Green-function theory of plasmons in two-dimensional semiconductor structures: Zero magnetic field

M. S. Kushwaha
Instituto de Física, Universidad Autónoma de Puebla, Apdo. Post. J-48, Puebla 72570, Mexico

B. Djafari-Rouhani
Laboratoire de Dynamique et structure des Matériaux Moleculaires, CNRS, UFR DE Physique, Universite de Lille-I, 59655 Villeneuve d'Ascq Cédex, France

(Received 18 November 1996; revised manuscript received 11 August 1997)

A theoretical investigation has been made of the plasmon excitations in various two-dimensional (2D) semiconductor heterostructures in the framework of a Green-function (or response function) theory. The plasmon excitations in the periodic and nonperiodic systems are implicitly defined by the electromagnetic fields that are localized at and decay exponentially away from the interfaces. The Green-function theory generalized to be applicable to the 2D systems enables one to derive explicit expressions for the corresponding response functions (associated with the electromagnetic fields), which can in turn be used to calculate almost all physical properties of the systems at hand. A rigorous analytical diagnosis of the general results for all the systems investigated here leads one to reproduce exactly the previously well-established results obtained within a different theoretical framework. The elegance of the theory lies in its simplicity and the compact form of the desired results. The impact and relevance of the analytical results have been discussed briefly. [S0163-1829(98)05016-4]

I. INTRODUCTION

The seminal paper of Esaki and Tsu[1] laid the foundation of a field that is now becoming known as the ''physics and fabrication of the systems of reduced dimensionality'' and represents nearly 50% of the efforts devoted to semiconductor physics worldwide.[2] In this paper, the authors speculated that a periodic modulation of the composition or doping of a semiconductor at a length scale shorter than the electron mean free path would result in a folding of the Brillouin zone into minizones showing strong dispersion effects leading to exotic electronic and optical properties different from those found in the bulk. Early attempts on such quantized structures were focused on layered structures that confine charge carriers to two dimensions, quantum wells, for example. The original proposal had included two kinds of superlattices: compositional and doping. Compositional superlattices consist of alternating layers of two different semiconductors. The compositional variation modulates the electronic potential on a length scale shorter than the electron mean free path. The doping superlattices consist of alternating *n*- and *p*-type layers of a single semiconductor. Electric fields generated by the charged dopants modulate the electronic potential. The keystone to the designed electronic and/or optical properties in the superlattice systems is the band-gap discontinuity of the consecutive layers in the unit cell of the superstructures.

In this paper, we will confine ourselves to the compositional superlattices that have seen relatively wide interest, both theoretical and experimental, in the recent past. These are known as type-I and type-II superlattices. Type-I superlattices are typified by a GaAs-$Al_xGa_{1-x}As$ system, in which the band gap of GaAs is smaller than, and lies within, that of $Al_xGa_{1-x}As$, giving rise to the band-gap discontinuities in both the valence and conduction bands of the resultant superstructure. The simplest model of the type-I superlattice that we will be concerned with is the low-temperature periodic system of a two-dimensional electron gas (2DEG). Type-II superlattices are typified by the InAs-GaSb system, in which the conduction-band minimum of InAs is lower than the valence-band maximum of GaSb, leading to a transfer of electrons from one (GaSb) layer to the other (InAs) layer and resulting into a spatial separation of electrons and holes in the adjacent potential wells, with the formation of electrons and hole subbands (or minibands). For our purpose it is sufficient to consider type-II superlattices as a periodic arrangement of alternating 2DEG and two-dimensional hole gas (2DHG).

Initial theoretical investigations into these man-made semiconductors focused on various types of collective excitations, such as phonons, magnons, plasmons, polarons, and magnetoplasmons. The literature reveals that the elementary excitations in these superstructures have most often been treated in the framework of conventional theories,[3] such as the random phase approximation (RPA), the hydrodynamical model, or the transfer matrix method using electrodynamics with appropriate electromagnetic boundary conditions. The present work embarks on an investigation of the response of the heterointerfaces in these systems, using a Green-function (or response function) theory in a compact form. In a way it is the generalization of Dobrzynski's interface response theory[4] (IRT) to the 2D systems. The Green functions or response functions in the IRT are calculated as functions of *bulk* response functions of each subsystem and of the *interface* response operators. These operators are shown to be the linear superposition of the responses to a cleavage operator of the corresponding ideal free surfaces of all subsystems and of the responses to the coupling operator of all interfaces. The resultant response functions can then be made use of to derive, literally speaking, any physical property of the system at hand. They play a crucial role in the theories of light scattering (both Raman and Brillouin), as well as in various other physical phenomena.[5] The elegance of the present theory lies particularly in its simplicity. It is important to note that our accomplishment lies in presenting analytical solutions and thus physical insight into a rather complex problem. The illustrative analytical diagnoses have been made at all stages to reproduce the well established results and hence to embolden our confidence in the adequacy of the theoretical development. The computation is deferred to a future work.

This paper is organized as follows. In Sec. II we derive the bulk response function for an infinite semiconductor. In Sec. III we analyze the infinite medium limited by a ''black-box surface'' and thus calculate the surface response function. Section IV is devoted to the calculation of the response functions for a black-box slab (i.e., an infinite semiconductor limited by two black-box surfaces) in the limit that the thickness of the slab approaches zero. This applies to the case of a 2DEG bounded by two identical or nonidentical dielectric media. In Sec. V we study the case of double inversion layers, which could be considered a buiding block of the type-II superlattice. Section VI deals with an infinite type-II superlattice. The case of a truncated (semi-infinite) type-II superlattice is worked out in Sec. VII. All the results in Secs. VI and VII are shown, with a formal trick, to be reducible to those valid for the type-I superlattice. Finally, we comment, in Sec. VIII, on how the IRT in its compact form has been able to reproduce exactly the previously reported well-established results and discuss briefly the implications of the response functions derived in the framework of IRT for the systems considered in the present work.

II. AN INFINITE SEMICONDUCTING MEDIUM

First we describe the geometry at hand. We consider the electromagnetic waves propagating along the $\hat{y}$ axis with angular frequency ω and wave vector $q \equiv q_y$. The $\hat{x}$ component of the wave vector may be taken to be zero without loss of generality. The plasma waves, here as well as in the latter part of the work, will be assumed to observe the spatial localization along the $\hat{z}$ axis. This refers to the fact that the $\hat{y}$-$\hat{z}$ plane is the sagittal plane.

After eliminating the magnetic field variable **B** from the Maxwell's curl-field equations, we obtain the wave-field equation in terms of the macroscopic electric field vector **E**,

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{\nabla}) - q_0^2\epsilon\mathbf{E} = 0. \tag{2.1}$$

Here the dielectric function ϵ is a scalar, since the system we are concerned with is not subjected to any external magnetostatic field and the physical system is assumed to be isotropic. Also, note that we are interested in the nonmagnetic materials, so that $\mathbf{B}\equiv\mathbf{H}$ in the Maxwell's curl-field equations. In Eq. (2.1) $q_0 = \omega/c$ is the vacuum wave vector, c being the velocity of light in vacuum. We will take the spatial and temporal dependence of the fields of the form of $e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$. In the present situation, Eq. (2.1) can thus be cast in the form

$$\begin{bmatrix} q_0^2\epsilon - q^2 - \partial_z^2 & 0 & 0 \\ 0 & q_0^2\epsilon + \partial_z^2 & -iq\partial_z \\ 0 & -iq\partial_z & q_0^2\epsilon - q^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{2.2}$$

Here $\partial_z \equiv \partial/\partial_z$. Since we are interested in the TM (p-polarized modes), Eq. (2.2) essentially takes the form

$$\begin{bmatrix} q_0^2\epsilon + \partial_z^2 & -iq\partial_z \\ -iq\partial_z & q_0^2\epsilon - q^2 \end{bmatrix} \begin{bmatrix} E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{2.3}$$

The condition of nontrivial solutions of such a set of the linear equations yields

$$-q_z^2 \equiv \alpha^2 = q^2 - q_0^2\epsilon(\omega), \tag{2.4}$$

where $\epsilon(\omega) = \epsilon_L(1-\omega_p^2/\omega^2)$ is the local dielectric function; $\omega_p = (4\pi n_0 e^2/m^*\epsilon_L)^{1/2}$ is the screened plasma frequency and ϵ_L the background dielectric constant. Here α refers to the decay constant in the medium concerned. Employing the appropriate Green function (or response function) $G(z,z')$, Eq. (2.3) may be written as

$$\begin{bmatrix} q_0^2\epsilon + \partial_z^2 & -iq\partial_z \\ -iq\partial_z & q_0^2\epsilon - q^2 \end{bmatrix} \begin{bmatrix} G_{yy} & G_{yz} \\ G_{zy} & G_{zz} \end{bmatrix} = \delta(z-z')\overleftrightarrow{\mathbf{I}}, \tag{2.5}$$

where $\overleftrightarrow{\mathbf{I}}$ is 2×2 unit matrix and $G_{ij}(z,z')$ refers to the (i,j)th element of the bulk response function $\overleftrightarrow{\mathbf{G}}(z,z')$. Solving Eq. (2.5) yields

$$G_{yy}(z,z') = \frac{\alpha}{2q_0^2\epsilon}\, e^{-\alpha|z-z'|}, \tag{2.6a}$$

$$G_{zy}(z,z') = i\,\frac{q}{2q_0^2\epsilon}\,\mathrm{sgn}(z-z')e^{-\alpha|z-z'|}, \tag{2.6b}$$

$$G_{yz}(z,z') = i\,\frac{q}{2q_0^2\epsilon}\,\mathrm{sgn}(z-z')e^{-\alpha|z-z'|}, \tag{2.6c}$$

$$G_{zz}(z,z') = \frac{1}{2q_0^2\alpha\epsilon}\,[2\alpha\delta(z-z') - q^2 e^{-\alpha|z-z'|}]. \tag{2.6d}$$

In Eqs. (2.5) and (2.6) $\delta(z-z')$ is the Dirac delta function and $G_{ij}(z,z')$ are the elements of the bulk response function, which will be made use of in the following sections.

