Electronic-transport properties of tight-binding multiring systems

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(Received 1 December 1997)

Systems consisting of open mesoscopic rings threaded by a magnetic flux Φ and connected in parallel and in series are studied within the tight-binding model. A recursion method, which allows one to deal with systems consisting of a large number of rings, is found for the calculation of transmission amplitutes. The numerical results show that for systems with rings connected in series and in parallel, the quantum coherence effect plays quite different roles. We found that as the number of rings increases, the magnetic flux Φ will progressively block the transmission through the system for parallel multiring systems (PRS), but not for the serial multiring systems (SRS). In the absence of magnetic flux, the transmission coefficient T in PRS decreases only slightly when the ring number N increases, while T in SRS decreases rapidly with an increasing number of rings. [S0163-1829(98)05120-0]

I. INTRODUCTION

In the last decade, rapid progress has been made in area of mesoscopic physics. Quantum transport in mesoscopic systems has been extensively studied both experimentally and theoretically.[1–22] For mesoscopic systems at very low temperatures the scattering due to phonons, which is a dephasing scattering, is significantly suppressed and the phase-coherence length of electrons becomes large compared to the system dimension. The scattering in the systems can then be modeled as phase coherent elastic scatterings. Furthermore, if we consider the electron as a free particle, an idealized sample becomes an electron waveguide, which assumes that the electron transport through the system is perfectly ballistic. In recent years, there are many works devoted to the study of the electronic properties of mesoscopic systems within the framework of the waveguide theory.[10,13–19] Along this line, the theoretical work to date has largely focused on the problems related to an isolated ring, and to open rings connected via leads to electronic reservoirs together with a magnetic flux Φ through the rings. For an isolated ring, the persistent current has been the focus of attention. The idea is based on the possibility that the electron wave function may extend coherently over the whole circumference of the ring, and elastic scatterings, finite temperature, and weak inelastic scatterings do not destroy the coherence. As for the open ring systems, the important problem is to study the relationship between the transmission coefficient T and incident electron energy and its wave vector. The electron reservoirs in the open-ring systems act as the source of energy dissipation or irreversibility, and all scattering processes in the leads and rings are assumed to be elastic. Based on the wave-guide theory, Xia[14] has studied the Aharonov-Bohm (AB) effect in an open single ring by calculating the transmission and reflection amplitudes as functions of the magnetic flux, the arm length, and the wave vector. Singha Deo and Jayannavar[16,17] have studied the quantum transport properties of serial stub or loop structures and the band formation in these geometries as well as the persistent current of an open ring. Takai and Ohta[19] have published a series of papers investigating similar problems in the presence of both an electrostatic potential and magnetic flux. Cahay, Bandyopadyay, and Grubin[10] have also studied the AB-type conductance oscillations in the presence of either a magnetic flux or an electrostatic potential. On the other hand, Wu and Mahler[13] have developed the quantum-network theory of transport, by which the transmission probability for an open AB-type ring with an arbitrary form factor has been studied in detail. All of the work stated above are based on the wave-guide theory, and they have presented a clear physical picture for the problems studied. On the other hand, it is well known that the tight-binding model is more flexible in theoretical treatments than the wave-guide theory as disorderness can be introduced readily and the band-structure effects are included.[23,24] However, work in the literature along this line is relatively few up to now. One of the reasons may be that the tight-binding method is slightly more complicated to deal with. Entin-Wohlman, Hartzstein, and Imry[9] and Kowal *et al.*[12] have studied the electronic transport properties of an open single ring. Aldea, Gartner, and Corcotoi[15] studied the same problems using the Green's function method. Liu and co-workers have investigated the persistent current of an isolated disordered ring,[20] the effects of spin interaction on the persistent current,[21] as well as the electronic transport properties of parallel double-ring system[22] in which the two rings are threaded by different magnetic fluxes.

In the present work, we study the electronic-transport properties within the tight-binding approach of the two basic structures in open multiring systems, i.e., the parallel multiring system (PRS) and serial multiring system (SRS). The structures are assumed to consist of identical rings threaded by a magnetic flux connected either in parallel or in series. Furthermore, we assume that each ring contains four sites

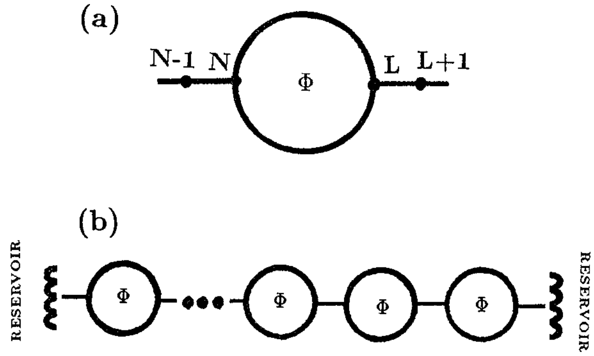

FIG. 1. (a) A single ring coupled to two reservoirs via two leads. (b) Multirings threaded by magnetic flux Φ, connected in series, and coupled to two reservoirs via ideal leads.

