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Scattering by time-periodic potentials in one dimension and its influence on electronic transport

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We investigate electron scattering by time-periodic potentials of finite range in one dimension. A general scattering potential is approximated by a sequence of rectangular barriers, and can be handled within a multiple-scattering approach. The scattering is described in terms of a scattering-amplitude operator which takes into account propagating as well as evanescent states. The general properties of this operator are discussed. Our approach permits us to study the transport in time-periodic mesoscopic systems. In particular, we investigate quantum-coherent pumping of electrons. [S0163-1829(98)11019-6]

I. INTRODUCTION

Quantum interference effects in mesoscopic systems have been extensively studied in the dc regime, where the scattering of electrons is due to static perturbations.[1] Whereas the description of scattering by time-independent potentials is straightforward, the calculation becomes much more difficult for time-dependent potentials, where inelastic contributions have to be considered.

Scattering by time-dependent potentials was reexamined recently in Ref. 2. In the present work, we discuss the case of time-periodic potentials which has been investigated in Refs. 3–8. Photon-assisted tunneling through quantum dots has been observed experimentally,[9,10] and has also been discussed theoretically including[11] or neglecting the potential variations due to Coulomb blocking.[12,13]

Time periodicity implies that incoming waves with energy E are inelastically scattered into sidebands with energies $E+n\omega, n\in\mathbb{Z}, \omega=2\pi/P$, P being the period of the potential. In order to calculate the respective scattering amplitudes, we adopt the procedure proposed in Refs. 4,6, and 7, i.e., we approximate the scattering potential by a sequence of time-periodic rectangular barriers. We first solve the problem for an arbitrary rectangular barrier. The scattering matrix for the overall potential is then obtained using a multiple-scattering approach which is based on the calculated generalized scattering matrices comprising propagating as well as evanescent waves. In this way we avoid the rather awkward law of error propagation inherent to the transfer-matrix technique (see, e.g., Ref. 14) employed in Refs. 4 and 6.

Our approach is used to describe the electron transport through time-periodic mesoscopic systems. We consider a sample connected to two electron reservoirs with the same chemical potential. In contrast to the case of static barriers, the scattering is generally not left-right symmetric when dynamic barriers are present. This left-right asymmetry of the scattering probabilities induces a net current between the reservoirs, i.e., the system acts as an electron pump, the pumping efficiency depending strongly on the chemical potential.

II. THEORETICAL APPROACH

A. Potential

We consider electron scattering by a short-range potential $V(x,t)$ in one dimension, and periodic in time with period P. In this case, a particle moving freely for $t\rightarrow-\infty$ outside the potential region is scattered by the potential, and moves again freely after scattering for $t\rightarrow+\infty$, i.e., the asymptotic condition is satisfied.[15] It is worthwile to note that we use an effective one-electron approach, and therefore the time-periodic potential corresponds to the screened potential and should not be confounded with the external potential.

Let $S=(x_0,x_N]\subset\mathbb{R}$ be the support of $V(x,t)$, and $L=(-\infty,x_0]\subset\mathbb{R}$ and $R=(x_N,\infty)\subset\mathbb{R}$ the free-evolution regions (Fig. 1). The final state for an incoming electron with energy E is described by a superposition of all free scattering states with energies $E_n=E+n\omega$, $n\in\mathbb{Z}$, $\omega=2\pi/P$ being the angular frequency of the potential.[2] In one dimension, the scattering states are plane waves. For an incoming wave at energy E, the corresponding reflection and transmission amplitudes $R_{n0}(E)$ and $T_{n0}(E)$ can be calculated from the matching conditions at the junctions x_0 and x_N for the wave function $\psi(x,t)$ and its derivative $\partial_x\psi(x,t)$. Negative energies $E_n<0$ correspond to evanescent wave functions which describe the situation near the scattering potential.

Following Refs. 4, 6, and 7, we represent $V(x,t)$ by a sequence of time-periodic rectangular potentials on finite intervals, as shown in Fig. 1,

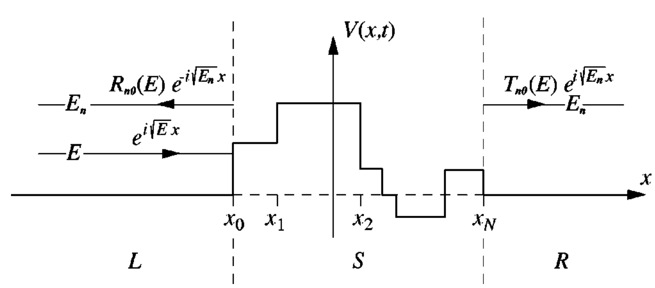

FIG. 1. Scattering by a time-periodic potential.

$$V(x,t)=\sum_{n=1}^{N} V^{(n)}(t)C^{(n)}(x), \tag{1}$$

with

$$C^{(n)}(x)=\begin{cases}1 & \text{if } x\in(x_{n-1},x_n],\ x_{n-1}<x_n,\ n=1,\dots,N\\ 0 & \text{otherwise.}\end{cases} \tag{2}$$

B. Time-periodic rectangular barrier

For a single rectangular barrier with periodic time dependence, the Schrödinger equation is

$$i\partial_t\psi(x,t)=-\partial_x^2\psi(x,t)+V(x,t)\psi(x,t), \tag{3}$$

where we have set $\hbar=1$ and $m=\frac{1}{2}$. We use Floquet's theorem to write the solution as

$$\psi(x,t)=e^{-iEt}\varphi(x,t)\quad \text{with}\ i\partial_t\varphi=-\partial_x^2\varphi+[V(x,t)-E]\varphi \tag{4}$$

and $\varphi(x,t)=\varphi(x,t+P)$. At this point, E is an arbitrary real number.

The potential is given by

$$V(x,t)=V(x,t+P)=\begin{cases}V_0+V_1(t) & \text{if } x\in S=(x_0,x_0+\Delta x]\\ 0 & \text{otherwise,}\end{cases} \tag{5}$$

with $V_0=(1/P)\int_0^P V(x,t)\,dt$, $x\in S$, and $\int_0^P V_1(t)\,dt=0$. Thus for $x\in S$, Equation (4) becomes

$$i\partial_t\varphi(x,t)=-\partial_x^2\varphi(x,t)+[V_0+V_1(t)-E]\varphi(x,t). \tag{6}$$

Equation (6) is separable, so we may use

$$\varphi(x,t)=f(t)g(x) \tag{7}$$

with $f(0)=1$. This leads to the equations

$$-\partial_x^2 g(x)+V_0 g(x)=\eta g(x), \tag{8}$$

$$i\partial_t f(t)-V_1(t)f(t)=(\eta-E)f(t), \tag{9}$$

with the solutions

$$g(x)=e^{\pm ik_S x},\quad k_S^2=\eta-V_0, \tag{10}$$

$$f(t)=e^{-i(\eta-E)t}\exp\left(-i\int_0^t V_1(t')\,dt'\right). \tag{11}$$

$V_1(t)$ and $f(t)$ are periodic in time with the same period P. This implies $\eta-E=m\omega$, $m\in\mathbb{Z}$. The solution to Eq. (6) is thus

$$\varphi_m(x,t)=e^{\pm ik_S(m)x}e^{-im\omega t}\exp\left(-i\int_0^t V_1(t')\,dt'\right), \tag{12}$$

with $k_S(m)^2=E_m-V_0$. Inserting the Fourier expansion

$$\exp\left(-i\int_0^t V_1(t')\,dt'\right)=\sum_{\nu=-\infty}^{\infty}F_\nu e^{-i\nu\omega t}, \tag{13}$$

we obtain

$$\varphi_m(x,t)=e^{\pm ik_S(m)x}\sum_n F_{n-m}e^{-in\omega t},\quad m\in\mathbb{Z},\ x\in S. \tag{14}$$

