Stochastic model of plasma waves for a simple band structure in semiconductors

M. Zakari*
Departament de Física, Universitat Autònoma de Barcelona, 08193 Bellaterra, Catalonia, Spain
(Received 25 July 1997; revised manuscript received 7 November 1997)

We consider the application of a stochastic model of two-layer systems to a simple band structure in semiconductors. The telegrapher's equation for the probability density is recovered and the source term is expressed as a function of the electron and hole concentrations. We derive the dispersion relation and we discuss its correction terms with respect to the purely telegrapher's description in fermion systems, for example, a photoexcited electron-hole plasma in semiconductors [S0163-1829(98)02916-6]

I. INTRODUCTION

Considerable literature on physical processes leading to the telegrapher's equation for the probability density or for temperature is presently available. Such an equation is the one akin to Maxwell-Cattaneo's transport equation for the diffusion (or probability) flux or the heat flux combined with the mass or the energy balance equations.[1–5] On the basis of information theory a hydrodynamic model for dissipative systems has been derived,[1,2,6] which seems to give a good tool for a mesoscopic description, as, for instance, the one used in the framework of extended irreversible thermodynamics (EIT).[6–8] This thermodynamic theory considers the usual dissipative fluxes or some higher-order fluxes as additional independent variables in a generalized entropy overcoming some paradoxes of local equilibrium theory, as the resulting infinite velocity of propagation of thermal and viscous signals and the negativeness of entropy production for some intervals of thermal or plasma waves.[6–8]

The random walk or the two-layer system is a simple model whose particles jump between two states in discrete times in a one-dimensional lattice. Such a model has been used in several domains of mathematical and physical sciences.[9–16] Our aim in the present work is to apply it to a simple band structure in semiconductors at three dimensions. The model was introduced as a model of diffusion in a number of biological and physical situations.[10] Some generalizations of this model taking into account inertial effects, as the persistent random walk, have been used in the so-called Taylor dispersion and turbulent diffusion,[11,12,5] by Godoy to generalize the Landauer coefficient for the diffusion of electrons in one dimension at 0 K,[16] by Kramers[14] in a Fokker-Plank equation in the presence of absorbing boundaries, and by Boughaleb and Gouyet[15] to generalize the lattice gas model to the Kramers regime. In many situations the source terms are neglected and the evolution equation is described by the telegrapher's equation.[4,13] In this paper we include not only the dissipative source, but also the Coulomb interaction.

Stochastic diffusion processes for discrete systems have been applied to several thermophysical systems.[9,17] In our case, the first layer is the conduction band, containing mobile electrons, and the second one is the valence band of mobile holes. The drift-diffusion equation[18] has been used particularly to describe hot electrons, and some collision processes, but the model suffers some criticisms at short times. Let us mention that good tools may be obtained by taking into account the full band structure in semiconductor materials. This subject is one of the relevant topics in the field of some nonequilibrium theories in the spirit to include the hyperbolic theory (nonparabolic bands) in the microscopic phenomena.[19] Furthermore, hydrodynamic models[20–22] obtained from higher moments of the Boltzmann transport equation have been used to simulate microelectronic devices. In the present paper we consider the semiconductors from another point of view. Our aim is to apply the stochastic model of the two-layer system to semiconductors, which leads to a microscopic approach to describe plasma waves in nonequilibrium situations.

We assume that the motion of particles in the energy bands of semiconductors is similar to the so-called persistent random walk;[5,9] in our case, the electrons move in the conduction band with velocity $\mathbf{v}_e(\mathbf{k})$ and the holes in the valence band with the velocity $\mathbf{v}_h(\mathbf{k})$. For example, in the parabolic approximation $\mathbf{v}_e(\mathbf{k})=\hbar\mathbf{k}/m_e^*$ and $\mathbf{v}_h(\mathbf{k})=\hbar\mathbf{k}/m_h^*$, where $\mathbf{k}$ is the wave vector and m_e^* and m_h^* are the effective masses of electron and hole, respectively. This parabolic theory is well known in the literature, but it is not accepted at high frequencies and low temperature, because fast phenomena appear and the particle mass varies with the microscopic energy.[19] In order to give a good analysis, one has to include the hyperbolic energy-momentum relation and to take into account several energy levels. This latter case will not be considered in the present paper; we limit ourselves to apply a stochastic two-layer model to semiconductor materials with simple bands, where the velocity of carriers does not depend on the position.

The paper is organized as follows: In Sec. II we present the mathematic formalism to be used, and in continuation we discuss the evolution equations from EIT. In Sec. III we reproduce the telegrapher's description, which corresponds in our case to the diffusion of electron and holes in semiconductors, without recombinations. In Secs. IV and V we take into account the dissipative source and the Coulomb interaction, respectively, and we compare our results for the dispersion relation and the propagation velocity with the ones corresponding to the purely telegrapher's equation. In the last section we give some concluding remarks.

