Electron-electron interaction effect on the conductivity and the Hall conductivity of weakly disordered electron systems

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The effect of the electron-electron Coulomb interaction on the conductivity and Hall conductivity of weakly disordered ($T\tau > 1$, where τ is the electron mean free path) three- and two-dimensional electron systems is studied. We find that (i) temperature-dependent interaction corrections to the impurity resistivity and the Hall coefficient are positive in three and two dimensions; (ii) in two dimensions, gapless plasmons and particle-hole excitations both contribute to the electron-electron-impurity interference correction, to the resistivity; and (iii) in two-dimensional electron systems such as GaAs heterojunctions, the electron-electron interaction gives the leading temperature-dependent correction to the impurity conductivity and the Hall conductivity more important than the corresponding corrections from the piezoelectric electron-phonon interaction.
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I. INTRODUCTION

It is well known that interference between electron-electron and electron-impurity interactions leads to numerous anomalies in the low-temperature properties of impure electron systems.[1] Such anomalies originate from the diffusion motion of electrons, and come from the region of small momentum and energy transfers $ql \ll 1$ and $\omega\tau \ll 1$, where $l = v_F\tau$ is the electron mean free path, and τ is elastic electron-impurity relaxation time. The above conditions are satisfied for temperatures $T < 1/\tau$.

The deformation electron-phonon interaction affects the low-temperature conductivity differently. As shown in Ref. 2, the interference between the deformation electron-phonon and electron-impurity interactions leads to an important temperature-dependent contribution to the impurity conductivity σ not in the diffusion region but in the short-wave region, $q_T l \gg 1$, where $q_T = T/\mu$ is the thermal phonon wave vector and μ is the sound velocity. This effect was experimentally studied in Refs. 3 and 4.

Now we are going to study the contribution to the conductivity from the interference between electron-electron and electron-impurity interactions in the short-wave region, $ql \gg 1$ and $\omega\tau \gg 1$, which corresponds to $T \gg 1/\tau$. It is expected that in typical metals this correction to the conductivity $\delta^{e\text{-}e}\sigma$ is less important that the corresponding correction from the electron-phonon impurity interference $\delta^{e\text{-ph}}\sigma$ due to the relative smallness of the electron-electron interaction. However, we expect that, in semiconductors with a small Fermi energy, and especially in the two-dimensional case, the interference correction to the conductivity $\delta^{e\text{-}e}\sigma$ will be important. In addition, in semiconductors without the inversion center, the piezoelectric electron-phonon interaction dominates over the deformation one in low-temperature electron kinetics. Thus we extend the analysis of electron-phonon-impurity interference[2] for the case of piezoelectric electron-phonon interaction in semiconductor two-dimensional systems such as GaAs heterostructures.

Then we study electron-electron interaction effects on the impurity Hall conductivity in weak magnetic fields, a problem which has never been studied before, to our knowledge. For the similar problem of localization and interaction effects on the Hall conductivity in the diffusion regime,[5,6] the linear-response method was applied. This method requires working with vector potentials, and the gauge invariance must be maintained. In addition, the electron-hole asymmetry must also be taken into account. For these reasons, the calculations were very involved. We will apply the quantum kinetic equation method, where we deal with real electric and magnetic fields, which for our purpose is more convenient than the linear-response method.

Another motivation for the present work is to study the role of gapless two-dimensional plasmons in the electron kinetics. Recently, it was shown[7] that two-dimensional plasmons lead to a nonanalytical structure of the electron density of states. In the present work we show that two-dimensional gapless plasmons are important for electron-electron-impurity interference corrections to the conductivity and the Hall conductivity.

II. INTERFERENCE CORRECTIONS TO THE CONDUCTIVITY

In this section, we develop a formalism for calculating corrections to the conductivity from interference of the electron-electron and electron-impurity interactions. The piezoelectric electron-phonon interaction will be considered in a separate section. We apply the quantum kinetic equation method based on the Keldysh diagrammatic technique,[2,8] where, in addition to the retarded and advanced Green's functions

$$G_0^R(P) = [G_0^A(P)]^* = \frac{1}{\epsilon - \xi_p + i/2\tau},$$

$$\xi_p = \frac{p^2 - p_F^2}{2m}, \quad P = (\mathbf{p}, \epsilon), \tag{1}$$

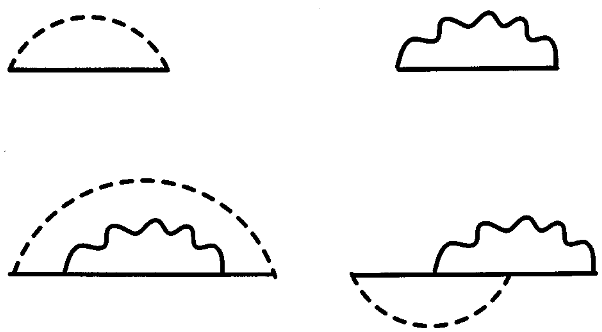

FIG. 1. Diagrams of the electron self-energy.

where τ is the elastic scattering time, a function G^C is introduced. In the first order in the nonuniformity, G^C is defined by the equation

$$G^C(P)=S(P)[G^A(P)-G^R(P)]+\delta G^C(P),$$

$$\delta G^C(P)=\frac{i}{2}\{S_0(\epsilon),G^A(P)+G^R(P)\}, \tag{2}$$

where the Poisson brackets in the electric and magnetic fields are[8,9]

$$\{A,B\}_E=e\mathbf{E}\left(\frac{\partial A}{\partial \epsilon}\frac{\partial B}{\partial \mathbf{p}}-\frac{\partial B}{\partial \epsilon}\frac{\partial A}{\partial p}\right), \tag{3}$$

$$\{A,B\}_H=\frac{e}{c}\mathbf{H}\cdot\left(\frac{\partial A}{\partial \mathbf{p}}\times\frac{\partial B}{\partial p}\right). \tag{4}$$

