delta\mathcal{E}=-\frac{e^2}{2\pi\varepsilon\ell_H^2}\int\frac{e^{-\rho^2/2\ell_H^2}}{[(\vec{\rho}+\vec{\rho}_0)^2+d^2]^{1/2}}d^2\rho. \qquad (10)$$

Putting $\rho_0=0$, we find the pair binding energy as a function of ℓ_H and d:

$$\delta\mathcal{E}=-\left(\frac{\pi}{2}\right)^{1/2}\frac{e^2}{\varepsilon\ell_H}\exp\left(\frac{d^2}{2\ell_H^2}\right)\left[1-\Phi\left(\frac{d^2}{2\ell_H^2}\right)\right], \qquad (11)$$

where $\Phi(x)=(2/\sqrt{\pi})\int_0^x\exp(-t^2)dt$. This energy does not depend on the masses of an electron and a hole. Below, we will be interested in the case when $d\ll\ell_H$.

With $d\ll\ell_H$ from Eq. (10), it follows that

$$\delta\mathcal{E}=-\left(\frac{\pi}{2}\right)^{1/2}\frac{e^2}{\varepsilon\ell_H}I_0\left(\frac{\rho_0^2}{4\ell_H^2}\right)\exp\left\{-\frac{\rho_0^2}{4\ell_H^2}\right\}, \qquad (12)$$

where I_0 is the Bessel function.

With small momenta $\vec{P}$ and electric fields $E_{\rm ext}$ (more precisely, with $\rho_0\ll\ell_H$) the part of the pair energy depending on the momentum $\vec{P}$ and velocity $\vec{u}$ is given by the previous expression (8); however, the mass M_H from Eq. (9) should be (at $d\ll\ell_H$) changed for

$$M_H=\frac{4\varepsilon\hbar^2}{\sqrt{2\pi}e^2}\frac{1}{\ell_H}\equiv m\frac{4}{\sqrt{2\pi}}\frac{a_0}{\ell_H}. \qquad (13)$$

Thus in the both cases in a strong magnetic field and electric field perpendicular to it, the energy of an electron-hole pair is given by expression (8). It follows from this that due to the Coulomb interaction a pair acquires a finite (and not infinite) transverse mass M_H when it moves in the structure plane. It is useful to note that for conducting layers of small thickness the mass M_H as well as the energy $\delta\mathcal{E}$ from Eq. (11) do not depend on the masses of an electron and a hole.

Now I would like to discuss a question which has a fundamental significance for the problem of the superfluidity of electron-hole pairs. As was first shown by Guseinov and Keldysh,[15] interband transitions always occurring in real systems lead to the appearance in the Hamiltonian of the terms

$$\sum_{\vec{k}}\ [T_{ab}\hat{a}(\vec{k})\hat{b}(-\vec{k})+T_{ab}^{*}\ \hat{b}^{\dagger}(\vec{k})\hat{a}^{\dagger}(-\vec{k})], \qquad (14)$$

where $\hat{a}$, $\hat{a}^{\dagger}$ and $\hat{b}$, $\hat{b}^{\dagger}$ are operators of electron and hole creation and annihilation. These terms lift the phase degeneration of the order parameter, and lead to the appearance of a gap in the perturbation spectrum, which, in contrast to the case of superconductors, is rigidly linked with the lattice but not with current carriers. As a result, the current state becomes impossible, and, with account taken of interband transitions, the system transfers to a dielectric, but not a superfluid state.

For systems without spatially separated carriers the matrix elements T_{ab} are determined by the potential of the atom interaction in the lattice, and they cannot be varied by the experimenter's wish. If electrons and holes are spatially separated, then the interband transitions coincide with the interlayer ones, and the matrix elements T_{ab} depend exponentially on the thickness of the dielectric layer, d, which separates the layers with the electron and hole conductivity. Since the Coulomb interaction of electrons and holes decreases with the growth of d according to a power law, it is easy to find such a thickness d to leave the binding energy sufficiently large while the interband transitions become negligibly small. Thus the genuine superfluidity of electron-hole pairs can occur only in systems with PSSEH. In what follows we shall assume that interband transitions are completely absent.