III. AN INTERFACE BETWEEN TWO MEDIA

We now consider a semiconducting medium limited by a black-box surface (BBS) at $z=0$. By BBS we mean an entirely opaque surface through which electromagnetic fields cannot propagate. Conceptually this is achieved by stressing that c (the vacuum speed of light) and ϵ (the dielectric function) vanish for $z\leqslant 0$. As such, we write the Maxwell's curl-field equations for the semiconducting medium ($z>0$) limited by a black-box surface as

$$\theta(z)[c\boldsymbol{\nabla}\times\mathbf{E}] + \dot{\mathbf{B}} = 0, \tag{3.1}$$

$$\theta(z)[c\boldsymbol{\nabla}\times\mathbf{B} - \dot{\mathbf{D}}] = 4\pi J. \tag{3.2}$$

The overdot on **B** and **D** refers to the time derivative of the respective quantities and $\theta(z)$ is the step function. Eliminating **B** from Eqs. (3.1) and (3.2) and performing all differentiations provides us with

$$\theta(z)[\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{E}) - q_0^2\epsilon E] + \delta(z)\overleftrightarrow{\mathbf{V}}(\vec{r})\mathbf{E} = 0, \tag{3.3}$$

where the so-called black-box cleavage operator $\overleftrightarrow{\mathbf{V}}(\mathbf{r})$ is defined as a 3×3 matrix.[4] It is noteworthy that $\overleftrightarrow{\mathbf{V}}(\mathbf{r})$ has to have the opposite sign if one considers the complementary (in the infinite space) medium. $\overleftrightarrow{\mathbf{V}}(\mathbf{r})$ in the present configuration takes the form

$$\overleftrightarrow{\mathbf{V}}(\mathbf{r})=\begin{bmatrix}\partial_z & 0 & 0\\ 0 & \partial_z & -iq\\ 0 & 0 & 0\end{bmatrix}. \tag{3.4}$$

As such, the part of Eq. (3.3) concerned with the TM waves is written in terms of the surface response function $\overleftrightarrow{\mathbf{g}}(z,z')$ as

$$\theta(z)\begin{bmatrix}q_0^2\epsilon+\partial_z^2 & -iq\partial_z\\ -iq\partial_z & q_0^2\epsilon-q^2\end{bmatrix}\begin{bmatrix}g_{yy} & g_{yz}\\ g_{zy} & g_{zz}\end{bmatrix}+\delta(z)\begin{bmatrix}\partial_z & -iq\\ 0 & 0\end{bmatrix}\times\begin{bmatrix}g_{yy} & g_{yz}\\ g_{zy} & g_{zz}\end{bmatrix}=\delta(z-z')\overleftrightarrow{\mathbf{I}}. \tag{3.5}$$

This, after some algebra, yields

$$g_{yz}(z,z')=-\frac{iq}{q_0^2\epsilon-q^2}\,\partial_{z'}g_{yy}(z,z'), \tag{3.6a}$$

$$g_{zy}(z,z')=\frac{iq}{q_0^2\epsilon-q^2}\,\partial_z g_{yy}(z,z'), \tag{3.6b}$$

$$g_{zz}(z,z')=\frac{iq}{q_0^2\epsilon-q^2}\,g_{yz}(z,z')+\frac{\delta(z-z')}{q_0^2\epsilon-q^2}. \tag{3.6c}$$

Then one obtains, from Eq. (3.5),

$$C[\theta(z)(\partial_z^2-\alpha^2)g_{yy}+\delta(z)\partial_z g_{yy}]=\delta(z-z'), \tag{3.7}$$

where $C=-q_0^2\epsilon/\alpha^2$. It should be pointed out that we will henceforth consider only the $\hat{y}$-$\hat{y}$ component of the Green function. Now the response operator at the surface ($z>0\Rightarrow$positive half space) of a black-box crystal is written as

$$A_s(0,z')=V(z)G(z,z')\big|_{z=0}=-\tfrac{1}{2}e^{-\alpha|z'|} \tag{3.8}$$

and define

$$\Delta_s(0,0)=1+A_s(0,0)=\tfrac{1}{2}. \tag{3.9}$$

The inverse of the surface response function is given by

$$g_s^{-1}(0,0)=\Delta_s(0,0)G^{-1}(0,0)=-\alpha C \tag{3.10}$$

and the complete surface response function

$$g_s(z,z')=G(z,z')-G(z,0)\Delta_s^{-1}(0,0)A_s(0,z')=-\frac{1}{2\alpha C}[e^{-\alpha|z-z'|}+e^{-\alpha(z+z')}]. \tag{3.11}$$

Let us now write Eq. (3.5) in the negative half space ($z<0$) by changing the sign of the cleavage operator $\overleftrightarrow{\mathbf{V}}$. We get, corresponding to Eq. (3.7),

$$C[\theta(z)(\partial_z^2-\alpha^2)g_{yy}-\delta(z)\partial_z g_{yy}]=\delta(z-z'). \tag{3.12}$$

Then following a procedure analogous to Eqs. (3.8)–(3.11) leaves us with

$$g_s(z,z')=G(z,z')-G(z,0)\Delta_s^{-1}(0,0)A_s(0,z')=-\frac{1}{2\alpha C}[e^{-\alpha|z-z'|}+e^{+\alpha(z+z')}]. \tag{3.13}$$

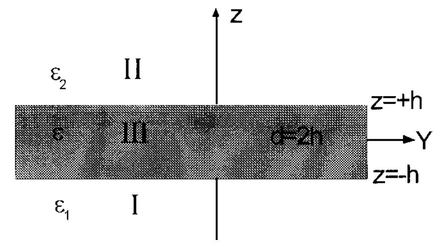

FIG. 1. The schematics of the semiconducting film (shaded region) limited by two black-box surfaces ($-h\leqslant z\leqslant+h$) before taking the limit $d(=2h)\rightarrow0$.

In Eqs. (3.11) and (3.13) $g_s(z,z')$ are the corresponding complete response functions. Assuming now that the negative and positive half spaces are filled, respectively, by materials 1 and 2, we calculate

$$g_I^{-1}(0,0)=g_{s1}^{-1}(0,0)+g_{s2}^{-2}(0,0)=-(\alpha_1C_1+\alpha_2C_2). \tag{3.14}$$

Here $g_I^{-1}(0,0)$ is the inverse of the response function in the interface space. All other elements of $\overleftrightarrow{\mathbf{g}}$ can be obtained from[4]

$$g_I(z,z')=G(z,z')-G(z,0)G^{-1}(0,0)G(0,z')+G(z,0)G^{-1}(0,0)g_I(0,0)G^{-1}(0,0)G(0,z'). \tag{3.15}$$

Equation (3.15) allows us to study the four different situations, i.e., $z,z'>0$; $z,z'<0$; $z>0$; $z'<0$, and $z<0$; $z'>0$.

The interface plasmons are describable via the dispersion relation obtained through $\det|g_I^{-1}(0,0)|=0$ that yields a well-established result[6] specified by

$$\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}=0. \tag{3.16}$$

IV. A FILM BOUNDED BY TWO MEDIA

We now consider a semiconducting film limited by two parallel black-box surfaces (see Fig. 1), such that $-h\leqslant z\leqslant+h$. The basic formalism for this black-box slab can easily be generalized by making use of the concepts developed for the semi-infinite black-box surfaces in the preceding section. However, this is not our end point. We will intend to take the limit $d(=2h$, the thickness of the film$)\rightarrow0$, which implies a 2DEG virtually limited by two unidentical semiconductors or dielectrics. We will specify that situation by assuming $d\rightarrow0$ and $\epsilon\rightarrow\infty$ but $\epsilon d\rightarrow$ finite in medium III of Fig. 1. This then leads us to define the following physical approximations to be imposed: $\alpha^2\rightarrow-\infty$, $\alpha^2d^2\rightarrow0$, $\alpha/\epsilon\rightarrow0$, $\alpha^2/\epsilon\rightarrow-q_0^2$ and hence

$$\epsilon d\rightarrow4\pi\chi_e\Rightarrow\alpha^2d\rightarrow-4\pi q_0^2\chi_e, \tag{4.1}$$

where $\chi_e=-(n_se^2/m^*\omega^2)$ is the 2D polarizability function, which is related to the conductivity (σ) such that $\sigma=-i\omega\chi_e$, with $n_s\equiv n_0d$ being the surface carrier concentration in the resulting 2D sheet. Equation (4.1) will play a very important role in the obtention of the desired results in the remaining part of this paper.

