Anisotropic transport of quantum Hall meron-pair excitations

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Double-layer quantum Hall systems at total filling factor $\nu_T = 1$ can exhibit a commensurate-incommensurate phase transition driven by a magnetic field $B_\parallel$ oriented parallel to the layers. Within the commensurate phase, the lowest charge excitations are believed to be *linearly confined meron pairs*, which are energetically favored to align with $B_\parallel$. In order to investigate this interesting object, we propose a gated double-layer Hall bar experiment in which $B_\parallel$ can be rotated with respect to the direction of a constriction. We demonstrate the strong angle-dependent transport due to the anisotropic nature of linearly confined meron pairs and discuss how it would be manifested in experiment. [S0163-1829(98)07503-1]

The quantum Hall effect has served as a canonical example of a strongly correlated, two-dimensional electron system. Since the strong perpendicular magnetic field $B_\perp$ quenches the kinetic energy of the electrons, the electron-electron correlations become crucial. These correlations can manifest themselves in a variety of *topological charges*: patterns or textures in the spin or isospin of the two-dimensional (2D) electrons. In the case of a single-layer quantum Hall system, Sondhi *et al.* have shown that even when the Zeeman gap vanishes, the quantum Hall effect survives at some filling factors including $\nu = 1$.[1,2] They argued that in this case the lowest-energy charged excitations are Skyrmions, topological excitations consisting of dimples in the electron spin distribution that involve multiple spin flips.[1–4] Subsequently Barrett *et al.* performed NMR Knight shift measurements and clearly demonstrated the existence of multiple spin-flip excitations that may be a manifestation of these Skyrmionic excitations.[5–7]

The closely related double-layer quantum Hall systems (DLQHS's) have also drawn much theoretical and experimental attention.[2,8–13] When the distance d between the layers is comparable to the mean intralayer particle spacing, strong interlayer correlations induce a many-body quantum Hall effect at $\nu_T = 1/m$ (m is an odd integer), where ν_T is the total filling factor. The layer degrees of freedom (i.e., upper or lower) can be viewed as a spin-$\frac{1}{2}$ *isospin* variable and the description of DLQHS's can be mapped onto itinerant quantum ferromagnets.[2,8,9,14] We emphasize that the real spins are fully spin polarized along $B_\perp$ due to the Zeeman gap and the excitations consist of variations in the isospin.[15] It has been argued that for the experimentally relevant set of parameters, linearly confined meron pairs (LCMP's) are the lowest-energy charged excitations. They too are genuine topological excitations made from textures in the isospin distribution[9,16] as shown in Fig. 1. In contrast to Skyrmions, these excitations cannot be seen via NMR Knight shift measurements because they do not couple to the nuclear magnetic moment. Such excitations are notoriously difficult to observe and have been seen only indirectly via transport measurements.[12]

In this paper we propose a method of investigating topological excitations by forcing them to pass through a narrow channel. As the constriction is approached, details of the excitations can be measured. This method can be used on systems in which it is hard to couple with the relevant isospin (e.g., valley Skyrmions in silicon 2D electronic systems.[17]) Here we concentrate on the LCMP: By noticing that the orientation of the LCMP tends to follow the direction of the magnetic field $B_\parallel$ applied parallel to the layers, we demonstrate the strong transport anisotropy depending on the relative angle between the gate and $B_\parallel$. The method can be generalized to other classes of topological objects.[18]

Following the magnetic analogy, we map the layer degrees of freedom onto a $S = \frac{1}{2}$ isospin variable, where an electron in the upper layer has an isospin $|\uparrow\rangle$ and one in the lower layer has an isospin $|\downarrow\rangle$.[2] A local charge imbalance between the two layers corresponds to $\langle S_z(\mathbf{r})\rangle \neq 0$ in that region. Such charge fluctuations between the layers have an energy gap at long wavelengths and are suppressed. Thus, while the true spin of the electron points in the $\hat{z}$ direction, the isospin is forced to lie in the $\hat{x}$-$\hat{y}$ plane in isospin space, so the system is equivalent to a quantum XY ferromagnet. Using the spin texture state ansatz $|\theta(\mathbf{r})\rangle = (|\uparrow\rangle + e^{i\theta(\mathbf{r})}|\downarrow\rangle)/2$, we obtain the energy functional $E[\theta]$,