II. MATHEMATIC FORMALISM

Since in most physical applications one cannot distinguish between right- and left-moving particles, one can use the density probabilities to find electrons $P_e(\mathbf{r},t,\mathbf{k})$ or holes $P_h(\mathbf{r},t,\mathbf{k})$, in the position $\mathbf{r}$ with the wave vector $\mathbf{k}$, at time t, under the hypothesis that the total probability and the particle flux take the form

$$P(\mathbf{r},t,\mathbf{k})=P_e(\mathbf{r},t,\mathbf{k})+P_h(\mathbf{r},t,\mathbf{k}), \tag{1}$$

$$\mathbf{J}(\mathbf{r},t,\mathbf{k})=\mathbf{v}_e(\mathbf{k})P_e(\mathbf{r},t,\mathbf{k})+\mathbf{v}_h(\mathbf{k})P_h(\mathbf{r},t,\mathbf{k}). \tag{2}$$

According to Eqs. (1) and (2) the probability to find electrons $P_e(\mathbf{r},t,\mathbf{k})$ and holes $P_h(\mathbf{r},t,\mathbf{k})$ in the position $\mathbf{r}$, at time t, with wave vector $\mathbf{k}$ may be written in terms of P and $\mathbf{J}$ as

$$P_e(\mathbf{r},t,\mathbf{k})=[\mathbf{J}(\mathbf{r},t,\mathbf{k})-\mathbf{v}_h(\mathbf{k})P(\mathbf{r},t,\mathbf{k})]\cdot\frac{\mathbf{v}_e(\mathbf{k})-\mathbf{v}_h(\mathbf{k})}{|\mathbf{v}_e(\mathbf{k})-\mathbf{v}_h(\mathbf{k})|^2}, \tag{3}$$

$$P_h(\mathbf{r},t,\mathbf{k})=-[\mathbf{J}(\mathbf{r},t,\mathbf{k})-\mathbf{v}_e(\mathbf{k})P(\mathbf{r},t,\mathbf{k})]\cdot\frac{\mathbf{v}_e(\mathbf{k})-\mathbf{v}_h(\mathbf{k})}{|\mathbf{v}_e(\mathbf{k})-\mathbf{v}_h(\mathbf{k})|^2}. \tag{4}$$

The evolution equations of probability densities for electrons and holes in the presence of Coulomb interaction are respectively described by

$$\begin{aligned}\frac{\partial P_e(\mathbf{r},t,\mathbf{k})}{\partial t}&+\mathbf{v}_e(\mathbf{k})\cdot\nabla_r P_e(\mathbf{r},t,\mathbf{k})\\ &=-\sum_{k'}[r_{ee}(\mathbf{k},\mathbf{k}')P_e(\mathbf{r},t,\mathbf{k}')+r_{eh}(\mathbf{k},\mathbf{k}')P_h(\mathbf{r},t,\mathbf{k}')]\\ &\quad+\int K_{eh}(\mathbf{r}-\mathbf{r}',\mathbf{k})\sum_{k'}[P_e(\mathbf{r}',t,\mathbf{k}')+P_h(\mathbf{r}',t,\mathbf{k}')]d\mathbf{r}',\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial P_h(\mathbf{r},t,\mathbf{k})}{\partial t}&+\mathbf{v}_h(\mathbf{k})\cdot\nabla_r P_h(\mathbf{r},t,\mathbf{k})\\ &=-\sum_{k'}[r_{he}(\mathbf{k},\mathbf{k}')P_e(\mathbf{r},t,\mathbf{k}')+r_{hh}(\mathbf{k},\mathbf{k}')P_h(\mathbf{r},t,\mathbf{k}')]\\ &\quad+\int K_{he}(\mathbf{r}-\mathbf{r}',\mathbf{k})\sum_{k'}[P_h(\mathbf{r}',t,\mathbf{k}')+P_e(\mathbf{r}',t,\mathbf{k}')]d\mathbf{r}',\end{aligned} \tag{6}$$

where $r_{ij}(\mathbf{k},\mathbf{k}')$ are the microscopic rates of transition probability between two energy levels (i,j stand for electrons and holes), and $K_{eh}(\mathbf{r}-\mathbf{r}',\mathbf{k})$ is the microscopic kernel associated with the Coulomb potential $V(\mathbf{r}-\mathbf{r}')$ of carrier-carrier interaction. Due to the exchange between electron and hole representations, one can assume that $K_{eh}=-K_{he}$. Furthermore, the detailed balance is assumed, $r_{ij}(\mathbf{k},\mathbf{k}')=r_{ij}\delta(\mathbf{k}-\mathbf{k}')$. Hereafter, we will consider a macroscopic situation at three-dimensional system, i.e., $P(\mathbf{r},t)=\Sigma_k\Sigma_a P_a(\mathbf{r},t,\mathbf{k})$ and $\mathbf{J}(\mathbf{r},t)=\Sigma_k\Sigma_a\mathbf{J}_a(\mathbf{r},t,\mathbf{k})$, $a=e$ for electron and $a=h$ for holes.