The function $S(P)$ plays the role of the electron distribution function. In equilibrium, $S=S_0=-\tanh(\epsilon/T)$. In the presence of the electric and weak magnetic fields (the quantization of the electron levels is neglected), S is determined from the following quantum transport equation:

$$e(\mathbf{v}\cdot\mathbf{E})\frac{\partial S}{\partial \epsilon}+\frac{e}{c}(\mathbf{v}\times\mathbf{H})\frac{\partial S}{\partial \mathbf{p}}=I_{e\text{-imp}}+I_{e\text{-}e}+I_{e\text{-}e\text{-imp}}+I'_{e\text{-}e\text{-imp}}, \tag{5}$$

where $I_{e\text{-imp}}$ and $I_{e\text{-}e}$ are the collision integrals corresponding to the electron-impurity and the electron-electron interactions, and $I_{e\text{-}e\text{-imp}}$ and $I'_{e\text{-}e\text{-imp}}$ are the interference collision integrals containing both electron-electron and electron-impurity interactions; they will be described in detail below. All collision integrals are expressed in terms of the corresponding self-energies by the equation

$$I(S)=I^0(S)+\delta I(S),\quad I^0=-i[\Sigma^C-S(\Sigma^A-\Sigma^R)], \tag{6}$$

$$\delta I=-i[\delta\Sigma^C-S_0(\delta\Sigma^A-\delta\Sigma^R)]+\tfrac{1}{2}\{\Sigma^A+\Sigma^R,S_0\}, \tag{7}$$

where $\delta\Sigma$ is the correction in Poisson brackets form. In our case, $\delta\Sigma$ is obtained by taking into account the correction δG^C in the expressions for Σ.

The collision integral $I_{e\text{-imp}}$ that corresponds to the first electron self-energy diagram of Fig. 1 is chosen in the simplest form:

$$I_{e\text{-imp}}=\frac{2}{\pi\nu\tau}\int\frac{d\mathbf{k}}{(2\pi)^3}[S(\mathbf{k},\epsilon)-S(\mathbf{p},\epsilon)]\mathrm{Im}\,G_0^A(\mathbf{k},\epsilon)$$
$$=\frac{S_0(\epsilon)-S(\epsilon)}{\tau}. \tag{8}$$

Constructing the electron-electron collision integrals $I_{e\text{-}e}$ and $I_{e\text{-}e\text{-imp}}$, we need the advanced electron self-energy, the second diagram of Fig. 1,

$$\Sigma^A_{e\text{-}e}(P)=\frac{i}{2}\int\frac{d^4Q}{(2\pi)^4}$$
$$\times[V^R(Q)G^C(P+Q)+V^C(Q)G_0^A(P+Q)], \tag{9}$$

where $V^C=2i\,\mathrm{Im}[V^R(Q)][2N(\omega)+1]$, and $V^R(Q)$ is the retarded electron-electron potential which will be discussed later, and $N(\omega)$ is the Bose distribution function.

The first interference collision integral $I_{e\text{-}e\text{-imp}}$ corresponds to the correction to the electron density of states in $I_{e\text{-imp}}$ in the form $\delta_{e\text{-}e}G^A=(G_0^A)^2\Sigma^A_{e\text{-}e}$ (the third self-energy diagram of Fig. 1),

$$I_{e\text{-}e\text{-imp}}=\frac{2}{\pi\nu\tau}\int\frac{d\mathbf{k}}{(2\pi)^3}[S(\mathbf{k},\epsilon)$$
$$-S(\mathbf{p},\epsilon)]\mathrm{Im}\{[G_0^A(\mathbf{k},\epsilon)]^2\Sigma^A_{e\text{-}e}(\mathbf{k},\epsilon)\}. \tag{10}$$

The second interference collision integral $I'_{e\text{-}e\text{-imp}}$ corresponds to the electron self-energy diagram with the impurity vertex correction, the fourth diagram of Fig. 1.

Calculating the interaction corrections to the conductivity, we drop the magnetic field term in Eq. (5). Assuming the electron-impurity scattering is a dominant momentum relaxation process, we solve Eq. (5) by iteration: $S=S_0+\phi_0^E+\Sigma_i\phi_i^E$, $i=1,2,\ldots$. For the first correction ϕ_0^E, we keep only $I_{e\text{-imp}}$ in Eq. (5), and find

$$\phi_0^E(P)=-e\tau(\mathbf{v}\cdot\mathbf{E})\frac{\delta S_0(\epsilon)}{\delta\epsilon}. \tag{11}$$

The corrections ϕ_i^E correspond to the other collision integrals in Eq. (5) which include the effects of the electron-electron interaction. Below, we describe all these corrections in detail. The first correction ϕ_1^E is

$$\phi_1^E=\tau[I_{e\text{-}e\text{-imp}}(\phi_0^E)]. \tag{12}$$

The next corrections ϕ_2^E and $\phi_{2'}^E$ contain terms in the collision integral $I_{e\text{-}e}$ in the form of the the Poisson brackets [see Eq. (7)],

$$\phi_2^E=-i\tau[\delta\Sigma^C_{e\text{-}e}-S_0(\delta\Sigma^A_{e\text{-}e}-\delta\Sigma^R_{e\text{-}e})], \tag{13}$$

$$\phi_{2'}^E=\frac{\tau}{2}\{\Sigma^A_{e\text{-}e}(S_0)+\Sigma^R_{e\text{-}e}(S_0),S_0(\epsilon)\}_E. \tag{14}$$

Note that contribution of $\delta\Sigma^C_{e\text{-}e}$ term to ϕ_2^E is canceled out by contribution to $\phi_{2'}^E$ from the V^C terms in $\Sigma^A_{e\text{-}e}$ and $\Sigma^R_{e\text{-}e}$.