For each E, we define a set of modes

$$M_E=\{E_n|E_n=E+n\omega\}_{n\in\mathbb{Z}}, \tag{15}$$

with $E_0=E$. The solution in the region S is a superposition of all modes in the set M_E. In the regions L and R, the potential $V(x,t)$ is zero, and the solutions to Eq. (6) are given by the plane waves

$$\varphi_n(x,t)=e^{\pm ik(n)x}e^{-in\omega t},\quad x\in L\cup R,\ n\in\mathbb{Z}, \tag{16}$$

where

$$k^2(n)=E_n. \tag{17}$$

We consider an incoming wave from the left with energy $E=E_0=k^2(0)$. Introducing the reflection and transmission amplitudes $R_{n0}(E)$ and $T_{n0}(E)$ in mode E_n, the solution $\varphi(x,t)$ to Eq. (4) can be written as

$$\varphi(x,t)=\begin{cases}e^{ik(0)x}+\sum_n R_{n0}e^{-ik(n)x}e^{-in\omega t}, & x\in L\\ \sum_{m,n}(\alpha_{m0}e^{ik_S(m)x}+\beta_{m0}e^{-ik_S(m)x})F_{n-m}e^{-in\omega t}, & x\in S\\ \sum_n T_{n0}e^{ik(n)x}e^{-in\omega t}, & x\in R,\end{cases} \tag{18}$$

with the convention $k(n),k_S(n)\in\mathbb{R}_+\cup i\mathbb{R}_+$, $n\in\mathbb{Z}$. The reflection and transmission amplitudes R_{n0} and T_{n0} are defined by the matching conditions for the wave function $\psi(x,t)$ and its derivative $\partial_x\psi(x,t)$ at x_0 and $x_1=x_0+\Delta x$. Because of the unicity of the Fourier expansion, each mode can be treated separately. We obtain

$$e^{ik(n)x_0}\delta_{n0}+R_{n0}e^{-ik(n)x_0}=\sum_m(\alpha_{m0}e^{ik_S(m)x_0}+\beta_{m0}e^{-ik_S(m)x_0})F_{n-m}, \tag{19}$$

$$k(n)(e^{ik(n)x_0}\delta_{n0}-R_{n0}e^{-ik(n)x_0})$$

$$=\sum_m k_S(m)(\alpha_{m0}e^{ik_S(m)x_0}-\beta_{m0}e^{-ik_S(m)x_0})F_{n-m}, \quad (20)$$

$$\sum_m (\alpha_{m0}e^{ik_S(m)(x_0+\Delta x)}+\beta_{m0}e^{-ik_S(m)(x_0+\Delta x)})F_{n-m} = T_{n0}e^{ik(n)(x_0+\Delta x)}, \quad (21)$$

$$\sum_m k_S(m)(\alpha_{m0}e^{ik_S(m)(x_0+\Delta x)}-\beta_{m0}e^{-ik_S(m)(x_0+\Delta x)})F_{n-m} = k(n)T_{n0}e^{ik(n)(x_0+\Delta x)} \quad (22)$$

for all $n\in\mathbb{Z}$. For an incoming wave with energy $E_{n'}=E+n'\omega$, we just have to replace the index 0 by n'. Defining the linear operators R, T, α, and β by their respective matrix elements $R_{nn'}$, $T_{nn'}$, $\alpha_{mn'}$, and $\beta_{mn'}$, and introducing the operators

$$K_{mn}=k(n)\,\delta_{mn}, \quad (23)$$

$$(K_S)_{mn}=k_S(n)\,\delta_{mn}, \quad (24)$$

$$(L_L)_{mn}=e^{ik(n)x_0}\delta_{mn}, \quad (25)$$

$$(L_R)_{mn}=e^{ik(n)(x_0+\Delta x)}\delta_{mn}, \quad (26)$$

$$(L_S)_{mn}=e^{ik_S(n)x_0}\delta_{mn}, \quad (27)$$

$$(L_\Delta)_{mn}=e^{ik_S(n)\Delta x}\delta_{mn}, \quad (28)$$

$$F_{mn}=F_{m-n}, \quad (29)$$

Eqs. (19)–(22) can be written as

$$L_L+L_L^{-1}R=F(L_S\alpha+L_S^{-1}\beta), \quad (30)$$

$$K(L_L-L_L^{-1}R)=FK_S(L_S\alpha-L_S^{-1}\beta), \quad (31)$$

$$F(L_SL_\Delta\alpha+L_S^{-1}L_\Delta^{-1}\beta)=L_RT, \quad (32)$$

$$FK_S(L_SL_\Delta\alpha-L_S^{-1}L_\Delta^{-1}\beta)=KL_RT. \quad (33)$$

Using the identity (see Appendix A)

$$F^\dagger F=\mathbb{I}, \quad (34)$$

where $F^\dagger$ is the Hermitian conjugate of F, we multiply Eqs. (30)–(33) on the left by $F^\dagger$ and on the right by L_L^{-1}, and eliminate the operators α and β to obtain two independent equations

$$[(L_\Delta-\mathbb{I})K_SF^\dagger-(L_\Delta+\mathbb{I})F^\dagger K](\tilde{T}+\tilde{R}) = (\mathbb{I}-L_\Delta)K_SF^\dagger-(\mathbb{I}+L_\Delta)F^\dagger K, \quad (35)$$

$$[(L_\Delta+\mathbb{I})K_SF^\dagger-(L_\Delta-\mathbb{I})F^\dagger K](\tilde{T}-\tilde{R}) = (\mathbb{I}+L_\Delta)K_SF^\dagger-(\mathbb{I}-L_\Delta)F^\dagger K, \quad (36)$$

with

$$R_\rightarrow=L_L\tilde{R}L_L, \quad (37)$$

$$T_\rightarrow=L_R^{-1}\tilde{T}L_L. \quad (38)$$

The arrow indicates incoming waves from the left.

For incoming waves from the right, we obtain the same Eqs. (35) and (36), and the corresponding operators $R_\leftarrow$ and $T_\leftarrow$ are

$$R_\leftarrow=L_R^{-1}\tilde{R}L_R^{-1}, \quad (39)$$

$$T_\leftarrow=L_L\tilde{T}L_R^{-1}. \quad (40)$$

$\tilde{R}$ and $\tilde{T}$ are independent of the position x_0 of the barrier, and are associated with a specific set of modes M_E.

C. Multibarrier scattering

For a rectangular barrier, the position-independent operators $\tilde{R}$ and $\tilde{T}$ are identical for incoming waves from the left or from the right. This left-right symmetry is no longer expected for the potential given by Eq. (1). We thus describe the scattering by the scattering-amplitude operator

$$S=\begin{pmatrix}\tilde{R}_\rightarrow & \tilde{T}_\leftarrow\\ \tilde{T}_\rightarrow & \tilde{R}_\leftarrow\end{pmatrix}. \quad (41)$$

The reflection and transmission amplitudes are given by

$$R_\rightarrow=L_L\tilde{R}_\rightarrow L_L, \quad (42)$$

$$T_\rightarrow=L_R^{-1}\tilde{T}_\rightarrow L_L, \quad (43)$$

$$R_\leftarrow=L_R^{-1}\tilde{R}_\leftarrow L_R^{-1}, \quad (44)$$

$$T_\leftarrow=L_L\tilde{T}_\leftarrow L_R^{-1}, \quad (45)$$

with

$$(L_L)_{mn}=e^{ik(n)x_0}\delta_{mn}, \quad (46)$$

$$(L_R)_{mn}=e^{ik(n)x_N}\delta_{mn}. \quad (47)$$

Let us consider two potentials $V^{\mathrm{I}}(x,t)$ and $V^{\mathrm{II}}(x,t)$ with the same time-periodicity and separated in space by a distance d. Suppose that the operators