The response operator is written, with the aid of Eqs. (3.11) and (3.13), as

$$A_s(z,z)=\begin{cases} -\frac{1}{2}e^{-\alpha(h+z)}, & z=-h \\ -\frac{1}{2}e^{-\alpha(h-z)}, & z=+h. \end{cases} \tag{4.2}$$

Therefore

$$\overleftrightarrow{\Delta}_s(M,M)=\overleftrightarrow{\mathbf{I}}+\overleftrightarrow{\mathbf{A}}_s(M,M)=\begin{bmatrix} \frac{1}{2} & -\frac{1}{2}e^{-\alpha d} \\ -\frac{1}{2}e^{-\alpha d} & \frac{1}{2} \end{bmatrix}, \tag{4.3}$$

where $d=2h$ is the film thickness, and the bulk response function

$$\overleftrightarrow{\mathbf{G}}(M,M)=-\frac{1}{2\alpha C}\begin{bmatrix} 1 & e^{-\alpha d} \\ e^{-\alpha d} & 1 \end{bmatrix}. \tag{4.4}$$

Then

$$\overleftrightarrow{\mathbf{g}}_s^{-1}(M,M)=\overleftrightarrow{\Delta}_s(M,M)\overleftrightarrow{\mathbf{G}}^{-1}(M,M) = -\frac{\alpha C}{\sinh(\alpha d)}\begin{bmatrix} \cosh(\alpha d) & -1 \\ -1 & \cosh(\alpha d) \end{bmatrix}. \tag{4.5}$$

Now, if we consider a real film bounded by two unidentical media, say 1 and 2, respectively, in the negative and positive half spaces, the inverse of the total response function in the interface space (M,M) is given by

$$\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)=\overleftrightarrow{\mathbf{g}}_{s1}^{-1}(M,M)+\overleftrightarrow{\mathbf{g}}_s^{-1}(M,M)+\overleftrightarrow{\mathbf{g}}_{s2}^{-1}(M,M)=\begin{bmatrix} -\alpha_1 C_1-\alpha C\coth(\theta) & \alpha C\,\mathrm{csch}(\theta) \\ \alpha C\,\mathrm{csch}(\theta) & -\alpha_2 C_2-\alpha C\coth(\theta) \end{bmatrix}, \tag{4.6}$$

where $\theta=\alpha d$, $C_i=-q_0^2\epsilon_i/\alpha_i^2$, and $i\equiv 1$, 2 is the suffix assigned to the quantities in the media I and II of Fig. 1. The complete response functions in the whole space are obtained from[4]

$$\overleftrightarrow{\mathbf{g}}_f(D,D)=\overleftrightarrow{\mathbf{G}}(D,D)-\overleftrightarrow{\mathbf{G}}(D,M)\overleftrightarrow{\mathbf{G}}^{-1}(M,M)\overleftrightarrow{\mathbf{G}}^{-1}(M,D) +\overleftrightarrow{\mathbf{G}}(D,M)\overleftrightarrow{\mathbf{G}}^{-1}(M,M)\overleftrightarrow{\mathbf{g}}_f(M,M)\overleftrightarrow{\mathbf{G}}^{-1} \times(M,M)\overleftrightarrow{\mathbf{G}}(M,D), \tag{4.7}$$

where $\overleftrightarrow{\mathbf{g}}_f(M,M)$ is the inverse of $\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)$. Remember, we are finally interested in this section in calculating $\overleftrightarrow{\mathbf{g}}_f(D,D)$ when the points z,z' belong to either medium I or medium II, but not to medium III. This is because we ultimately intend to eliminate the medium III by taking the limit $d\to 0$ ($\Rightarrow$2DEG): In view of this, it is necessary that we write $\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)$ and hence $\overleftrightarrow{\mathbf{g}}_f(M,M)$ within the said limits. Making use of the series expansion

$$\coth(\theta)=\frac{1}{\theta}+\frac{\theta}{3}-\frac{\theta^3}{45}+\cdots,$$

$$\mathrm{csch}(\theta)=\frac{1}{\theta}-\frac{\theta}{6}+\frac{7\theta^3}{360}-\cdots, \tag{4.8}$$

we write $\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)$, Eq. (4.6), to the first order (in correction) as

$$\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)=q_0^2\begin{bmatrix} \frac{\epsilon}{\alpha^2 d}+\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon d}{3}\right) & -\frac{\epsilon}{\alpha^2 d}+\frac{\epsilon d}{6} \\ -\frac{\epsilon}{\alpha^2 d}+\frac{\epsilon d}{6} & \frac{\epsilon}{\alpha^2 d}+\left(\frac{\epsilon_2}{\alpha_2}+\frac{\epsilon d}{3}\right) \end{bmatrix}. \tag{4.9}$$

In view of the limits imposed, the first term in each of the four elements predominates. As such,

$$\det\left|\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)\right|=q_0^4\left(\frac{\epsilon}{\alpha^2 d}\right)\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}+4\pi\chi_e\right). \tag{4.10}$$

Then to first order (in correction) the inverse of $\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)$ is found to be

$$\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)=\frac{1}{q_0^2\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}+4\pi\chi_e\right)}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}. \tag{4.11}$$

This implies that the determinant of $\overleftrightarrow{\mathbf{g}}_f(M,M)=0$. This is, however, least troublesome for any part of the calculations. What is important is to make use of the proper series expansion in $\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)$ in order to have $\det|\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)|\neq 0$. The plasmon dispersion relation specified by $\det|\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)| =0$ is given, from Eq. (4.10),

$$\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}+4\pi\chi_e=0. \tag{4.12}$$

This represents the plasma modes of a single 2DEG layer sandwiched between two semiconductors or dielectrics characterized by the dielectric constants ϵ_1 and ϵ_2, and is a standard result [see, for example, Eq. (8) of Kushwaha[7]].

It is important to note that the meaning of the above approximation, literally speaking, is that for a single film approaching the limit of a 2DEG (or 2DHG), one can approximate,

$$\overleftrightarrow{\mathbf{g}}_s^{-1}(M,M)=q_0^2\begin{bmatrix} \frac{\epsilon}{\alpha}\coth(\theta) & -\frac{\epsilon}{\alpha}\mathrm{csch}(\theta) \\ -\frac{\epsilon}{\alpha}\mathrm{csch}(\theta) & \frac{\epsilon}{\alpha}\coth(\theta) \end{bmatrix}, \tag{4.13}$$

in Eq. (4.5), by

$$\overleftrightarrow{\mathbf{g}}_s^{-1}(M,M)=q_0^2\begin{bmatrix} \frac{\epsilon}{\alpha\theta}\left(1+\frac{\theta^2}{3}\right) & -\frac{\epsilon}{\alpha\theta}\left(1-\frac{\theta^2}{6}\right) \\ -\frac{\epsilon}{\alpha\theta}\left(1-\frac{\theta^2}{6}\right) & \frac{\epsilon}{\alpha\theta}\left(1+\frac{\theta^2}{3}\right) \end{bmatrix}, \tag{4.14}$$

where the correction term is *important and necessary* at the stage of calculating the determinant of $\overleftrightarrow{\mathbf{g}}_f^{-1}(M,M)$. The concept and consequence of this statement will be encountered at many places in the following sections.

Now, the Green functions for media I and II virtually separated by a 2DEG can be easily written down from Eq. (4.7). The result is as follows:

(i) $z,z' \in$ medium I:

$$g_f(z,z')=\frac{\alpha_1}{2q_0^2\epsilon_1}\left[e^{-\alpha_1|z-z'|} + \frac{\epsilon_1/\alpha_1-\epsilon_2/\alpha_2-4\pi\chi_e}{\epsilon_1/\alpha_1+\epsilon_2/\alpha_2+4\pi\chi_e}\, e^{\alpha_1(z+z')}\right]; \tag{4.15}$$

(ii) $z,z' \in$ medium II:

$$g_f(z,z')=\frac{\alpha_2}{2q_0^2\epsilon_2}\left[e^{-\alpha_2|z-z'|} + \frac{\epsilon_2/\alpha_2-\epsilon_1/\alpha_1-4\pi\chi_e}{\epsilon_1/\alpha_1+\epsilon_2/\alpha_2+4\pi\chi_e}\, e^{-\alpha_2(z+z'-2d)}\right]; \tag{4.16}$$

(iii) $z \in$ medium I and $z' \in$ medium II:

$$g_f(z,z')=\frac{1}{q_0^2}\,\frac{1}{\epsilon_1/\alpha_1+\epsilon_2/\alpha_2+4\pi\chi_e}\, e^{\alpha_1 z}e^{-\alpha_2(z'-d)}; \tag{4.17}$$

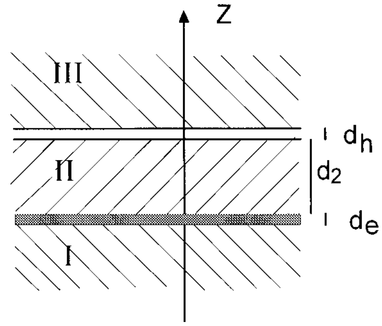

FIG. 2. The schematics of a double-inversion-layer system—a 2DEG (shaded region) and a 2DHG (blank region) separated by medium II of thickness d_2. The symbols d_e and d_h refer to the thickness of 2DEG and 2DHG layers, respectively.

(iv) $z \in$ medium II and $z' \in$ medium I:

$$g_f(z,z')=\frac{1}{q_0^2}\,\frac{1}{\epsilon_1/\alpha_1+\epsilon_2/\alpha_2+4\pi\chi_e}\, e^{\alpha_1 z'}e^{-\alpha_2(z-d)}. \tag{4.18}$$

Within the framework of IRT, one can build up two new systems depicting semi-infinite and finite systems. These systems can further serve the purpose of our building blocks to construct a double inversion layer and a unit cell of type-II superlattice systems.

V. DOUBLE INVERSION LAYERS

To be precise, we start with a system, as depicted in Fig. 2, with four interfaces delinking the five media. Ultimately, we will go to the limit $d_e,d_h\rightarrow 0$, which implies a double inversion layer system. The inverse of the response function of such a system, prior to taking the above-mentioned limits, can be written by adding $\overleftrightarrow{\mathbf{g}}_{si}^{-1}(M,M)$ of the different layers and the semi-infinite media. The result is

$$\overleftrightarrow{\mathbf{g}}_d^{-1}(M,M)=q_0^2\begin{pmatrix} -\alpha_1C_1-\alpha_eC_e\coth(\theta_e) & \alpha_eC_e\,\mathrm{csch}(\theta_e) & 0 & 0 \\ \alpha_eC_e\,\mathrm{csch}(\theta_e) & -\alpha_eC_e\coth(\theta_e)-\alpha_2C_2\coth(\theta_2) & \alpha_2C_2\,\mathrm{csch}(\theta_2) & 0 \\ 0 & \alpha_2C_2\,\mathrm{csch}(\theta_2) & -\alpha_2C_2\coth(\theta_e)-\alpha_hC_h\coth(\theta_h) & \alpha_hC_h\,\mathrm{csch}(\theta_h) \\ 0 & 0 & \alpha_hC_h\,\mathrm{csch}(\theta_h) & -\alpha_hC_h\coth(\theta_h)-\alpha_3C_3 \end{pmatrix}, \tag{5.1}$$

where $C_i=-q_0^2\epsilon_i/\alpha_i^2$, $i\equiv 1,e,h,2,3$, and $\theta_j=\alpha_j d_j$; $j\equiv e,h,2$. It is straightforward and simple algebra to calculate the inverse and the determinant of $\overleftrightarrow{\mathbf{g}}_d^{-1}(M,M)$. The pertinent question at this stage is whether to take the limits $d_e,d_h\rightarrow 0$ before or after calculating the inverse of $\overleftrightarrow{\mathbf{g}}_d^{-1}(M,M)$. A careful analytical diagnosis, however, proves that the two alternatives turn out to be exactly identical.