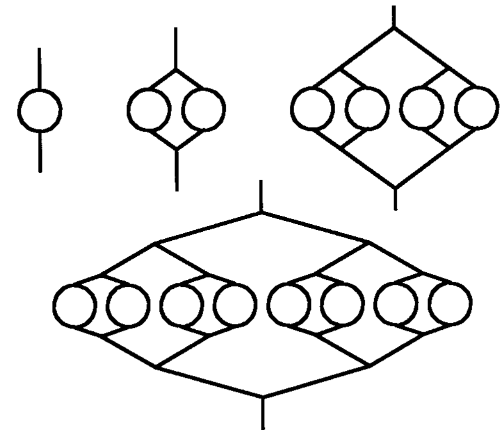

FIG. 2. Parallel multiring system in which only the rings are assumed to be threaded by a magnetic flux Φ. All of the rings are assumed to be identical.

that symmetrically distribute in the upper and lower arms. The main purpose is to investigate the behavior of the transmission coefficient T as the incident electron energy E and the magnetic flux Φ, which penetrates the mesoscopic rings, are varied. It is well known that the electronic transport properties of mesoscopic systems have a ''fingerprint'' character. For a tiny change of the system structure the transport behavior will largely change. Therefore, if the upper and lower arms of the rings are unsymmetrical, the transport behavior will be totally different.[13] Detailed results are given in three-dimensional plots of T against E and Φ. It is found that due to the coherence effect, the transmission coefficient T vanishes in both the PRS and SRS geometries when the flux Φ is closed to the value $\Phi_0/2$, where $\Phi_0 = hc/e$ is the fundamental flux quantum, regardless of the number of rings in the system. For the PRS geometry, the region corresponding to $T=0$ in the E-Φ space progressively increases as the number of rings increases, while it is not the case for the SRS geometry. Numerical results show that for the PRS, the transmission coefficient T drops to nearly zero for almost any value of the flux Φ when the number of rings is increased to such large number as $N=4096$. It means that the magnetic flux completely blocks out the electronic transport. This is an interesting quantum phenomenon. In the absence of magnetic flux, we have investigated the dependence of the transmission coefficient on the incident electron energy E for both the PRS and SRS. Our results show that as the number of rings increases in the SRS, T decreases rapidly. For the PRS, the transmission coefficient T decreases only slightly even for N increases to the value 1024. These results are discussed within a reasonable physical picture.

The paper is organized as follows. In Sec. II, we give the formalism for calculating the transmission coefficient of mesoscopic multiring systems. Numerical results for the transmission coefficient are presented in Sec. III. The effects of a magnetic flux Φ through the rings are studied and the sensitivity of the transmission coefficient to the number of rings is investigated. A summary is given in Sec. IV.

II. TRANSMISSION COEFFICIENT OF MESOSCOPIC MULTIRING SYSTEMS

We consider two basic configurations of mesoscopic multiring systems as shown in Figs. 1 and 2. The former shows a multiring system connected in series (SRS), and the latter a multiring system connected in parallel (PRS). In both cases, we assume that only the rings are threaded by a magnetic flux Φ. These two configurations form the basis of forming more complicated systems. In the following, we present the explicit expressions for calculating the transmission and reflection amplitudes of an open-single ring. Based on these expressions, the formulas for calculating the corresponding quantities in the two multiring configurations are derived. A recursive scheme is then proposed for systems with larger number of rings.

For an open single ring, i.e., a ring connected via two leads to two electron reservoirs, we assume that the leads and rings are composed of one-dimensional ordered chains with on-site energies ε_n and transfer integral J between nearest-neighboring sites. Denoting the electron energy by E, and the projection of the wave function on the nth site by ψ_n, the tight-binding equation can be written as

$$(\varepsilon_n - E)\psi_n = \sum_{n'} J_{n,n'}\psi_{n+n'}, \tag{1}$$

where the sum runs over the nearest neighbors of site n. For a single ring threaded by a magnetic flux Φ and connected via two leads to electron reservoirs [Fig. 1(a)], the reflection amplitude r and the transmission amplitude t are given by[12,22]

$$r = e^{2iqN}c - \frac{2K}{d}\,[b\cos\phi + a - e^{i\psi}(b^2-a^2)(b-a)], \tag{2}$$

and

$$t = 2K\,\frac{\cos(\phi/2)}{d}\,[(b-a)^2 - e^{-i\psi}], \tag{3}$$

where

$$\phi=2\pi\frac{\Phi}{\Phi_0},$$

$$\psi=2q(L-N)=qS,$$

$$a=e^{-2iqN}\left[\frac{2iJ\sin q}{D}-1\right],$$

$$b=e^{-2iqN}\frac{2iJ\sin q}{D},$$

$$c=e^{2iqN}\left[\frac{2iJ\sin q}{D}-1\right],$$

$$\sqrt{K}=\frac{2iJ\sin q}{D},$$

$$D=E-\varepsilon_n+3Je^{iq},$$

$$q=\arccos\left(\frac{-E+\varepsilon_n}{2J}\right),$$

$$d=2b^2\cos\phi-e^{-i\psi}-(b^2-a^2)^2e^{i\psi}+2a^2.$$

Here, N and L are the sites on the right and left sides of the ring at which the ring is coupled to the reservoirs via the leads and S is the circumference of the ring. Equations (2) and (3) can be applied to handle the multiring systems.