The contribution from $\delta\Sigma^A_{e\text{-}e}$ and $\delta\Sigma^R_{e\text{-}e}$ terms in Eq. (13) is

$$\phi_2^E = \tau i S_0(\delta\Sigma^A_{e\text{-}e} - \delta\Sigma^R_{e\text{-}e})$$
$$= i\tau S_0(\epsilon)\frac{i}{2}\int \frac{dQ}{(2\pi)^4} 2i \ \mathrm{Im}\ V^R(Q)\,\delta G^C(P+Q). \tag{15}$$

According to Eq. (2),

$$\delta G^C(P+Q) = \frac{i}{2}\frac{\partial S(\epsilon+\omega)}{\partial\epsilon} e\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot\mathbf{E}\{[G_0^A(P+Q)]^2 + [G_0^R(P+Q)]^2\}, \tag{16}$$

and thus

$$\phi_2^E = \tau S_0(\epsilon)\int \frac{dQ}{(2\pi)^4}\ \mathrm{Im}\ D^R(Q)\frac{\partial S(\epsilon+\omega)}{\partial\epsilon} \times e\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot\mathbf{E}\ \mathrm{Re}[G_0^A(P+Q)]^2. \tag{17}$$

If, according to Eq. (9), we include in $\Sigma^A_{e\text{-}e}$ and $\Sigma^R_{e\text{-}e}$ only terms with V^R and V^A, we obtain

$$\phi^E_{2'} = \frac{\partial S_0(\epsilon)}{\partial\epsilon}\int \frac{dQ}{(2\pi)^4}\tau^2 e\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot\mathbf{E}\ \mathrm{Re}[V(Q)] \times S_0(\epsilon+\omega)\mathrm{Im}[G_0^A(P+Q)]^2 \tag{18}$$

As seen from Eq. (18), the correction $\phi^E_{2'}$ is proportional to the real part of the potential. It may be shown that such terms, corresponding to the renormalization effects,[10] are less important than terms proportional to the imaginary part of the potential; thus ϕ_2' will be dropped.

The next correction ϕ_3^E is associated with the interference collision integral, $I'_{e\text{-}e\text{-imp}}$,

$$\phi_3^E = \tau[I'_{e\text{-}e\text{-imp}}(\mathrm{eq}) + I'_{e\text{-}e\text{-imp}}(\mathrm{noneq})], \tag{19}$$

where collision integrals $I'_{e\text{-}e\text{-imp}}(\mathrm{eq}) + I'_{e\text{-}e\text{-imp}}(\mathrm{noneq})$ corresponding to the equilibrium and nonequilibrium vertex functions Γ and $\delta\Gamma$, are derived following Ref. 2,

$$I'_{e\text{-}e\text{-imp}}(\mathrm{eq}) = 2\int \frac{dQ}{(2\pi)^4}\phi_0^E(P)\Gamma(q)S_0(\epsilon+\omega) \times \mathrm{Re}[G_0^A(P+Q)V^A(Q)], \tag{20}$$

$$I'_{e\text{-}e\text{-imp}}(\mathrm{noneq}) = i\int \frac{dQ}{(2\pi)^4} E\,\delta\Gamma(q)S_0(\epsilon+\omega) \times \frac{\partial S(\epsilon)}{\partial\epsilon}\ \mathrm{Im}[G_0^A(P+Q)V^A(Q)], \tag{21}$$

where the vertex functions for $qv_F \gg \omega$ are

$$\Gamma(q) = \frac{1}{\pi\nu\tau}\int \frac{d^3p}{(2\pi)^3} G_0^A(P)G_0^R(P+Q) = \frac{\pi}{2ql}, \tag{22}$$

$$E\,\delta\Gamma(q) = \frac{1}{\pi\nu\tau}\int \frac{d^3p}{(2\pi)^3} G_0^A(P)G_0^R(P+Q)(-e\tau\mathbf{v}\cdot\mathbf{E}) = ie\frac{\mathbf{E}\cdot\mathbf{q}}{q^2}. \tag{23}$$

The electric current and the correction to the electric current due to the electron-electron interaction are given by the equations

$$\mathbf{J} = -i\int \frac{dP}{(2\pi)^4} e\mathbf{v}G^C(P), \tag{24}$$

$$\delta\mathbf{J} = \delta\sigma\mathbf{E} = 2\int \frac{dP}{(2\pi)^4} e\mathbf{v}(\phi_0^E\ \mathrm{Im}\{[G_0^A(P)]^2\Sigma^A_{e\text{-}e}(S_0)\} + S_0\mathrm{Im}\{[G_0^A(P)]^2\Sigma^A_{e\text{-}e}(\phi_0)\} + (\phi_1^E+\phi_2^E+\phi_3^E) \times \mathrm{Im}[G_0^A(P)]), \tag{25}$$

where $\delta\sigma$ is the correction to the impurity conductivity due to the electron-electron interaction. The first and third terms in Eq. (25) mutually cancel out. The second term gives the following correction to the conductivity:

$$\delta_1^{e\text{-}e}\sigma = 2e^2\tau\int \frac{dP}{(2\pi)^4} \times \int \frac{dQ}{(2\pi)^4}\mathbf{v}\cdot\mathbf{n}\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot\mathbf{n}S_0(\epsilon)\frac{\partial S(\epsilon+\omega)}{\partial\epsilon} \times \mathrm{Im}\{[G_0^A(P)]^2\mathrm{Im}\ G_0^A(P+Q)V^R(Q)\}, \tag{26}$$

where n is a unit vector. There are two choices how to get the imaginary part in the right-hand side of Eq. (26),

$$\mathrm{Im}\{[G_0^A(P)]^2\mathrm{Im}\ G_0^A(P+Q)V^R(Q)\} = \mathrm{Im}[G_0^A(P)]^2\mathrm{Im}\ G_0^A(P+Q)\mathrm{Re}\ V^R(Q) + \mathrm{Re}[G_0^A(P)]^2\mathrm{Im}\ G_0^A(P+Q)\mathrm{Im}\ V^R(Q). \tag{27}$$

The first term proportional to the real part of the potential will be dropped. Keeping the second term, we have

$$\delta_1^{e\text{-}e}\sigma = 2e^2\tau\int \frac{dP}{(2\pi)^4} \times \int \frac{dQ}{(2\pi)^4}\mathbf{v}\cdot\mathbf{n}\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot\mathbf{n}S_0(\epsilon)\frac{\partial S(\epsilon+\omega)}{\partial\epsilon} \times \mathrm{Im}\ V^R(Q)\mathrm{Re}[G_0^A(P)]^2\ \mathrm{Im}\ G_0^A(P+Q). \tag{28}$$

Equation (28) can be rewritten in the following way:

$$\delta_1^{e\text{-}e}\sigma = 2e^2\tau\int \frac{dP}{(2\pi)^4} \times \int \frac{dQ}{(2\pi)^4}\mathbf{v}\cdot\mathbf{n}\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot nS_0(\epsilon)\frac{\partial S(\epsilon+\omega)}{\partial\epsilon} \times \mathrm{Im}\ V^R(Q)\tfrac{1}{2}\ \mathrm{Im}\{[G_0^A(P+Q)][G_0^R(P)]^2\}. \tag{29}$$

The ϕ_2^E term gives the following correction to the conductivity:

$$\delta_2^{e\text{-}e}\sigma = 2e^2\tau\int \frac{dP}{(2\pi)^4} \times\int \frac{dQ}{(2\pi)^4}\,\mathbf{v}\cdot\mathbf{n}\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot\mathbf{n}S_0(\epsilon)\frac{\partial S(\epsilon+\omega)}{\partial\epsilon} \times \mathrm{Im}\,V^R(Q)\mathrm{Im}\,G_0^A(P)\mathrm{Re}[G_0^A(P+Q)]^2. \quad (30)$$

Again we can rewrite Eq. (30) as

$$\delta_2^{e\text{-}e}\sigma = 2e^2\tau\int \frac{dP}{(2\pi)^4} \times\int \frac{dQ}{(2\pi)^4}\,\mathbf{v}\cdot\mathbf{n}\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot\mathbf{n}S_0(\epsilon)\,\frac{\partial S(\epsilon+\omega)}{\partial\epsilon} \times \mathrm{Im}\,V^R(Q)\tfrac{1}{2}\,\mathrm{Im}\{G_0^A(P)[G_0^R(P+Q)]^2\}. \quad (31)$$

Comparing Eqs. (29) and (31), we see that $\delta_1^{e\text{-}e}\sigma = \delta_2^{e\text{-}e}\sigma$.

The collision integrals with equilibrium and nonequilibrium vertex functions, Eqs. (20) and (21), give the following corrections to the conductivity

$$\delta_3^{e\text{-}e}\sigma = (e\tau)^2\int \frac{dP}{(2\pi)^4} \times\int \frac{dQ}{(2\pi)^4}\,(\mathbf{v}\cdot\mathbf{n})^2 S_0(\epsilon+\omega)\,\frac{\partial S(\epsilon)}{\partial\epsilon}\,\Gamma(q) \times \mathrm{Im}\,V^R(Q)\mathrm{Re}[G_0^A(P)G_0^R(P+Q)], \quad (32)$$

$$\delta_4^{e\text{-}e}\sigma = i2e\tau\int \frac{dP}{(2\pi)^4} \times\int \frac{dQ}{(2\pi)^4}\,\mathbf{v}\cdot\mathbf{n}S_0(\epsilon+\omega)\,\frac{\partial S(\epsilon)}{\partial\epsilon}\,\delta\Gamma(q) \times \mathrm{Im}\,V^R(Q)\mathrm{Im}[G_0^A(P)G_0^R(P+Q)], \quad (33)$$

In the next sections we will calculate the corrections $\delta_{1\text{-}4}^{e\text{-}e}\sigma$ in three and two dimensions.

III. THREE-DIMENSIONAL CASE

The screened Coulomb electron-electron interaction is

$$V^R(Q) = \frac{V_0(q)}{1-V_0(q)P^R(Q)}, \quad (34)$$

where in three dimensions the nonscreened potential is $V_0^{(3)}(q) = 4\pi e^2/q^2\epsilon$, where ϵ is the static dielectric constant, and the polarization operator for $qv_F > \omega$ is

$$P_3^R(Q) = -\nu_3\left(1+\frac{i\pi\omega}{2qv_F}\right). \quad (35)$$

Thus

$$V^R(Q) = \frac{1}{\nu_3}\,\frac{\kappa_3^2}{q^2+\kappa_3^2(1+i\pi\omega/2qv_F)}, \quad (36)$$

where $\kappa_3^2 = 4\pi e^2\nu_3/\epsilon$ and $\nu_3 = mp_F/\pi^2$.