Let us first take the limit $d_e,d_h\rightarrow 0$ and consequently write Eq. (5.1) to the first order in corrections. Then we calculate the $\det|\overleftrightarrow{\mathbf{g}}_d^{-1}(M,M)|$ to the first order. In view of the limits imposed, the most significant and predominant terms are those proportional to $\epsilon_i/\alpha_i^2 d_i (i\equiv e,h)$. As such, we obtain

$$\det|\overleftrightarrow{\mathbf{g}}_d^{-1}(M,M)|=q_0^8\,\frac{\epsilon_e}{\alpha_e^2 d_e}\,\frac{\epsilon_h}{\alpha_h^2 d_h}\left\{\left[\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}\coth(\theta_2)+4\pi\chi_e\right]\left[\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_2}{\alpha_2}\coth(\theta_2)+4\pi\chi_h\right]-\left[\frac{\epsilon_2}{\alpha_2}\operatorname{csch}(\theta_2)\right]^2\right\}. \tag{5.2}$$

The dispersion relation defined by $\det|\overleftrightarrow{\mathbf{g}}_d^{-1}(M,M)|=0$ is given by

$$\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}\coth(\theta_2)+4\pi\chi_e\right)\left(\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_2}{\alpha_2}\coth(\theta_2)+4\pi\chi_h\right)-\left(\frac{\epsilon_2}{\alpha_2}\operatorname{csch}(\theta_2)\right)^2=0. \tag{5.3}$$

This is exactly the same as Eq. (7) of Kushwaha,[7] and thus represents the plasmon dispersion relation in the double-inversion-layer system. Equation (5.3) subjected to the limit $d_2\to\infty$ reproduces two independent plasmon modes supported by 2DEG and 2DHG [see Eqs. (8) in Kushwaha[7]].

Now we calculate the inverse of $\overleftrightarrow{\mathbf{g}}_d^{-1}(M,M)$. The result is

$$\overleftrightarrow{g}_d(M,M)=\frac{1}{q_0^2\Delta}\begin{bmatrix}
\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_2}{\alpha_2}ctnh(\theta_2)+4\pi\chi_h & \frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_2}{\alpha_2}ctnh(\theta_2)+4\pi\chi_h & \frac{\epsilon_2}{\alpha_2}csch(\theta_2) & \frac{\epsilon_2}{\alpha_2}csch(\theta_2)\\
\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_2}{\alpha_2}ctnh(\theta_2)+4\pi\chi_h & \frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_2}{\alpha_2}ctnh(\theta_2)+4\pi\chi_h & \frac{\epsilon_2}{\alpha_2}csch(\theta_2) & \frac{\epsilon_2}{\alpha_2}csch(\theta_2)\\
\frac{\epsilon_2}{\alpha_2}csch(\theta_2) & \frac{\epsilon_2}{\alpha_2}csch(\theta_2) & \frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}ctnh(\theta_2)+4\pi\chi_e & \frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}ctnh(\theta_2)+4\pi\chi_e\\
\frac{\epsilon_2}{\alpha_2}csch(\theta_2) & \frac{\epsilon_2}{\alpha_2}csch(\theta_2) & \frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}ctnh(\theta_2)+4\pi\chi_e & \frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}ctnh(\theta_2)+4\pi\chi_e
\end{bmatrix} \tag{5.4}$$

where

$$\Delta=\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}\coth(\theta_2)+4\pi\chi_e\right)\left(\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_2}{\alpha_2}\coth(\theta_2)+4\pi\chi_h\right)-\left(\frac{\epsilon_2}{\alpha_2}\operatorname{csch}(\theta_2)\right)^2. \tag{5.5}$$

The complete response function in the whole space is now obtained from Eq. (4.7) with $\overleftrightarrow{\mathbf{g}}_f(M,M)$ replaced by $\overleftrightarrow{\mathbf{g}}_d(M,M)$, which allows us to study many situations, such as, for example, $z,z'\in$ I; $z,z'\in$ II; $z,z'\in$ III; $z(z')\in$ I, $z'(z)\in$ II; $z(z')\in$ II; $z'(z)\in$ III; $z(z')\in$ I, $z'(z)\in$ III, etc. For instance, for $z,z'\in$ I, we obtain

$$g_d(z,z')=\frac{\alpha_1}{2q_0^2\epsilon_1}\left[e^{-\alpha_1|z-z'|}+\frac{1}{\Delta}\left\{\left(\frac{\epsilon_1}{\alpha_1}-\frac{\epsilon_2}{\alpha_2}\coth(\theta_2)-4\pi\chi_e\right)\left(\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_2}{\alpha_2}\coth(\theta_2)+4\pi\chi_h\right)+\left[\frac{\epsilon_2}{\alpha_2}\operatorname{csch}(\theta_2)\right]^2\right\}e^{\alpha_1(z+z')}\right]. \tag{5.6}$$

Similarly, one can write down the complete response functions in other situations. We do not expand on this simple writing and leap ahead to the case of an infinite, periodic superstructure.

VI. AN INFINITE TYPE-II SUPERLATTICE

We consider a four-layer superlattice (see Fig. 3). Out of these four layers in the unit cell, we will finally take the limit $d_e,d_h\to 0$, and medium I≡II. The resulting superstructure will then represent a type-II superlattice in which the alternating 2DEG and 2DHG are embedded in a dielectric medium I≡II.

To start with, each layer of width d_i is labeled by the index $i(\equiv 1,2,e,h)$ within the unit cell designated by an index n. All the interfaces are taken to be parallel to the $\hat{x}$-$\hat{y}$ plane. This means that the $\hat{z}$ axis is the superlattice axis that observes the periodicity with a period $D=d_h+d_1+d_e+d_2$. We replace the $\hat{z}$ coordinate by two variables: (m,z) such that $m\equiv n,i$; $-\infty<n<+\infty$. The equivalent notations are $m\equiv(n,i)\equiv i+Nn$; N being the number of slabs within the unit cell. Also, there are two different ways to label one and the same interface:

$$(m,\overline{1})\equiv\begin{cases}(n,i,\overline{1})\equiv(n,i-1,1) & \text{if } i\neq 1\\ (n,1,\overline{1})\equiv(n-1,N,1) & \text{if } i=1.\end{cases}$$

The response function $\overleftrightarrow{\mathbf{g}}^{-1}(M,M)$ for an infinite superlattice is an infinite ''tridiagonal'' matrix. With the aid of the Bloch theorem, we can use the space $(\tilde{M},k\equiv q_z)$ instead of the infinite space (M). The space $(\tilde{M})$ contains only *four* states (i.e., four interfaces) in the unit cell. In Fig. 3, the four states are labeled by the encircled numbers 1, 2, 3, and 4. As such, we write

$$\overleftrightarrow{\mathbf{g}}^{-1}(\tilde{M},\tilde{M};k)=q_0^2\begin{pmatrix}\frac{\epsilon_h}{\alpha_h}\coth(\theta_h)+\frac{\epsilon_2}{\alpha_2}\coth(\theta_2) & -\frac{\epsilon_h}{\alpha_h}\mathrm{csch}(\theta_h) & 0 & -\frac{\epsilon_2}{\alpha_2}\mathrm{csch}(\theta_2)e^{-i\phi}\\ -\frac{\epsilon_h}{\alpha_h}\mathrm{csch}(\theta_h) & \frac{\epsilon_h}{\alpha_h}\coth(\theta_h)+\frac{\epsilon_1}{\alpha_1}\coth(\theta_1) & -\frac{\epsilon_1}{\alpha_1}\mathrm{csch}(\theta_1) & 0\\ 0 & -\frac{\epsilon_1}{\alpha_1}\mathrm{csch}(\theta_1) & \frac{\epsilon_1}{\alpha_1}\coth(\theta_1)+\frac{\epsilon_e}{\alpha_e}\coth(\theta_e) & -\frac{\epsilon_e}{\alpha_e}\mathrm{csch}(\theta_e)\\ -\frac{\epsilon_2}{\alpha_2}\mathrm{csch}(\theta_2)e^{i\phi} & 0 & -\frac{\epsilon_e}{\alpha_e}\mathrm{csch}(\theta_e) & \frac{\epsilon_e}{\alpha_e}\mathrm{ctnh}(\theta_e)+\frac{\epsilon_2}{\alpha_2}\coth(\theta_2)\end{pmatrix} \tag{6.1}$$

with $\phi=kD$. We now impose the limits $d_e,d_h\to 0$ to calculate the inverse of $\overleftrightarrow{\mathbf{g}}^{-1}(\tilde{M},\tilde{M};k)$, which is, in principle, needed to write the complete response functions. We first determine the determinant of $\overleftrightarrow{\mathbf{g}}^{-1}(\tilde{M},\tilde{M};k)$ to the first order in correction. In view of the limits imposed, the most significant terms are the those proportional to $\epsilon_i/\alpha_i^2 d_i$ $(i\equiv e,h)$. As such, we write