Taking into account Eqs. (1) and (2), a summation of Eqs. (5) and (6), and after summation over k of the corresponding expression, the balance equation of carrier particles yields (see Appendix A)

$$\frac{\partial P(\mathbf{r},t)}{\partial t}+\nabla_{\mathbf{r}}\cdot\mathbf{J}(\mathbf{r},t)=-\mathbf{A}\cdot\mathbf{J}(\mathbf{r},t)-BP(\mathbf{r},t), \tag{7}$$

where

$$\mathbf{A}=[(r_{ee}+r_{he})-(r_{hh}+r_{eh})]\frac{\mathbf{u}_e-\mathbf{u}_h}{|\mathbf{u}_e-\mathbf{u}_h|^2}, \tag{8}$$

$$B=[(r_{hh}+r_{eh})\mathbf{u}_e-(r_{ee}+r_{he})\mathbf{u}_h]\cdot\frac{\mathbf{u}_e-\mathbf{u}_h}{|\mathbf{u}_e-\mathbf{u}_h|^2}, \tag{9}$$

where $u_{e(h)}=\langle v_{e(h)}(\mathbf{k})\rangle$ is the average velocity of electrons (e) and holes (h).

The physical situation of the system is described assuming the following restrictions: (i) At time t_0 ($t_0=0$) the extrinsic semiconductor $[P_e(\mathbf{r},t,\mathbf{k})\neq P_h(\mathbf{r},t,\mathbf{k})]$ is excited by an external source. The laser pulse is considered sufficiently intense to produce mobile independent electrons and holes. (ii) After a lapse of time, the system tries to return to the equilibrium state by electron-hole recombinations. In the relaxational approximation, the rapid recombination is proportional to P/τ_R, where τ_R is the relaxation time corresponding to the process. This means that for short times we can neglect a term $\mathbf{A}\cdot\mathbf{J}$, by taking $\mathbf{A}$ equal to zero, i.e., we suppose that $r_{ee}+r_{he}=r_{hh}+r_{eh}$, which agrees with the hypothesis of Eqs. (14)–(16). Then Eq. (7) reduces to

$$\frac{\partial P(\mathbf{r},t)}{\partial t}+\nabla_r\cdot\mathbf{J}(\mathbf{r},t)=-BP(\mathbf{r},t), \tag{10}$$

with $B^{-1}=1/(r_{ee}+r_{he})=\tau_R$ is the recombination relaxation time.

The equation for the flux, which is the sum of Eq. (5) multiplied by $[\mathbf{v}_e(\mathbf{k})]$ and Eq. (6) multiplied by $[\mathbf{v}_h(\mathbf{k})]$ and after summation over $\mathbf{k}$, takes the form (see Appendix B)

$$\begin{aligned}\tau\frac{\partial\mathbf{J}(\mathbf{r},t)}{\partial t}+\mathbf{J}(\mathbf{r},t)+D\nabla_r P(\mathbf{r},t)&=-\tau\mathbf{u}\nabla_r\cdot\mathbf{J}(\mathbf{r},t)+\tau\mathbf{E}P(\mathbf{r},t)\\ &\quad+\tau\mathbf{u}'\int K(\mathbf{r}-\mathbf{r}')P(\mathbf{r}',t)d\mathbf{r}',\end{aligned} \tag{11}$$

where $\mathbf{u}=\mathbf{u}_e+\mathbf{u}_h$, $\mathbf{u}'=\mathbf{u}_e-\mathbf{u}_h$, and the parameters C and $\mathbf{E}$ are expressed in Appendix B. The relaxation time τ and the diffusion coefficient D are identified as

$$\tau=\frac{1}{C}, \tag{12}$$

$$D=-\tau\mathbf{u}_e\mathbf{u}_h. \tag{13}$$

The left-hand side (LHS) of Eq. (11) corresponds to a Maxwell-Cattaneo equation for the flux $\mathbf{J}$, and the right-hand side (RHS) represents some nonlocal terms. These latter terms are the main aim of this paper to describe plasma waves in particular systems. Generally, the RHS of Eq. (11) contains not only the nonlocal effects but also nonlinear ones. When $u_e=-u_h=v$ (v constant), we recover the results of Ref. 9, which describes a thermodynamic and stochastic diffusion in two-layer systems.

For the sake of simplicity, we suppose that diagonal elements of $(r)_{ij}$ are equal and very important with respect to others; this means that the transmission coefficients are more important than the reflection ones, i.e., $r_{ee}=r_{hh}$, $r_{eh}=r_{he}$, and $r_{eh}\ll r_{ee}$. Therefore, the parameters of the system are simplified as

$$B=(r_{ee}+r_{he})\simeq\frac{1}{\tau}, \tag{14}$$

$$C=(r_{ee}-r_{eh})=\frac{1}{\tau}, \tag{15}$$

$$\mathbf{E}=-r_{eh}\mathbf{u}. \tag{16}$$

Taking into account Eq. (10), we can transform Eq. (11) in terms of the probability concentration as

$$\tau\frac{\partial^2 P(\mathbf{r},t)}{\partial t^2}+\frac{\partial P(\mathbf{r},t)}{\partial t}-D\nabla_r^2 P(\mathbf{r},t) =\tau\frac{\partial F(\mathbf{r},t)}{\partial t}+F(\mathbf{r},t)+\tau\mathbf{u}'\cdot\nabla_r \times\int K(\mathbf{r}-\mathbf{r}')P(\mathbf{r}',t)d\mathbf{r}', \tag{17}$$

where the dissipative source $F(\mathbf{r},t)$ is identified as

$$F(\mathbf{r},t)=-BP(\mathbf{r},t)-\mathbf{u}\cdot\nabla_r P(\mathbf{r},t). \tag{18}$$

From the evolution equation (17) some concluding remarks can be drawn, particularly in the reaction-diffusion systems. The LHS of Eq. (17) corresponds to the telegrapher's equation, which has been used to describe the particle diffusion in many physical situations, and the RHS may be decomposed in two parts: the first two terms represent a particle reaction or a particle supplier described by the source F in neutral systems, and the last one corresponds to the Coulomb interaction.