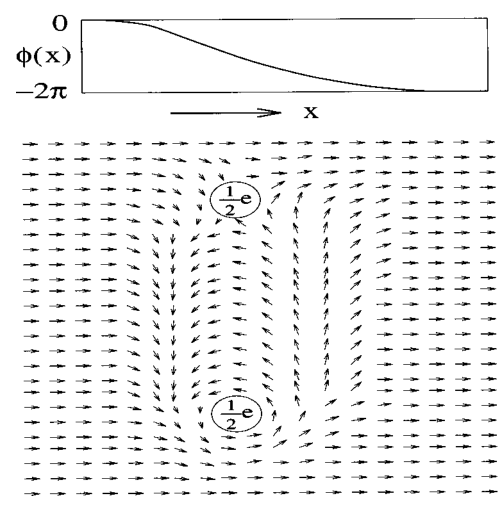

FIG. 1. Isospin configuration of a finite-length meron pair excitation: The arrows represent the isospin orientations. Merons with the opposite vorticity and the same charge $\frac{1}{2}e$ are located at both ends of domain wall. The inset shows the solution profile of $\phi(x)$ along a line passing between the meron pair.

$$E[\theta]=\int d^2r\left\{\frac{1}{2}\ \rho_s(\nabla\theta)^2-\frac{t}{2\pi\ell_B^2}\cos\theta(\mathbf{r})\right\},\qquad(1)$$

where $\theta(\mathbf{r})$ represents the isospin orientation in the plane, ρ_s is the isospin stiffness, t stands for an interlayer tunneling amplitude, and $\ell_B=(\hbar c/|e|B_\perp)^{1/2}$.

The first term indicates that the isospins tend to be parallel to each other, which minimizes the overlap of the electrons due to the Pauli exclusion principle and so minimizes the Coulomb exchange energy. The second term reflects the fact that the tunneling term lowers the energy of electrons in a symmetric superposition of the two layer states, which corresponds to $\langle S_x\rangle=\frac{1}{2}$. This favors orienting the isospin along the $\hat{x}$ direction in isospin space. Hence the ground state of this Hamiltonian will be the Slater determinant of the completely filled symmetric state.

The presence of an in-plane magnetic field $B_\parallel$ makes the physics of DLQHS's much more intriguing. Suppose $B_\parallel$ is parallel to the $\hat{y}$ coordinate. Choosing the gauge potential $\mathbf{A}=(0,B_\perp x,-B_\parallel x)$, we see that the tunneling matrix element from one layer to the other will pick up an Aharanov-Bohm phase e^{iQx}, with $Q=dB_\parallel/\ell_B^2B_\perp$, and that this phase rotates or ''tumbles'' as we move in x. The imposition of $B_\parallel$ leaves the orbital degrees of freedom intact and we obtain the following energy functional for the ansatz given above:

$$E[\theta]=\int d^2r\left\{\frac{1}{2}\ \rho_s(\nabla\theta)^2-\mathbf{h}(\mathbf{r})\cdot\mathbf{m}(\mathbf{r})\right\},\qquad(2)$$

where the fictitious magnetic field $\mathbf{h}(\mathbf{r})=(t/2\pi\ell_B^2)(\cos Qx,\sin Qx)$ tumbles along the $\hat{x}$ coordinate with a period $2\pi/Q$, which couples to the isospin $\mathbf{m}(\mathbf{r})=(\cos\theta(\mathbf{r}),\sin\theta(\mathbf{r}))$. This is the well-known Pokrovsky-Talapov model, which exhibits a highly collective commensurate-incommensurate transition.[19] For small Q and/or small ρ_s, the phase tracks the tumbling field, so that $\theta(\mathbf{r})=Qx$. As $B_\parallel$ increases, the local field tumbles too rapidly and a continuous phase transition to an incommensurate state with broken translation symmetry occurs.[9,19] The ground-state energy of the commensurate state is given by

$$\frac{E_0[Q]}{A}=\frac{1}{2}\rho_sQ^2-\frac{t}{2\pi\ell_B^2}.\qquad(3)$$

For the clarity of further discussion, we introduce a phase field $\phi(\mathbf{r})=\theta(\mathbf{r})-Qx$, which represents an isospin orientation measured with respect to the direction of $\mathbf{h}(\mathbf{r})$. For the commensurate state, the isospin is parallel to $\mathbf{h}(\mathbf{r})$ yielding $\phi(\mathbf{r})=0$. The energy functional for the generic excited states can be written in terms of the $\phi(\mathbf{r})$ field as