First, we consider the SRS case. In order to obtain the total transmission coefficient of N connected rings in series, we divide the system into two subsystems at a site n, which is usually chosen to be the site in the center of a lead connecting two neighboring rings. The subsystems are then referred to as the left subsystem and the right subsystem. In this work t and r are used to denote the transmission and reflection amplitudes for an incident wave from the left, while t' and r' are used for the transmission and reflection amplitudes of an incident wave from the right. We denote the transfer matrix of the entire system by τ. In general, τ can be expressed as[12]

$$\tau=\frac{1}{t'}\begin{pmatrix} tt'-rr' & r' \\ -r & 1\end{pmatrix}. \tag{4}$$

On the other hand, if we denote the transfer matrix of the left subsystem by τ_L, and that of the right by τ_R, the transfer matrix τ of the whole system is given by

$$\tau=\tau_R\tau_L. \tag{5}$$

Applying Eq. (4) for the left and right systems and substituting the results into Eq. (5), we obtain the total transmission and reflection amplitudes in terms of t_R, t_L, t'_R, t'_L, r_R, r_L, r'_R, and r'_L, where the subscripts label the left and right systems, as

$$t=\frac{t_Rt_L}{1-r_Rr'_L}, \tag{6}$$

$$r=\frac{(t_Lt'_L-r_Lr'_L)r_R+r'_L}{1-r_Rr_L}. \tag{7}$$

It should be pointed out that t, t', r, and r' satisfy the following relations:

$$t'=t, \quad r'=re^{-2iq(N+L)}. \tag{8}$$

For two rings connected in series forming a two-ring system, Eqs. (6)–(8) together with Eqs. (2) and (3) can be applied to calculate the total transmission and reflection amplitudes of the system if one ring is taken as the left subsystem and the other as the right subsystem. Similarly, if we take the above double-ring system as a whole to be the right subsystem, and add another single ring from the left as the left subsystem, we can obtain the total transmission and reflection amplitudes of a three-ring system by using Eqs. (6)–(8). Iterating the procedure and by repeatedly using Eqs. (6)–(8), the total transmission and reflection amplitudes of an N-ring system can be obtained by connecting one ring to an $(N-1)$-ring system. The transmission coefficient T and reflection coefficient R are then obtained from the corresponding amplitudes. This iterative scheme can be easily implemented numerically.

Next, we consider the PRS. In a previous paper,[22] we presented the formulation for calculating the transmission coefficient T of an open double-ring system connected in parallel and threaded by magnetic fluxes Φ_1 and Φ_2, respectively. The main idea is that the double-ring system can be reduced to a single-ring system by representing the two rings together with the flux through them in a double-ring system as scatterers. The transmission and reflection amplitudes of the scatterers are assigned so as to model the effects of the rings. The double-ring system can then be treated within the formulation of a single ring.[4,22] This approach can be readily extended to the parallel multiring systems as shown in Fig. 1(b) with a large number of rings. For example, if we represent the double-ring system by an effective scatterer, the parallel four-ring system shown in Fig. 1(b) can be reduced to a single ring with the effective scatterers in the two arms. Repeating the above procedure allows one to generate a hierarchy of parallel multiring system with progressively large number of rings. If we take the double-ring system as the first generation, then the ith generation of this hierarchy consists of $N_i=2^i$ rings. Calculations in these PRS are made possible by the observation that there exists simple recursive relations relating the transmission and reflection amplitudes of the $(i-1)$th generation and the ith generation. Hence, the calculations for a PRS simply amounts to iterating the recursive relations. For a parallel double-ring system threaded by the same magnetic flux Φ, i.e., the first generation of PRS, we obtain the following formulas for calculating the transmission and reflection amplitudes t and r:

$$t=\frac{1}{t_0|A|}\left[(P_{22}+P_{12})(t_0^2-r_0^2e^{i\theta}-r_0e^{-2iqL})-(P_{11}+P_{21})\times(r_0e^{i\theta}+e^{-2iqL})\right], \tag{9}$$

$$r=-1-\frac{1}{|A|}(P_{11}+P_{21}-P_{22}-P_{12}), \tag{10}$$

where t_0 and r_0 are the transmission and reflection amplitudes of a single ring, and can be calculated using Eqs. (2)

and (3). In Eq. (9), $|A|=P_{11}P_{22}-P_{12}P_{21}$ and P_{ij} are the elements of the 2×2 matrix given by

$$P_{11}=\frac{1}{b}-\frac{M_{11}}{bt_0^2},$$

$$P_{12}=\frac{-a}{b}-\frac{M_{12}}{bt_0^2}=-P_{21},$$

$$P_{22}=\frac{a+b}{b}-\frac{M_{22}}{bt_0^2},$$

with

$$M_{11}=-r_0^2e^{-2iqL}-2ar_0(t_0^2-r_0^2e^{i\theta})+(a+b)e^{2iqL}(t_0^2-r_0^2e^{i\theta})^2,$$

$$M_{12}=r_0e^{-2iqL}+a(t_0^2-2r_0^2e^{i\theta})+(a+b)e^{2iqL}(t_0^2-r_0^2e^{i\theta})r_0e^{i\theta},$$