III. TELEGRAPHER'S DESCRIPTION

In order to compare with the results of the next sections, we reproduce, briefly, an analytical solution and physical description of the well-known telegrapher's equation.[4,9,17] Recently Godoy and García-Colín showed that this equation is in general not valid in two and three dimensions for crystalline solids[23] by using a second-order Markov process in phase space. In our case lateral scattering probabilities are neglected, then Fick's and Maxwell-Cattaneo's laws are satisfied in two-dimensional (2D) and 3D. In fact, when the RHS in Eq. (17) is neglected, or in other words, in the limit in which very long wavelengths predominate, we obtain the purely telegrapher's equation, and its dispersion relation in the (ω,Q) space is written as

$$\omega^2+\frac{i}{\tau}\omega-\frac{D}{\tau}Q^2=0, \tag{19}$$

where ω is the thermal wave or plasma wave frequency and $\mathbf{Q}$ is the wave vector. Analytical solution to this latter equation takes the form

$$\omega_\pm=-\frac{i}{2\tau}\pm\frac{1}{2\tau}\sqrt{-1+4D\tau Q^2}. \tag{20}$$

For small values of Q $(Q<\frac{1}{2}\sqrt{1/\tau D})$, the frequency is purely imaginary. But, for higher values $(Q\geqslant\frac{1}{2}\sqrt{1/\tau D})$, the real part of Eq. (11) is derived as

$$\omega_\pm=\pm\frac{1}{2\tau}\sqrt{-1+4D\tau Q^2}. \tag{21}$$

This equation describes the propagation of thermal pulses or plasma waves in the context of Maxwell-Cattaneo. If the relaxation time tends to infinity, the propagation velocity $v_p\rightarrow\sqrt{D/\tau}=\sqrt{|\mathbf{u}_e\mathbf{u}_h|}$, which is finite. In Secs. IV and V we will see how the dispersion relation and the propagation velocity should be modified by the dissipative source and Coulomb interaction effect, respectively. A similar equation to Eq. (17) without dissipative source has been derived from information theory to study second sound in photoexcited plasma in semiconductors.[1,2] Furthermore, if we replace the concentration probability by the quasitemperature of carrier system, Eq. (17) implies a damped propagation in nonequilibrium study. When the relaxation time is equal to zero (classical theory), the propagation velocity is infinite. Then we recover the classical transport description, namely, Fourier's law for the heat flux and Fick's law for the diffusion flux.

IV. DISSIPATIVE SOURCE

The evolution equation of diffusion with dissipative source or reaction-diffusion systems is governed by the conservation law and the relaxational relation with memory.[24,25] Here, we want to take into account additional nonlocal effects of dissipative source F [i.e. in this case $V(x)=0$, and $F=-BP(\mathbf{r},t)-\mathbf{u}\cdot\nabla_r P(\mathbf{r},t)$]. The dispersion relation corresponding to this situation, described by the evolution equation (17), takes the form

$$\omega^2+\left[\frac{i}{\tau}+iB+\mathbf{u}\cdot\mathbf{Q}\right]\omega-\left[\frac{D}{\tau}Q^2+\frac{B}{\tau}-\frac{i}{\tau}\mathbf{u}\cdot\mathbf{Q}\right]=0, \tag{22}$$

and its analytical solution is written as

$$\omega_\pm=-\frac{1}{2}\left[i\left(\frac{1}{\tau}+B\right)+\mathbf{u}\cdot\mathbf{Q}\right]\pm\frac{1}{2}\left\{\left[i\left(\frac{1}{\tau}+B\right)+\mathbf{u}\cdot\mathbf{Q}\right]^2+\frac{4}{\tau}[DQ^2+B-i\mathbf{u}\cdot\mathbf{Q}]\right\}^{1/2}. \tag{23}$$

We observe that for vanishing dissipative source F (i.e., $B=0$ and $\mathbf{u}=\mathbf{0}$), we recover the telegrapher's description, which corresponds to the diffusion of particles in two-layer systems, with velocity v and $-v$ in each energy level, respectively.[9]