$$\det|g^{-1}(\tilde{M},\tilde{M};k)|=q_0^8\,\frac{\epsilon_e}{\alpha_e^2 d_e}\,\frac{\epsilon_h}{\alpha_h^2 d_h}\,\Delta_{SL}, \tag{6.2}$$

where

$$\Delta_{SL}=-2\,\frac{\epsilon_1}{\alpha_1}\,\frac{\epsilon_2}{\alpha_2}\,\frac{1}{S_1S_2}\left\{\cos(\phi)-\left[C_1C_2+\frac{1}{2}\,(4\pi\chi_e+4\pi\chi_h)\left(\frac{\alpha_2}{\epsilon_2}\,C_1S_2+\frac{\alpha_1}{\epsilon_1}\,C_2S_1\right)+\frac{1}{2}\,S_1S_2\left(\frac{\alpha_1\alpha_2}{\epsilon_1\epsilon_2}\,4\pi\chi_e4\pi\chi_h+\frac{\epsilon_1}{\alpha_1}\,\frac{\alpha_2}{\epsilon_2}+\frac{\epsilon_2}{\alpha_2}\,\frac{\alpha_1}{\epsilon_1}\right)\right]\right\}, \tag{6.3}$$

where the symbols C_i and S_i stand for $C_i=\cosh(\theta_i)$ and $S_i=\sinh(\theta_i)$. In the general situation considered hitherto, the dispersion relation for the collective (bulk) excitations in the type-II superlattice is given by $\Delta_{SL}=0$, which implies that the middle bracketed terms, in Eq. (6.3), equated to zero yields the desired dispersion relation.

Let us now consider the special case when the material layers I and II are identical. That means that $\epsilon_1=\epsilon_2=\epsilon$, $\alpha_1=\alpha_2=\alpha$, $d_1=d_2=d$; $\theta_1=\theta_2=\theta\Rightarrow S_1=S_2=S$, $C_1=C_2=C$, and $\phi=kD=2kd$. As such, Eq. (6.3) simplifies to

$$\Delta_{SL}=-2\left(\frac{\epsilon}{\alpha}\right)^2\frac{1}{S^2}\left\{\cos(2kd)-\left[\cosh(2\theta)+\frac{\alpha}{\epsilon}\,\sinh(2\theta)\times(2\pi\chi_e+2\pi\chi_h)+2\sinh^2(\theta)\left(\frac{\alpha}{\epsilon}\right)^2 2\pi\chi_e 2\pi\chi_h\right]\right\}. \tag{6.4}$$

The middle bracketed terms equated to zero yields, after some algebraic manipulation,

$$\left(1+\frac{2\pi\alpha}{\epsilon}\,\chi_e S_e\right)\left(1+\frac{2\pi\alpha}{\epsilon}\,\chi_h S_e\right)-\left(\frac{2\pi\alpha}{\epsilon}\right)^2\chi_e\chi_h S_h^2=0, \tag{6.5}$$

where the structure factors S_e and S_h are defined as follows:

$$S_e=\frac{\sinh(2\alpha d)}{\cosh(2\alpha d)-\cos(2kd)},\quad S_h=\frac{2\sinh(\alpha d)\cos(kd)}{\cosh(2\alpha d)-\cos(2kd)}. \tag{6.6}$$

Equation (6.5) is exactly identical to Eq. (11) of Kushwaha,[8] which was derived there by making use of the transfer matrix method employing messy boundary conditions. The apparent difference of sign in Eq. (6.5) and Eq. (11) of Kushwaha owes to the definitions of χ_e and χ_h.

A formal trick to derive the dispersion relation for the collective (bulk) modes for the type-I superlattice is to substitute $d'=2d$ and $\chi_h=0$ in Eq. (6.5). The result is

$$1+\frac{2\pi\alpha}{\epsilon}\chi_e S_e'=0 \tag{6.7}$$

where the structure factor S_e' is now defined as

$$S_e'=\frac{\sinh(\alpha d')}{\cosh(\alpha d')-\cos(kd')}. \tag{6.8}$$

Equation (6.7) is the desired dispersion relation for the type-I superlattice, and is exactly the same as Eq. (26) of Kushwaha.[8] Note that the period of the type-I superlattice is d'.

Next, we calculate the inverse of $\overleftrightarrow{\mathbf{g}}^{-1}(\tilde{M},\tilde{M},k)$. Taking the adjoint of the matrix of cofactors and dividing the resulting matrix by the determinant of $\overleftrightarrow{\mathbf{g}}^{-1}(\tilde{M},\tilde{M},k)$, Eq. (6.2), leaves us with

$$\overleftrightarrow{g}(\tilde{M},\tilde{M},k)=\frac{1}{-2[\cos(\phi)-\eta]}\begin{pmatrix} \frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_e q_0^2\frac{S_1S_2}{F_1F_2} & \frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_e q_0^2\frac{S_1S_2}{F_1F_2} & \frac{S_2}{F_2}+\frac{S_1}{F_1}e^{-i\phi} & \frac{S_2}{F_2}+\frac{S_1}{F_1}e^{-i\phi} \\ \frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_e q_0^2\frac{S_1S_2}{F_1F_2} & \frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_e q_0^2\frac{S_1S_2}{F_1F_2} & \frac{S_2}{F_2}+\frac{S_1}{F_1}e^{-i\phi} & \frac{S_2}{F_2}+\frac{S_1}{F_1}e^{-i\phi} \\ \frac{S_2}{F_2}+\frac{S_1}{F_1}e^{i\phi} & \frac{S_2}{F_2}+\frac{s_1}{F_1}e^{i\phi} & \frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_h q_0^2\frac{S_1S_2}{F_1F_2} & \frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_h q_0^2\frac{S_1S_2}{F_1F_2} \\ \frac{S_2}{F_2}+\frac{S_1}{F_1}e^{i\phi} & \frac{S_2}{F_2}+\frac{S_1}{F_1}e^{i\phi} & \frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_h q_0^2\frac{S_1S_2}{F_1F_2} & \frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_h q_0^2\frac{S_1S_2}{F_1F_2} \end{pmatrix} \tag{6.9}$$

where $F_j=q_0^2\epsilon_j/\alpha_j$ and

$$\eta=C_1C_2+\frac{1}{2}(4\pi\chi_e q_0^2+4\pi\chi_h q_0^2)\left(\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}\right)+\frac{1}{2}S_1S_2\left(\frac{1}{F_1F_2}4\pi\chi_e q_0^2 4\pi\chi_h q_0^2+\frac{F_1}{F_2}+\frac{F_2}{F_1}\right). \tag{6.10}$$

In the present form Eq. (6.9) will help draw a direct parallel between the following results and some earlier published ones.

After Fourier analyzing $\overleftrightarrow{\mathbf{g}}(\tilde{M},\tilde{M},k)$ and making use of the identity

$$\frac{1}{2}\int dk\,\frac{e^{iNkD}}{\cos(kD)-\eta}=\frac{\pi}{D}\,\frac{t^{|N|+1}}{t^2-1}, \tag{6.11}$$

where

$$t=\begin{cases}\eta-\sqrt{\eta^2-1}, & \eta>1\\ \eta\pm i\sqrt{1-\eta^2}, & -1<\eta<1\\ \eta+\sqrt{\eta^2-1}, & \eta<1,\end{cases} \tag{6.12}$$

we write

$$g(n,I,\overline{1};n',I,\overline{1})=-\left[\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_e q_0^2\frac{S_1S_2}{F_1F_2}\right]\frac{t^{|n-n'|+1}}{t^2-1}, \tag{6.13a}$$

$$g(n,I,\bar{1};n',I,1)=-\left[\frac{S_2}{F_2}\,t^{|n-n'|+1}+\frac{S_1}{F_1}\,t^{|n-n'-1|+1}\right]\frac{1}{t^2-1}, \tag{6.13b}$$

$$g(n,I,1;n',I,\bar{1})=-\left[\frac{S_2}{F_2}\,t^{|n-n'|+1}+\frac{S_1}{F_1}\,t^{|n-n'+1|+1}\right]\frac{1}{t^2-1}, \tag{6.13c}$$

$$g(n,I,1;n',I,1)=-\left[\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_h q_0^2\,\frac{S_1S_2}{F_1F_2}\right]\frac{t^{|n-n'|+1}}{t^2-1} \tag{6.13d}$$

or, in matrix form,

$$\overleftrightarrow{\mathbf{g}}(M_m,M_{m'})=-\frac{t}{t^2-1}\begin{bmatrix}\left(\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_e q_0^2\,\frac{S_1S_2}{F_1F_2}\right)t^{|n-n'|} & \frac{S_2}{F_2}\,t^{|n-n'|}+\frac{S_1}{F_1}\,t^{|n-n'-1|}\\ \frac{S_2}{F_2}\,t^{|n-n'|}+\frac{S_1}{F_1}\,t^{|n-n'+1|} & \left(\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_h q_0^2\,\frac{S_1S_2}{F_1F_2}\right)t^{|n-n'|}\end{bmatrix}. \tag{6.14}$$

Here $m\equiv n$,I; and $m'\equiv n'$,I. The other elements, for example, $\overleftrightarrow{\mathbf{g}}(M_{n\mathrm{II}};M_{n'\mathrm{II}})$, $\overleftrightarrow{\mathbf{g}}(M_{n\mathrm{I}};M_{n'\mathrm{II}})$, and $\overleftrightarrow{\mathbf{g}}(M_{n,\mathrm{II}};M_{n',\mathrm{I}})$, can be obtained by noticing the ''periodic transformation rules (PTR)'': $(n,\mathrm{II},1)\equiv(n,\mathrm{I},1)$ and $(n,\mathrm{II},1)\equiv(n+1,\mathrm{I},1)$.

