

Electron localization effects on the low-temperature high-field magnetoresistivity of three-dimensional amorphous superconductors

A. V. Samoilov and N.-C. Yeh

Department of Physics 114-36, California Institute of Technology, Pasadena, California 91125

C. C. Tsuei

IBM, Thomas Watson Research Center, Yorktown Heights, New York 10598

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The electrical resistivity ρ of three-dimensional amorphous superconducting films a -Mo₃Si and a -Nb₃Ge is measured in magnetic fields $\mu_0 H$ up to 30 T. At low temperatures and at magnetic fields above the upper critical field H_{c2} , ρ is temperature independent and decreases as a function of magnetic field. This field dependence is consistent with localization theory in the high-field limit [$\mu_0 H \gg \hbar/(4eL_\phi^2)$, where L_ϕ is the phase-coherence length]. Above the superconducting transition temperature T_c , the temperature dependence of the conductivity is consistent with inelastic scattering processes which are destructive to the phase coherence for electron localization, thereby allowing estimates for $L_\phi(T)$. The Hall effect data on a -Mo₃Si, in conjunction with the resistivity data, allow the determination of the carrier concentration and mean free path. The upper critical field is comparable to (in a -Mo₃Si) and significantly larger than (in a -Nb₃Ge) the Clogston-Chandrasekhar paramagnetic limit. This phenomenon is discussed in the context of electron localization. [S0163-1829(98)01702-0]

I. INTRODUCTION

The possibility of observing negative magnetoresistance due to the suppression of electron localization and hence an enhancement in the electrical conductivity (σ) of three-dimensional (3D) disordered metals is an interesting long-standing issue which has not been well explored experimentally.^{1,2} In contrast to the inactivity in the studies of 3D disordered metals, a number of experiments have been done on 3D disordered semiconductors,³⁻⁵ and the results are found to be in good agreement with the localization theory by Kawabata.² The reason for more studies of the localization in semiconductors than in metals is largely due to the smaller magnitude of the negative magnetoresistance in the latter. In other words, the higher conductivity σ and the predicted universal enhancement of the conductivity in high fields, $\Delta\sigma$ (see below), conspire to reduce the magnitude of $\Delta\sigma/\sigma$ and therefore make measurements in metals more difficult.

On the other hand, it is known that the electron-electron interaction results in a positive contribution to the magnetoresistivity^{1,6} which, in disordered semiconductors, generally predominates over the localization term which yields a negative magnetoresistance. Hence, the localization-related behavior becomes more difficult to infer directly.³ In this context experiments on metals are more advantageous for revealing the effects of localization, due to the stronger screening of the electron-electron interaction. One such example is the observation of a negative magnetoresistance in aluminum granular films.⁷

In addition to the negative magnetoresistance in 3D metals, the temperature dependence of the resistivity ρ in 3D disordered superconductors at low temperatures and high magnetic fields is another interesting issue. The question re-

garding whether the resistivity continues to increase on cooling, similarly to the diverging behavior of ρ in 2D superconductors at high fields,⁸ or saturates at low temperatures has not been addressed experimentally.

In this paper, we present an experimental investigation of the electron transport properties of homogeneous amorphous superconducting films of Mo₃Si and Nb₃Ge, under applied magnetic fields (H) up to 30 T and at temperatures (T) down to 35 mK. We find that both temperature and field dependences of the resistivity ρ can be qualitatively described by the localization theory.^{1,2} In addition, we report detailed studies of the upper critical field H_{c2} in both compounds and find that the low-temperature H_{c2} behavior disagrees with conventional theory⁹ involving the paramagnetic effect in dirty superconductors. This result may be qualitatively described in terms of a diverging paramagnetic limit in disordered superconductors.

II. EXPERIMENT

The samples used in this work are three 1700-Å-thick a -Mo₃Si films and a 200-Å-thick a -Nb₃Ge film, all deposited on cold sapphire substrates (held at 77 K) using rf sputtering.¹⁰ The homogeneity of the amorphous nature of these films is confirmed with x-ray diffraction. Tunneling studies in a -Mo₃Si (Ref. 11) reveal a BCS-like energy gap Δ , with $2\Delta/k_B T_c \approx 3.5$ (k_B is the Boltzmann constant, and T_c is the superconducting transition temperature). The zero-field T_c values for a -Mo₃Si and a -Nb₃Ge are 7.9 K and 2.9 K, with transition widths 20 mK and 80 mK, respectively. Most experiments reported here were carried out at the National High Magnetic Field Laboratory (NHMFL) (Tallahassee, FL), on two samples, one a -Mo₃Si and the other a -Nb₃Ge. At the NHMFL, a 20 T superconducting solenoid is used for measurements below 0.6 K and a 30 T resistive