From Eq. (23) and taking into account Eqs. (14)–(16), we derive the real part of the frequency as

$$\omega_{\pm}(\text{real}) = -\tfrac{1}{2}\mathbf{u}\cdot\mathbf{Q} \pm \tfrac{1}{2}\sqrt{[u^2\cos^2(\theta)+4|\mathbf{u}_e\cdot\mathbf{u}_h]Q^2}, \tag{24}$$

where $\mathbf{u}\cdot\mathbf{Q}=uQ\,\cos(\theta)$. Due to the restrictions, Eqs. (14)–(16), this frequency expression does not depend on the relaxation time. When the average velocity $\mathbf{u}$ and the wave vector take the same direction $[\cos(\theta)=1]$, the physical situation is dominated by the individual excitations of electrons or holes,

$$\omega^2_{\text{pl},e} = u_e^2 Q^2, \tag{25}$$

$$\omega^2_{\text{pl},h} = u_h^2 Q^2, \tag{26}$$

where $\mathbf{u}_{e(h)} = \langle \mathbf{v}_{e(h)} \rangle$ is the average random velocity of electrons (e) and holes (h). In a general situation $[\cos(\theta)\neq 0]$, and taking into account Eq. (24), the propagation velocity is modified as $v_p = \tfrac{1}{2}|u\,\cos(\theta) + \sqrt{u^2\cos^2(\theta)+4|\mathbf{u}_e\cdot\mathbf{u}_h|}|$. As mentioned above, for vanishing electric potential and dissipative source, we recover the results of the well-known telegrapher's description.

V. COULOMB INTERACTION INCLUDED

In this section we treat the Coulomb interaction considered in the evolution equation (11). Taking into account Fourier transformation, the dispersion relation of Eq. (17) in (ω,Q) space takes the form

$$\omega^2 + \left[\frac{i}{\tau} + iB + \mathbf{u}\cdot\mathbf{Q}\right]\omega - \frac{1}{\tau}[DQ^2 + B - i\mathbf{u}\cdot\mathbf{Q}] + i\mathbf{u}'\cdot\mathbf{Q}\widetilde{K}(\mathbf{Q}) = 0, \tag{27}$$

where $\widetilde{K}(\mathbf{Q}) = \alpha\widetilde{V}(\mathbf{Q})$ is the Fourier transformation of the kernel expression $K(\mathbf{r})$, corresponding to the electric potential $V(\mathbf{r})$ and α is a parameter to be determined. The frequency is written as

$$\omega_{\pm} = -\frac{1}{2}\left(\frac{2i}{\tau} + \mathbf{u}\cdot\mathbf{Q}\right) \pm \frac{1}{2}\left\{\left(\frac{2i}{\tau} + \mathbf{u}\cdot\mathbf{Q}\right)^2 + 4\left[\frac{D}{\tau} + \frac{B}{\tau} - \frac{i}{\tau}\mathbf{u}\cdot\mathbf{Q} - i\mathbf{u}'\cdot\mathbf{Q}\widetilde{K}(\mathbf{Q})\right]\right\}^{1/2}. \tag{28}$$

For vanishing values of electric potential $\widetilde{V}(\mathbf{Q})$ we recover Eq. (22). Furthermore, for neglected dissipative source $F(\mathbf{r},t)$ and vanishing electric potential, we obtain the telegrapher's description, Eq. (19). Using Eqs. (14)–(16), the real part of ω is given by

$$\omega_{\pm}(\text{real}) = -\tfrac{1}{2}\mathbf{u}\cdot\mathbf{Q} \pm \sqrt{\omega^2_{\text{op}} + \{[\tfrac{1}{2}u\,\cos(\theta)]^2 + D/\tau\}Q^2}. \tag{29}$$

Compared with the results of Ref. 1 for the expression of optical plasma

$$\omega^2_{\text{op}} = \frac{n}{m}Q^2\widetilde{V}(\mathbf{Q}), \tag{30}$$

we can identify the parameter α and the kernel expression as

$$\alpha = i\,\frac{n}{m}\,\frac{Q^2}{|\mathbf{u}\cdot\mathbf{Q}|}, \quad \widetilde{\mathbf{K}}(\mathbf{Q}) = \alpha\widetilde{V}(\mathbf{Q}), \tag{31}$$

where $n=n_e+n_h$ is the total particle number and $m=m_e+m_h$ is the sum of electron and hole effective masses. Putting in order Eq. (29) for the expression of ω, we obtain

$$[\omega(\text{real}) + \tfrac{1}{2}\mathbf{u}\cdot Q]^2 = \omega^2_{\text{op}} + v^2_{\text{th}}Q^2, \tag{32}$$

with $v_{\text{th}} = \sqrt{\tfrac{1}{4}|\mathbf{u}|^2\cos^2(\theta) + |\mathbf{u}_e\cdot\mathbf{u}_h|}$ the effective thermal speed.

Recently Bingham, Mendarca, and Dawson[26] presented a general description of the nonlinear dispersion relation of electron plasma waves in a plasma with intense radiation, using the Kinetic equation of the Klimontovich type for the photons or plasmons. Our result for the dispersion relation obtained from a stochastic model for photoexcited electron-hole in semiconductors, without coupling with the photons beam, agrees with that of Ref. 26. Such a result seems to underline the connection between Kinetic theory and the present stochastic model.