$$M_{21}=-M_{12},$$

$$M_{22}=e^{-2iqL}-2ar_0e^{i\theta}-(a+b)e^{2iqL}r_0^2e^{i2\theta},$$

where $\theta=-2q(N+L)$. Following the iteration scheme for the PRS hierarchy, we have the following recursive relations for the transmission and reflection amplitudes between the $(i-1)$th and the ith generations:

$$t_i=\frac{1}{t_{i-1}|A|}\left[(P_{22}^{(i-1)}+P_{12}^{(i-1)})(t_{i-1}^2-r_{i-1}^2e^{i\theta}-r_{i-1}e^{-2iqL})-(P_{11}^{(i-1)}+P_{21}^{(i-1)})(r_{i-1}e^{i\theta}+e^{-2iqL})\right] \quad (11)$$

and

$$r_i=-1-\frac{1}{|A|}(P_{11}^{(i-1)}+P_{21}^{(i-1)}-P_{12}^{(i-1)}-P_{22}^{(i-1)}), \quad (12)$$

where $P_{ij}^{(i-1)}$ are expressed in terms of the transmission and reflection amplitudes of the $(i-1)$th generation of the hierarchy.

III. NUMERICAL RESULTS AND DISCUSSION

A. Transport properties of parallel multiring systems

The formalism can be easily implemented numerically and results for both PRS and SRS are obtained. In our calculations the on-site energies are chosen to be $\epsilon_n=0$ and the transfer integrals $J=-1.0$. Each ring is taken to consist of four sites,[12,22] one on each arm of the ring, and one on each of the two conjunctions to the leads. The length of the lead connecting neighboring rings is chosen to be one lattice unit. The plausibility of this assumption of ''one lattice unit'' lead comes mainly from such a fact that for the tight-binding model the spacing of site does not necessarily equal the spacing of atom. It means that the length of two neighboring sites is a modelized distance, and can be a piece of atom chain. In fact, in the derivation of transmission coefficient formulas used in the present paper we have considered that the sites mean a finite piece of the wire longer than the screening length as well as electron wavelength, and performed an averaging over a wavelength (Ref. 12). If we assume varied length of lead connecting ring clusters, on one hand, the results would be definitely different from the present ones, but on the other hand, the difference only reflects the effect of the lattice wave phase gained in the connecting leads on the electronic-transport behavior. Therefore, this difference is not essential. But, this varied length would bring in big technical difficulty; we cannot use the simple recursive formulas (11) and (12) for the transmission and reflection amplitudes between the $(i-1)$th and ith generation ring systems anymore. In this case, to perform a simulation for a system with 4096 rings will be a tedious and difficult work. Furthermore, even though we can also choose a united lead with finite but not varied length to connect the neighboring rings, compared with the case of ''one lattice unit'' lead the difference of numerical results also comes only from the effect of the lattice wave phase gained in the leads. In these two cases, it seems to us, there is no essential difference. For these reasons, the assumption of ''one lattice unit'' of lead may be crude, but may also be acceptable. To check the

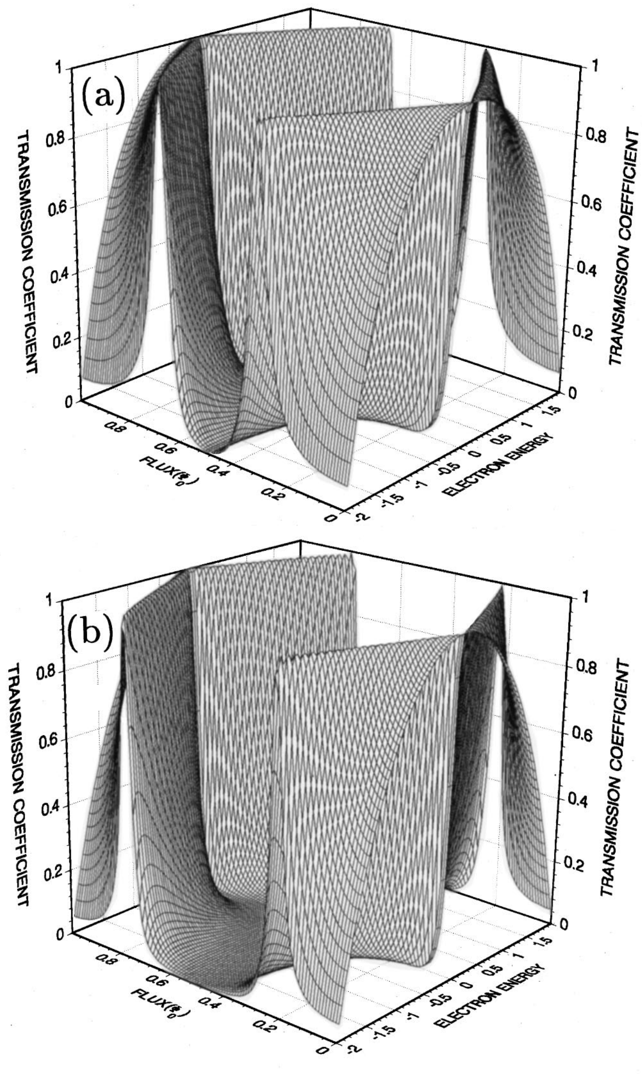

FIG. 3. Transmission coefficient T as a function of magnetic flux Φ and incident electron energy E for parallel multiring systems with (a) two rings and (b) four rings.