In the absence of interband transitions, the lifetime of electron-hole pairs is unlimited, and, since the pairs are bosons, they can pass to a superfluid state on lowering the temperature. Due to the two-dimensional nature of the system considered this transition will occur via the Berezinskii-Kosterlitz-Thouless mechanism. The transition temperature T_c obeys the equation

$$T_c=\frac{\pi}{2}\frac{\hbar^2 n_s(T_c)}{M_H}, \tag{15}$$

where $n_s(T_c)$ is the superfluid density of pairs. The mass M_H, not M, enters into this expression, since, as follows from the dispersion law (8), it is the former mass that determines the dynamic of electron-hole pairs in the structure plane. That is why the planar vortex energy (and, hence, T_c) depend on the quantity M_H. In the low density limit considered to estimate T_c, one can change the superfluid density $n_s(T_c)$ in Eq. (15) for the total density of pairs n.

A crystallization of pairs with the formation of a dipole crystal can compete with a transition of electron-hole pairs to a superfluid state. The crystallization point T_m is determined by an interaction of pairs between themselves that is the dipole-dipole one for a low density of pairs: $V(\vec{r}_1-\vec{r}_2)=e^2d^2/\varepsilon|\vec{r}_1-\vec{r}_2|^3$. If one neglects quantum effects, then for the crystallization point T_m one can apply the estimate $T_m\approx V(n^{-1/2})$, i.e.,

$$T_m=\frac{e^2d^2n^{3/2}}{\varepsilon}. \tag{16}$$

We could obtain the same estimate using the Kosterlitz-Thouless melting criterion for two-dimensional systems.

Taking into account that the density of pairs is $n=\nu/2\pi\ell_H^2$, where ν is the filling factor of the layers, one can easily find from the inequality $T_m<T_c$ the condition for the existence of the temperature range where the pairs are superfluid:

$$d^2<\frac{\pi}{2}\frac{m}{M_H}\left(\frac{2\pi}{\nu}\right)^{1/2}a_0\ell_H. \tag{17}$$

One should keep in mind that the quantum effects omitted may lower the temperature T_m considerably, and therefore the condition for the existence of a superfluid phase actually may not be so strict.

Below the temperature of a superfluid transition, one can describe the behavior of a superfluid component with an order parameter. From the dispersion law of pair (8), it follows that the part of the energy depending on the phase of the order parameter φ has the form

$$\mathcal{E}=\int\left[\frac{n_s}{2M_H}(\hbar\nabla\varphi)^2+n_s\left(1-\frac{M}{M_H}\right)\hbar\nabla\varphi\cdot\vec{u}\right]d^2\rho. \tag{18}$$

It follows from this expression that for the corresponding magnitude and direction of the vector $\vec{u}\sim\vec{E}_{\rm ext}\times\vec{H}$, the appearance of flows of pairs is advantageous. This question is not quite trivial, and it deserves a more detailed treatment. Below we will consider it in the case where the p-i-n structure under study is a disc of radius R.

First, let us assume a potential difference U to be applied between the disc center and its edge. Then a radial electric field will appear in the disc related to the potential difference by the expression $E_\rho=U/\rho\ \ln(R/r)$, where r is the size of the electric contact at the disc center. The field E_ρ together with the magnetic field H_z leads to a gradient $\nabla\varphi$ with only the tangent component nonvanishing $(\nabla\varphi)_\tau$. On going along the closed contour, the phase φ can experience changes only by $2\pi s$, where s is an integer including zero; therefore we have $(\nabla\varphi)_\tau=s/\rho$. As a result, the energy of the system of pairs will be equal to (at $T=0$)

$$\mathcal{E}=\frac{\pi n}{M_H}\left\{\left[\hbar s+(M_H-M)\frac{c}{H}\frac{U}{\ln R/r}\right]^2-\left(M_H\frac{c}{H}\frac{U}{\ln R/r}\right)^2\right\}\ln\frac{R}{r}. \tag{19}$$

