Resonant and nonresonant polarons in bulk InSb

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The theory of polarons in weakly polar InSb-type narrow-gap semiconductors in the presence of a magnetic field is developed, taking into account resonant and nonresonant contributions to the electron Green function and a nonparabolicity of the conduction band both in the energies and wave functions. An ansatz for the nonresonant correction to the Landau energies is proposed, and shown to lead to good results for upper and lower polaron branches. It is demonstrated that the band nonparabolicity weakens the effective electron–optic-phonon interaction. Experimental results of various authors concerning the cyclotron and the combined resonances in bulk InSb in regions of resonant polarons are described. The theory accounts very well for the available data. [S0163-1829(98)02219-X]

I. INTRODUCTION

Resonant polarons in semiconductors, since their discovery in magneto-optical properties of InSb by Johnson and Larsen,[1] have been the subject of numerous experimental and theoretical investigations. The resonant behavior in the vicinity of $\hbar\omega_c \approx \hbar\omega_L$, where the higher Landau level splits into upper and lower polaron branches, is of particular interest in weakly polar semiconductors, as it allows one to determine directly the polar coupling constant α. The polaron problem offers some theoretical difficulties concerning an instability of the upper branch, a choice of the adequate formalism, and an importance of the nonresonant polaron correction to energies. As a result, there exist in the literature quite a few misunderstandings on the subject.

Harper[2] explained a broadening of the cyclotron resonance line above the resonance $\hbar\omega_c = \hbar\omega_L$, observed by Summers, Harper, and Smith.[3] Using the Green-function formalism, Korovin and Pavlov[4] and Nakayama[5] showed that a pinning behavior of the upper branch is due to the imaginary part of the self-energy. (The Wigner-Brillouin perturbation theory, which implicitly assumes the imaginary part of the self-energy to vanish, leads to a nonphysical behavior of the upper branch.[6]) Swierkowski and Zawadzki[7] considered separately resonant and nonresonant contributions to the polaron energy, and showed that the ''offset'' effect near the resonance (i.e., an energy shift of $\alpha\hbar\omega_L$ between lower and upper polaron branches), claimed by some authors,[8,9] does not exist. This was confirmed in a more complete theory by Pfeffer and Zawadzki.[10]

A proper incorporation of the nonresonant polaron correction into the resonant behavior turned out to be a nontrivial task. Lindemann *et al.*, in Ref. 11, considering the lower polaron branch, proposed to shift all levels by the Raileigh-Schrödinger correction to the lowest Landau level. This results in the so-called improved Wigner-Brillouin (IWB) perturbation theory. The same procedure was adopted by Peeters and Devreese.[12] Various improvements of the Green-function description were discussed by Pfeffer and Zawadzki,[10] and it was shown that Lindemann *et al.* ansatz leads to good results for the resonant polaron behavior in GaAs (both lower and upper branches).

The influence of band nonparabolicity on the polaron behavior has to be taken into account in narrow-gap semiconductors. The position of the resonance $\epsilon_1 - \epsilon_0 \approx \hbar\omega_L$ is directly affected by the structure of the conduction band via the dependence of the Landau energies ϵ_n on the magnetic field. The nonparabolicity causes the resonance to occur at higher fields, which was recognized from the beginning.[8] However, in narrow-gap materials the structure of the wave functions also enters the polaron problem via the matrix elements of the Frohlich interaction. This was first included by Swierkowski and Zawadzki[7] (cf. also Swierkowski *et al.*, Ref. 13). Pfeffer and Zawadzki[14] described an observable case of resonant interband polar interaction in the zero-gap material $Hg_{1-x}Mn_xTe$, in which the initial electron state is a conduction Landau level, while the final state is a heavy-hole Landau level. Das Sarma and Mason,[15] considering the $B = 0$ case and neglecting the proper structure of the wave functions, predicted nonresonant large corrections due to band nonparabolicity. Larsen,[16] taking into account the proper band structure, showed that the corrections are considerably smaller. Huant and Karrai,[17] considering the $B \neq 0$ case within a three-level $\mathbf{k}\cdot\mathbf{p}$ model for the conduction band of InSb, showed that in the limit of $B \to 0$ the nonresonant polaron corrections related to the band nonparabolicity have a negligible influence on the effective mass at the band edge. This result could be expected, since in weakly polar materials the nonresonant polaron contributions are very small to begin with, and the band at the edge is parabolic.

The resonant polarons in InSb were also considered in two-dimensional systems.[18,19] The upper polaron branch in such systems can be described without recourse to the Green-function formalism, since, because of the absence of a k_z dependence of the Landau levels, the real phonon emission does not occur also above the resonance (cf. below).

In the present paper, we consider the influence of band nonparabolicity in InSb (both concerning the energies and the wave functions) within the Green-function formalism, which allows us to treat correctly both lower and upper polaron branches. This is particularly important in narrow-gap materials since, due to the smallness of the effective mass, the resonance condition $\hbar\omega_c \approx \hbar\omega_L$ is reached at low magnetic fields ($B_{\rm res} \approx 3.7$ T for InSb). We pay due attention to

the nonresonant polaron contributions, generalizing and simplifying the procedure proposed in Ref. 11. The formalism is applied to various available experiments in InSb, which until the present has been regarded to be the model narrow-gap semiconductor.

II. THEORY

In this section we present a general formulation of the polaron problem in InSb-type narrow-gap materials. The electronic band structure of such semiconductors in the presence of an external magnetic field is described, and the Frohlich electron-phonon interaction is introduced. Then the nonresonant and resonant polaron corrections to the electron self-energy are derived.