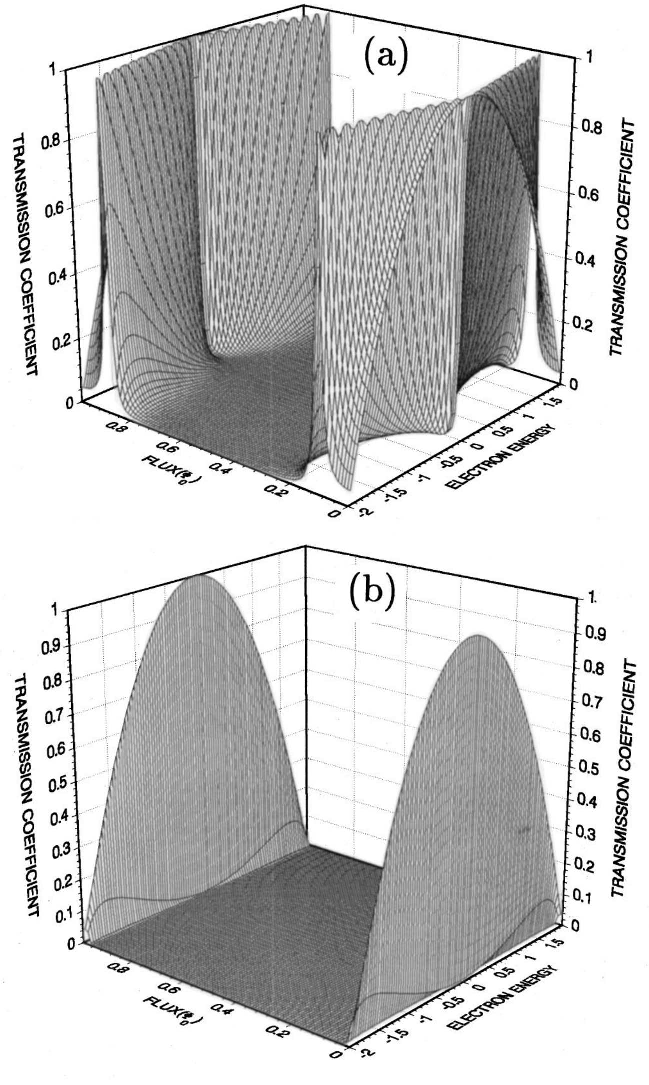

FIG. 4. Transmission coefficient T as a function of magnetic flux Φ and incident electron energy E for parallel multiring systems with (a) 16 rings, and (b) 4096 rings. Note that the region in the E-Φ plane corresponding to vanishing transmission increases as the number of rings increases.

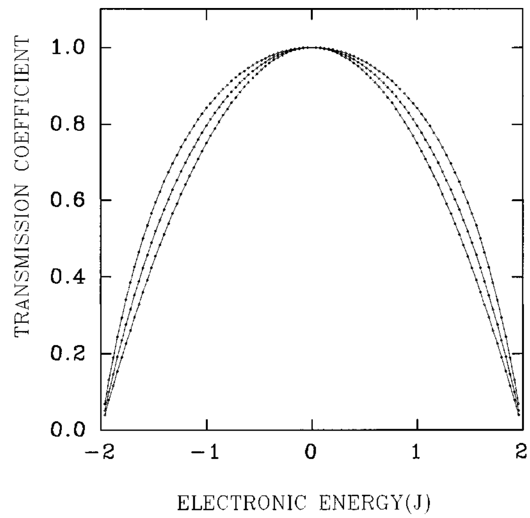

FIG. 5. Transmission coefficient T for parallel multiring systems as a function of incident energy in the absence of magnetic flux, i.e., $\Phi=0$. The three curves from top to bottom correspond to systems with 2 rings, 16 rings, and 1024 rings, respectively. The transmission coefficient decreases only slightly as the number of rings increases in PRS.

accuracy of our numerical calculations, we check at every intermediate stage of the calculation that the criterion $|t|^2+|r|^2=1$ for the transmission and reflection coefficients is satisfied to a tolerance of 10^{-14}. This accuracy enables us to examine with confidence the transport properties of the two fundamental configurations of a multiring systems.