$$S^{\mathrm{I}}=\begin{pmatrix}\tilde{R}^{\mathrm{I}}_\rightarrow & \tilde{T}^{\mathrm{I}}_\leftarrow\\ \tilde{T}^{\mathrm{I}}_\rightarrow & \tilde{R}^{\mathrm{I}}_\leftarrow\end{pmatrix}, \quad S^{\mathrm{II}}=\begin{pmatrix}\tilde{R}^{\mathrm{II}}_\rightarrow & \tilde{T}^{\mathrm{II}}_\leftarrow\\ \tilde{T}^{\mathrm{II}}_\rightarrow & \tilde{R}^{\mathrm{II}}_\leftarrow\end{pmatrix} \quad (48)$$

are known. Instead of using the transfer-matrix technique[4,6] to calculate the overall scattering-amplitude operator S, we connect modules I and II and sum directly the contributions due to multiple scattering. We obtain

$$\tilde{R}_\rightarrow=\tilde{R}^{\mathrm{I}}_\rightarrow+\tilde{T}^{\mathrm{I}}_\leftarrow(\mathbb{I}-L_d\tilde{R}^{\mathrm{II}}_\rightarrow L_d\tilde{R}^{\mathrm{I}}_\leftarrow)^{-1}L_d\tilde{R}^{\mathrm{II}}_\rightarrow L_d\tilde{T}^{\mathrm{I}}_\rightarrow, \quad (49)$$

$$\tilde{T}_\rightarrow=\tilde{T}^{\mathrm{II}}_\rightarrow(\mathbb{I}-L_d\tilde{R}^{\mathrm{I}}_\leftarrow L_d\tilde{R}^{\mathrm{II}}_\rightarrow)^{-1}L_d\tilde{T}^{\mathrm{I}}_\rightarrow, \quad (50)$$

$$\tilde{R}_\leftarrow=\tilde{R}^{\mathrm{II}}_\leftarrow+\tilde{T}^{\mathrm{II}}_\rightarrow(\mathbb{I}-L_d\tilde{R}^{\mathrm{I}}_\leftarrow L_d\tilde{R}^{\mathrm{II}}_\rightarrow)^{-1}L_d\tilde{R}^{\mathrm{I}}_\leftarrow L_d\tilde{T}^{\mathrm{II}}_\leftarrow, \quad (51)$$

$$\tilde{T}_{\leftarrow}=\tilde{T}^{\mathrm{I}}_{\leftarrow}(\mathbb{I}-L_d\tilde{R}^{\mathrm{II}}_{\rightarrow}L_d\tilde{R}^{I}_{\leftarrow})^{-1}L_d\tilde{T}^{\mathrm{II}}_{\leftarrow}, \tag{52}$$

with

$$(L_d)_{mn}=e^{ik(n)d}\delta_{mn}. \tag{53}$$

Iterating this procedure, we obtain the scattering-amplitude operator S for an arbitrary sequence of rectangular barriers with the same time periodicity [Eq. (1)]. Equations (49)–(52) generalize the corresponding relations for the case of static barriers derived in Ref. 14.

For an incoming wave from the left at energy E, the mean current per period P through a barrier is (see Appendix B 1)

$$j_{\rightarrow}(E)=\sum_{E_n>0} j_{\rightarrow,n}(E), \tag{54}$$

where

$$j_{\rightarrow,n}(E)=2\sqrt{E_n}|\tilde{T}_{\rightarrow,n0}(E)|^2 \tag{55}$$

is the contribution of mode $E_n>0$. The corresponding equations for an incoming wave from the right are

$$j_{\leftarrow}(E)=\sum_{E_n>0} j_{\leftarrow,n}(E), \tag{56}$$

$$j_{\leftarrow,n}(E)=2\sqrt{E_n}|\tilde{T}_{\leftarrow,n0}(E)|^2. \tag{57}$$

D. Properties of the scattering-amplitude operator S

Shifting the reference energy E by $\lambda\omega$, $\lambda\in\mathbb{Z}$, we have, obviously

$$\tilde{R}_{mn}(E)=\tilde{R}_{m-\lambda,n-\lambda}(E+\lambda\omega), \tag{58}$$

$$\tilde{T}_{mn}(E)=\tilde{T}_{m-\lambda,n-\lambda}(E+\lambda\omega). \tag{59}$$

The scattering-amplitude operator S satisfies (see Appendix B 1)

$$S^\dagger\begin{pmatrix} K+K^\dagger & 0\\ 0 & K+K^\dagger\end{pmatrix}S =\begin{pmatrix} K+K^\dagger & 0\\ 0 & K+K^\dagger\end{pmatrix}+S^\dagger\begin{pmatrix} K-K^\dagger & 0\\ 0 & K-K^\dagger\end{pmatrix} -\begin{pmatrix} K-K^\dagger & 0\\ 0 & K-K^\dagger\end{pmatrix}S, \tag{60}$$

where we use multiplication by blocks. Equation (60) generalizes the unitarity relation for the standard scattering matrix. The last two terms contribute only to the properties of evanescent modes. Considering only propagating modes, from Eq. (60) we obtain

$$S_p^\dagger\begin{pmatrix} K_p & 0\\ 0 & K_p\end{pmatrix}S_p=\begin{pmatrix} K_p & 0\\ 0 & K_p\end{pmatrix}, \tag{61}$$

where the index p stays for the restriction of the respective operator to propagating modes. This relation was already obtained in Ref. 4. With the standard scattering matrix

$$\sigma=\begin{pmatrix} \rho_{\rightarrow} & \tau_{\leftarrow}\\ \tau_{\rightarrow} & \rho_{\leftarrow}\end{pmatrix}=\begin{pmatrix} \sqrt{K_p} & 0\\ 0 & \sqrt{K_p}\end{pmatrix}S_p\begin{pmatrix} \sqrt{K_p} & 0\\ 0 & \sqrt{K_p}\end{pmatrix}^{-1}, \tag{62}$$

we obtain the unitarity relation

$$\sigma^\dagger\sigma=\sigma\sigma^\dagger=\mathbb{I}. \tag{63}$$

In particular, the element nn in the upper left block of Eq. (63) is

$$(\rho^\dagger_{\rightarrow}\rho_{\rightarrow})_{nn}+(\tau^\dagger_{\rightarrow}\tau_{\rightarrow})_{nn}=1, \tag{64}$$

or

$$\sum_{E_m>0} 2\sqrt{E_m}[|\tilde{R}_{\rightarrow,mn}(E)|^2+|\tilde{T}_{\rightarrow,mn}(E)|^2]=2\sqrt{E_n}, \tag{65}$$

which expresses the conservation of the mean current for waves incoming from the left. We note that Eq. (65) implies

$$j_{\rightarrow}(E_n)\leqslant 2\sqrt{E_n}, \tag{66}$$

i.e., the mean current is reduced in presence of barriers.

From Eq. (63), it also follows that

$$\tau^\dagger_{\rightarrow}\tau_{\rightarrow}-\tau_{\leftarrow}\tau^\dagger_{\leftarrow}=\rho_{\rightarrow}\rho^\dagger_{\rightarrow}-\rho^\dagger_{\rightarrow}\rho_{\rightarrow} \tag{67}$$

and, since $\mathrm{Tr}\,(\rho^\dagger_{\rightarrow}\rho_{\rightarrow})<\infty$ (see Appendix B 2), we have

$$\mathrm{Tr}\,(\tau^\dagger_{\rightarrow}\tau_{\rightarrow}-\tau^\dagger_{\leftarrow}\tau_{\leftarrow})=\mathrm{Tr}\,(\rho_{\rightarrow}\rho^\dagger_{\rightarrow}-\rho^\dagger_{\rightarrow}\rho_{\rightarrow})=0. \tag{68}$$

For potentials with time-inversion symmetry $V(x,t)=V(x,-t)$, the scattering-amplitude operator S can be shown to satisfy

$$\begin{pmatrix} K & 0\\ 0 & K\end{pmatrix}S=S^T\begin{pmatrix} K & 0\\ 0 & K\end{pmatrix}, \tag{69}$$

where S^T is the transpose of S as indicated in Appendix B 3. In particular, the element mn in the upper left block of Eq. (69) gives, for $E_m, E_n>0$,

$$\frac{\sqrt{E_m}}{\sqrt{E_n}}|\tilde{R}_{\rightarrow,mn}(E)|^2=\frac{\sqrt{E_n}}{\sqrt{E_m}}|\tilde{R}_{\rightarrow,nm}(E)|^2, \tag{70}$$

which expresses the detailed equilibrium in the reflection probabilities, i.e., the reflected current in mode E_m for an incoming unit current in mode E_n is equal to the reflected current in mode E_n for an incoming unit current in mode E_m. Similarly, the element mn in the lower left block of Eq. (69) gives, for $E_m, E_n>0$,

$$\frac{\sqrt{E_m}}{\sqrt{E_n}}|\tilde{T}_{\rightarrow,mn}(E)|^2=\frac{\sqrt{E_n}}{\sqrt{E_m}}|\tilde{T}_{\leftarrow,nm}(E)|^2 \tag{71}$$

i.e., the transmitted current in mode E_m for a unit current in mode E_n incoming from the left is equal to the transmitted current in mode E_n for a unit current in mode E_m incoming from the right.