$$E_A[\phi]=\rho_sQL_y\Delta\phi+\int d^2r\left\{\frac{1}{2}\ \rho_s(\nabla\phi)^2-\frac{t}{2\pi\ell_B^2}\times[1-\cos\phi(\mathbf{r})]\right\},\qquad(4)$$

where L_y is the system size along the $\hat{y}$ direction and $\Delta\phi\equiv\phi(x=\infty)-\phi(x=-\infty)$, the number of full rotations the isospin makes with respect to the tumbling field $\mathbf{h}(\mathbf{r})$, is a *topological* charge.[20]

These rotations or phase slips with $\Delta\phi=\pm2\pi$ occur over a finite width and can be viewed as domain walls.[19] If the domain-wall string soliton (DWSS) has finite length, its end points will be localized excitations in the isospin texture called merons. The domain wall serves to link the meron pair so that they are linearly confined, as shown in Fig. 1. It has been argued that the LCMP is the lowest-energy *charged* excitation of the DLQHS within the commensurate phase.[2,9,15]

Suppose we have a LCMP extending over the system size and making an angle α with $B_\parallel$. Since the second term of Eq.(4) is invariant under spatial rotations (that is, rotations in $\mathbf{r}$), dependence of the LCMP activation energy on its orientation in the plane comes entirely from the first term, which depends on L_y—the projected length to the $\hat{y}$ direction. The first term can be viewed as a chemical potential for the LCMP with $\mu_D=-2\pi\rho_sQL_y$. We notice that for the nontopological excitations with $\Delta\phi=0$, the activation energy has *no* angle-dependence. For the sake of simplicity, we can take the LCMP to be aligned along the $\hat{y}$ axis. The analytical solution for the profile of the domain wall is well-known and given by $\phi(x)=-4\tan^{-1}[\exp((2\pi\rho_s/t)^{1/2}x/\ell_B)]$.[19] Based on this solution, the energy T_0 of the LCMP per unit length is given to be $(4t/\pi\ell_B)(2\pi\rho_s/t)^{1/2}$. Hence the string tension is given by

$$T[\alpha,B_\parallel]=T_0\left\{\left(1-\frac{B_\parallel}{B_\parallel^c}\right)+\frac{B_\parallel}{B_\parallel^c}(1-\cos\alpha)\right\},\qquad(5)$$

where $B_\parallel^c=B_\perp(4\ell_B/\pi d)(t/2\pi\rho_s)^{1/2}$. When $B_\parallel>B_\parallel^c$, it is energetically favorable to create DWSS's of infinite length (in other words, the LCMP's become unbound) making a phase transition to an incommensurate phase.[8,9,12] Within the commensurate phase $B_\parallel<B_\parallel^c$, the activation energy of the DWSS increases linearly with the length.

Since the merons carry charge $\pm\frac{1}{2}e$ depending on the vorticity and core-spin configurations,[2] one can construct a finite-energy charged excitation by attaching two merons with the same charge and opposite vorticity. The activation energy E_{LCMP} of the LCMP with the length R and the relative angle α with respect to $B_\parallel$ can be determined by balancing the Coulomb repulsion and the linear string tension

$$E_{\mathrm{LCMP}}=T(\alpha,B_\parallel)R+\frac{e^2}{4\epsilon R}+2E_{\mathrm{MC}},\qquad(6)$$

where ϵ is the dielectric constant and E_{MC} represents the meron core energy obtained by integrating out the short-distance degrees of freedom. Equation (6) is optimized when the LCMP is oriented along $B_\parallel$, that is, $\alpha=0$. The equilibrium distance R_c is given by

$$R_c=\left(\frac{e^2}{4\epsilon T(\alpha=0,Q)}\right)^{1/2}\propto(1-\rho)^{-1/2},\qquad(7)$$

where $\rho=B_\parallel/B_\parallel^c$ is a magnetic field measured in units of critical value $B_\parallel^c$. It is amusing to note that the rotations of $B_\parallel$ can be used as a knob to orient the LCMP. As $B_\parallel$ increases, the string tension decreases as $1-\rho$ and the length of the LCMP increases with $(1-\rho)^{-1/2}$. At finite temperature, one needs to take into account the effect of thermal fluctuations