VI. CONCLUSIONS

The last equation shows the corrections to the dispersion relation in fermion systems. Let us mention that the above results for the acoustic and optic plasma are observed and discussed in the literature.[27,28] Analogous development has been used for an extended quantum hydrodynamic approach[29] in other contexts to demonstrate the necessity of EIT at high frequency. In this paper we introduced a dissipative source [Eq. (18)], and Coulomb interaction corresponding to the electric potential in the evolution equation (11), which was neglected in many previous works. In Sec. V, as in Sec. IV, we derived a plasma frequency and a speed of propagation. Then, we compared our results with respect to the telegrapher's equation. This means that nonlinear and nonlocal effects are included in the theory of EIT by the generalized Maxwell-Cattaneo equation (11) or modified telegrapher's equation (17).

In summary, we applied a stochastic model of two energy levels for a photoexcited electron-hole plasma in semiconductors, and we discussed the effects of dissipative source and Coulomb interaction in the dispersion relation and random (or thermal) speed of propagation. Our corrections are notable, particularly in the reaction-diffusion systems and in the microscopic approximation in semiconductors.

ACKNOWLEDGMENTS

The author is indebted to Professor R. Luzzi and Professor A. R. Vasconcellos for helpful discussions and the warm hospitality extended to him by their group at the Unicamp University of Campinas. He also thanks Professor D. Jou and Dr. J. Camacho of the Autonomous University of Barcelona for valuable discussions on the topic of the stochastic model in two-layer systems. Financial support from the Generalitat de Catalunya (Ref. 1996BEAI300306) is also acknowledged.

This work was partially supported by DGICYT of the Spanish Ministry of Education and Science under Grant No. PB-94/0718.

APPENDIX A: DERIVATION OF NONCONSERVATION EQUATION (7)

In the Boltzmann-like equations (5) and (6) a detailed balance principle is assumed, i.e., $r_{ij}(\mathbf{k},\mathbf{k}')=r_{ij}\delta(\mathbf{k}-\mathbf{k}')$. This means that

$$\sum_{k'}[r_{ee}(\mathbf{k},\mathbf{k}')P_e(\mathbf{r},t,\mathbf{k}')+r_{eh}(\mathbf{k},\mathbf{k}')P_h(\mathbf{r},t,\mathbf{k}')]=r_{ee}P_e(\mathbf{r},t,\mathbf{k})+r_{eh}P_h(\mathbf{r},t,\mathbf{k}), \quad (A1)$$

$$\sum_{k'}[r_{he}(\mathbf{k},\mathbf{k}')P_e(\mathbf{r},t,\mathbf{k}')+r_{hh}(\mathbf{k},\mathbf{k}')P_{\mathrm{h}}(\mathbf{r},t,\mathbf{k}')]=r_{he}P_e(\mathbf{r},t,\mathbf{k})+r_{hh}P_h(\mathbf{r},t,\mathbf{k}). \quad (A2)$$

Furthermore, if we assume that electron and hole speed $\mathbf{v}_a(\mathbf{k})$ do not depend explicitly on the position $\mathbf{r}$, then we can rewrite the evolution equations (5) and (6) in the following forms:

$$\frac{\partial P_e(\mathbf{r},t,\mathbf{k})}{\partial t}+\nabla_r\cdot[\mathbf{v}_e(\mathbf{k})P_e(\mathbf{r},t,\mathbf{k})]=-[r_{ee}P_e(\mathbf{r},t,\mathbf{k})+r_{eh}P_h(\mathbf{r},t,\mathbf{k})]+\int K_{eh}(\mathbf{r}-\mathbf{r}',\mathbf{k})\sum_{k'}P(\mathbf{r}',t,\mathbf{k}')d\mathbf{r}', \quad (A3)$$

$$\frac{\partial P_h(\mathbf{r},t,\mathbf{k})}{\partial t}+\nabla_r\cdot[\mathbf{v}_h(\mathbf{k})P_h(\mathbf{r},t,\mathbf{k})]=-[r_{he}P_e(\mathbf{r},t,\mathbf{k})+r_{hh}P_h(\mathbf{r},t,\mathbf{k})]+\int K_{he}(\mathbf{r}-\mathbf{r}',\mathbf{k})\sum_{k'}P(\mathbf{r}',t,\mathbf{k}')d\mathbf{r}'. \quad (A4)$$

Due to the exchange between electron and hole representations, we can assume that their corresponding microscopic kernels are equal with opposite kernels are equal with opposite sign, i.e., $K_{eh}=-K_{he}=K$. The summation of Eqs. (A3) and (A4) is written as

$$\frac{\partial}{\partial t}[P_e(\mathbf{r},t,\mathbf{k})+P_h(\mathbf{r},t,\mathbf{k})]+\nabla_r\cdot[\mathbf{v}_e(\mathbf{k})P_e(\mathbf{r},t,\mathbf{k})+\mathbf{v}_h(\mathbf{k})P_h(\mathbf{r},t,\mathbf{k})]=-(r_{ee}+r_{he})_e(\mathbf{r},t,\mathbf{k})-(r_{eh}+r_{hh})_h(\mathbf{r},t,\mathbf{k}). \quad (A5)$$