Band structure and Frohlich interaction

The initial Hamiltonian for the problem reads

$$H=P^2/2m_0+V_0(r)+H_{so}+H_F+H_{\rm ph}, \tag{1}$$

where m_0 is the free-electron mass, $\mathbf{P}=\mathbf{p}+e\mathbf{A}$ is the kinetic momentum, $\mathbf{A}$ is the vector potential of magnetic field, V_0 is the periodic potential of the lattice, $H_{\rm so}$ is the spin-orbit interaction, H_F is the Frohlich interaction (assumed weak), and $H_{\rm ph}$ is the free-phonon term. The first three terms describe the band structure of the material in the presence of a magnetic field.

We describe the electron energies and wave functions in InSb in the presence of a magnetic field within the three-level $\mathbf{P}\cdot\mathbf{p}$ model, which explicitly takes into account the Γ_6 conduction level, separated by the energy gap ϵ_g from the Γ_8 (degenerate) valence level, which is in turn split by the spin-orbit energy Δ from the Γ_7 valence level. The resulting $\mathbf{P}\cdot\mathbf{p}$ set of eight coupled differential equations can be solved exactly.[20] For the electron energies satisfying the condition $\epsilon \ll \epsilon_g+2\Delta/3$ one obtains the Landau-level (LL) energies in the form[21]

$$\epsilon_n^{\pm}(k_z)=-\frac{\epsilon_g}{2}+\left[\left(\frac{\epsilon_g}{2}\right)^2+\epsilon_g D_n^{\pm}(k_z)\right]^{1/2}, \tag{2}$$

where

$$D_n^{\pm}(k_z)=\hbar\omega_c\left(n+\frac{1}{2}\right)\pm\frac{1}{2}g_0^*\mu_B B+\frac{\hbar^2k_z^2}{2m_0^*}, \tag{3}$$

in which $\omega_c=eB/m_0^*$ is the cyclotron frequency, m_0^* is the effective mass at the band edge, $\mu_B=e\hbar/2m_0$ is the Bohr magneton, g_0^* is the spin Lande factor at the band edge, and $\pm$ signs correspond to effective spin-up and spin-down states. The nonparabolicity of the conduction band is expressed by the square root in Eq. (2).

Specifying the Landau gauge for the vector potential of the magnetic field, $\mathbf{A}=(-By,0,0,)$, one looks for solutions (the envelope functions) in the form $f(\mathbf{r})=\exp(ik_x x+ik_z z)\phi(y)$. The wave functions for the above range of energies are (cf. Ref. 21)

$$\begin{aligned}\varphi_n^+ &= a_n^+\phi_n u_1+\frac{\sqrt{3}}{2}b_n^+\left(\frac{\hbar\omega_c n}{D_n^+}\right)^{1/2}\phi_{n-1}u_3\\ &+\frac{1}{2}b_n^+\left(\frac{\hbar\omega_c(n+1)}{D_n^+}\right)^{1/2}\phi_{n+1}u_5\\ &-\frac{1}{\sqrt{2}}b_n^+\frac{\hbar k_z}{\sqrt{D_n^+ m_0^*}}\phi_n u_6,\end{aligned} \tag{4}$$

$$\begin{aligned}\varphi_n^- &= a_n^-\phi_n u_2+\frac{\sqrt{3}}{2}b_n^-\left(\frac{\hbar\omega_c(n+1)}{D_n^-}\right)^{1/2}\phi_{n+1}u_4\\ &-\frac{1}{\sqrt{2}}b_n^-\frac{\hbar k_z}{\sqrt{D_n^- m_0^*}}\phi_n u_5-\frac{1}{2}b_n^-\left(\frac{\hbar\omega_c n}{D_n^-}\right)^{1/2}\phi_{n-1}u_6,\end{aligned} \tag{5}$$

where $\phi_n[(y-k_xL^2)/L]$ are the harmonic-oscillator functions, $1/L^2=eB/\hbar$ defines the magnetic radius, and the coefficients are

$$(a_n^{\pm})^2=\frac{\epsilon_g+\epsilon_n^{\pm}}{\epsilon_g+2\epsilon_n^{\pm}},\quad (b_n^{\pm})^2=\frac{\epsilon_n^{\pm}}{\epsilon_g+2\epsilon_n^{\pm}}. \tag{6}$$

The Luttinger-Kohn amplitudes are defined in Ref. 21, but for our purposes it will suffice to know that they possess the periodicity of the lattice and are orthonormal within the unit cell.

The structure of the wave functions, as expressed by Eqs. (4), (5), and (6), must be taken into account together with the band nonparabolicity in a consistent polaron theory for narrow-gap semiconductors.

The Frohlich polar electron-phonon coupling is taken in the standard form

$$H_F=C\frac{1}{q}(e^{i\mathbf{q}\cdot\mathbf{r}}b_{\mathbf{q}}+e^{-i\mathbf{q}\cdot\mathbf{r}}b_{\mathbf{q}}^+), \tag{7}$$

where $C=[\alpha\sqrt{2}\hbar(\hbar\omega_L)^{3/2}2\pi/V\sqrt{m_0^*}]^{1/2}$. Here α is the polar constant, $\mathbf{q}$ is the phonon wave vector, $\hbar\omega_L$ is the energy of the longitudinal-optic phonon, and V is the crystal volume. Since in a narrow-gap semiconductor the electron mass depends in general on its energy, the problem arises as to whether the constant C does not depend on electron energy. However, the definition of the polar coupling constant α involves proportionality to $\sqrt{m_0^*}$ (cf. Mahan, Ref. 22), so that in reality the strength of electron-phonon interaction does not depend on the mass. [In fact, employing the initial formulation (1) it would make more sense to use in the definitions of α and of C the free-electron mass m_0, but this would lead to unusual values of α, so we keep the standard definition which, as stated above, does not influence the final results.]