The combined contribution of $\delta_1^{e\text{-}e}\sigma$ and $\delta_2^{e\text{-}e}\sigma$ is

$$\delta_{1,2}^{e\text{-}e}\sigma = -\frac{e^2\nu_3 v_F^2\tau}{12\pi^4}\int_{q_0}^{2p_F} dq\; q^2 \times\int d\omega\; f(\omega/T)I_3(q)\mathrm{Im}\,V^R(q,\omega), \quad (37)$$

where

$$f(\omega/T) = \tfrac{1}{2}\int d\epsilon\,\tanh\left(\frac{\epsilon+\omega}{2T}\right)\frac{\partial}{\partial\epsilon}\tanh\left(\frac{\epsilon}{2T}\right) = \frac{\partial}{\partial\omega}\left[\omega\,\coth\left(\frac{\omega}{2T}\right)\right], \quad (38)$$

and

$$I_3(q) = \mathrm{Im}\int \frac{d\Omega_q}{4\pi}\int d\xi_p G_0^R(P)[G_0^A(P+Q)]^2, \quad (39)$$

and Ω_q means angular variables of vector $\mathbf{q}$. Performing integration for $ql \gg 1$ and $\omega\tau \gg 1$, we have

$$I_3(q) = -\pi\;\mathrm{Re}\int_{-1}^{1}\frac{dx}{(qv_F x-\omega+i/\tau)^2} = \frac{2\pi}{(qv_F)^2}, \qquad q > q_0 = |\omega|/v_F. \quad (40)$$

Performing the remaining calculations, we have

$$\frac{\delta_{1,2}^{e\text{-}e}\sigma}{\sigma_3} = \frac{\pi^2}{72}\left(\frac{T}{\epsilon_F}\right)^2 \times\left\{\ln\left(\frac{4\epsilon_F}{T}\right)-\ln\left[1+\left(\frac{2p_F}{\kappa_3}\right)^2\right]-\frac{2p_F^2}{(2p_F)^2+\kappa_3^2}-1\right\}. \quad (41)$$

The corrections to the conductivity from equilibrium and nonequilibrium vertex corrections are

$$\delta_3^{e\text{-}e}\sigma = -\frac{\pi^2}{4}\,\delta_{1,2}^{e\text{-}e}\sigma, \qquad \delta_4^{e\text{-}e}\sigma = -\delta_{1,2}^{e\text{-}e}\sigma. \quad (42)$$

Thus the final result is

$$\frac{\delta^{e\text{-}e}\sigma}{\sigma_3} = -\frac{\pi^4}{288}\left(\frac{T}{\epsilon_F}\right)^2 \times\left\{\ln\left(\frac{4\epsilon_F}{T}\right)-\ln\left[1+\left(\frac{2p_F}{\kappa_3}\right)^2\right]-\frac{2p_F^2}{(2p_F)^2+\kappa_3^2}-1\right\}, \quad (43)$$

where $\sigma_3 = e^2\nu_3 v_F^2\tau/3$ is the Drude conductivity in three dimensions.

It is interesting to compare Eq. (43) with the correction to the conductivity from the interference of electron-impurity and the deformation electron-phonon interactions,[2]

$$\frac{\delta^{e\text{-ph}}\sigma}{\sigma_3} = \left[\frac{4}{3\pi^2}-\frac{1}{12}-\frac{8}{3\pi^2}\left(\frac{u_l}{u_t}\right)^3\right]\frac{\pi^4\beta_l T^2}{2\epsilon_F p_F u_l}, \quad (44)$$

where u_l and u_t are longitudinal and transverse sound velocities, and β_l is the electron-longitudinal phonon coupling constant. It is clear that in good metals, where $p_F u_l \ll \epsilon_F$, the correction $\delta^{e\text{-ph}}\sigma$ dominates, while in semiconductors, where the electron-phonon coupling constant is much smaller than in metals, the correction $\delta^{e\text{-}e}\sigma$ is more important. The case of the piezoelectric electron-phonon interaction will be considered in Sec. V.

IV. TWO-DIMENSIONAL CASE

The nonscreened Coulomb interaction in two dimensions is $V_0^{(2)} = 2\pi e^2/\epsilon_0 q$. For $ql \gg 1$ and $\omega\tau \gg 1$ the polarization operator in two dimensions is

$$P_2^R(Q) = 2\int \frac{d^2p}{(2\pi)^2}\frac{qv\cos\phi\,\delta(\xi_p)}{\omega - qv_F\cos\phi + i0} = -\nu_2\left(1 - \frac{\omega}{\sqrt{(\omega+i0)^2-(qv_F)^2}}\right). \tag{45}$$

In the particle-hole excitation region, $qv_F > \omega$,

$$P_2^R(Q) = -\nu_2(1 + i\omega/qv_F),$$

$$V^R(q) = \frac{1}{\nu_2}\frac{\kappa_2}{q+\kappa_2(1+i\omega/qv_F)}, \quad \kappa_2 = 2\pi e^2\nu_2/\epsilon, \tag{46}$$

where κ_2 is the two-dimensional Debye screening momentum, and $\nu_2 = m/\pi$.