Taking into account Eqs. (1) and (2), we can transform the last equation as

$$\frac{\partial}{\partial t}P(\mathbf{r},t,\mathbf{k})+\nabla_r\cdot\mathbf{J}(\mathbf{r},t,\mathbf{k})=-(r_{ee}+r_{he})P_e(\mathbf{r},t,\mathbf{k})-(r_{eh}+r_{hh})P_h(\mathbf{r},t,\mathbf{k}). \quad (A6)$$

Hereafter, we shall consider a macroscopic situation at three-dimensional system and we substitute the microscopic quantities by the macroscopic ones defined as

$$P(\mathbf{r},t)=\sum_k[P_e(\mathbf{r},t,\mathbf{k})+P_h(\mathbf{r},t,\mathbf{k})], \quad (A7)$$

$$\mathbf{J}(\mathbf{r},t)=\sum_k[\mathbf{v}_e(\mathbf{k})P_e(\mathbf{r},t,\mathbf{k})+\mathbf{v}_h(\mathbf{k})P_h(\mathbf{r},t,\mathbf{k})]=\mathbf{u}_eP_e(\mathbf{r},t)+\mathbf{u}_hP_h(\mathbf{r},t), \quad (A8)$$

where $\mathbf{u}_a=\langle\mathbf{v}_a\rangle$ is the average velocity. The summation of Eq. (A6) over k takes the form

$$\frac{\partial}{\partial t}P(\mathbf{r},t)+\nabla_r\cdot\mathbf{J}(\mathbf{r},t)=-[(r_{ee}+r_{he})-(r_{eh}+r_{hh})]\times\frac{\mathbf{u}_e-\mathbf{u}_h}{|\mathbf{u}_e-\mathbf{u}_h|^2}\mathbf{J}(\mathbf{r},t)-[(r_{hh}+r_{eh})\mathbf{u}_e-(r_{ee}+r_{he})\mathbf{u}_h]\cdot\frac{\mathbf{u}_e-\mathbf{u}_h}{|\mathbf{u}_e-\mathbf{u}_h|^2}P(\mathbf{r},t). \quad (A9)$$

Replacing in the right-hand side of Eq. (A9) the coefficients of $\mathbf{J}$ and P by $\mathbf{A}$ and B, respectively, we obtain a nonconservation equation (7).

APPENDIX B: DERIVATION OF EVOLUTION EQUATION (11)

The summation of evolution equation for electron Eq. (A3) multiplied by $\mathbf{v}_e(\mathbf{k})$ and the corresponding one for holes, Eq. (A4), multiplied by $\mathbf{v}_h(\mathbf{k})$ takes the form

$$\frac{\partial\mathbf{J}(\mathbf{r},t,\mathbf{k})}{\partial t}+\nabla_r[v_e^2(\mathbf{k})P_e(\mathbf{r},t,\mathbf{k})+v_h^2(\mathbf{k})P_h(\mathbf{r},t,\mathbf{k})]=-[\mathbf{v}_e(\mathbf{k})r_{ee}+\mathbf{v}_h(\mathbf{k})r_{he}]P_e(\mathbf{r},t,\mathbf{k})-[\mathbf{v}_e(\mathbf{k})r_{he}+\mathbf{v}_h(\mathbf{k})r_{hh}]P_h(\mathbf{r},t,\mathbf{k})+[\mathbf{v}_e(\mathbf{k})-\mathbf{v}_h(\mathbf{k})]\times\int K_{eh}(\mathbf{r}-\mathbf{r}',\mathbf{k})\sum_{k'}P(\mathbf{r}',t,\mathbf{k}')]d\mathbf{r}'. \quad (B1)$$

We use the following definitions well known in the literature:

$$\mathbf{u}_aP_a(\mathbf{r},t)=\sum_k\mathbf{v}_a(\mathbf{k})P_a(\mathbf{r},t,\mathbf{k}), \quad (B2)$$

$$\mathbf{u}_a^2P_a(\mathbf{r},t)=\mathrm{Var}(\mathbf{v}_a)(\mathbf{k})+\sum_k\mathbf{v}_a^2(\mathbf{k})P_a(\mathbf{r},t,\mathbf{k}), \quad (B3)$$

where $\mathrm{Var}(\mathbf{v}_a)$ is the variance of $\mathbf{v}_a$, a refers to electrons or holes. From information theory $\mathrm{Var}(\mathbf{v}_a)=\frac{3}{2}m_an_ak_\beta T_a$, with m_a, n_a, and T_a the mass, density, and absolute temperature of particle a, respectively. Assuming that $\nabla_r[\mathrm{Var}(\mathbf{v}_e)+\mathrm{Var}(\mathbf{v}_h)]=0$, and the microscopic kernels are equal with opposite signe $K_{eh}=-K_{he}=K$, as mentioned in Appendix A, the summation over k of Eq. (B1) is written as