Now it is necessary to conform the bulk response function, Eq. (2.6a), according to the geometrical configuration used in the superlattice system. This requires replacing z and z' by $zd_i/2$ and $z'd_i/2$, respectively; whence one can calculate $\overleftrightarrow{\mathbf{G}}_i(M,M)$, $\overleftrightarrow{\mathbf{G}}_{\perp}^{-1}(M,M)$, and hence $G_i(z,1)$, $G_i(z,\bar{1})$, $G_i(1,z')$, and $G_i(1,z')$, which have to be used later.

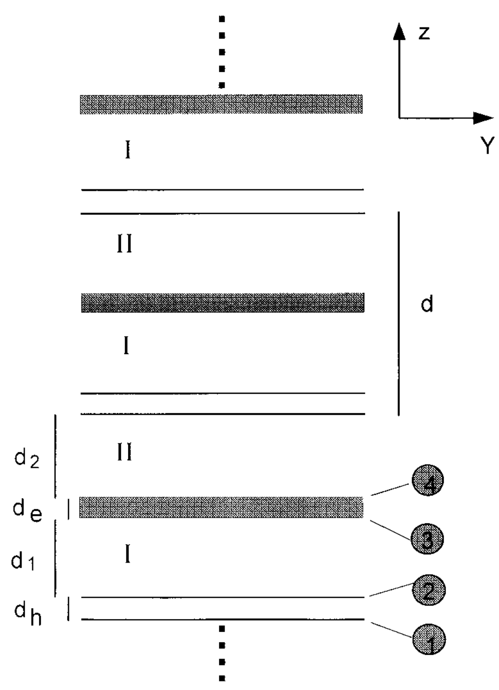

FIG. 3. The schematics of an infinite type-II superlattice system. The symbol d_i ($i\equiv1,2,e,h$; with $1,2,e,h$ referring to the I, II, shaded, and blank layers) stands for the thickness of the respective layer. Encircled numbers refer to the four states (i.e., four interfaces) in the unit cell belonging to the reduced space $\tilde{M}$. $D=d_h+d_1+d_e+d_2$ is the period of the superstructure. The letter $\hat{z}$ refers to the superlattice axis and $\hat{y}$ to the direction of propagation.

We now have everything at hand to write the complete response function in the whole space of eventually $N(=2)$-layer superlattice $\hat{g}(m,z;m',z')$ to be defined by[4]

$$\begin{aligned}\hat{g}(m,z;m',z')=\delta_{mm'}\Bigg\{&G_m(z,z')-[G_m(z,\bar{1}),G_m(z,1)]\\ &\times\overleftrightarrow{\mathbf{G}}_m^{-1}(M_m,M_m)\begin{bmatrix}G_m(\bar{1},z')\\ G_m(1,z')\end{bmatrix}\Bigg\}\\ &+[G_m(z,\bar{1}),G_m(z,1)]\overleftrightarrow{\mathbf{G}}_m^{-1}(M_m,M_m)\overleftrightarrow{\mathbf{g}}\\ &\times(M_m,M_{m'})\overleftrightarrow{\mathbf{G}}_{m'}^{-1}(M_{m'},M_{m'})\\ &\times\begin{bmatrix}G_{m'}(\bar{1},z')\\ G_{m'}(1,z')\end{bmatrix},\end{aligned} \tag{6.15}$$

which, after a few algebraic steps, leads us to write

$$\begin{aligned}\hat{g}(m,z;m',z')=\delta_{mm'}\Bigg\{&\frac{1}{2F_m}\,e^{(\theta_m/2)|z-z'|}-\frac{1}{2F_mS_m}\\ &\times\left[e^{-(\theta_m/2)(1+z)}\sinh\left\{\frac{\theta_m}{2}\,(1-z')\right\}\right.\\ &\left.+e^{-(\theta_m/2)(1-z)}\,\sinh\left\{\frac{\theta_m}{2}\,(1+z')\right\}\right]\Bigg\}\\ &+\frac{1}{S_mS_{m'}}\left[\sinh\left\{\frac{\theta_m}{2}\,(1-z)\right\},\right.\\ \left.\sinh\left\{\frac{\theta_m}{2}\,(1+z)\right\}\right]&\overleftrightarrow{\mathbf{g}}(M_m,M_m{}')\begin{bmatrix}\sinh\left\{\frac{\theta_{m'}}{2}\,(1-z')\right\}\\ \sinh\left\{\frac{\theta_{m'}}{2}\,(1+z')\right\}\end{bmatrix},\end{aligned} \tag{6.16}$$

where the 2×2 matrix $\overleftrightarrow{\mathbf{g}}(M_m,M_{m'})$ can be easily obtained through a careful diagnoses of Eqs. (6.14), using PTR. It should be pointed out that the formal analogy [considered between Eqs. (6.5) and (6.8)] leading to Eq. (6.7) can easily convert the results following Eq. (6.9) corresponding to

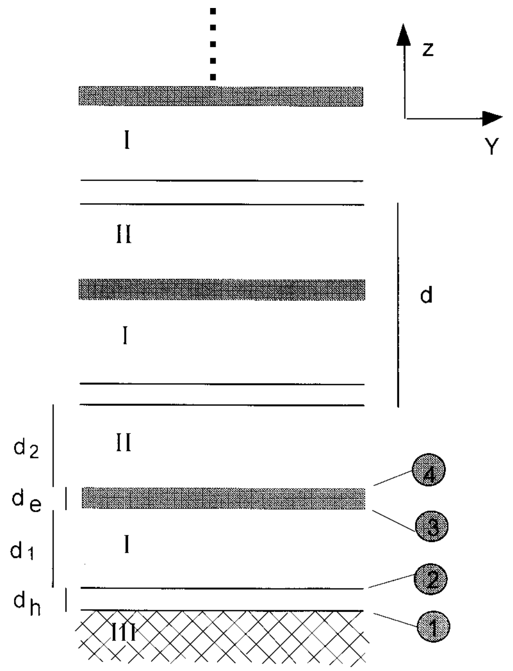

FIG. 4. The schematics of a truncated (at the surface $z=0$) type-II superlattice system. The truncation of an infinite superlattice (Fig. 3) results into a semi-infinite superstructure in which the region $-\infty \leqslant z \leqslant 0$ is filled with a medium III. The surface $z=0$ in the framework of IRT is defined by $(n,i,z)\equiv(0,\mathrm{I},\bar{1})\equiv(\bar{1},\mathrm{II},1)$.

those for the type-I superlattice. In what follows, we will concentrate on the practically realistic situation of truncated (semi-infinite) superlattices.

VII. A TRUNCATED TYPE-II SUPERLATTICE

We consider an infinite type-II superlattice truncated at an interface $z=0$, such that we have a semi-infinite superlattice with a different medium (III) in the region $-\infty \leqslant z \leqslant 0$ (Fig. 4). Let us assign a subscript 3 to the quantities in this medium. The surface $z=0[\equiv(0,\mathrm{I},\bar{1})\equiv(\bar{1},\mathrm{II},1)]$ perturbs an otherwise periodic superstructure. This perturbation in the framework of IRT is accounted for by a cleavage operator that removes the layer $m[\equiv(n,i)\equiv(\bar{1},\mathrm{II})]$ and a filling operator that fills in the negative half space $(-\infty \leqslant z \leqslant 0)$ with a medium III. The cleavage operator for the existing situation is defined as follows:

$$\overleftrightarrow{\mathbf{V}}_c(\bar{M},\bar{M})=\overleftrightarrow{\mathbf{V}}_f-\begin{bmatrix} g_{s2}^{-1}(\bar{1},\bar{1}) & g_{s2}^{-1}(\bar{1},1)\\ g_{s2}^{-1}(1,\bar{1}) & g_{s2}^{-1}(1,1)\end{bmatrix}$$

$$=\overleftrightarrow{\mathbf{V}}_f-q_0^2\begin{bmatrix} \frac{\epsilon_2}{\alpha_2}\frac{C_2}{S_2} & -\frac{\epsilon_2}{\alpha_2}\frac{1}{S_2}\\ -\frac{\epsilon_2}{\alpha_2}\frac{1}{S_2} & \frac{\epsilon_2}{\alpha_2}\frac{C_2}{S_2}\end{bmatrix}, \tag{7.1}$$

where the filling operator

$$\overleftrightarrow{\mathbf{V}}_f=q_0^2\begin{bmatrix} 0 & 0\\ 0 & \epsilon_3/\alpha_3\end{bmatrix}. \tag{7.2}$$

Equation (7.1) together with Eq. (7.2) can be cast in the form

$$\overleftrightarrow{\mathbf{V}}_c(\bar{M},\bar{M})=\begin{bmatrix} -C_2\frac{F_2}{S_2} & \frac{F_2}{S_2}\\ \frac{F_2}{S_2} & -C_2\frac{F_2}{S_2}+F_3\end{bmatrix}. \tag{7.3}$$

Here $\bar{M}$ is the set of the interface $(n=\bar{1},\ \mathrm{II},\ \pm 1)$. We are interested in the response operator whose elements are $A(\bar{1},\mathrm{II},\pm 1;M_{m'})$, where M_m is any of the interfaces of the truncated superlattice. The response operator is given by

$$A(\bar{1},\mathrm{II},1;M_{m'})=V_c(\bar{1},\mathrm{II},1;\bar{1},\mathrm{II},\bar{1})g(\bar{1},\mathrm{II},\bar{1};M_{m'})+V_c(\bar{1},\mathrm{II},1;\bar{1},\mathrm{II},1)g(\bar{1},\mathrm{II},1;M_{m'}). \tag{7.4}$$