Using the recursive relations Eqs. (11) and (12) of the PRS, we have studied the transport properties of systems with number of rings $N_i=2^i$ up to $i=12$. Typical plots of the transmission coefficient T against the electron energy E and magnetic flux Φ are shown in Fig. 3 for $i=1,2$ and in Fig. 4 for $i=4,12$. In the absence of magnetic flux, T has a single peak at $E=0$. For $\Phi\neq 0$, T has a double-peak structure as a function of E. It is due to the fact that the presence of a magnetic flux destroys the time-reversal symmetry and the paths going clockwise and counterclockwise over the ring have different phases. Another major feature is that for $\Phi=\Phi_0/2$, the transmission coefficient vanishes for all incident electron energy regardless of the number of rings in the system. This is consistent with the result for the transmission amplitude given in Eq. (3) for a single ring. This is a quantum-coherence effect and can be understood as follows. An incident electron wave in one junction, say site N, will separate into two traveling waves passing through the upper and lower arms of the ring into the other junction L, where they interference with each other. The amplitude squared of wave function at L reads[1,2]

$$|\psi|^2=|a_1e^{i\lambda_1}+a_2e^{i\lambda_2}|^2$$
$$=|a_1|^2+|a_2|^2+2|a_1^*a_2|\cos(\lambda_1-\lambda_2), \qquad (13)$$

where a_i $(i=1,2)$ are the amplitudes and λ_i $(i=1,2)$ are the phases for waves traveling through the upper and lower arms. In the case of a symmetric ring with two identical arms, the amplitudes are identical, while the phase difference is given by $(\lambda_1-\lambda_2)=e\Phi/\hbar c=2\pi\Phi/\Phi_0$. It is thus evident that if $\Phi=\Phi_0/2$, $|\psi|^2=0$. Note that the argument is independent of the incident electron energy and of the way that the rings are connected in a system. Hence, we expect this result to hold for both the PRS and SRS.

Another interesting feature exhibited in Figs. 3 and 4 is that, for given Φ, the region in the parameter space with vanishing transmission coefficient increases with increasing number of rings in PRS. For the 12th generation of the hierarchy with $N_{12}=4096$, for example, the transmission coefficient T (see Fig. 4) vanishes with almost any $\Phi\neq 0$. The magnetic flux, hence, completely blocked out the electronic transport. This result is at first sight unexpected. A plausible physical explanation is that destructive interference becomes the dominant effect as the system size increases. Increasing number of rings leads to many different path differences from one end of the system to another. This in turn leads to destructive interference at nearly continuous values of magnetic flux. Our numerical calculations show that for a system with 4096 rings (see Fig. 4), destructive interference occurs at nearly all finite values of Φ. We have also studied the

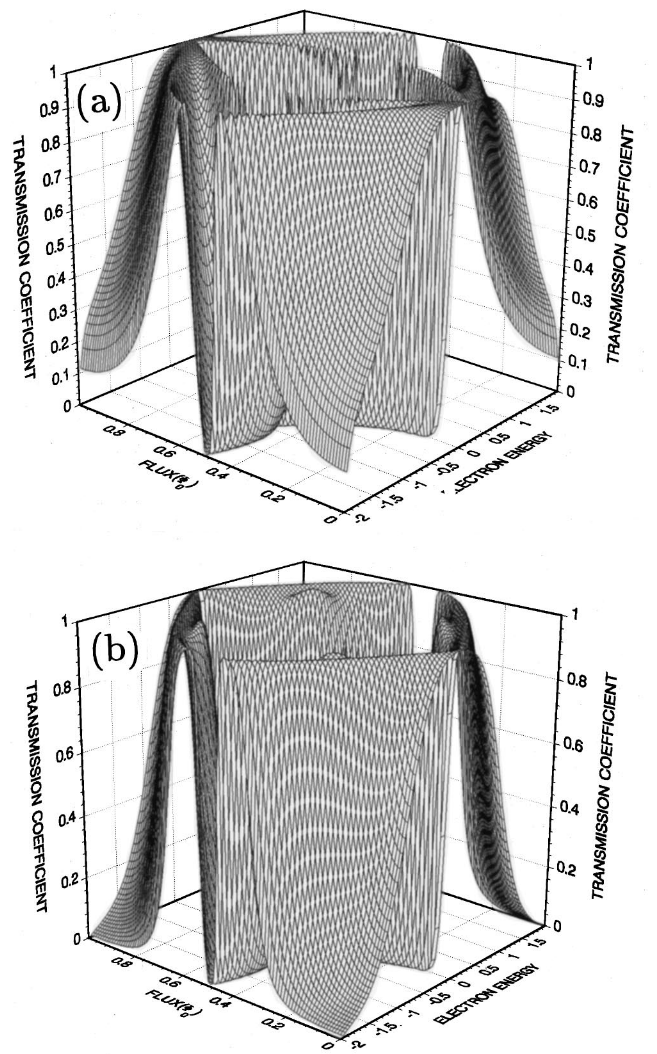

FIG. 6. Transmission coefficient T as a function of magnetic flux Φ and incident electron energy E for serial multiring systems with (a) two rings and (b) four rings.