III. RESULTS

A. Single time-periodic rectangular barrier

We consider a single rectangular barrier $V(x,t)=V_0+V_1\cos(\omega t+\varphi)$, $x\in S$. For this potential, Eq. (13) yields

$$\exp\left(-i\int_0^t V_1\cos(\omega t'+\varphi)\,dt'\right)=e^{i(V_1/\omega)\sin\varphi}\sum_{m=-\infty}^{\infty}e^{-im\varphi}J_m\left(\frac{V_1}{\omega}\right)e^{-im\omega t}, \tag{72}$$

where $J_m(z)$ is the Bessel function of the first kind of order m. The global phase factor $e^{i(V_1/\omega)\sin\varphi}$ can be dropped, so

$$F_m=e^{-im\varphi}J_m\left(\frac{V_1}{\omega}\right), \tag{73}$$

which gives the operator F defined in Eq. (29). With the definitions

$$J_{mn}=J_{m-n}\left(\frac{V_1}{\omega}\right), \tag{74}$$

$$\Phi_{mn}=e^{im\varphi}\delta_{mn}, \tag{75}$$

we obtain the factorization

$$F=\Phi^\dagger J\Phi. \tag{76}$$

The ratio V_1/ω determines the number of coupled modes. For $V_1/\omega=0$, we have $F=\mathbb{I}$, which corresponds to a static barrier.

The mean current does not depend on the direction of the incoming wave with energy E, since $\tilde{T}_{\rightarrow}=\tilde{T}_{\leftarrow}$ for a single rectangular barrier, i.e., we have

$$j_{\rightarrow,n}(E)=j_{\leftarrow,n}(E)=j_n(E) \tag{77}$$

and

$$j(E)=\sum_{E_n>0}j_n(E). \tag{78}$$

The energy dependence of the current through a static rectangular barrier $V_0=10$ of width $\Delta x=2.5$ is presented in Fig. 2. In this case the scattering is elastic, i.e., the modes are not coupled and only the term $n=0$ in Eq. (78) contributes to the current. The barrier is nearly opaque for $E<V_0$. The Fabry-Pérot oscillations found for $E>V_0$ disappear at large energies where the barrier becomes transparent.

For an oscillating rectangular barrier the electrons are inelastically scattered into the modes E_n, and each mode $E_n>0$ contributes to the current. This implies

$$n>-\frac{E}{\omega}. \tag{79}$$

The contributions $j_n(E)$, together with the mean current $j(E)$, are shown in Fig. 3 for $\omega=3, V_1/\omega=1, V_0=0$, and $\Delta x=2.5$. The small structures in $j(E)$ found for energies E close to $\lambda\omega$, $\lambda\in\mathbb{N}$, correspond to the change of the number of contributing modes described by Eq. (79). Regarding the mean current, the barrier is nearly transparent. It just distributes the mean current over the modes E_n. As shown in Appendix B 2, the asymptotic behavior of $j_{\pm1}(E)$ for $E\to\infty$ is given by

$$j_{\pm1}(E)\simeq\frac{(V_1\Delta x)^2}{8}\frac{1}{\sqrt{E}}. \tag{80}$$

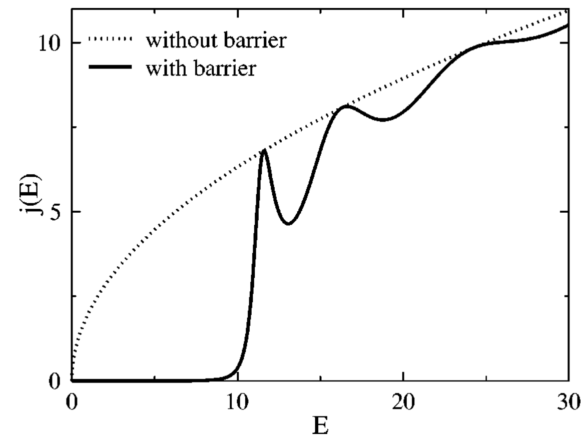

FIG. 2. Current through a static barrier as a function of the energy of the incoming electrons. The current in the absence of the barrier is given for reference (dotted line).

B. Composed barrier

For arbitrary static barriers in one dimension, the right side of Eq. (67) is zero, and the left-to-right and right-to-left mean currents, Eqs. (54) and (56), are equal (see also Ref. 16). This is no longer true for dynamic barriers. As an example, we take a composed barrier consisting of the two rectangular barriers considered in Sec. III A, the left being dynamic (I; parameters: $\omega=3, V_1/\omega=1$, $V_0=0$, and $\Delta x=2.5$) and the right being static (II; parameters: $V_0=10$ and $\Delta x=2.5$). The distance between the barriers is $d=0$. Using Eqs. (49)–(52), we calculate the scattering-amplitude operator S. Obviously, the mean current does not depend on the phase φ. The results are presented in Fig. 4. Corresponding

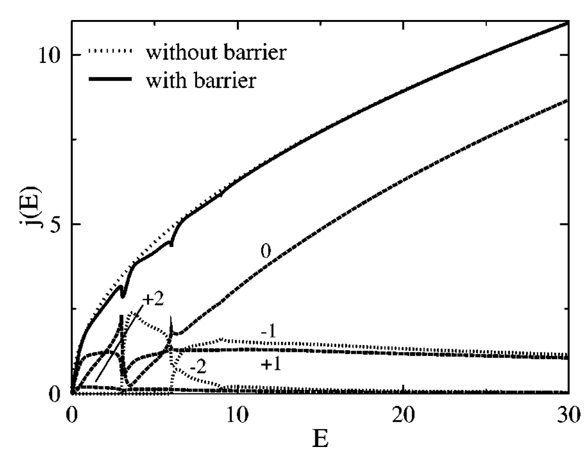

FIG. 3. Mean current through an oscillating barrier and the contributions of different modes E_n as a function of the energy of the incoming electrons. The current in the absence of the barrier is given for reference (dotted line). The labels n denote the modes E_n.