magnet is used for measurements above 0.4 K. The magnetic field is always perpendicular to the film surface. The four-probe lock-in technique at an ac-current frequency 13.1 Hz was employed. The Joule heating at low temperatures was limited to $Q=10^{-6}$ W/m². Although the Kapitza thermal boundary resistance R_K between a -Mo₃Si and a -Nb₃Ge and helium is not known, we take the largest value of $R_K\sim 0.1$ m² K⁴/W(1/T³) available in literature¹² for the boundary between ³He and a solid to estimate the upper limit of the overheating $\Delta T=R_K Q$. At $T=35$ mK, $\Delta T\approx 3$ mK.

Measurements at $T>1.4$ K and for $H<15.5$ T are performed at Ecole Polytechnique (France) and at Caltech on all samples, and results for all three a -Mo₃Si samples are found to be consistent.¹³

III. RESULTS

Before presenting the experimental results, it is worthwhile to first verify the dimensionality of our samples, in the context of both superconductivity and localization, by comparing the thickness of the samples with relevant length scales. For superconductivity, the length for comparison is the coherence length $\xi=[\hbar/(2e\mu_0 H_{c2})]^{1/2}$ (\hbar is the Planck constant, e is the electron charge, and μ_0 is the permeability of vacuum). From our H_{c2} data (see below), we obtain $\xi(0)=49$ Å and 66 Å for a -Mo₃Si and a -Nb₃Ge, respectively. Therefore, at low temperatures [where $\xi(T)<d$, d is the thickness of the sample] we expect the samples to be in the 3D regime. It is worth noting that Theunissen and Kes¹⁴ have studied the fluctuation conductivity in a -Nb₃Ge and a -MoGe films, and have found that the data on the films with thicknesses up to $10\xi(0)$ are better described by 2D than 3D scaling theory.¹⁵ However, this 2D scaling is observed in the vicinity of the transition temperature¹⁴ where the coherence length becomes comparable to or larger than the sample thickness. Therefore, the finding of Theunissen and Kes¹⁴ does not contradict our conjecture about the 3D character of superconductivity in our films at low temperatures.

Of more relevance to the main theme of the present paper is the dimensionality with respect to weak localization. In this case, the characteristic length is $\min[L_\phi, L_H]$, where $L_\phi=(D\tau_{in})^{1/2}$ is the phase-coherence length ($D=\frac{1}{3}v_F l$ is the diffusion coefficient, τ_{in} is the inelastic scattering time, v_F is the Fermi velocity, and l is the mean free path) and $L_H=[2\pi\hbar/(2e\mu_0 H)]^{1/2}$.¹ In the field range 10–30 T, $L_H=140$ – 80 Å. Consequently, at high magnetic fields (where $L_H<d$), our samples of both a -Mo₃Si and a -Nb₃Ge are in the 3D regime in the context of weak localization.

The representative R -vs- H curves (R is the resistance) for both a -Mo₃Si and a -Nb₃Ge are shown in Fig. 1 (top and bottom panels, respectively). The distance between the voltage contacts on the films is approximately equal to the film width, and so the resistivity $\rho\approx Rd$. In the normal state, $\rho\approx 110$ $\mu\Omega$ cm and 190 $\mu\Omega$ cm for a -Mo₃Si and a -Nb₃Ge, respectively. With decreasing temperature, the field-induced superconducting transition occurs at higher fields and becomes sharper.

In order to better demonstrate the decrease in the transition width with decreasing temperature, we shift the curve at $T=35$ mK for a -Mo₃Si along the field axis by $\mu_0\Delta H=-10.6$ T and that for a -Nb₃Ge by $\mu_0\Delta H=-4.4$ T, as