The integer s in this expression is determined from the requirement of the minimum for the energy $\mathcal{E}$ with fixed U and H. If U changes, then s remains unchanged within a certain range of U values, and then it becomes equal to $s+1$ (or $s-1$). This transition is associated with the appearance or disappearance of a quantized vortex with a center at the disc axis. One sees that $\mathcal{E}$ is a continuous function of U; however, the derivative $d\mathcal{E}/dU$ possesses discontinuities at the points where $(M_H-M)cU/\hbar H\ \ln R/r=s+\frac{1}{2}$. In the neighborhoods of these points the second derivative $d^2\mathcal{E}/dU^2$ must possess spikes.

Assume now that the p-i-n structure considered is placed into an electric field created by a charged round disc of radius a with $a\gg R$. If the distance h from the p-i-n structure to the charged disc satisfies the inequality $|h-a|\ll a$, and its center coincides with the center of the p-i-n structure, then the radial component of the electric field is equal to $E_\rho=Q\rho/4a^3$, where Q is the total charge of the disc (see, e.g., Ref. 16). In this case the energy of a system of pairs is equal to

$$\mathcal{E}=\int\left[\frac{n_s}{2M_H}(\hbar\nabla\varphi)^2-n_s\left(1-\frac{M_H}{M}\right)\frac{c}{H}\hat{z}\times\left(\frac{Q}{4a^3}\vec{\rho}\times\hbar\nabla\varphi\right)\right]d^2\rho. \tag{20}$$

Energy (20) coincides exactly with the energy of the superfluid liquid in a vessel rotating with the angular velocity $\Omega=Q(1-M/M_H)c/4Ha^3$. But it is well known that if the angular velocity exceeds a critical value $\Omega>\Omega_c\equiv\hbar\ \ln(R/\xi)/M_HR^2$, then quantized vortices arise in the liquid. Therefore, under the condition

$$Q>Q_c\equiv\frac{4\hbar}{M_HR^2}\frac{Ha^3}{(1-M/M_H)c}\ln\frac{R}{\xi}, \tag{21}$$

the planar vortices will arise in the system considered. In these vortices every electron-hole pair rotates as a whole in the structure plane, so that an electron supercurrent is accompanied by a hole supercurrent that is equal in value and opposite in direction to the former one. At the angular velocities $\Omega \gg \Omega_c$ the vortices will be distributed uniformly, and their density will be equal to $n_v = M_H \Omega / \pi\hbar$. Here we describe vortices at the macroscopic level. A microscopic description is not a simple task because in an electrically neutral system the velocity field of a vortex decreases according to a power law rather than exponentially and one should take into account the specimen boundaries.

Quantized vortices can also arise in the system with an arbitrary dependence of the electric field on coordinates, and one can show that for continuously distributed vortices their density is equal to

$$n_v = \frac{c(M_H - M)}{2\pi\hbar H} \left| \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \right|. \tag{22}$$

Let us give quantitative estimates for purely two-dimensional layers. On the field $H \approx 10^5$ G s, the magnetic length is $\ell_H \approx 80$ Å. As a result, at $\varepsilon = 10$ the mass $M_H \approx 9 \times 10^{-29}$ g [see Eq. (13)] and the temperature of a superfluid transition is $T_c \approx 33\nu$ K. For $\nu = \frac{1}{5}$ the temperature $T_c \approx 7$. If the radius of the charged disc is $a = 10$ cm, then the critical charge on the disc above which the quantized vortices arise is $Q_c \approx \frac{4}{3} \times 10^{-9}$ C. This charge creates the electric-field strength perpendicular to the disc and equal to $E^z_{\rm ext} \approx 24$ V/cm. The density of vortices is $n_v \sim \Omega / \Omega_c$.

In conclusion, we point out that, similar to the situation in superfluid ^{4}He, the most efficient tool for studying predicted quantum vortices is the second sound. It is in order to note a recent paper[17] in which a method of detection and measurement of quantum vorticity by scattering second sound off quantized vortices in superfluid helium has been suggested. We hope to consider this circle of problems in more details soon.

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