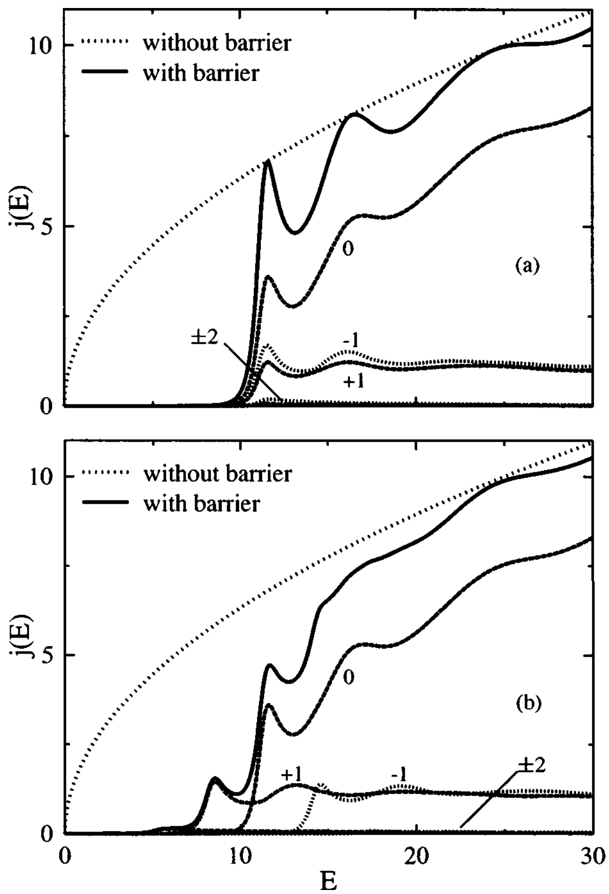

FIG. 4. Mean current through the composed barrier and the contributions of different modes E_n as a function of the energy of the electrons incoming (a) from the right and (b) from the left. The current in the absence of the barrier is given for reference (dotted line). The labels n denote the modes E_n.

to the small coupling parameter $V_1/\omega=1$, the mean current is essentially given by the contributions of modes $E_0=E$ and $E_{\pm 1}$.

Electrons incoming from the right with energy E have to pass first the static barrier II. This case is presented in Fig. 4(a). For $E<V_0^{\mathrm{II}}$, the electrons are reflected (see Fig. 2). For $E>V_0^{\mathrm{II}}$, practically all electrons passing the static barrier contribute to the mean current, since the oscillating barrier I is nearly transparent (see Fig. 3). Under these conditions, we may write

$$j_{\leftarrow,n}(E)\simeq 2\sqrt{E_n}|\tilde{T}^{\mathrm{I}}_{n0}(E)|^2|\tilde{T}^{\mathrm{II}}_{00}(E)|^2. \tag{81}$$

In our case, the electrons are just distributed over the modes $E_0=E$, $E_{\pm 1}$, and $E_{\pm 2}$ by the oscillating barrier.

Electrons incoming from the left are first scattered into modes E_n; only modes $E_n=E+n\omega>V_0^{\mathrm{II}}$, i.e., $n>(V_0^{\mathrm{II}}-E)/\omega$, contribute to the mean current, as can be seen in Fig. 4(b), with

$$j_{\rightarrow,n}(E)\simeq 2\sqrt{E_n}|\tilde{T}^{\mathrm{I}}_{n0}(E)|^2|\tilde{T}^{\mathrm{II}}_{nn}(E)|^2. \tag{82}$$

In particular for $V_0^{\mathrm{II}}-\omega<E<V_0^{\mathrm{II}}$, electrons scattered into mode E_{+1} may now pass the static barrier.

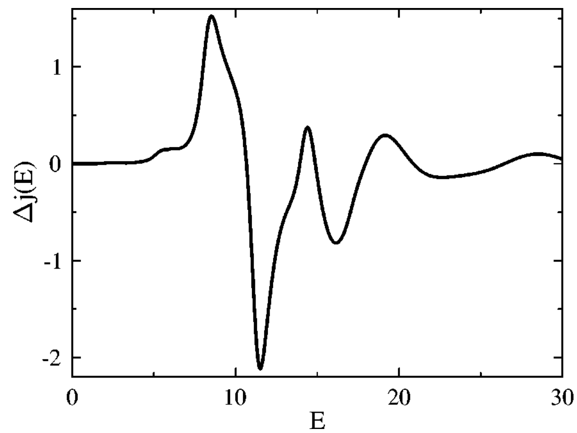

FIG. 5. Net mean current through the composed barrier as a function of the energy of the incoming electrons.

The left-right asymmetry of the scattering by the composed barrier is visualized in Fig. 5 where we show the net mean current

$$\Delta j(E)=j_{\rightarrow}(E)-j_{\leftarrow}(E). \tag{83}$$

The asymmetry is most important for $E\simeq V_0^{\mathrm{II}}$, and disappears with increasing E.

C. An electron pump

We consider a system of time-periodic barriers connected through wave guides to two electron reservoirs with the chemical potentials μ_1 and μ_2 with $\mu_1>\mu_2$ (Fig. 6). Assuming reflection-free ideal contacts, the total mean current through the system is given by

$$I(\mu_1,\mu_2)=\int_0^\infty dE\ g(E)[j_{\rightarrow}(E)f(E,\mu_1)-j_{\leftarrow}(E)f(E,\mu_2)], \tag{84}$$

where

$$g(E)=\frac{1}{2\pi\sqrt{E}} \tag{85}$$

is the density of electrons contributing to the current in one direction, and $f(E,\mu)$ is the Fermi-Dirac distribution. At zero temperature, the total mean current becomes

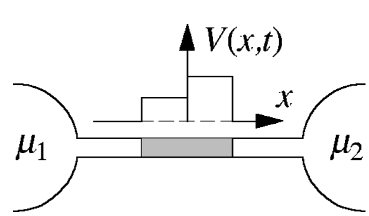

FIG. 6. A mesoscopic system connected to two electron reservoirs.

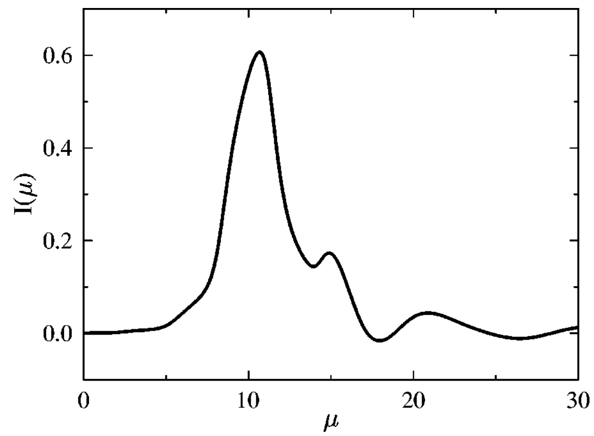

FIG. 7. Total mean current through the composed barrier as a function of the chemical potential.

$$I(\mu_1,\mu_2)=\int_0^{\mu_2}dE\,\frac{1}{\pi\sqrt{E}}\sum_{E_n>0}\sqrt{E_n}[|\tilde{T}_{\rightarrow,n0}(E)|^2-|\tilde{T}_{\leftarrow,n0}(E)|^2]+\int_{\mu_2}^{\mu_1}dE\,\frac{1}{\pi\sqrt{E}}\sum_{E_n>0}\sqrt{E_n}|\tilde{T}_{\rightarrow,n0}(E)|^2. \tag{86}$$

In the following we assume that the chemical potentials are equal, i.e., $\mu_1=\mu_2=\mu$. Then the second term in Eq. (86) is zero. In the case of an arbitrary static barrier, we have

$$|\tilde{T}_{\rightarrow,n0}(E)|^2=|\tilde{T}_{\leftarrow,n0}(E)|^2,\quad n\in\mathbb{Z}, \tag{87}$$

and therefore $I(\mu)=I(\mu,\mu)=0$. This is no longer true for dynamic barriers, as is demonstrated in Fig. 7 for our composed barrier discussed in Sec. III B. This system acts as an electron pump. The pumping efficiency is largest for chemical potentials μ close to the height V_0^{II} of the static barrier.

The total mean current $I(\mu)$ vanishes for large chemical potential since

$$\lim_{\mu\to\infty}I(\mu)=\lim_{\lambda\to\infty}\int_0^{\omega}dE\sum_{n=0}^{\lambda-1}\frac{1}{2\pi\sqrt{E_n}}[j_{\rightarrow}(E_n)-j_{\leftarrow}(E_n)]=\frac{1}{\pi}\int_0^{\omega}dE\ \mathrm{Tr}\,[\tau_{\rightarrow}^{\dagger}(E)\tau_{\rightarrow}(E)-\tau_{\leftarrow}^{\dagger}(E)\tau_{\leftarrow}(E)]=0 \tag{88}$$

according to Eq. (68).