In the plasmon region, $qv_F < \omega$,

$$P_2^R(Q) = \frac{\nu_2}{2}\left(\frac{qv_F}{\omega+i0}\right)^2, \tag{47}$$

and the screened potential $V(Q)$ has a plasmon pole with the spectrum $\omega = v_F(\kappa_2 q/2)^{1/2}$.

We start with the correction to the conductivity $\delta_{1,2}^{e\text{-}e}\sigma$,

$$\delta_{1,2}^{e\text{-}e}\sigma = \frac{e^2\nu_2 v_F^2\tau}{8\pi^3}\int_0^\infty dq\, q\int d\omega\, f(\omega/T) I_2(q)\,\mathrm{Im}\, V^R(q,\omega), \tag{48}$$

where

$$I_2(q) = \mathrm{Im}\int_0^{2\pi}\frac{d\phi}{2\pi}\int d\xi_p G_0^R(P)[G_0^A(P+Q)]^2 = -\mathrm{Re}\int_0^{2\pi}\frac{d\phi}{(qv_F\cos\phi - \omega + i/\tau)^2}. \tag{49}$$

It is easy to see that in two dimensions for $ql \gg 1$ and $\omega\tau \gg 1$, the real part of the last integral exists only for $\omega > qv_F$, and $I_2(q) = -2\pi/\omega^2$. Integrating the plasmon pole, we have

$$\nu_2\int_0^\infty dq\, q\,\mathrm{Im}\, V^R(Q) = \kappa_2\omega^2\,\mathrm{Im}\int_0^\infty\frac{dq}{(\omega+i0)^2 - \kappa_2 v_F^2 q/2} = -\frac{2\pi}{v_F^2}\frac{\omega^3}{|\omega|}. \tag{50}$$

Finally the correction to the conductivity $\delta_{1,2}^{e\text{-}e}\sigma$ is

$$\frac{\delta_{1,2}^{e\text{-}e}\sigma}{\sigma_2} = -\frac{2T}{\epsilon_F}, \quad \sigma_2 = e^2\nu_2 v_F^2\tau/2. \tag{51}$$

We note that unlike the three-dimensional case, the correction $\delta_{1,2}^{e\text{-}e}\sigma$ is negative in two dimensions. The reason for this is the following: according to Eq. (49), the function $I_2(q)$ in the plasmon region, $\omega > qv_F$, is negative, unlike $I_3(q)$, which is positive in the particle-hole excitation region $qv_F > \omega$.

For the other corrections, $\delta_3^{e\text{-}e}\sigma$ and $\delta_4^{e\text{-}e}\sigma$, the particle-hole excitation region $qv_F > \omega$ gives the main contribution. Calculating the equilibrium and nonequilibrium vertex corrections in two dimensions,

$$\Gamma(q) = \frac{1}{ql}, \quad E\,\delta\Gamma(q) = ie\frac{\mathbf{E}\cdot\mathbf{q}}{q^2}, \quad qv_F \gg \omega, \tag{52}$$

we find the corresponding corrections to the conductivity are

$$\frac{\delta_3^{e\text{-}e}\sigma}{\sigma_2} = -\frac{T}{2\epsilon_F}, \quad \frac{\delta_4^{e\text{-}e}\sigma}{\sigma_2} = -\frac{T}{4\epsilon_F}. \tag{53}$$

The combined contribution of all terms is

$$\frac{\delta^{e\text{-}e}\sigma}{\sigma_2} = -\frac{11}{4}\frac{T}{\epsilon_F}. \tag{54}$$

V. PIEZOELECTRIC ELECTRON-PHONON INTERACTION IN HETEROSTRUCTURES

Calculation of the interference correction $\delta^{e\text{-ph}}\sigma$ for the deformation electron-phonon interaction performed in Ref. 2 is complicated, because local charge disturbance is responsible for both the electron-phonon interaction and electron-impurity scattering. Therefore, to insure the local electroneutrality of the electron-ion system, the processes of inelastic electron-impurity scattering should be taken into account. In piezoelectric crystals, where the electron-phonon interaction stems from the coupling of electrons with a macroscopic electric field caused by the local elastic strain, the electroneutrality does not affect the piezoelectric coupling. As was shown recently,[11] in a piezoelectric crystal, local strain caused by an impurity is less important than local charge disturbance, and processes of inelastic electron-impurity scattering may be neglected. This fact allows us to treat the piezoelectric electron-phonon and electron-electron interactions on the same footing. We will concentrate on a two-dimensional electron gas in GaAs heterojunctions. In what follows, we treat phonons as bulk acoustic modes coupled to a local electronic density by virtue of the screened vertex

$$M_\lambda(\mathbf{Q}) = \frac{eh_{14}}{\epsilon(q,\omega)}\left(\frac{A_\lambda}{2\rho u_\lambda Q}\right)^{1/2},$$

$$A_l = \frac{9q_z^2q^4}{2Q^6}, \quad A_t = \frac{8q_z^4q^2+q^6}{4Q^6}, \tag{55}$$

where $\mathbf{Q} = (\mathbf{q}, q_z)$ is the three-dimensional phonon momentum, ρ is the bulk mass density of GaAs, and u_λ is longitudinal (l) or transverse (t) sound velocity. We will use the notation u for estimates, h_{14} is the only nonzero component of the piezoelectric constant, and $\epsilon(q,\omega)=1+V_0^{(2)}P_2(q,\omega)$ is the two-dimensional electron dielectric function.