We are interested in energies of the Landau state n at $k_z=0$, perturbed by the polar interaction with other Landau states, accompanied by a virtual or real emission of a longitudinal-optic phonon at low temperatures. If the emission of phonons is real, the lifetime of an electron in the initial state is finite. This requires the Green-function formalism, involving both real and imaginary parts of the electron self-energy. The electron energy is given by the maxima of the spectral function $A_n(E)=-(1/\pi)\mathrm{Im}\,G_n(E)$, where

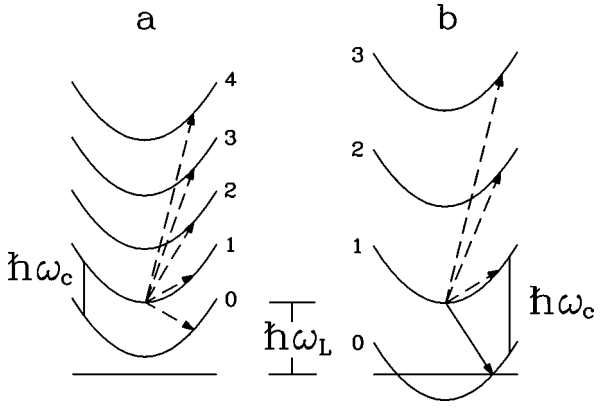

FIG. 1. Landau levels vs k_z: (a) below the polaron resonance $\hbar\omega_c<\hbar\omega_L$, and (b) above the polaron resonance $\hbar\omega_c>\hbar\omega_L$ (schematically). For $\hbar\omega_c<\hbar\omega_L$, all phonon emission transitions from the level $n=1$ at $k_z=0$ are virtual (dashed arrows). For $\hbar\omega_c>\hbar\omega_L$, the phonon emission to the $n=0$ level is real (solid arrow). At $\hbar\omega_c=\hbar\omega_L$, the real phonon emission is resonant.

$$G_n(E)=1\Big/\left[E+i\Gamma-\left(\epsilon_n+\sum\nolimits_n^r+\sum\nolimits_n^{\rm nr}\right)\right], \tag{8}$$

is the Green function. Here Γ is the phenomenological level broadening not related to optic phonons (discussed later), ϵ_n is the unperturbed electron energy, $\Sigma_n^{\rm nr}$ is the nonresonant correction, and Σ_n^r is the resonant correction to the energy. The situation is shown schematically in Fig. 1.

Nonresonant polaron correction

According to the second-order perturbation theory, the nonresonant correction to the nth LL at $k_z=0$ is

$$\sum\nolimits_n^{\rm nr}=\sum_{m\geqslant n}^{\infty}\sum_{\mathbf{q}}\frac{|M_{nm}(\mathbf{q})|^2}{E_n-[E_m(k_z)+\hbar\omega_L]}, \tag{9}$$

where $E_m(k_z)$ is the energy of the mth level and $M_{nm}(\mathbf{q})$ are the matrix elements of the Frohlich interaction (the phonon emission part).

One proves in a diagrammatical analysis of the polaron problem that in the self-energy calculation the energies E_m should be final energies, i.e., they should already include corrections due to the electron-phonon interaction.[23,24] We write, in general, $E_m=\epsilon_m+\delta_m$, where ϵ_m are the unperturbed energies and δ_m are the unknown corrections. For the lowest level of the sum of Eq. (9) (the most important one for the perturbation) one deals with δ_n, and we propose to take $\delta_m=\delta_n$ for all m. Since for the nth LL in question one can also write $E_n=\epsilon_n+\delta_n$, the above approximation leads to the cancellation of δ_n in the denominators of formula (9), and the latter becomes simply the Raleigh-Schrödinger (RS) perturbation series. Thus we obtain $\Sigma_n^{\rm nr}=\delta_n^{\rm RS}$. The RS theory is known to give the best results for the nonresonant perturbations.[22] The above ansatz generalizes the proposition of the authors of Ref. 11, who took $\delta_m=\delta_0^{\rm RS}$ for all m and n.

Further (only for the nonresonant part), we take the parabolic approximation to electron energies, i.e., $\epsilon_n^{\pm}(k_z)\approx D_n^{\pm}(k_z)$; cf. Eq. (3). We do the same for the wave functions, which amounts to assuming $\epsilon_n^{\pm}(k_z)\ll\epsilon_g$. This leaves only the first terms in Eqs. (4) and (5) with $a_n^{\pm}\approx 1$.

The matrix elements in Eq. (9) are calculated using the well-known formula for the harmonic-oscillator functions[25]

$$\begin{aligned}\langle m,k_x,k_z|e^{-i\mathbf{qr}}|n,0,0\rangle&=\delta_{0,k_z+q_z}\delta_{0,k_x+q_x}Ce^{i\varphi(m-n)}e^{iL^2q_xq_y/2}\\&\times(-Lq_\perp/\sqrt{2})^{m-n}\left(\frac{n!}{m!}\right)^{1/2}e^{-L^2q_\perp^2/4}\frac{1}{q}L_n^{m-n}(L^2q_\perp^2/2),\end{aligned} \tag{10}$$

where φ is the asimuthal angle, $q_\perp^2=q_x^2+q_y^2$ in the cylindric coordinate system, and $L_n^{m-n}(L^2q_\perp^2/2)$ are the associated Laguerre polynomials. Since the involved q values are rather small, the exponential function $\exp(-i\mathbf{q}\cdot\mathbf{r})$ is slowly varying within the unit cell, so that one can take the u_1 and u_2 periodic amplitudes in Eq. (4) and (5) out of the integral sign and use their normalization.