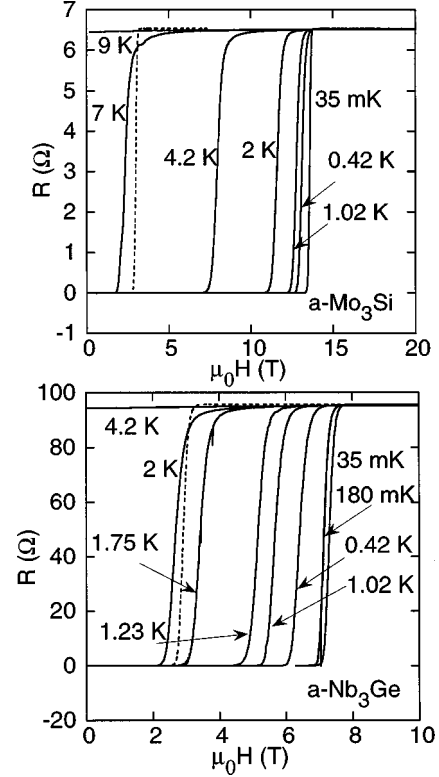


FIG. 1. The resistance (R) vs magnetic field (H) isotherms of a -Mo₃Si (top) and a -Nb₃Ge (bottom). The temperature of each isotherm is indicated near each curve. The dashed lines depict the isotherms for $T=35$ mK shifted along the H axis by $\mu_0\Delta H=-10.6$ T for a -Mo₃Si and by $\mu_0\Delta H=-4.4$ T for a -Nb₃Ge.

illustrated by the dashed lines in Fig. 1. Comparing the shifted R -vs- H curves with the higher-temperature isotherms taken at $T=7$ K and 2 K for a -Mo₃Si and a -Nb₃Ge, respectively, it is evident that the transition broadens with increasing temperature, although this broadening is much smaller than, for instance, that in high-temperature superconductors (HTSC's). In HTSC's, the higher operation temperatures, larger anisotropy, and smaller coherence length relative to those in conventional superconductors (such as a -Mo₃Si and a -Nb₃Ge) are known¹⁶ to yield significantly enhanced thermal fluctuations and reduced vortex pinning, hence a broad resistivity transition. (For an example of the comparison of the vortex dynamics in HTSC's and a -Mo₃Si, see Ref. 17.)

The uppermost (high-resistance) parts of the resistivity curves for a -Mo₃Si and a -Nb₃Ge are presented in Fig. 2 and Fig. 3. Shown in Fig. 2 are the isotherms $R(H)$, whereas Fig. 3 presents the $R(T)$ dependences at different fields. In the normal state, the resistance of the both samples increases monotonically with H (Fig. 2). Below the zero-field transition temperature $T_c(0)$, the field-induced superconducting to normal-state transition is followed by a resistivity decrease with increasing magnetic field up to the maximum available value of 30 T. With decreasing temperature, the decrease in resistivity at high fields becomes more pronounced.

From the resistance R -vs- H measurements, we construct the temperature dependences shown in Fig. 3. A magnetic field shifts the transition to lower temperatures [see data on $H_{c2}(T)$ in Fig. 4]. There is a well-defined field (13.8 T for a -Mo₃Si and 7.8 T for a -Nb₃Ge), at which the resistance no

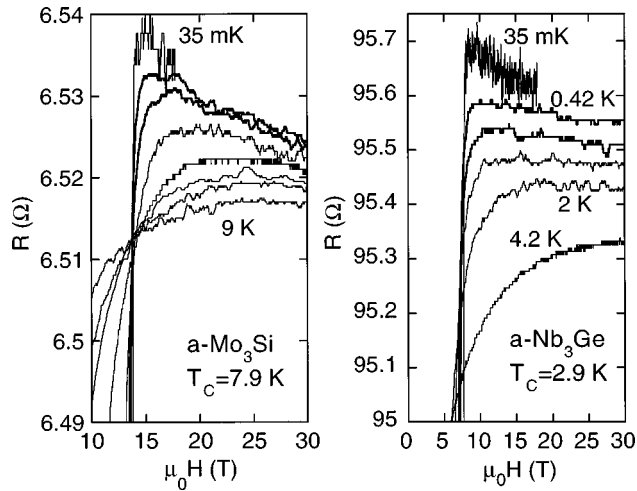


FIG. 2. The uppermost (high- R) parts of the isotherms of R for $a\text{-Mo}_3\text{Si}$ (left panel) and for $a\text{-Nb}_3\text{Ge}$ (right panel). The isotherms, in the left panel, correspond to temperatures $T=35$ mK, 0.42 K, 1.015 K, 2 K, 4.2 K, 5.5 K, 7 K, and 9 K. In the right panel, $T=35$ mK, 0.42 K, 0.83 K, 1.225 K, 2 K, and 4.2 K.

longer decreases with the decreasing temperature. (The large scattering of points at 13.8 T for $a\text{-Mo}_3\text{Si}$, Fig. 3, left panel, is due to a rapid change in the resistance near this field at low temperatures; see Fig. 2, left panel). Above this field, R increases monotonically upon cooling and eventually flattens at low T for both systems.