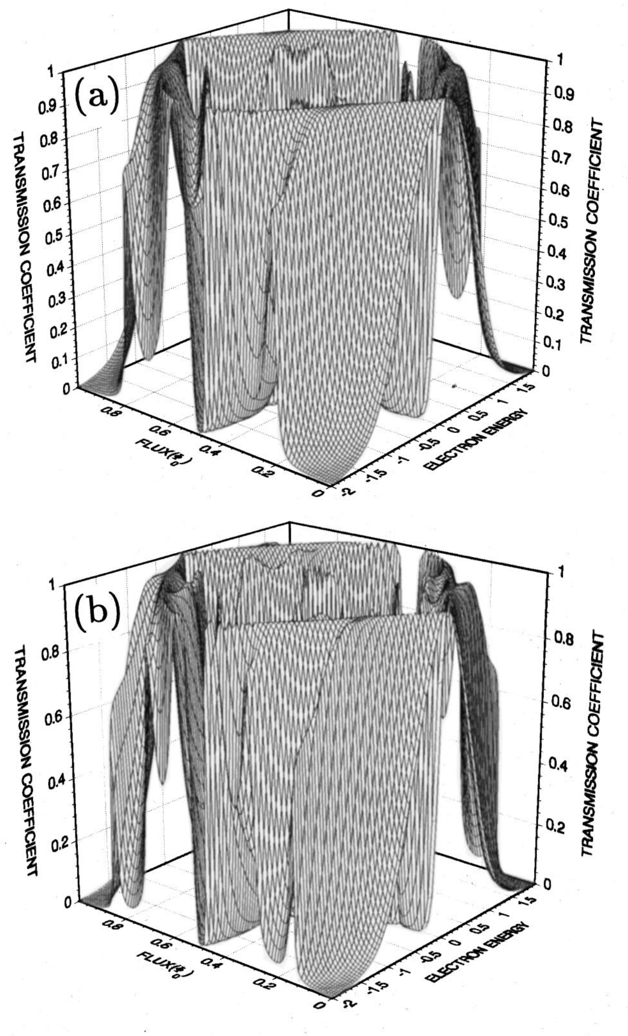

FIG. 7. Transmission coefficient T as a function of magnetic flux Φ and incident electron energy E for serial multiring systems with (a) six rings and (b) eight rings. Note the complicated structures in the transmission coefficient due to the coupling among the rings.

effect of increasing number of rings on the transmission coefficient for $\Phi=0$. Figure 5 shows the results for $N_i=2$, 4, 16, and 1024. It was found that the transmission coefficient decreases only slightly with respect to the result of the $N_1=2$ case as N_i increases with the maxima of the curves all coinciding. A simple-minded picture is that the wave, after entering the system, has equal amplitute passing through each ring in the PRS, and each identical ring has the same transmission amplitude. The total transmission amplitude is thus of the order of the transmission amplitute of a single-ring system. Therefore, for the $\Phi=0$ case the transmission coefficient is almost independent of the number of ring in the PRS. It should be pointed out that although the present PRS has some similarities to fractal systems and a Cayley tree,[25,26] the actual configurations of these systems are quite different.

B. Transport properties of serial multiring systems

Figures 6 and 7 show the transmission coefficient as a function of the electron energy and magnetic flux for rings connected in series for the number of rings equals to 2, 4, 6, and 8, respectively. The feature that $T=0$ at $\Phi=\Phi_0/2$ is common to both SRS and PRS. At other values of Φ, the transmission coefficient has four peaks on the T-E plane for a two-ring system [see Fig. 6(a)]. This should be contrasted with the two-peak structure in a two-ring system in parallel. The complexity of the structures in the transmission coefficient can be understood as the result of coherence between the rings as the lead connecting the rings plays the role of an additional scatterer. It will then be expected that as the number of rings increases, the structure of the transmission coefficient as a function of the electron energy becomes increasingly complicated as a result of the coherence among the large number of rings. It is in fact the case as shown in Figs. 6(b) and 7 for systems with four, six, and eight rings, respectively. For showing the relationship between the electron energy and its transmission coefficient, we have plotted the E vs T curves with $\Phi=0.1$, 0.3, $0.4\Phi_0$, respectively, for serial six-ring system in Fig. 8, which very well complement the three-dimensional (3D) plot shown in the Fig. 7(a). From Fig. 8, one can see some similarities with previous result based on the wave-guide theory.[19]