For $n=0$, formula (9) leads to

$$\begin{aligned}\sum\nolimits_0^{\rm nr}&=-\frac{CV}{(2\pi)^3}\sum_{m=0}^{\infty}\frac{1}{m!}\\&\times\int_0^{\infty}\frac{1}{q^2}\frac{(Lq_\perp/\sqrt{2})^{2m}e^{-L^2q_\perp^2/2}}{(-\epsilon_0+\epsilon_m+\hbar^2q_z^2/2m_0^*+\hbar\omega_L)}\,d\mathbf{q}.\end{aligned} \tag{11}$$

The summation can be performed analytically, and the integration is carried out in the spherical coordinate system. The final result is

$$\sum\nolimits_0^{\rm nr}=-\frac{\alpha\hbar\omega_L}{\sqrt{\pi\beta}}\int_0^{\infty}\frac{\exp(-t/\beta)\sqrt{\tau}g(\tau)}{\sqrt{t}\sqrt{t-1}}\,dt, \tag{12}$$

where $\beta=\hbar\omega_c/\hbar\omega_L$, $g(\tau)=\ln(\sqrt{\tau}+\sqrt{\tau-1})$ and $\tau=t/(1-e^{-t})$. Similarly, for $n=1$ one obtains

$$\begin{aligned}\sum\nolimits_1^{\rm nr}&=-\frac{\alpha\hbar\omega_L}{2\sqrt{\pi\beta}}\int_0^{\infty}e^{t(1-1/\beta)}\left[\frac{f(t)-\sqrt{t}}{t-1}+\frac{\sqrt{t}}{\tau-1}\right.\\&\left.+\frac{2(\tau-t)-t/(\tau-1)}{\sqrt{t\tau(\tau-1)}}g(\tau)\right]dt,\end{aligned} \tag{13}$$

where

$$f(t)=\begin{cases}\ln(\sqrt{t}+\sqrt{t-1})/\sqrt{t-1} & \text{for } t\geqslant 1\\ \arccos\sqrt{t}/\sqrt{1-t} & \text{for } t<1.\end{cases}$$

For $n=2$,

$$\sum{}_2^{\rm nr} = -\frac{\alpha\hbar\omega_L}{\sqrt{\pi\beta}}\int_0^\infty e^{t(1-1/\beta)}\left\{\frac{f(t)}{t-1}\left[\frac{15}{16(t-1)^2}-\frac{3(e^t-4)}{8(t-1)}+1\right]-\frac{\sqrt{t}}{t-1}\left[\frac{33-26t+8t^2}{16(t-1)^2}+\frac{(e^t-4)(2t-5)}{8(t-1)}+1\right]+\frac{\sqrt{t}}{\tau-1}\left[\frac{te^t(2\tau-5)}{8\tau(\tau-1)}+1\right]+\frac{\sqrt{t}g(\tau)}{\sqrt{\tau}(\tau-1)^{3/2}}\left[\frac{3te^t}{8\tau(\tau-1)}+\frac{\tau(\tau-1)}{te^t}-1\right]\right\}dt, \tag{14}$$

where $f(t)$ is defined as above.

In the limit of $B\to 0$, the complete nonresonant corrections for various LL's give the polaron ''binding energy'' $\Delta E=-\alpha\hbar\omega_L$ (the details are given in the Appendix). For $n=0$ and 1, this was shown by Larsen,[26] and for arbitrary n by Peeters and Devreese[12] (cf. also Huant and Karrai, Ref. 17). In weakly polar materials this shift is very small, and it has not been observed experimentally. The polaron corrections to the effective mass are discussed in the next subsection and in the Appendix.

Resonant polaron correction

The resonant polaron correction to the nth LL can occur for the phonon emission only by interaction with the lower levels $m<n$ (cf. Fig. 1). The second-order perturbation gives

$$\sum{}_n^r(E)=\sum_{m=0}^{n-1}\sum_{\mathbf{q}}\frac{|M_{nm}(\mathbf{q})|^2}{E+i\Gamma-[E_m(k_z)+\hbar\omega_c]}. \tag{15}$$

The phenomenological contribution $i\Gamma$ represents a residual scattering not related to optic phonons (always present in a crystal), which eliminates a nonphysical divergence at $E=E_m+\hbar\omega_L$, and gives a physically observed polaron behavior: at magnetic fields below the resonance, the lower branch is dominant, whereas above the resonance the upper branch takes over.[7]

As already mentioned in the previous subsection, the energies E_m in Eq. (9) should be final ones (i.e., they should already be corrected by the electron-phonon interaction). Thus for $n=1$ the sum in Eq. (15) involves one term $m=0$, in which the final energy includes only the nonresonant correction: $E_0(k_z)=\epsilon_0(k_z)+\delta_0^{\rm RS}$. For $n=2$, the sum involves two terms $m=0$ and 1, in which $E_0(k_z)$ is calculated as above, while the final energy $E_1(k_z)$ is calculated including both the nonresonant part $\delta_1^{\rm RS}$, as well as the resonant one $\delta_1^{\rm res}$. As follows from Eq. (8), the appropriate nonresonant corrections also appear in the Green function.