With the aid of PTR, Eq. (7.4) assumes the form

$$A(0,\mathrm{I},\bar{1};n',i',\bar{1})=V_c(\bar{1},\mathrm{II},1;\bar{1},\mathrm{II},\bar{1})g(\bar{1},\mathrm{I},1;n',i',\bar{1})+V_c(\bar{1},\mathrm{II},1;\bar{1},\mathrm{II},1)g(0,\mathrm{I},\bar{1};n',i',\bar{1}) \tag{7.5a}$$

and

$$A(0,\mathrm{I},\bar{1};n',i',1)=V_c(\bar{1},\mathrm{II},1;\bar{1},\mathrm{II},\bar{1})g(\bar{1},\mathrm{I},1;n',i',1)+V_c(\bar{1},\mathrm{II},1;\bar{1},\mathrm{II},1)g(0,\mathrm{I},\bar{1};n',i',1). \tag{7.5b}$$

From Eqs. (6.13), (7.3), and (7.5), we write (with $n,n'>0$)

$$A(0,\mathrm{I},\bar{1};n',\mathrm{I},\bar{1})\equiv\frac{t^{n'+1}}{t^2-1}Y_1, \tag{7.6}$$

where

$$Y_1=-t+C_1C_2+\frac{F_2}{F_2}S_1S_2+\frac{C_2S_1}{F_1}4\pi\chi_e q_0^2-F_3\left(\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_e q_0^2\frac{S_1S_2}{F_1F_2}\right). \tag{7.7}$$

Similarly,

$$A(0,\mathrm{I},\bar{1};n',\mathrm{I},1)\equiv\frac{t^{n'+1}}{t^2-1}Y_2, \tag{7.8}$$

where

$$Y_2=C_2-C_1t-\frac{S_1t}{F_1}4\pi\chi_h q_0^2-F_3\left(\frac{S_2}{F_2}+\frac{S_1t}{F_1}\right). \tag{7.9}$$

Combining Eqs. (7.6) and (7.8) yields

$$A(0,\mathrm{I},\bar{1};n',i',\bar{1})=\frac{t^{n'+1}}{t^2-1}Y_{i'};\quad i'\equiv 1,2. \tag{7.10}$$

In particular, the response operator in the truncating interface space (M_s,M_s) is given by

$$A(0,\mathrm{I},\bar{1};0,\mathrm{I},\bar{1})\equiv A(M_s,M_s)=\frac{t}{t^2-1}Y_1\equiv\frac{t}{t^2-1}(X-t), \tag{7.11}$$

where the symbol X is defined by $X=Y_1+t$ [see Eq. (7.7)]. Then

$$\Delta(M_s,M_s)=1+A(M_s,M_s)=\frac{t}{t^2-1}\frac{1}{Y_1}W, \quad (7.12)$$

where

$$W=X^2-2\eta X+1. \quad (7.13)$$

A rigorous but straightforward algebra leads us to obtain an explicit expression of W. The result is

$$\begin{aligned}W=&-\left(\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_e q_0^2\frac{S_1S_2}{F_1F_2}\right)\Bigg\{C_1S_2F_2+C_2S_1F_1\\&+C_1C_2(4\pi\chi_e q_0^2+4\pi\chi_h q_0^2)+\frac{F_2}{F_1}S_1S_2 4\pi\chi_h q_0^2\\&+\frac{C_1S_1}{F_1}4\pi\chi_e q_0^2\cdot 4\pi\chi_h q_0^2-F_3^2\left(\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}\right.\\&\left.+4\pi\chi_e q_0^2\frac{S_1S_2}{F_1F_2}\right)+F_3\Bigg[S_1S_2\left(\frac{F_2}{F_1}-\frac{F_1}{F_2}\right)\\&+\left(\frac{C_2S_1}{F_1}-\frac{C_1S_2}{F_2}\right)4\pi\chi_e q_0^2\\&-\left(\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_e q_0^2\frac{S_1S_2}{F_1F_2}\right)4\pi\chi_h q_0^2\Bigg]\Bigg\}. \quad (7.14)\end{aligned}$$

Since the first factor is independent of the surface, the second factor (inside the middle bracket) equated to zero yields the general dispersion relation for the plasmon polaritons in the truncated type-II superlattice.

In the limit of $\chi_e=0=\chi_h$, Eq. (7.14) reduces to the form

$$\begin{aligned}W=&-F_2\left(\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}\right)\Bigg[C_2S_1\left(\frac{F_1}{F_2}-\frac{F_3^2}{F_1F_2}\right)\\&+C_1S_2\left(1-\frac{F_2^3}{F_2^2}\right)-\frac{F_3}{F_2}\left(\frac{F_1}{F_2}-\frac{F_2}{F_1}\right)S_1S_2\Bigg]. \quad (7.15)\end{aligned}$$

This is exactly the same as Eq. (104) of Kushwaha and Djafari-Rouhani.[9]

Let us now look for the limit when medium I and medium II are identical [see the paragraph preceding Eq. (6.4)]. As a consequence of this limit the general dispersion relation for plasmon polaritons [i.e., the middle bracket in Eq. (7.14) equated to zero] assumes the following form:

$$\begin{aligned}&\left(1+\frac{4\pi\alpha}{\epsilon}\chi_h\coth(\theta')\right)\left(1+\frac{2\pi\alpha}{\epsilon}\chi_e\coth(\theta)\right)\\&-\left(\frac{2\pi\alpha}{\epsilon}\right)^2\chi_e x_h\operatorname{csch}^2(\theta)\\&=\left(\frac{\alpha\epsilon_3}{\epsilon\alpha_3}\right)\left(\frac{\alpha\epsilon_3}{\epsilon\alpha_3}+\frac{4\pi\alpha}{\epsilon}\chi_h\right)\\&\times\left(1+\frac{2\pi\alpha}{\epsilon}\chi_e\tanh(\theta)\right). \quad (7.16)\end{aligned}$$

Note that this is exactly the same as Eq. (22) of Kushwaha,[8] provided that we (i) interchange χ_e and χ_h and (ii) replace χ_i by $-\chi_i (i\equiv e,h)$. The point (ii) has the same origin as explained with reference to Eq. (6.5), whereas the point (i) refers to the difference that our superlattice in the present work terminates at the 2DHG layer instead of at the 2DEG layer as was the case in Ref. 8.

Substituting $\chi_e=0$ in Eq. (7.16) and then replacing χ_h by χ_e (see the remarks made in the preceding paragraph) yields

$$1+\frac{4\pi\alpha}{\epsilon}\chi_e\coth(\theta')=\left(\frac{\alpha\epsilon_3}{\epsilon\alpha_3}\right)\left(\frac{\alpha\epsilon_3}{\epsilon\alpha_3}+\frac{4\pi\alpha}{\epsilon}\chi_e\right). \quad (7.17)$$

This is exactly the same as Eq. (31) of Kushwaha[8] (with $\chi_e\to-\chi_e$) and represents the dispersion relation of plasmon polaritons in the truncated type-I superlattice to include the effect of retardation. Here $\theta'=\alpha d'=2\alpha d$; with d' as the period of infinite type-I superlattice.

Next, we calculate the basic elements of the Green function $\overleftrightarrow{\mathbf{d}}(M_m,M_{m'})$, which would eventually lead one to calculate the complete response function $\hat{d}(D,D)$ in the whole space. The interface response theory[4] provides us with

$$\begin{aligned}\overleftrightarrow{\mathbf{d}}(M_m,M_{m'})=&\overleftrightarrow{\mathbf{g}}(M_m,M_{m'})\\&-\mathbf{g}(M_m,M_s)\overleftrightarrow{\Delta}^{-1}(M_s,M_s)\overleftrightarrow{\mathbf{A}}(M_s,M_{m'}). \quad (7.18)\end{aligned}$$

This together with Eqs. (6.13), (7.10), and (7.12) gives us all the interface elements of the basic response function $\overleftrightarrow{\mathbf{d}}$ of the semi-infinite superlattice at hand. The results, for $n,n'>0$, are

$$d(n,\mathrm{I},\bar{1};n',\mathrm{I},\bar{1})=-\frac{t}{t^2-1}Z_1\left[t^{|n-n'|}-\frac{Y_1^2}{W}t^{n+n'}\right], \quad (7.19a)$$

$$\begin{aligned}d(n,\mathrm{I},\bar{1},n',\mathrm{I},1)=&-\frac{t}{t^2-1}\Bigg\{\left[\frac{S_2}{F_2}t^{|n-n'|}+\frac{S_1}{F_1}t^{|n-n'-1|}\right]\\&-Z_1\frac{Y_1Y_2}{W}t^{n+n'}\Bigg\}, \quad (7.19b)\end{aligned}$$

$$\begin{aligned}d(n,\mathrm{I},1;n',\mathrm{I},\bar{1})=&-\frac{t}{t^2-1}\Bigg\{\left[\frac{S_2}{F_2}t^{|n-n'|}+\frac{S_1}{F_1}t^{|n-n'+1|}\right]\\&-\left(\frac{S_2}{F_2}+\frac{S_1t}{F_1}\right)\frac{Y_1^2}{W}t^{n+n'}\Bigg\}, \quad (7.19c)\end{aligned}$$

$$\begin{aligned}d(n,\mathrm{I},1;n',\mathrm{I},1)=&-\frac{t}{t^2-1}\Bigg\{Z_2t^{|n-n'|}-\left(\frac{S_2}{F_2}+\frac{S_1t}{F_1}\right)\\&\times\frac{Y_1Y_2}{W}t^{n+n'}\Bigg\}, \quad (7.19d)\end{aligned}$$

where Z_i stands for

$$Z_{1(2)}=\frac{C_1S_2}{F_2}+\frac{C_2S_1}{F_1}+4\pi\chi_{e(h)}q_0^2\frac{S_1S_2}{F_1F_2}. \quad (7.20)$$