Equations (29)–(35) should be modified by assuming two-dimensional integration over the electron momentum, using three-dimensional integration over the phonon momentum, and substituting $\mathrm{Im}\, V^A(Q)$ for $\Sigma_\lambda |M_\lambda(\mathbf{Q})|^2 \,\mathrm{Im}\, D^A(Q,\omega)$, where the imaginary part of the phonon propagator is

$$\mathrm{Im}\, D^A(Q,\omega)=\pi\{\delta[\omega-\Omega_\lambda(Q)]-\delta[\omega+\Omega_\lambda(Q)]\}, \tag{56}$$

here $\Omega_\lambda(Q)$ is the phonon frequency. Calculating the contribution $\delta_{1,2}\sigma_{e\text{-ph-imp}}$, we notice that after integrating over ω the result is proportional to

$$I_2'(q)=-\mathrm{Re}\int_0^{2\pi}\frac{d\phi}{(qv_F\cos\phi-u_\lambda q+i/\tau)^2}=0, \tag{57}$$

because $v_F>u$. Thus, unlike the electron-electron interaction in two dimensions, where the plasmon region contributes to integral $I_2(q)$ [see Eq. (49)], the contribution $\delta_{1,2}^{e\text{-ph}}\sigma$ is zero. Calculating the other contributions $\delta_{3,4}^{e\text{-ph}}\sigma$, we note that the dielectric function may be taken in the static limit, $\epsilon(q,\omega)=1+\kappa_2/q$. There are two regimes of screening; strong screening for $T<T_1$, and weak screening for $T_1<T$, where $T_1=\kappa_2 u$ for GaAs, and $T_1\approx 0.5$ K. Calculations show that for $T_1<T$, the correction $\delta_{3,4}^{e\text{-ph}}\sigma$ is temperature independent, while for lower temperatures, $T<T_1$,

$$\frac{\delta_{3,4}^{e\text{-ph}}\sigma}{\sigma_2}=-\frac{C_3}{4}\,\frac{(eh_{14})^2}{\rho u_l^3}\left(\frac{T}{\kappa_2 v_F}\right)^2, \tag{58}$$

where, for GaAs,

$$C_3=\frac{1}{4}\int_0^1 dx\left[9x^2(1-x^2)^2+\left(\frac{u_t}{u_l}\right)^3[8x^4(1-x^2)-(1-x^2)^6]\right]=1.2. \tag{59}$$

Comparing Eqs. (59) and (54), we see that $\delta^{e\text{-}e}\sigma$ dominates at low temperatures. In the three-dimensional case, using Eqs. (34) and (35) for the dielectric function, we can show that for $T<T_1$ the corresponding correction $\delta^{e\text{-ph}}\sigma/\sigma_3$ acquires an additional small factor $(T/\kappa_3 u_l)^2$, and as a result the interference correction from the electron-electron interaction $\delta^{e\text{-}e}\sigma$ [see Eq. (43)] dominates in three dimensions as well.

VI. INTERFERENCE CORRECTIONS TO THE HALL CONDUCTIVITY

In the presence of crossed electric and magnetic fields, the iteration solution of Eq. (6) is $S=S_0+\phi_0^E+\phi_0^H+\Sigma_i\phi_i^H$, where $i=1,2,\ldots$. The correction ϕ_0^E is defined by Eq. (11). The next correction ϕ_0^H depends on both the electric and magnetic fields:

$$\phi_0^H(\mathbf{p},\epsilon)=-\tau\frac{e}{c}(\mathbf{v}\times\mathbf{H})\frac{\partial(S_0+\phi_0^E)}{\partial\mathbf{p}}=-\frac{e^2\tau^2}{cm}\mathbf{v}\cdot(\mathbf{E}\times\mathbf{H})\frac{\partial S_0}{\partial\epsilon}. \tag{60}$$

The other corrections ϕ_i^H include the effects of the electron-electron interaction. The first of them, ϕ_1^H, is obtained by taking into account nonequilibrium function ϕ_0^H in Eq. (10),

$$\phi_1^H=\tau[I_{e\text{-}e\text{-imp}}(\phi_0^H)]. \tag{61}$$

The next corrections are obtained from Eq. (7) where the Poisson brackets are taken in the magnetic field form $\{A,B\}_H$,

$$\phi_2^H=-i\tau[\delta\Sigma_{e\text{-}e}^C-S_0(\delta\Sigma_{e\text{-}e}^A-\delta\Sigma_{e\text{-}e}^R)], \tag{62}$$

$$\phi_{2'}^H=\frac{\tau}{2}\{\Sigma_{e\text{-}e}^A(\phi_0^E)+\Sigma_{e\text{-}e}^R(\phi_0^E),S_0+\phi_0^E\}_H. \tag{63}$$

The following calculations are similar to that presented in Sec. II for the correction to the conductivity; e.g., ϕ_2^H is satisfied by Eq. (15), where δG^C is substituted for

$$\begin{aligned}\delta G^C(P+Q)&=\frac{i}{2}\{S_0(\epsilon+\omega)+\phi_0^E(P+Q),G_0^A(P+Q)+G_0^R(P+Q)\}_H\\&=\tau\frac{i}{2}\frac{\partial S_0(\epsilon+\omega)}{\partial\epsilon}\frac{e^2}{c}\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot(\mathbf{E}\times\mathbf{H})\\&\quad\times\{[G_0^A(P+Q)]^2+[G_0^R(P+Q)]^2\}.\end{aligned} \tag{64}$$

Thus

$$\phi_2^H=\tau^2 S_0(\epsilon)\int\frac{dQ}{(2\pi)^4}\,\mathrm{Im}\,V^R(Q)\,\frac{\partial S(\epsilon+\omega)}{\partial\epsilon}\frac{e^2}{c}\times\left(\mathbf{v}+\frac{\mathbf{q}}{m}\right)\cdot(\mathbf{E}\times\mathbf{H})\mathrm{Re}[G_0^A(P+Q)]^2. \tag{65}$$

We see that Eq. (65) may be obtained from Eq. (17) by substituting vector $\mathbf{E}$ for $(e/c)\mathbf{E}\times\mathbf{H}$. It may be checked that the correction ϕ_3^H may be obtained from Eqs. (19)–(23) by the same substitution.