In the resonant part of the polaron correction (which gives the main observable effect) we include the real band structure of InSb, both in the energies and the wave functions. In the energies it amounts to taking formulas (2) and (3) for $\epsilon_m(k_z)$. When calculating the matrix elements $M_{nm}(\mathbf{q})$ with the complete wave functions (4) and (5), one takes into account that the interaction part $\exp(-i\mathbf{q}\cdot\mathbf{r})$ as well as the envelope functions in Eqs. (4) and (5) are slowly varying within the unit cell, so the integrals over the crystal volume may be broken into those involving the slowly varying functions ϕ and quickly varying periodic Luttinger-Kohn amplitudes u_i. One can then use the orthonormality relations for u_i, and they disappear from the final results. The matrix elements of $\exp(-i\mathbf{q}\cdot\mathbf{r})$ between various harmonic oscillators are calculated according to Eq. (10). It turns out that the structure of the wave functions (4) and (5) leads to the decrease of the matrix elements, as compared to those in which only the first terms in Eqs. (4) and (5) are accounted for. In more physical terms, the band nonparabolicity leads to weakening of the electron-phonon interaction. This feature has already been noticed in the calculation of electron mobility in InSb; cf. Ref. 27. [It should be emphasized that this result is in contradiction with the conclusions of Huant and Karrai (Ref. 17) who stated that the band nonparabolicity makes the effect of electron-phonon interaction stronger.] In the calculation we neglect the matrix elements between the different spin states, as given by Eqs. (4) and (5). They are nonzero (cf. Ref. 28), but are much smaller then those between the states of the same spin.

After some manipulation, the resonant part for $n=1$ (both lower and upper polaron branches) is obtained in the form (spin is included)

$$\sum{}_{1\pm}^r = -\left[\frac{\alpha\hbar\omega_L}{\sqrt{\beta}}\times\int_0^\infty\frac{x^2e^{-x^2}(\gamma^\pm+x+i\eta^\pm)(A^\pm-B^\pm x^2)^2F^\pm dx}{(\gamma^\pm+x)^2+(\eta^\pm)^2}+(\eta^\pm+i\gamma^\pm)\Gamma\right]\frac{1}{\Omega^\pm}. \tag{16}$$

Here $x=Lq_\perp\sqrt{2}$, $\Omega^\pm=\gamma^\pm-i\eta^\pm$, and $S^\pm=\Gamma F^\pm/\hbar\omega_c$, where

$$\gamma^\pm=\{[(T^\pm)^2+(S^\pm)^2]^{1/2}-T^\pm/2\}^{1/2},$$

$$\eta^\pm=\{[(T^\pm)^2+(S^\pm)^2]^{1/2}+T^\pm/2\}^{1/2},$$

in which

$$F^\pm=(E+\epsilon_0^\pm-\delta_0^{\rm RS}+\epsilon_g-\hbar\omega_L)/\epsilon_g,$$

$$T^\pm=(E-\epsilon_0^\pm-\delta_0^{\rm RS}-\hbar\omega_L)F^\pm/\hbar\omega_c.$$

Further,

$$A^\pm=a_0^\pm a_1^\pm+2B_1^\pm,$$

where

$$B^+=b_0^+b_1^+\hbar\omega_c/4\sqrt{D_0^+D_1^+},$$

$$B^-=b_0^-b_1^-3\hbar\omega_c/4\sqrt{D_0^-D_1^-},$$

in which $a_m^\pm$, $b_m^\pm$, and $D_m^\pm$ are given in Eqs. (6) and (3).

For the level $n=2$ the resonant correction comes from both lower levels $m=0$ and $m=1$. We have $\Sigma_2^r=R_0^\pm+R_1^\pm$, in which the first resonant part is

$$R_0^{\pm}=-\left[\frac{\alpha\hbar\omega_L}{2\sqrt{\beta}}\right.$$
$$\times\int_0^{\infty}\frac{x^4e^{-x^2}(\gamma_0^{\pm}+x+i\eta_0^{\pm})(A_0^{\pm}-B_0^{\pm}x^2)^2F_0^{\pm}dx}{(\gamma_0^{\pm}+x)^2+(\eta_0^{\pm})^2}$$
$$\left.+(\eta_0^{\pm}+i\gamma_0^{\pm})\Gamma\right]\frac{1}{\Omega_0^{\pm}}. \tag{17}$$

Here $\Omega^{\pm}=\gamma_0^{\pm}-i\eta_0^{\pm}$ and $S_0^{\pm}=\Gamma F_0^{\pm}/\hbar\omega_c$, where

$$\gamma_0^{\pm}=\{[(T_0^{\pm})^2+(S_0^{\pm})^2]^{1/2}-T_0^{\pm}/2\}^{1/2},$$
$$\eta_0^{\pm}=\{[(T_0^{\pm})^2+(S_0^{\pm})^2]^{1/2}+T_0^{\pm}/2\}^{1/2},$$

in which

$$F_0^{\pm}=(E+\epsilon_0^{\pm}-\delta_0^{\mathrm{RS}}+\epsilon_g-\hbar\omega_L)/\epsilon_g,$$
$$T_0^{\pm}=(E-\epsilon_0^{\pm}-\delta_0^{\mathrm{RS}}-\hbar\omega_L)F_0^{\pm}/\hbar\omega_c.$$

Further,

$$A_0^{\pm}=a_0^{\pm}a_2^{\pm}+3B_0^{\pm},$$

where

$$B_0^{+}=b_0^{+}b_2^{+}\hbar\omega_c/4\sqrt{D_0^{+}D_2^{+}},$$
$$B_0^{-}=3b_0^{-}b_2^{-}\hbar\omega_c/4\sqrt{D_0^{-}D_2^{-}}.$$

The second resonant part is

$$R_1^{\pm}=-\left[\frac{\alpha\hbar\omega_L}{2\sqrt{\beta}}\int_0^{\infty}\right.$$
$$\times\frac{x^2e^{-x^2}(\gamma_1^{\pm}+x+i\eta_1^{\pm})(A_1^{\pm}-B_1^{\pm}x^2+C_1^{\pm}x^4)^2F_1^{\pm}dx}{(\gamma_1^{\pm}+x)^2+(\eta_1^{\pm})^2}$$
$$\left.+(\eta_1^{\pm}+i\gamma_1^{\pm})\Gamma\right]\frac{1}{\Omega_1^{\pm}}. \tag{18}$$