$$\frac{\partial \mathbf{J}(\mathbf{r},t)}{\partial t}+\nabla_r[(\mathbf{u}_e+\mathbf{u}_h)\mathbf{J}(\mathbf{r},t)-\mathbf{u}_e\mathbf{u}_h P(\mathbf{r},t)] = -C\mathbf{J}(\mathbf{r},t)+\mathbf{E}P(\mathbf{r},t)+\mathbf{u}'\int K(\mathbf{r}-\mathbf{r}')P(\mathbf{r}',t)d\mathbf{r}', \tag{B4}$$

where $\mathbf{u}=\mathbf{u}_e+\mathbf{u}_h$, $\mathbf{u}'=\mathbf{u}_e-\mathbf{u}_h$, and the parameters C and $\mathbf{E}$ take the following expressions:

$$C=[(r_{ee}\mathbf{u}_e+r_{he}\mathbf{u}_h)-(r_{eh}\mathbf{u}_e+r_{hh}\mathbf{u}_h)]\cdot\frac{\mathbf{u}_e-\mathbf{u}_h}{|\mathbf{u}_e-\mathbf{u}_h|^2}, \tag{B5}$$

$$\mathbf{E}=[\mathbf{u}_h\cdot(r_{ee}\mathbf{u}_e+r_{he}\mathbf{u}_h)-\mathbf{u}_e\cdot(r_{eh}\mathbf{u}_e+r_{hh}\mathbf{u}_h)]\frac{\mathbf{u}_e-\mathbf{u}_h}{|\mathbf{u}_e-\mathbf{u}_h|^2}. \tag{B6}$$

After multiplying by $1/C$ and putting in order Eq. (B4) we obtain Eq. (11), namely, a generalized Maxwell-Cattaneo equation.

*Electronic address: mohamed@ulises.uab.es

[1] A. R. Vasconcellos, R. Luzzi, and A. S. Esperidião, Phys. Rev. B **52**, 5021 (1995).
[2] A. R. Vasconcellos, R. Luzzi, D. Jou, and J. Casas-Vazquez, Phys. Rev. B **52**, 5030 (1995).
[3] B. Vick and M. N. Özisik, J. Heat Transfer, **105**, 902 (1983).
[4] J. Masoliver, J. M. Porrà, and G. H. Weiss, Phys. Rev. A **45**, 2222 (1992); **46**, 3574(E) (1992).
[5] C. Van Den Broeck, Physica A **168**, 677 (1990).
[6] L. S. Garcia-Colin, A. R. Vasconcellos, and R. Luzzi, J. Non-Equilib. Thermodyn. **19**, 24 (1994).
[7] D. Jou, J. Casas-Vázquez, and G. Lebon, *Extended Irreversible Thermodynamics* (Springer, Berlin, 1993) (second revised and enlarged edition, Springer, Berlin 1996); also, Rep. Prog. Phys. **51**, 1105 (1988).
[8] I. Müller and T. Ruggeri, *Extended Thermodynamics* (Springer, New York, 1993).
[9] J. Camacho and M. Zakari, Phys. Rev. E **50**, 4233 (1994).
[10] R. Fürth, Ann. Phys. (Leipzig) **53**, 117 (1917).
[11] G. I. Taylor, Proc. London Math. Soc. **20**, 196 (1921).
[12] G. I. Taylor, Proc. R. Soc. London, Ser. A **219**, 186 (1953); **223**, 446 (1954).
[13] S. Goldstein, Q. J. Mech. Appl. Math. **4**, 129 (1951).
[14] H. Kramers, Physica (Amsterdam) **7**, 284 (1940).
[15] Y. Boughaleb and J. F. Gouyet, Solid State Ionics **9**, 10 (1983).
[16] S. Godoy, J. Chem. Phys. **94**, 6214 (1990).
[17] D. D. Joseph and L. Preziosi, Rev. Mod. Phys. **61**, 41 (1989); **62**, 375 (1990).
[18] W. Hansch, *The Drift-Diffusion Equation and its Applications in the MOSFET Modeling* (Springer, Wien, 1991).
[19] C. Jacoboni and L. Reggiani, Rev. Mod. Phys. **55**, 645 (1983).
[20] S. Selberherr, *Analysis and Simulation of Semiconductor Devices* (Springer-Verlag, Wien, 1984).
[21] A. M. Anile and S. Pennisi, Phys. Rev. B **46**, 13 186 (1992).
[22] A. M. Anile and O. Muscato, Phys. Rev. B **51**, 16 728 (1995).
[23] S. Godoy and L. S. García-Colín Phys. Rev. E **55**, 2127 (1997).
[24] S. A. Rice, *Diffusion-Controlled Reaction* (Elsevier, Amsterdam, 1985).
[25] H. Larralde and M. Araujo, Phys. Rev. A **46**, 855 (1992).
[26] R. Bingham, J. T. Mendonça, and J. M. Dawson, Phys. Rev. Lett. **78**, 247 (1997).
[27] P. A. Wolff, in *Raman Scattering Spectra in Solids I*, edited by G. B. Wright (Springer, New York, 1969).
[28] A. Pinczuk, J. Shah, and P. A. Wolff, Phys. Rev. Lett. **47**, 1487 (1981).
[29] A. R. Vasconcellos, R. Luzzi, D. Jou, and J. Casas-Vázquez, Physica A **212**, 357 (1994).