Calculating the Hall conductivity, we assume that the magnetic field is directed along the z axis, and that the electric field is directed along the x axis. The Hall current J_y is proportional to $\mathbf{E}\times\mathbf{H}$. For the system of noninteracting electrons we use Eq. (24), and take the first term in Eq. (2) for G^C where we include the nonequilibrium function ϕ_0^H in $S(P)$. As a result, we obtain the Hall conductivity of noninteracting electrons,

$$\sigma_{xy}=\frac{J_y}{E}=2e\int\frac{d^4P}{(2\pi)^4}\,v_y\phi_0^H\mathrm{Im}\,G_0^A(P)=\Omega\tau\sigma_3, \tag{66}$$

where $\Omega=eH/mc$ is the cyclotron frequency and σ_3 is the Drude conductivity.

The correction to the Hall current is

$$\Delta J_y = 2e \int \frac{dP}{(2\pi)^4} v_y \{ \phi_0^H \mathrm{Im}[(G_0^A)^2 \Sigma_{e\text{-}e}^A(S_0)] + S_0 \mathrm{Im}[(G_0^A)^2 \Sigma_{e\text{-}e}^A(\phi_0^H) + (\phi_1^H + \phi_2^H + \phi_3^H) \mathrm{Im}\ G_0^A]\}. \tag{67}$$

It may be checked that the corrections to the Hall conductivity $\delta_{1-4}\sigma_{xy}$ are defined by Eqs. (28)–(33), with additional factor eH/c and the unit vector **n** directed along **E**×**H**. Thus in any dimensions we have

$$\frac{\delta^{e\text{-}e}\sigma_{xy}}{\sigma_{xy}} = \frac{\delta^{e\text{-}e}\sigma}{\sigma}. \tag{68}$$

Consequently, for the Hall coefficient R_H, we have the relation

$$R_H = \frac{\sigma_{xy}}{\sigma^2 H}, \quad \frac{\delta^{e\text{-}e}R_H}{R_H} = \frac{\delta^{e\text{-}e}\sigma_{xy}}{\sigma} - 2\frac{\delta^{e\text{-}e}\sigma}{\sigma} = -\frac{\delta^{e\text{-}e}\sigma}{\sigma}. \tag{69}$$

VII. CONCLUSIONS

The temperature correction to the conductivity and the Hall conductivity due to interference of the Coulomb electron-electron interaction and electron-impurity scattering in the short-wave regime $T > 1/\tau$ is calculated. We find that the interference corrections to the conductivity and the Hall conductivity are negative in three and two dimensions. The corresponding correction to the resistivity is positive, and therefore it does not lead to a minimum in resistivity typical of the long-wave (diffusion) regime.

In two dimensions, both two-dimensional plasmons and particle-hole excitations are equally important. In two dimensions, plasmons are gapless with the spectrum $\omega = v_F(\kappa_2 q/2)^{1/2}$, and they play a role similar to the diffusion mode $\omega = iDq^2$ in the long-wave regime,[1] $ql \ll 1$ and $\omega\tau \ll 1$, and D is the diffusion coefficient.

The effect of interference of the piezoelectric electron-phonon and electron-impurity interactions on the conductivity in semiconductor heterostructures is also studied. We find that in semiconductors with small Fermi energy, the low-temperature dependence of the conductivity is determined by the electron-electron interaction rather than the electron-phonon interaction. It is worth mentioning that a well-known proof that the electron-electron interaction does not contribute to the conductivity[12] does not hold in the impure case for the interference electron-electron-impurity correction to the conductivity in both the diffusion region[1] and the short-wave region considered in the present work.

In conclusion, we repeat that the correction $\delta^{e\text{-}e}\sigma$ studied in the present paper is due to interference of the electron-electron and electron-impurity interactions, and thus the effect exists only in disordered electron systems. The condition that the electron-impurity scattering is a dominant momentum relaxation process means that the temperature-dependent correction to the conductivity is smaller than the Drude conductivity. This condition restricts the effect from high temperatures and clean materials, where direct electron-phonon scattering is a dominant momentum relaxation process.[13]

In strongly disordered samples, the low-temperature resistivity associated with the weak localization and interaction effects under condition $T\tau < 1$ exhibits the minimum in resistivity. In metals, the position of the minimum is defined by competition between the above-mentioned weak localization and electron-electron interactions,[1] with electron-phonon-impurity interference[2] at $Tl \gg u$, which were observed in Refs. 3 and 4. As shown in the present paper, in strongly disordered semiconductors the minimum in resistivity corresponds to a crossover from a low-temperature or strongly disordered regime $T\tau < 1$ of the electron-electron interaction, to a high-temperature or weakly disordered regime $T\tau > 1$. While the low-temperature resistivity of extremely clean two-dimensional semiconductors associated with the electron-phonon interaction was measured recently,[14] we are not aware of measurements similar to Refs. 3 and 4 for disordered semiconductors.

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