Here $\Omega^{\pm}=\gamma_1^{\pm}-i\eta_1^{\pm}$, $S_1^{\pm}=\Gamma F_1^{\pm}/\hbar\omega_c$, where

$$\gamma_1^{\pm}=\{[(T_1^{\pm})^2+(S_1^{\pm})^2]^{1/2}-T_1^{\pm}/2\}^{1/2},$$
$$\eta_1^{\pm}=\{[(T_1^{\pm})^2+(S_1^{\pm})^2]^{1/2}+T_1^{\pm}/2\}^{1/2},$$

in which

$$F_1^{\pm}=(E+\epsilon_1^{\pm}-\delta_1^{\mathrm{res}}-\delta_1^{\mathrm{RS}}+\epsilon_g-\hbar\omega_L)/\epsilon_g,$$
$$T_1^{\pm}=(E-\epsilon_1^{\pm}-\delta_1^{\mathrm{res}}-\delta_1^{\mathrm{RS}}-\hbar\omega_L)F_1^{\pm}/\hbar\omega_c.$$

Further,

$$A_1^{+}=2B_1^{+},$$
$$A_1^{-}=2a_1^{-}a_2^{-}+5b_1^{-}b_2^{-}\hbar\omega_c/\sqrt{D_1^{-}D_2^{-}},$$

where

$$B_1^{+}=a_1^{+}a_2^{+}+3b_1^{+}b_2^{+}\hbar\omega_c/2\sqrt{D_1^{+}D_2^{+}},$$
$$B_1^{-}=a_1^{-}a_2^{-}+9b_1^{-}b_2^{-}\hbar\omega_c/2\sqrt{D_1^{-}D_2^{-}},$$
$$C_1^{+}=b_1^{+}b_2^{+}\hbar\omega_c/4\sqrt{D_1^{+}D_2^{+}},$$
$$C_1^{-}=3b_1^{-}b_2^{-}\hbar\omega_c/4\sqrt{D_1^{-}D_2^{-}}.$$

The limiting case of the parabolic energy band is obtained, taking in the above formulas $a_n^{\pm}=1$, $b_n^{\pm}=0$, and $\epsilon_g=\infty$.

The polaron energies for LL $n=1$ are obtained as maxima of the spectral density function

$$A_1^{\pm}(E)=-\frac{1}{\pi}\frac{\Gamma_1^{\pm}}{(E-\epsilon_1^{\pm}-\Delta_1^{\pm})^2+(\Gamma_1^{\pm})^2}, \tag{19}$$

where $\Delta_1^{\pm}$ and $\Gamma_1^{\pm}$ are real and imaginary parts of the self-energy, as given by Eq. (8). The spectral density function for LL $n=2$ is obtained in a similar way.

In the practical description of the data one first uses the relation $m_{\mathrm{pol}}^*=m_b^*/(1-\alpha/6)$ in order to determine the band effective mass m_b^* from the experimental mass at the band edge (m_b^* being a hypothetical mass related to the band structure; cf. the Appendix). Then the band parameters are adjusted to describe the band mass. Finally, one includes the nonresonant and resonant polaron corrections. This automatically gives the theoretical mass at the band edge equal to its experimental value $m_{\mathrm{pol}}^*=m_0^*$. In InSb the nonresonant polaron correction $\alpha/6$ is very small compared to unity.

III. RESULTS

In various experiments performed on InSb, one deals with different initial and final electron states coupled by the phonon emission. In the cyclotron resonance at low temperatures, the initial state is 1^+ and the final state is 0^+. In this direct arrangement one cannot easily follow the resonant region, since the optical absorption near the resonance $\hbar\omega=\epsilon_1^+-\epsilon_0^+\approx\hbar\omega_L$ is dominated by the reststrahlen band. The upper polaron branch is much wider than the lower one. As a result, the precision of the data above the resonance is not as good as that below.

To calculate the unperturbed energies in the conduction band we use the model of Pidgeon and Brown,[29] which includes, in addition to the three levels, contributions of other bands within the k^2 terms. In the spherical approximation (which we use) this model is slightly more precise than the three-level model (cf. Ref. 30), but it gives the same structure of the wave functions as that used above [cf. Eqs. (4), (5), and (6)], so that our derivations are valid without any changes. We use the following band parameters for InSb: $E_g=0.2352$ eV, $\Delta=0.803$ eV, $\gamma_1=3.25$, $\overline{\gamma}=0.35$, $\kappa=-1.3$, $N_1=-0.55$, and $F=-0.2$. As to the value of $E_{P_0}=2m_0P_0^2/\hbar^2$, we adjust it slightly to describe different experiments (see below).

The cyclotron resonance data may be presented in two equivalent ways. One either plots directly the cyclotron energy $E_1^+-E_0^+$ or the cyclotron effective mass m_0^*, defined as $\hbar eB/m_0^*=E_1^+-E_0^+$. For the lower polaron branch, the saturation of the energy $E_1^+-E_0^+$ near the value $\hbar\omega_L$ for mag-

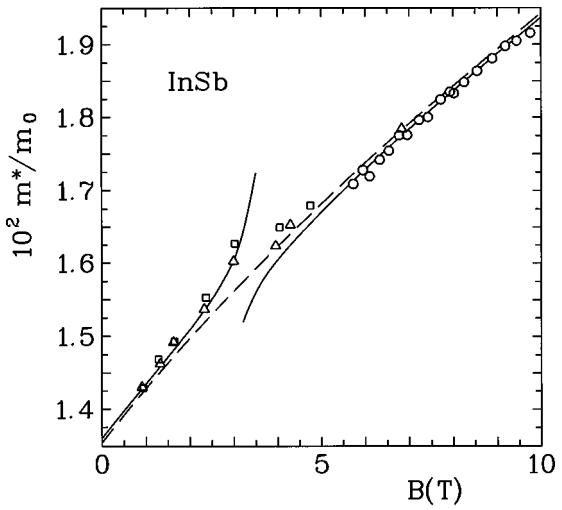

FIG. 2. Cyclotron mass of conduction electrons in InSb (spin-up transitions) vs magnetic field. Experimental data: circles are after McCombe, Bishop, and Kaplan (Ref. 31), squares after Huant *et al.* (Ref. 32), and triangles after Zawadzki, Klahn, and Merkt (Ref. 33). The dashed line is calculated ignoring the electron-phonon interaction. The solid lines are calculated including polaron corrections with the use of $\hbar\omega_L = 23.7$ meV and $\alpha = 0.02$.