IV. CONCLUSION

We have presented an efficient method to describe the scattering by time-periodic potentials. The solution is expressed in terms of a scattering-amplitude operator S. The general properties of S have been given. We emphasize that this operator treats propagating as well as evanescent waves, in contrast to the usual scattering matrix, which describes only the propagating solutions. The inclusion of the evanescent waves allows us to obtain the scattering solution for a sequence of barriers in terms of the scattering-amplitude operators for each barrier in the sequence. We have shown that, in contrast to the case of static barriers, generally no left-right symmetry of the scattering properties can be expected when dynamic barriers are present.

Using these results, we have described the coherent electron transport in a dynamic mesoscopic system connected to two electron reservoirs with the same chemical potential. The left-right asymmetry leads to a net current between the reservoirs. The pumping efficiency depends strongly on the relative position of the chemical potential and the threshold energy of the static barriers, and vanishes in the limit of large chemical potentials.

We emphasize that our results are obtained in the coherent limit. This should be distinguished from the situation discussed in Refs. 9–13, where coupling between quantum dots is rather small and relaxation in the dots becomes important. We note also that the pumping effect discussed here is different from the one in Ref. 13, where the pumping is caused by the Pauli exclusion principle.

ACKNOWLEDGMENT

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APPENDIX A: IDENTITY $F^{\dagger}F=\mathbb{I}$

Let $f(t)=e^{-i\nu\omega t}\exp[-i\int_0^t V_1(t')\,dt']$, $\nu\in\mathbb{Z}$, Eq. (11). We have

$$\frac{1}{P}\int_0^P f^*(t)f(t)e^{-im\omega t}\,dt=\frac{1}{P}\int_0^P e^{-im\omega t}\,dt=\delta_{0m}. \tag{A1}$$

With the Fourier expansion of $f(t)$, we obtain

$$\frac{1}{P}\int_0^P f^*(t)f(t)e^{-im\omega t}\,dt=\frac{1}{P}\int_0^P\sum_{n,n'}F_n^*F_{n'}e^{i(n-n'-m)\omega t}=(F^{\dagger}F)_{0m}, \tag{A2}$$

and thus $F^{\dagger}F=\mathbb{I}$.

APPENDIX B: PROPERTIES OF THE SCATTERING-AMPLITUDE OPERATOR S

1. Generalized continuity equation

Let $e^{-iEt}g(x,t)$ and $e^{-iEt}h(x,t)$ be two solutions to the Schrödinger equation (3) for the set of modes

$$M_E=\{E_n|E_n=E+n\omega,\ n\in\mathbb{Z}\}. \tag{B1}$$

The time-periodic functions g and h are solutions of Eq. (4). Multiplying the equation for h by g^* and the equation for g^* by h, we obtain, after subtraction, the generalized continuity equation

$$i\partial_t(g^*h)=-\partial_x(g^*\partial_x h-\partial_x g^*h). \tag{B2}$$

Integration over a time period P leads to

$$\partial_x\frac{1}{P}\int_0^P dt\ (g^*\partial_x h-\partial_x g^*h)=0. \tag{B3}$$

Therefore,

$$-\frac{i}{P}\int_0^P dt\ (g^*\partial_x h-\partial_x g^* h)=C_{g,h} \tag{B4}$$

is a constant in space associated with the solutions $e^{-iEt}g(x,t)$ and $e^{-iEt}h(x,t)$. In particular, for $g=h$, Eq. (B4) expresses the conservation of the mean current. Considering two sets of solutions $\{g_m\}_{m\in\mathbb{Z}}$ and $\{h_{m'}\}_{m'\in\mathbb{Z}}$ expressed in terms of the operators G and H as

$$g_m(x,t)=\sum_{n\in\mathbb{Z}} G_{nm}(x)e^{-in\omega t}, \tag{B5}$$

$$h_{m'}(x,t)=\sum_{n'\in\mathbb{Z}} H_{n'm'}(x)e^{-in'\omega t}, \tag{B6}$$

we can write Eq. (B4) as

$$-i(G^\dagger\partial_x H-\partial_x G^\dagger H)_{mm'}=C_{mm'} \tag{B7}$$

or

$$G^\dagger\partial_x H-\partial_x G^\dagger H=iC, \tag{B8}$$

with the operator C independent of x.

Defining the operator $L(x)$ by

$$L_{mn}(x)=e^{ik(n)x}\delta_{mn}, \tag{B9}$$

we obtain, for waves incoming from the left, the set of solutions of Eq. (4):

$$A(x)=\begin{cases} L(x)+L^{-1}(x)R_\rightarrow, & x\in L\\ L(x)T_\rightarrow, & x\in R.\end{cases} \tag{B10}$$

The corresponding spatial derivatives are given by

$$-i\partial_x A(x)=\begin{cases} K[L(x)-L^{-1}(x)R_\rightarrow], & x\in L\\ KL(x)T_\rightarrow, & x\in R.\end{cases} \tag{B11}$$

For waves incoming from the right, the respective operators are

$$B(x)=\begin{cases} L^{-1}(x)T_\leftarrow, & x\in L\\ L^{-1}(x)+L(x)R_\leftarrow, & x\in R\end{cases} \tag{B12}$$

and

$$i\partial_x B(x)=\begin{cases} KL^{-1}(x)T_\leftarrow, & x\in L\\ K[L^{-1}(x)-L(x)R_\leftarrow], & x\in R.\end{cases} \tag{B13}$$

For $G=H=A(x)$, Eq. (B8) implies, with multiplication from the left by $(L_L^{-1})^\dagger$ and from the right by L_L^{-1},

$$\begin{aligned} C_{A,A}&=(AL_L^{-1})^\dagger(-i\partial_x AL_L^{-1})+(-i\partial_x AL_L^{-1})^\dagger(AL_L^{-1})\\ &\overset{x=x_0}{=}(\mathbb{I}+\tilde{R}^\dagger_\rightarrow)K(\mathbb{I}-\tilde{R}_\rightarrow)+(\mathbb{I}-\tilde{R}^\dagger_\rightarrow)K^\dagger(\mathbb{I}+\tilde{R}_\rightarrow)\\ &\overset{x=x_N}{=}\tilde{T}^\dagger_\rightarrow K\tilde{T}_\rightarrow+\tilde{T}^\dagger_\rightarrow K^\dagger\tilde{T}_\rightarrow, \end{aligned} \tag{B14}$$

where we have used Eqs. (42) and (43). In particular, $(C_{A,A})_{00}$ is the mean current for an incoming wave in mode $E_0=E$,

$$\begin{aligned} j_\rightarrow(E)&=(\tilde{T}^\dagger_\rightarrow(K+K^\dagger)\tilde{T}_\rightarrow)_{00}=(2K_p\tau^\dagger_\rightarrow\tau_\rightarrow)_{00}\\ &=\sum_{E_n>0}2\sqrt{E_n}|(\tilde{T}_{\rightarrow,n0}(E)|^2, \end{aligned} \tag{B15}$$

which proves Eq. (54). Equation (B14) leads to

$$\begin{aligned} &\tilde{R}^\dagger_\rightarrow(K+K^\dagger)\tilde{R}_\rightarrow+\tilde{T}^\dagger_\rightarrow(K+K^\dagger)\tilde{T}_\rightarrow\\ &\quad=K+K^\dagger+\tilde{R}^\dagger_\rightarrow(K-K^\dagger)-(K-K^\dagger)\tilde{R}_\rightarrow. \end{aligned} \tag{B16}$$

Similarly, for $G=H=B(x)$, we obtain, multiplying from the left by $L_R^\dagger$ and from the right by L_R,

$$\begin{aligned} &\tilde{R}^\dagger_\leftarrow(K+K^\dagger)\tilde{R}_\leftarrow+\tilde{T}^\dagger_\leftarrow(K+K^\dagger)\tilde{T}_\leftarrow\\ &\quad=K+K^\dagger+\tilde{R}^\dagger_\leftarrow(K-K^\dagger)-(K-K^\dagger)\tilde{R}_\leftarrow. \end{aligned} \tag{B17}$$

For $G=B(x)$ and $H=A(x)$ we obtain, multiplying from the left by $L_R^\dagger$ and from the right by L_L^{-1},

$$\begin{aligned} &\tilde{T}^\dagger_\leftarrow(K+K^\dagger)\tilde{R}_\rightarrow+\tilde{R}^\dagger_\leftarrow(K+K^\dagger)\tilde{T}_\rightarrow\\ &\quad=\tilde{T}^\dagger_\leftarrow(K-K^\dagger)-(K-K^\dagger)\tilde{T}_\rightarrow. \end{aligned} \tag{B18}$$

Finally, for $G=A(x)$ and $H=B(x)$ we obtain the Hermitian conjugate of Eq. (B18). Using block by block multiplication, we obtain Eq. (60).