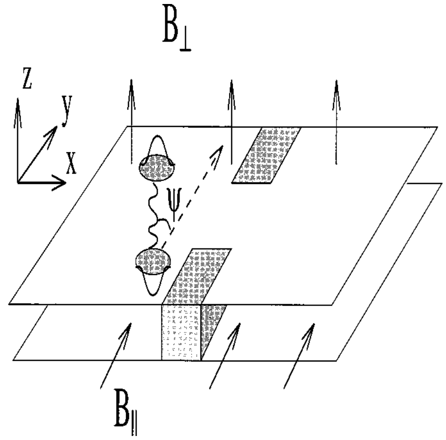

FIG. 2. Schematic diagram of a gated Hall bar: The domain-wall string soliton is a linearly confined meron pair. $B_\parallel$ is the magnetic field applied parallel to the layer and $B_\perp$ is a strong perpendicular magnetic field. ψ is the relative angle between $B_\parallel$ and the constriction. Depending on ψ and $B_\parallel$, the meron pair will either easily pass through the gated region or be blocked.

that can distort this object via stretching *or* rotation. The energy cost $\Delta E(\alpha,R,B_\parallel)$ of small fluctuations over the optimal solution of the LCMP is given by

$$\Delta E[\alpha,R,B_\parallel]\cong\frac{1}{2}\kappa_R\left(1-\frac{R}{R_c}\right)^2+\frac{1}{2}\kappa_\alpha\alpha^2, \qquad (8)$$

where the spring constants $\kappa_R=e^2/2\epsilon R_c\propto(1-\rho)^{1/2}$ and $\kappa_\alpha=\rho T_0 R_c\propto\rho/(1-\rho)^{1/2}$. In order to detect this interesting object, we propose a gated Hall bar experiment where the relative orientation of the constriction with respect to $B_\parallel$ can be varied. In Fig. 2, we have shown a quantum Hall bar that is gated in the middle by putting metallic gates in both layers. The constriction has a channel width W, which can be varied by adjusting the gate voltage. For simplicity, the channel is assumed to have a ''hard wall'' that prevents the transport of charge carriers. The $\nu_T=1$ state can be considered as a vacuum of the LCMP. The perpendicular magnetic field $B_\perp$ is applied so that the total filling factor ν_T of the system is slightly away from 1. Since the lowest charge excitations are argued to be the LCMP, the ground state of the system will have LCMP's. Since the LCMP tends to be parallel to $B_\parallel$, the transport through a constriction will have a strong dependence on the relative angle ψ between $B_\parallel$ and the constriction. The transport probability $T_{\mathrm{tr}}(B_\parallel,\psi)$ of the LCMP passing through a narrow constriction is given by

$$T_{\mathrm{tr}}(B_\parallel,\psi)\sim|\mathcal{T}|^2\int_0^{2\pi}d\alpha\int_0^{W_e/|\cos(\psi-\alpha)|}dR \times[W_e-R|\cos(\psi-\alpha)|]e^{-\beta\Delta E(\alpha,R,B_\parallel)}, \qquad (9)$$

where ψ is a relative angle between the gate and $B_\parallel$, β is inverse temperature, and $|\mathcal{T}|^2$ is a transmission coefficient. Since the channel width W should be larger than the meron core size R_{MC}, which is estimated to be about $2\ell_B$,[21,22] the effective channel width W_e is set equal to $W-2R_{\mathrm{MC}}$. We assume that $|\mathcal{T}|^2$ has *no* angle dependence.[23] As $B_\parallel$ approaches $B_\parallel^c$, κ_R vanishes as $(1-\rho)^{1/2}$ and κ_α diverges as $\rho/(1-\rho)^{1/2}$. In this limit, $T_{\mathrm{tr}}(B_\parallel,\psi)$ can be obtained analytically

$$T_{\mathrm{tr}}(B_\parallel,\psi)\sim|\mathcal{T}|^2W_e^2\frac{(1-\rho)^{1/4}}{\cos\psi}\frac{(k_BT)^{1/2}}{(T_0e^2/4\epsilon)^{1/4}}. \qquad (10)$$