netic fields above the resonance is equivalent to a strong increase of the cyclotron mass. For the upper branch, the saturation of the energy $E_1^+ - E_0^+$ near the value $\hbar\omega_L$ for fields below the resonance corresponds to a strong decrease of the mass.

In Fig. 2, we show the results of three different experiments on the cyclotron resonance in InSb. Near the polaron resonance $\epsilon_1^+ - \epsilon_0^+ \approx \hbar\omega_L$, the cyclotron mass for the lower branch shows the strong increase mentioned above. The dashed line indicates the increase of the mass resulting from the band nonparabolicity. This line is calculated taking the value of $E_{P_0} = 23.19$ eV, which results in the band-edge mass $m_b^* = 0.01355 m_0$ and the Lande factor $g^* = -51.04$. The solid lines are calculated including the nonresonant and resonant polaron corrections with the use of the polar constant $\alpha = 0.02$ and the optic-phonon energy $\hbar\omega_L = 23.7$ meV. The last value is somewhat lower than the generally accepted energy 24.4 meV, but we find that it is needed to describe all the data of our interest. It can be seen that the theory accounts well for the experimental results although, due to the reststrahlen band, there are no data points in the region of actual resonance.

This deficiency is overcome in the data of McCombe and Kaplan,[34] who investigated the combined resonance transition $0^+ \rightarrow 1^-$. This allows one to follow the 1^- Landau level through the resonant range $\epsilon_1^- - \epsilon_0^- \approx \hbar\omega_L$ without the reststrahlen problems, since the optical energy $\hbar\omega = \hbar\omega_c + \hbar\omega_s$ (where $\hbar\omega_s$ is the spin-flip contribution) is higher than the reststrahlen energy $\hbar\omega_L$; see Fig. 3. The dashed line indicates the unperturbed energy $\epsilon_1^- - \epsilon_0^+$. This energy is sensitive to the spin splitting and, in order to obtain the best fit, we had to use the value of $E_{P_0} = 21.92$ eV, which gives the mass $m_b^* = 0.01433 m_0$ and the spin factor $g_0^* = -48.26$. The other band parameters are the same as those given above.

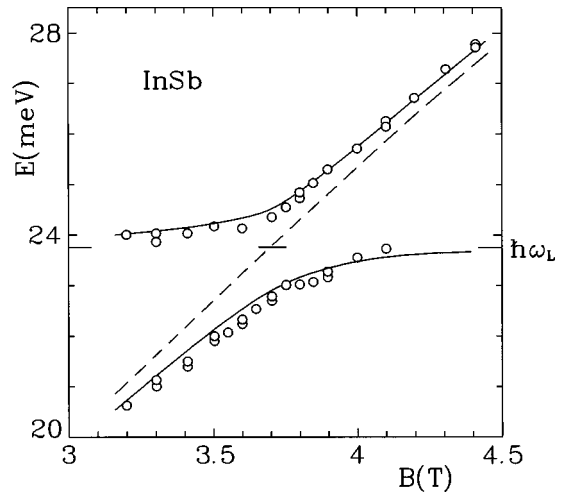

FIG. 3. Cyclotron energy for conduction electrons in InSb (spin-down transitions) vs magnetic field. Circles are the experimental data of McCombe and Kaplan (Ref. 34). The dashed line is calculated ignoring the electron-phonon interaction. The solid lines are calculated including polaron corrections. It can be seen that the upper and lower polaron branches saturate at the optic-phonon energy of $\hbar\omega_L = 23.7$ meV.

The solid lines are calculated including polaron corrections for the 1^- level. We emphasize that this is the most complete data of McCombe and Kaplan,[34] which require the optic-phonon energy of $\hbar\omega_L = 23.7$ meV, as explicitly seen in Fig. 3.

Finally, Fig. 4 shows the results of Koteles and Datars[35] for the cyclotron resonance (CR) masses in InSb, taken at the temperature of $T = 48$ K, which allowed the authors to observe four different CR transitions. The strongest polaron effect is seen for the $1^+ \rightarrow 2^+$ transition, involving the resonant polaron of $\epsilon_2^+ - \epsilon_0^+ \approx \hbar\omega_L$, which occurs at about half of the field corresponding to the resonance $\epsilon_1^+ - \epsilon_0^+ \approx \hbar\omega_L$. The transition $1^- \rightarrow 2^-$ is also affected by the resonance $\epsilon_2^- - \epsilon_1^- \approx \hbar\omega_L$, but to a lesser degree since, due to band's