2. Asymptotic behavior of S at large energies

Using the Fourier expansions

$$V_1(t)=\sum_n V_n e^{-in\omega t} \tag{B19}$$

$$\exp\left(-i\int_0^t V_1(t')\ dt'\right)=\sum_n F_n e^{-in\omega t}, \tag{B20}$$

we define the Hermitian operator $V=V^\dagger$ and the unitary operator F by their matrix elements

$$V_{mn}=V_{m-n},\quad F_{mn}=F_{m-n}. \tag{B21}$$

Together with the operator N,

$$N_{mn}=m\delta_{mn}, \tag{B22}$$

we rewrite Eq. (9) as

$$\omega NF-VF=\omega FN. \tag{B23}$$

Let us consider a sum of potentials with time average zero,

$$U_1(t)=V_1(t)+W_1(t). \tag{B24}$$

In the same way as in Eq. (B21), we define, for each potential,

$$V_1(t)\mapsto(V,F), \tag{B25}$$

$$W_1(t)\mapsto(W,H), \tag{B26}$$

$$U_1(t)\mapsto(U,G). \tag{B27}$$

From Eqs. (B19) and (B20), we obtain

$$U=V+W \quad \text{and} \quad G=FH=HF. \tag{B28}$$

With the choice $W_1(t)=V_1(t)$, we have $U=2V$ and $G=F^2$, and therefore

$$\omega NF^2-2VF^2=\omega F^2 N. \tag{B29}$$

Using the unitarity of F, from Eqs. (B23) and (B29) we obtain

$$\omega N-V=\omega FNF^\dagger,$$

$$\omega FNF^\dagger-FVF^\dagger=\omega F^2N(F^\dagger)^2,$$

$$\omega N-V-FVF^\dagger=\omega N-2V,$$

$$V=FVF^\dagger,$$

$$VF=FV. \tag{B30}$$

The operators V and F commute and with $E\mathbb{I}+\omega N=K^2$ [see Eq. (17)], Eq. (B23) becomes

$$F^\dagger K^2 F=K^2+V. \tag{B31}$$

Introducing the operators

$$X=F^\dagger(\tilde{T}+\tilde{R})F, \tag{B32}$$

$$Y=F^\dagger(\tilde{T}-\tilde{R})F, \tag{B33}$$

we write Eqs. (35) and (36) as

$$[(L_\Delta-\mathbb{I})K_S-(L_\Delta+\mathbb{I})F^\dagger KF]X$$
$$=(\mathbb{I}-L_\Delta)K_S-(\mathbb{I}+L_\Delta)F^\dagger KF, \tag{B34}$$

$$[(L_\Delta+\mathbb{I})K_S-(L_\Delta-\mathbb{I})F^\dagger KF]Y$$
$$=(\mathbb{I}+L_\Delta)K_S-(\mathbb{I}-L_\Delta)F^\dagger KF. \tag{B35}$$

Equation (B31) implies

$$F^\dagger KF=(F^\dagger K^2F)^{1/2}=(K^2+V)^{1/2}. \tag{B36}$$

In order to calculate $(K^2+V)^{1/2}$, we have to estimate the matrix element of the commutator $K^{-1}V-VK^{-1}$,

$$(K^{-1}V-VK^{-1})_{mn}=\frac{(n-m)\omega}{[k(m)+k(n)]k(m)k(n)}V_{m-n}. \tag{B37}$$

In the following, we assume that the number of nonzero Fourier components in Eq. (B19) is finite, and we consider large energies $E=E_0=k^2(0)=k^2$ of the incoming wave. Then Eq. (B37) can be written as

$$K^{-1}V-VK^{-1}=O(k^{-3}), \tag{B38}$$

and we may approximate

$$F^\dagger KF\approx K[\mathbb{I}+\tfrac{1}{2}VK^{-2}+O(k^{-4})]$$
$$\approx K+\tfrac{1}{2}VK^{-1}+O(k^{-3}). \tag{B39}$$

We now solve Eqs. (B34) and (B35) to the second order of k^{-1}. Defining the operators

$$A=(L_\Delta-\mathbb{I})K_S-(L_\Delta+\mathbb{I})K, \tag{B40}$$

$$B=(\mathbb{I}-L_\Delta)K_S-(\mathbb{I}+L_\Delta)K, \tag{B41}$$

$$C=(L_\Delta+\mathbb{I})K_S-(L_\Delta-\mathbb{I})K, \tag{B42}$$

$$D=(\mathbb{I}+L_\Delta)K_S-(\mathbb{I}-L_\Delta)K, \tag{B43}$$

which correspond to a static rectangular barrier, we rewrite Eqs. (B34) and (B35) as

$$[A-\tfrac{1}{2}(L_\Delta+\mathbb{I})VK^{-1}+O(k^{-3})][X_0+X_1+O(k^{-3})]$$
$$=B-\tfrac{1}{2}(\mathbb{I}+L_\Delta)VK^{-1}+O(k^{-3}), \tag{B44}$$

$$[C-\tfrac{1}{2}(L_\Delta-\mathbb{I})VK^{-1}+O(k^{-3})][Y_0+Y_1+O(k^{-3})]$$
$$=D-\tfrac{1}{2}(\mathbb{I}-L_\Delta)VK^{-1}+O(k^{-3}). \tag{B45}$$

X_0 and Y_0 are the solutions for the static barrier described by

$$\tilde{T}_{\text{static}}=\tfrac{1}{2}(X_0+Y_0), \tag{B46}$$

$$\tilde{R}_{\text{static}}=\tfrac{1}{2}(X_0-Y_0). \tag{B47}$$

We obtain

$$F^\dagger\tilde{T}F=\tfrac{1}{2}(X+Y)$$
$$=\tilde{T}_{\text{static}}+\tfrac{1}{2}[(L_\Delta+\mathbb{I})A^{-1}VK^{-1}A^{-1}(\mathbb{I}-L_\Delta)$$
$$+(L_\Delta-\mathbb{I})C^{-1}VK^{-1}C^{-1}(\mathbb{I}+L_\Delta)]K_S+O(k^{-3}), \tag{B48}$$

$$F^\dagger\tilde{R}F=\tfrac{1}{2}(X-Y)$$
$$=\tilde{R}_{\text{static}}+\tfrac{1}{2}[(L_\Delta+\mathbb{I})A^{-1}VK^{-1}A^{-1}(\mathbb{I}-L_\Delta)$$
$$-(L_\Delta-\mathbb{I})C^{-1}VK^{-1}C^{-1}(\mathbb{I}+L_\Delta)]K_S+O(k^{-3}). \tag{B49}$$

At high energies, the static part of the potential becomes negligible, i.e., $K_S\approx K$, and

$$\tilde{T}_{\text{static}}=L_\Delta, \tag{B50}$$

$$\tilde{R}_{\text{static}}=0, \tag{B51}$$

and thus

$$F^\dagger\tilde{T}F=L_\Delta+\tfrac{1}{4}(L_\Delta V-VL_\Delta)K^{-2}+O(k^{-3}), \tag{B52}$$

$$F^\dagger\tilde{R}F=\tfrac{1}{4}(V-L_\Delta VL_\Delta)K^{-2}+O(k^{-3}). \tag{B53}$$