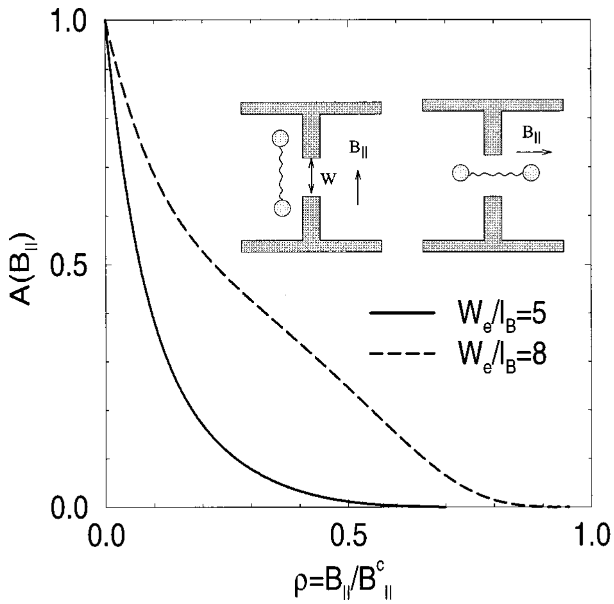

FIG. 3. Transport anisotropy of linearly confined meron pair excitations: The effective width of constriction W_e is chosen to be $5\ell_B, 8\ell_B$. $\mathcal{A}(B_\parallel)$ is plotted as a function of $\rho=B_\parallel/B_\parallel^c$ at $T\sim300$ mK.

Note the strong angle dependence of $T_{\mathrm{tr}}(B_\parallel,\psi)\propto1/\cos\psi$. At $\psi=0$, the LCMP tends to be parallel to the constriction. Since the LCMP that is larger than the narrow channel cannot easily pass through it, the transport probability rapidly decreases as $B_\parallel$ gets to $B_\parallel^c$, where the length of the LCMP becomes very large. Presumably a quantum Hall edge state transport through a constriction will persist *even* near $B_\parallel^c$.[24,25] We speculate that it is independent of ψ up to the leading order in the coupling between the bulk and edge states contributing mainly to the angle-independent background signal.[18] If we rotate the field by $\pi/2$, the LCMP's tend to be oriented perpendicular to the constriction, which will strongly enhance the transport probability. We define $\mathcal{A}(B_\parallel)$ to be the ratio of $T_{\mathrm{tr}}(B_\parallel,\psi=0)$ to $T_{\mathrm{tr}}(B_\parallel,\psi=\pi/2)$,

$$\mathcal{A}(B_\parallel)\equiv\frac{T_{\mathrm{tr}}(B_\parallel,\psi=0)}{T_{\mathrm{tr}}(B_\parallel,\psi=\pi/2)}, \qquad (11)$$

which measures the transport anisotropy. Based on the experiment by Murphy *et al.*,[12] we have chosen the following set of parameters: the Coulomb energy $e^2/\epsilon\ell_B\cong130$ K and $t\cong0.5$ K. The isospin stiffness ρ_s is estimated to be about 0.5 K and the string tension T_0 is about 1.6 K.[2,15] At $B_\parallel/B_\parallel^c=0.9$, the length of the LCMP is estimated to be about $15\ell_B\sim1400$ Å. We have chosen two values of W_e to be $5\ell_B, 8\ell_B$ and the temperature is set to be 300 mK.

Figure 3 shows $\mathcal{A}(B_\parallel)$ as a function of ρ. We notice that at $B_\parallel=0$, transport is isotropic as expected since the anisotropy is due to a finite $B_\parallel$. As $B_\parallel$ increases and approaches

$B_\parallel^c$, the anisotropy drastically increases, which we believe can be a clear signature to identify the LCMP. This anisotropy can only be seen below a certain temperature $T_{\rm KT}$. In the absence of tunneling, we expect the Kosterlitz-Thouless (KT) phase transition to occur at $T_{\rm KT}$.[26,2] Above $T_{\rm KT}$, there will be many free merons that carry a charge $\pm\frac{1}{2}e$. Finite tunneling converts the KT transition into a crossover due to the explicitly broken U(1) symmetry. Hence our picture of the linearly confined meron pairs as the lowest charged excitations holds below $T_{\rm KT}$. The transition temperature $T_{\rm KT}$ is estimated to be about $(\pi/2)\rho_s \sim 0.6$ K.[27] We notice that the temperature dependence of $\mathcal{A}(B_\parallel)$ is weak well below the transition.