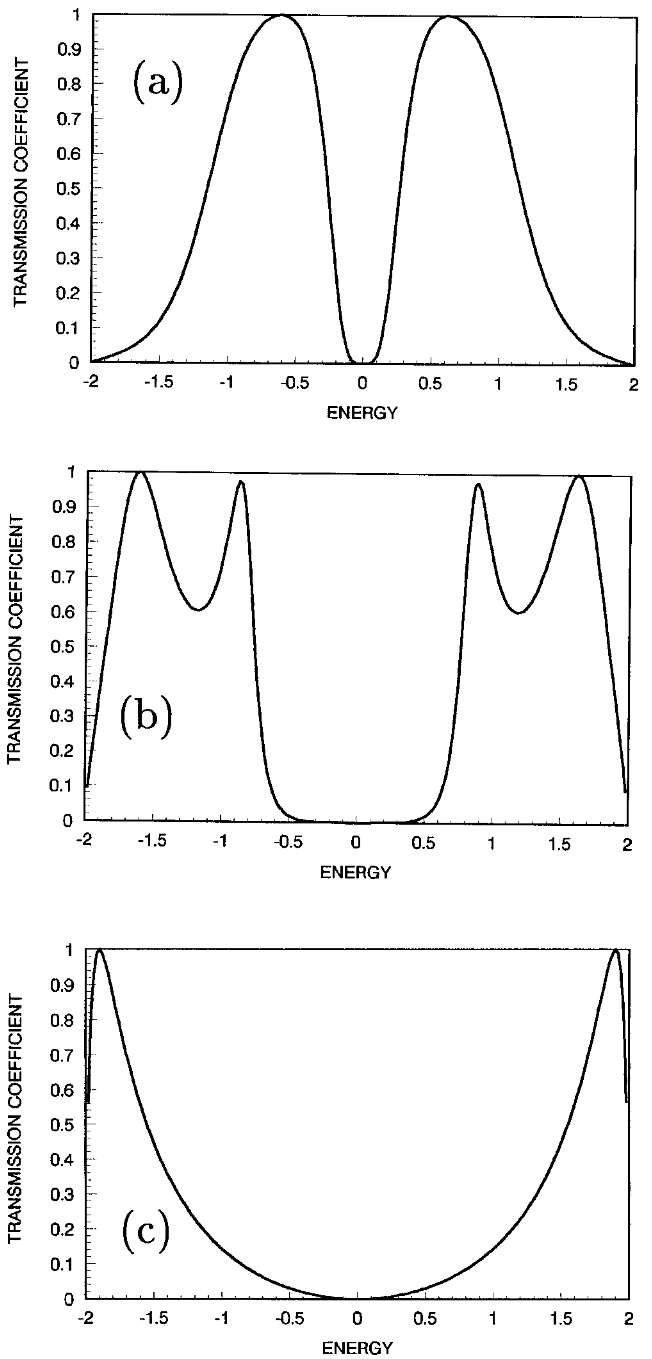

FIG. 8. Transmission coefficient T vs electron energy E for serial six-ring systems with (a) flux $\Phi=0.1\Phi_0$, (b) $\Phi=0.3\Phi_0$, and (c) $\Phi=0.4\Phi_0$. The relationship between T and E is clearly shown.

It is also interesting to study, as analogous to Fig. 5 for the PRS, the transmission at $\Phi=0$ for SRS as the number of rings is varied. Figure 9 shows the results for five different values of the number of rings in the system. The transmission coefficient is much more sensitive to the number of rings in SRS than in PRS. For $N_i=2$, there is a broad region in the middle of the band with an appreciable transmission coefficient. As the number of rings increases, the region with large transmission shrinks. A simple-minded argument is that the total transmission amplitute in SRS is of the order of the product of the transmission amplitutes of each individual ring. For energies at which the transmission amplitude $t(E)$ for a single ring is not so close to unity, the product of a large number of $t(E)$ is much smaller than $t(E)$ and leads to the sensitivity of the transmission to the number of rings in the system.

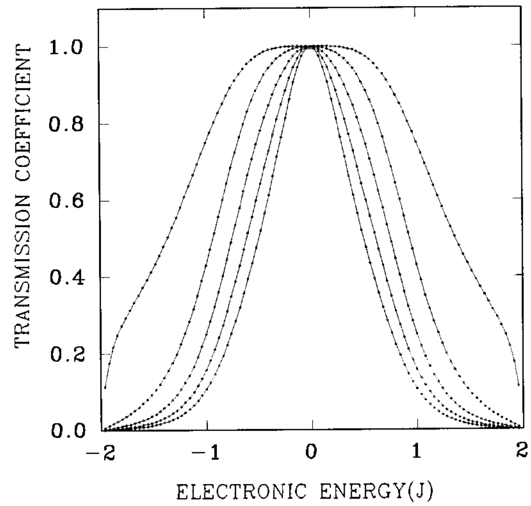

FIG. 9. Transmission coefficient T for serial multiring systems as a function of incident energy in the absence of magnetic flux, i.e., $\Phi=0$. The five curves from top to bottom correspond to systems with two rings, four rings, six rings, eight rings, and ten rings, respectively. Note that the transmission coefficient depends sensitively on the number of rings in SRS.

IV. SUMMARY

A recursive scheme is used to calculate the transmission coefficient of systems with a large number of mesoscopic rings threaded by a magnetic flux connected either in parallel or in series. The transmission coefficient is studied, for different numbers of rings in the system, as a function of the incident electron energy and magnetic flux. At $\Phi=\Phi_0/2$, the transmission vanishes for both the parallel and serial multiring systems. In PRS, the region of the E-Φ space corresponding to vanishing transmission increases as the number of rings increases. In SRS, the coupling between serial rings leads to complicated structures in the transmission coefficient as the number of rings increases. At $\Phi=0$, the transmission decreases only slightly with respect to that in a two-ring system as the number of rings increases in PRS, while the transmission decreases much more sensitively with the number of rings in SRS. When suitably generalized, the present calculation scheme can be extended to treat systems with a large number of mesoscopic rings connected in more complicated configurations.

ACKNOWLEDGMENTS

One of us (Y.L.) acknowledges support by the National Natural Science Foundation of China and Guangdong Provincial Natural Science Foundation of China. Work at the Chinese University of Hong Kong was supported in part by the Research Grants Council of the Hong Kong Government under Project Nos. CUHK 461/95P and CUHK 4191/97P. Y.L. would like to thank the Department of Physics at CUHK for the hospitality extended to him during his visit.

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