The matrix element of the commutator $L_\Delta V-VL_\Delta$ is

$$(L_\Delta V-VL_\Delta)_{mn}=iL_{\Delta,nn}\frac{(m-n)\omega}{k(m)+k(n)}\Delta xV_{m-n}+O(k^{-2}) \tag{B54}$$

or

$$L_\Delta V-VL_\Delta=O(k^{-1}). \tag{B55}$$

It follows that

$$F^\dagger \tilde{T} F = L_\Delta + O(k^{-3}), \quad \text{(B56)}$$

$$F^\dagger \tilde{R} F = \frac{V}{4}(\mathbb{I} - L_\Delta^2)K^{-2} + O(k^{-3}), \quad \text{(B57)}$$

and finally, with Eq. (B30),

$$\tilde{T} = F L_\Delta F^\dagger + O(k^{-3}) = O(k^0), \quad \text{(B58)}$$

$$\tilde{R} = \frac{V}{4} F(\mathbb{I} - L_\Delta^2)K^{-2}F^\dagger + O(k^{-3}) = O(k^{-2}) \quad \text{(B59)}$$

for a dynamic rectangular barrier and large energies $E = k^2$.

From Eqs. (B58), (B59), and (49) we also have

$$\tilde{R}_\rightarrow = O(k^{-2}) \quad \text{(B60)}$$

for a sequence of barriers. Therefore, constructing the reflection part $\rho_\rightarrow$ of the usual scattering matrix according to Eq. (62), we see that

$$\lim_{n\to\infty} (\rho_\rightarrow^\dagger \rho_\rightarrow)_{nn} = 0, \quad \text{(B61)}$$

i.e., the barrier becomes transparent for an incoming wave with high energy. Moreover, as $(\rho_\rightarrow^\dagger \rho_\rightarrow)_{nn} = O(n^{-2})$ decreases rapidly when $n \to \infty$, we have

$$\text{Tr}\,(\rho_\rightarrow^\dagger \rho_\rightarrow) < \infty, \quad \text{(B62)}$$

which proves Eq. (68).

Using Eq. (B15), we obtain the mean current to first order in k^{-1},

$$[\tilde{T}^\dagger (K + K^\dagger)\tilde{T}]_{00} = [F L_\Delta^\dagger F^\dagger (K + K^\dagger) F L_\Delta F^\dagger]_{00} + O(k^{-2}). \quad \text{(B63)}$$

With the expansion

$$L_\Delta = e^{ik\Delta x}\left(\mathbb{I} + i\frac{\Delta x \omega}{2k}N - \frac{\Delta x^2 \omega^2}{8k^2}N^2\right) + O(k^{-3}), \quad \text{(B64)}$$

we obtain

$$\tilde{T} = e^{ik\Delta x}\left(\mathbb{I} + i\frac{\Delta x}{2k}F\omega N F^\dagger - \frac{\Delta x^2}{8k^2}F\omega^2 N^2 F^\dagger\right) + O(k^{-3}). \quad \text{(B65)}$$

Using Eq. (B23), we have, finally,

$$\tilde{T} = e^{ik\Delta x}\mathbb{I} + i\frac{\Delta x e^{ik\Delta x}}{2k}(\omega N - V) - \frac{\Delta x^2 e^{ik\Delta x}}{8k^2}(\omega N - V)^2 + O(k^{-3}). \quad \text{(B66)}$$

Thus the contribution of a sideband $n \neq 0$ to the mean current is

$$j_n(E) = 2\sqrt{E_n}|\tilde{T}_{n0}|^2 = \frac{\Delta x^2}{2\sqrt{E}}|V_{n0}|^2 + O(E^{-1}), \quad n \neq 0. \quad \text{(B67)}$$

The main contribution to the mean current is

$$j_0(E) = 2\sqrt{E} - \frac{\Delta x^2}{2\sqrt{E}}(V^2)_{00} + O(E^{-1}). \quad \text{(B68)}$$

From

$$(V^2)_{00} = \sum_n |V_{n0}|^2, \quad \text{(B69)}$$

it follows that

$$\sum_n j_n(E) + O(E^{-1}) = 2\sqrt{E}, \quad \text{(B70)}$$

i.e., electrons incoming at high energies are not reflected by the barrier.

In the particular case $V(t) = V_0 + V_1 \cos(\omega t + \varphi)$, the operator V is described by

$$V_{mn} = \begin{cases} \frac{V_1}{2} & \text{for } |m-n| = 1 \\ 0 & \text{otherwise,} \end{cases} \quad \text{(B71)}$$

which yields (see also Fig. 3)

$$j_n(E) = \frac{\Delta x^2 V_1^2}{8\sqrt{E}} + O(E^{-1}), \quad n \neq 0. \quad \text{(B72)}$$

3. Time-reversal invariant potential

Let g and h be two solutions of Eq. (4) for the set of modes

$$M_E = \{E_n | E_n = E + n\omega, \ n \in \mathbb{Z}\}. \quad \text{(B73)}$$

For a time-reversal invariant potential $V(x,-t) = V(x,t)$, we define $\tilde{g}(t) = g(-t)$, satisfying

$$-i\partial_t \tilde{g} = -\partial_x^2 \tilde{g} + [V(x,t) - E]\tilde{g}. \quad \text{(B74)}$$

Multiplying the equation for h by $\tilde{g}$ and the equation for $\tilde{g}$ by h, we obtain, after subtraction,

$$i\partial_t(\tilde{g}h) = -\partial_x(\tilde{g}\partial_x h - \partial_x \tilde{g} h). \quad \text{(B75)}$$

Using the same procedure as in Appendix B 1, we finally obtain Eq. (69).

[1] R. Landauer, Z. Phys. B **68**, 217 (1987).
[2] D. S. Saraga and M. Sassoli de Bianchi, Helv. Phys. Acta **70**, 751 (1997).
[3] M. Büttiker and R. Landauer, Phys. Scr. **32**, 429 (1985).
[4] D. D. Coon and H. C. Liu, J. Appl. Phys. **58**, 2230 (1985); Phys. Scr. **32**, 429 (1985).
[5] A. Pimpale, S. Holloway, and R. J. Smith, J. Phys. A **24**, 3533 (1991).
[6] M. Wagner, Phys. Rev. B **49**, 16 544 (1994); Phys. Rev. A **51**, 798 (1995); Phys. Rev. Lett. **76**, 4010 (1996).
[7] O. A. Tkachenko, D. G. Baksheyev, and V. A. Tkachenko, Phys. Rev. B **54**, 13 452 (1996).

[8] J. A. Støvneng and E. H. Hauge, J. Stat. Phys. **57**, 841 (1989).
[9] L. P. Kouwenhoven, S. Jauhar, J. Orenstein, P. L. McEuen, Y. Nagamune, J. Motohisa, and H. Sakaki, Phys. Rev. Lett. **73**, 3443 (1994).
[10] K. Tsukagoshi, K. Nakazato, H. Ahmed, and K. Gamo, Phys. Rev. B **56**, 3972 (1997).
[11] L. P. Kouwenhoven, S. Jauhar, K. McCormick, D. Dixon, P. L. McEuen, Yu. V. Nazarov, N. C. van der Vaart, and C. T. Foxon, Phys. Rev. B **50**, 2019 (1994).
[12] Qing-feng Sun and Tsung-han Lin, Phys. Rev. B **56**, 3591 (1997).
[13] F. Hekking and Yu. V. Nazarov, Phys. Rev. B **44**, 9110 (1991).
[14] G. Burmeister, K. Maschke, and M. Schreiber, Phys. Rev. B **47**, 7095 (1993).
[15] W. O. Amrein, J. M. Jauch, and K. B. Sinha, *Scattering Theory in Quantum Mechanics* (Benjamin, Reading, MA, 1977).
[16] A. Messiah, *Mécanique Quantique* (Dunod, Paris, 1969).