To summarize, we propose that topological charges in 2D electronic systems can be probed by a gate geometry. This is especially important if the excitation is based upon an isospin that couples poorly to most experimentally controllable parameters. We have shown that for the case of linearly confined meron pairs there is a strong transport anisotropy due to its topological nature. In order to detect this fascinating object, we propose a transport experiment through a narrow constriction with a variable angle between the constriction and the parallel magnetic field $B_\parallel$. We have clearly demonstrated that the transport has a strong angular dependence as $B_\parallel$ gets closer to the critical value $B_\parallel^c$. In other cases parameters such as the size, energy, and stiffness of the topological excitation might be probed.

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[1] S.L. Sondhi, A. Karlhede, S.A. Kivelson, and E.H. Rezayi, Phys. Rev. B **47**, 16 419 (1993).

[2] K. Moon, H. Mori, Kun Yang, S.M. Girvin, A.H. MacDonald, L. Zheng, D. Yoshioka, and Shou-Cheng Zhang, Phys. Rev. B **51**, 5138 (1995).

[3] H.A. Fertig, L. Brey, R. Côté, and A.H. MacDonald, Phys. Rev. B **50**, 11 018 (1994).

[4] J.K. Jain and X.G. Wu, Phys. Rev. B **49**, 5085 (1994).

[5] S.E. Barrett, G. Dabbagh, L.N. Pfeiffer, K.W. West, and Z. Tycko, Phys. Rev. Lett. **74**, 5112 (1995).

[6] A. Schmeller, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **75**, 4290 (1995).

[7] E.H. Aifer, B.B. Goldberg, and D.A. Broido, Phys. Rev. Lett. **76**, 680 (1996).

[8] Kun Yang, K. Moon, L. Zheng, A.H. MacDonald, S.M. Girvin, D. Yoshioka, and Shou-Cheng Zhang, Phys. Rev. Lett. **72**, 732 (1994).

[9] Kun Yang, K. Moon, Lotfi Belkhir, H. Mori, S.M. Girvin, A.H. MacDonald, L. Zheng, and D. Yoshioka, Phys. Rev. B **54**, 11 644 (1996).

[10] S.M. Girvin and A.H. MacDonald, in *Novel Quantum Liquids in Low-Dimensional Semiconductor Structures*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1996).

[11] Z.F. Ezawa and A. Iwazaki, Int. J. Mod. Phys. B **19**, 3205 (1992).

[12] S.Q. Murphy, J.P. Eisenstein, G.S. Boebinger, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **72**, 728 (1994).

[13] T.S. Lay, Y.W. Suen, H.C. Manoharan, X. Ying, M.B. Santos, and M. Shayegan, Phys. Rev. B **50**, 17 725 (1994).

[14] Tin-Lun Ho, Phys. Rev. Lett. **73**, 874 (1994).

[15] Kyungsun Moon, Phys. Rev. Lett. **78**, 3741 (1997).

[16] N. Read, Phys. Rev. B **52**, 1926 (1995).

[17] M. Rasolt, B.I. Halperin, and D. Vanderbilt, Phys. Rev. Lett. **57**, 126 (1986).

[18] K. Moon and K. Mullen (unpublished).

[19] Per Bak, Rep. Prog. Phys. **45**, 587 (1982); Marcel den Nijs in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J.L. Lebowitz (Academic Press, New York, 1988), Vol. 12, pp. 219–333.

[20] *Aspects of Symmetry* (Cambridge University Press, New York, 1985).

[21] K. Yang and A.H. MacDonald, Phys. Rev. B **51**, 17 247 (1995).

[22] L. Brey, H.A. Fertig, R. Côté, and A.H. MacDonald, Phys. Rev. B **54**, 16 888 (1996).

[23] The effect of intrinsic impurity might give a natural angle dependence. However, we believe that it is negligible compared to the transport anisotropy due to the constriction. J.P. Eisenstein (private communication).

[24] K. Moon, H. Yi, C.L. Kane, S.M. Girvin, and M.P.A. Fisher, Phys. Rev. Lett. **71**, 4381 (1993); K. Moon and S.M. Girvin, Phys. Rev. B **54**, 4448 (1996).

[25] F.P. Milliken, C.P. Umbach, and R.A. Webb, Solid State Commun. **97**, 309 (1996); A.M. Chang, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **77**, 2538 (1996).

[26] X.G. Wen and A. Zee, Phys. Rev. Lett. **69**, 1811 (1992).

[27] The observed collapse of the activation gap at around 0.5 K in spite of much larger activation energy on the order of 10 K indicates that the charged excitation indeed has a many-body origin 12.