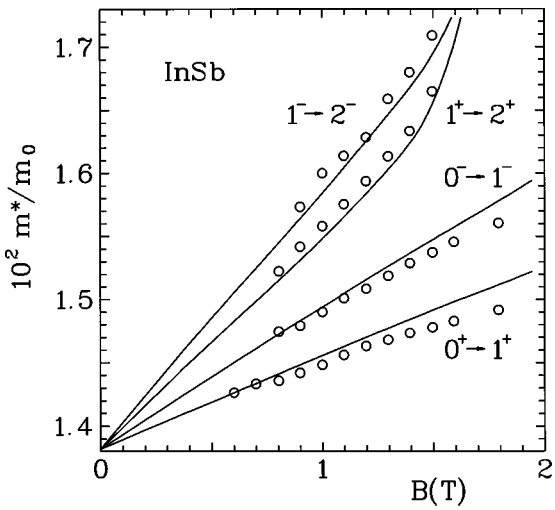

FIG. 4. Cyclotron masses of conduction electrons in InSb at $T = 48$ K for four cyclotron resonance transitions vs magnetic field. Circles are experimental data of Koteles and Datars (Ref. 35). Solid lines are calculated including the polaron corrections.

nonparabolicity, for a given magnetic field the energy $\epsilon_2^- - \epsilon_1^-$ is smaller than $\epsilon_2^+ - \epsilon_1^+$, so that at $B=1.6$ T one is further away from the resonance. The solid lines are calculated including polaron effects and taking $E_{P_0}=22.82$ eV, which corresponds to $m_b^*=0.013\,77 m_0$ and $g^*=-50.22$. (It is known that with increasing temperature the effective mass of electrons in InSb decreases.) The description of the data is good apart from small discrepancies for $0^+ \rightarrow 1^+$ and $0^- \rightarrow 1^-$ transitions, not related to polarons. In their original theoretical description, Koteles and Datars[35] used the Wigner Brillouin perturbation theory, neglecting the band nonparabolicity and the nonresonant polaron corrections. This oversimplified treatment forced the authors to employ the much too high value of $\alpha=0.041$. The resulting description is good for the two lower transitions and poor for the two higher ones.

In summary, we described resonant and nonresonant polaron corrections to the Landau energies in bulk InSb, and applied the results to various existing experimental data. The theory takes into account the nonparabolic band structure of the material both in the electron energies and the wave functions. It is found that the structure of the wave functions makes the effect of the electron-phonon interaction weaker. The theoretical analysis is applied to the observable polaron behavior of the Landau levels: 1^+, 1^-, 2^+, and 2^-. We achieve a very good description of the polaron behavior in all cases. The band parameters of InSb have to be slightly adjusted to describe the nonperturbed energies of Landau levels, which suggest that various experimental data are not completely consistent with each other.

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APPENDIX

Here we consider numerical values of the nonresonant polaron corrections to the Landau energies, and draw some conclusions. The entries S_0^t, S_1, and S_2 quote the values of strictly nonresonant parts as given by Eqs. (12), (13), and (14), respectively.

First, we observe that in the region of magnetic fields corresponding to the polaron resonance $\hbar\omega_c \approx \hbar\omega_L$, the nonresonant corrections do not undergo any jumps. As a consequence, the energy behavior near the resonance is completely governed by the resonant contribution. Thus the ''offset'' effect, i.e., an upward shift $\Delta E=\alpha\hbar\omega_L$ of the upper polaron branch with respect to the lower branch in the vicinity of resonance, does not exist (cf. Refs. 8 and 9). The nonresonant polaron shifts depend only weakly on magnetic field (cf. Table I and Ref. 10).

TABLE I. Calculated nonresonant polaron corrections to the Landau levels $n=0$, 1, and 2 at $k_z=0$ vs $\beta=\hbar\omega_c/\hbar\omega_L$. In all cases $\Delta E_n=-\alpha\hbar\omega_L S_n$. The entries S_0^t, S_1, and S_2 are nonresonant corrections given by Eqs. (12), (13), and (14), as used for the calculation of resonant behavior. The entries S_0^t, S_1^t, and S_2^t are total level shifts in the vicinity of $B\approx 0$, which also include the interaction with the level $n=0$ (for S_1^t), and with the levels $n=0$ and 1 (for S_2^t); see text.

β	S_0^t	S_1	S_1^t	S_2	S_2^t
0.001	1.000 08	0.9867	1.000 25	0.978	1.0004
0.01	1.0008	0.962	1.0025	0.940	1.0042
0.05	1.004	0.929	1.0129	0.889	1.0220
0.1	1.008	0.912	1.0265	0.861	1.0467
0.3	1.024	0.887		0.819	
0.5	1.040	0.880		0.804	
0.7	1.055	0.881		0.798	
0.9	1.070	0.884		0.796	
1.0	1.077	0.886		0.796	
1.1	1.084	0.888		0.797	
1.3	1.098	0.894		0.799	
1.5	1.111	0.900		0.803	
1.7	1.124	0.906		0.807	
1.9	1.136	0.915		0.812	
2.0	1.142	0.916		0.814	

Second, it can be seen that in the limit of $B\rightarrow 0$ the S_0^t correction (which contains the interaction with all higher levels) correctly converges to unity. This corresponds to the lowering of the energy by $\Delta E=-\alpha\hbar\omega_L$. On the other hand, for S_1 and S_2 this is not the case. The reason is that in the expression for S_1 one omits the interaction with the $n=0$ level, while in that for S_2 one omits the interactions with $n=0$ and 1 levels. The entries S_1^t and S_2^t include the omitted contributions, which in the limit of $B\rightarrow 0$ are nonresonant. It can be seen that S_1^t and S_2^t converge to unity, as they should. As a function of magnetic field the level $n=1$ is shifted downwards more than the level $n=0$. The same may be said about $n=2$ and 1 levels. This reflects the polaron correction to the effective mass. Comparing the numerical values one obtains that the corrected mass is $m_{\rm pol}^*=m_b^*/(1-\alpha/6)$. This agrees with the results of analytical calculations[12,17] carried out for the limit of $B\rightarrow 0$. It should be mentioned that, in spite of the fact that in the resonant parts we include the nonresonant shifts of the levels, for $\alpha<0.2$, the quantities S_1^t and S_2^t depend negligibly on the value of α.

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