In a compact form, Eqs. (7.19) can be cast in the form

$$\overleftrightarrow{\mathbf{d}}(M_m,M_{m'})=\begin{bmatrix} d(n,\mathrm{I},\overline{1};n',\mathrm{I},\overline{1}) & d(n,\mathrm{I},\overline{1};n',\mathrm{I},1) \\ d(n,\mathrm{I},1;n',\mathrm{I},\overline{1}) & d(n,\mathrm{I},1;n',\mathrm{I},1)\end{bmatrix}. \tag{7.21}$$

Here $m\equiv n,\mathrm{I}$ and $m'\equiv n',\mathrm{I}$. The other elements, for example, $\overleftrightarrow{\mathbf{d}}(M_{n,\mathrm{II}};M_{n',\mathrm{II}})$, $\overleftrightarrow{\mathbf{d}}(M_{n,\mathrm{I}};M_{n',\mathrm{II}})$, and $\overleftrightarrow{\mathbf{d}}(M_{n,\mathrm{II}};M_{n',\mathrm{I}})$, can be written with the aid of the PTR as stated before. With these results in hand, the application of the following general equation, analogous to Eq. (6.15),

$$\begin{aligned}\hat{d}(m,z;m',z') = \delta_{mm'}\Bigg\{ & G_m(z,z')-[G_m(z,\overline{1}),G(z,1)] \\ & \times\overleftrightarrow{\mathbf{G}}_m^{-1}(M_m,M_m)\begin{bmatrix}G_m(\overline{1},z')\\ G_m(1,z')\end{bmatrix}\Bigg\} \\ & +[G_m(z,\overline{1}),G_m(z,1)]\overleftrightarrow{\mathbf{G}}_m^{-1}(M_m,M_m)\overleftrightarrow{\mathbf{d}} \\ & \times(M_m,M_{m'})\overleftrightarrow{\mathbf{G}}_{m'}^{-1}(M_{m'},M_{m'}) \\ & \times\begin{bmatrix}G_{m'}(\overline{1},z')\\ G_{m'}(1,z)\end{bmatrix},\end{aligned} \tag{7.22}$$

provides us with the expression of the complete response function for the semi-infinite type-II superlattice systems. In Eq. (7.22), just as in Eq. (6.15), $\overleftrightarrow{\mathbf{G}}_m^{-1}(M_m,M_m)$ is the matrix inverse of the bulk response function $\overleftrightarrow{\mathbf{G}}_m(M_m,M_m)$. The elements of the rectangular matrices are calculated similar to those in Eq. (6.16). As such, the complete response function for the truncated type-II superlattice can be written as

$$\begin{aligned}\hat{d}(m,z;m',z') = \delta_{mm'}\Bigg\{ & \frac{1}{2F_m}\,e^{-(\theta_m/2)|z-z'|}-\frac{1}{2F_mS_m} \\ & \times\Bigg[e^{-(\theta_m/2)(1+z)}\sinh\Big\{\frac{\theta_m}{2}\,(1-z')\Big\} \\ & +e^{-(\theta_m/2)(1-z)}\sinh\Big\{\frac{\theta_m}{2}\,(1+z')\Big\}\Bigg]\Bigg\} \\ & +\frac{1}{S_mS_{m'}}\Bigg[\sinh\Big\{\frac{\theta_m}{2}\,(1-z)\Big\}, \\ \sinh\Big\{\frac{\theta_m}{2}\,(1+z)\Big\}\overleftrightarrow{\mathbf{d}}(M_m,M_{m'}) & \begin{bmatrix}\sinh\Big\{\frac{\theta_{m'}}{2}\,(1-z')\Big\}\\ \sinh\Big\{\frac{\theta_{m'}}{2}\,(1+z')\Big\}\end{bmatrix}.\end{aligned} \tag{7.23}$$

Since we already know the bulk response function, its inverse, and other specific forms, the problem that remains is the determination of $\overleftrightarrow{\mathbf{d}}(M_m,M_{m'})$ in order to calculate the complete response function $\hat{d}$. This is albeit a rigorous but not a very complicated problem once the $\overleftrightarrow{\mathbf{g}}(M_m,M_{m'})$ of an infinite superlattice are known. Equation (7.23) as such corresponds to the situation when both of the points (z,z') lie inside the superstructure. There are, however, other situations, for example, where both points lie in the homogeneous medium $(-\infty\leqslant z\leqslant 0)$, and where one point is inside the superlattice and other in the homogeneous medium. In such cases, Eq. (7.23) assumes essentially different forms depending upon the bulk response functions conformed to the respective situations. The reader is reminded of the fact that the above response function $\hat{d}$ is only the $\hat{y}$-$\hat{y}$ element. The other *three* elements can be easily obtained by making use of the constitutive relations [see, for example, Eqs. (3.6)], which are interweaved in Eq. (3.5). This remark is also valid for $\hat{g}$ in Sec. VI as well as for other response functions in Secs. III–V. Finally, we should point out that the same analogy as used to obtain Eq. (7.17) can easily lead one to reduce the results following Eq. (7.18) corresponding to those for truncated type-I superstructure.

VIII. CONCLUDING REMARKS

In this section, as stated earlier, we comment on the analytical formulation of IRT generalized to several 2D semiconductor structures. One of the notable features of IRT is that its framework, for any composite layered structure, does not require the messy electromagnetic boundary conditions and it represents the required results in an elegantly compact form. We particularly refer to analytical results obtained in Eqs. (3.16), (4.12), (5.3), (6.5), (6.7), (7.16), and (7.17). The obtainment of these previously well-established results for the respective geometries not only emboldens our confidence in the framework of IRT but also demonstrates how readily they can be arrived at. It is worth mentioning that most of the analytical results in the present work are independent of any particular model. For instance, one can always include the collisional damping and spatial dispersion by allowing the imaginary parts in the dielectric function and by making use of the hydrodynamical model, respectively. One can also incorporate the frequency dependence of the background dielectric constants, which allows the coupling of plasmons to optical phonons. It is worthwhile to do numerical computation by treating the frequency or propagation vector as a complex variable. Doing so will help one investigate the lifetime and propagation length of the plasmons in a given system. The calculation of the inverse penetration depth (λ) may lead one to determine the limitations of certain experimental techniques; for example, low-energy electron spectroscopy, which may not serve a useful purpose if λ is very small.

Besides studying the numerous propagation characteristics of the plasmons in superlattice systems, we are basically motivated in the framework of IRT to study the local and/or total density of states (DOS) of these modes. The DOS of the concerned modes in a given system can be obtained directly from the imaginary part of the respective response functions. Let us focus on the plasmon polaritons in the truncated superlattice systems. In these systems the accumulation of interfaces gives rise to peculiar electromagnetic modes distributed as continuous frequency (bulk) bands. The truncation of the superlattice system at the surface modifies the DOS of these modes, as compared to the mode density of an otherwise truly periodic system. In particular, one finds the isolated branches appearing above, below, and between the bulk bands. The specific spatial location of these plasmon-polariton modes in the ω-q space greatly depends on the ratio of the background dielectric constants of the truncating medium and the first inner layer in the system. The knowledge of the DOS provides complete information on the allowed plasmon excitations as a function of frequency or propagation vector at any depth in the superlattice systems. As a result, one can elucidate the infrared optical experiments performed on the moderately thick-layered superstructures.

The IRT generalized to be applicable to the 2D semiconductor structures in this work is, to our knowledge, the first theoretical formulation of its kind for investigating plasmon excitations in numerous composite systems. We currently have this response theory for similar structures subjected to an external magnetic field in the Voigt geometry at hand and the results will be reported shortly. The illustrative numerical examples of the present work are deferred to a future publication.

ACKNOWLEDGMENTS

One of us (B.D.R.) would like to acknowledge the hospitality of IFUAP and financial assistance from CONACyT (Mexico) during the short visit to IFUAP. The work of M.S.K. was partially supported by CONACyT, Grant No. 2373-PE. M.S.K. expresses his gratitude to the ''Ministere de l'Education, de la Researche et de la Technologie,'' and to the ''Conseil Regional Nord-Pas de Calais'' for financial support.

[1] L. Esaki and R. Tsu, IBM J. Res. Dev. **14**, 61 (1970).

[2] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1882).

[3] A. Tselis and J. J. Quinn, Phys. Rev. B **29**, 3318 (1984); J. K. Jain and P. B. Allen, *ibid.* **32**, 997 (1985); R. D. King-Smith and J. C. Inkson, *ibid.* **33**, 5489 (1986); R. F. Wallis and J. J. Quinn, *ibid.* **38**, 4205 (1988); M. S. Kushwaha, *ibid.* **41**, 5602 (1990); K. I. Golden and G. Kalman, *ibid.* **45**, 5834 (1992); M. S. Kushwaha, J. Appl. Phys. **73**, 792 (1993); K. I. Golden and G. Kalman, Phys. Rev. B **52**, 14 719 (1995).

[4] L. Dobrzynski, Surf. Sci. Rep. **6**, 119 (1986); **11**, 139 (1990); M. L. Bah, A. Akjouj, and L. Dobrzynski, *ibid.* **16**, 95 (1992).

[5] M. G. Cottam and A. A. Maradudin, in *Surface Excitations, Modern Problems in Condensed Mather Sciences Vol. 9*, edited by A. A. Maradudin and V. M. Agranovich (North-Holland, Amsterdam, 1986), p.1; see also M. Babiker, N. C. Constantinou, and M. G. Cottam, J. Phys. C **20**, 4581 (1987); M. S. Kushwaha and B. Djafari-Rouhani, Surf. Sci. **244**, 336 (1991); **268**, 457 (1992).

[6] K. L. Kliewer and R. Fuchs, Adv. Chem. Phys. **27**, 355 (1974).

[7] M. S. Kushwaha, Solid State Commun. **67**, 993 (1988).

[8] M. S. Kushwaha, J. Appl. Phys. **62**, 1895 (1987); **65**, 3303(E) (1989).

[9] M. S. Kushwaha and B. Djafari-Rouhani, Phys. Rev. B **43**, 9021 (1991).