Figure 5 presents the resistivity data at temperatures above T_c . In both systems, the magnetoresistance decreases rapidly with increasing temperature, and at $T=30.2$ K there is practically no magnetic field dependence in the resistivity. It is interesting to note that in $a\text{-Mo}_3\text{Si}$ (Fig. 5, left panel) there is a distinct change of slope in the R -vs- H isotherms for $T=13.2$ K and 16.4 K at a field $\mu_0 H \approx 13\text{--}14$ T. Below this field, the resistivity increases with increasing field, and above this field, the resistivity is almost field independent. Interestingly, this crossover field nearly coincides with a characteristic field where the low-temperature resistance is T independent (see the isotherm at $\mu_0 H \approx 13.8$ T, Fig. 3, left panel). This crossover field observed at $T > T_c$ is also com-

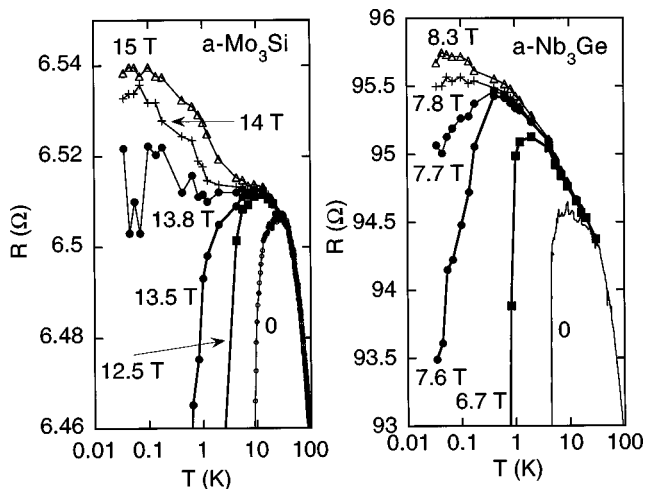


FIG. 3. The uppermost (high- R) parts of the isomagnetic curves of R for $a\text{-Mo}_3\text{Si}$ (left panel) and for $a\text{-Nb}_3\text{Ge}$ (right panel).

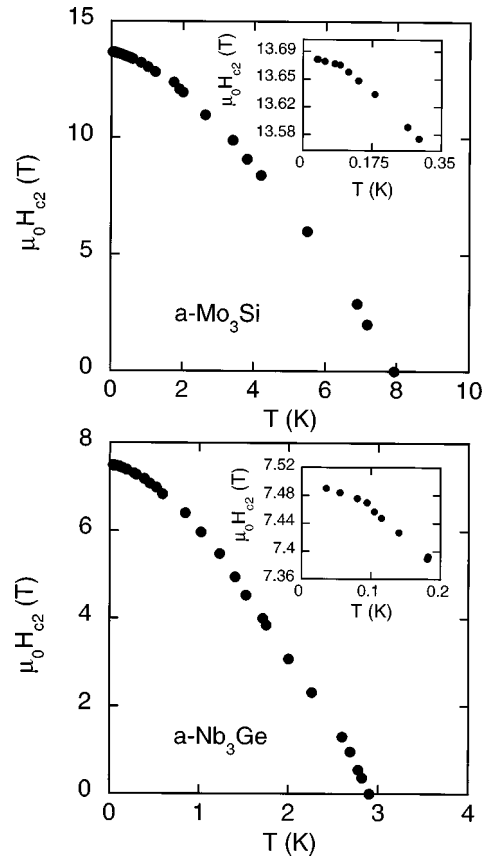


FIG. 4. The upper critical field (H_{c2}) vs temperature (T) for $a\text{-Mo}_3\text{Si}$ (top) and $a\text{-Nb}_3\text{Ge}$ (bottom). H_{c2} is determined using the criterion $\rho=0.9\rho_n$ (ρ_n is the normal-state resistivity). Shown in the insets are the low-temperature parts of the $H_{c2}(T)$ curves.

parable to the zero-temperature upper critical field ($\mu_0 H_{c2} \approx 13.7$ T, Fig. 4, upper panel, inset). In Nb_3Ge (Fig. 5, right panel), on the other hand, no sharp features are observed in the magnetic field dependences of R near $\mu_0 H_{c2}(T=0) \approx 7.5$ T, although some slower increase in R can be seen near $\mu_0 H_{c2}(T=0)$ at $T=7.0$ K and 9.0 K. Unlike in $a\text{-}$

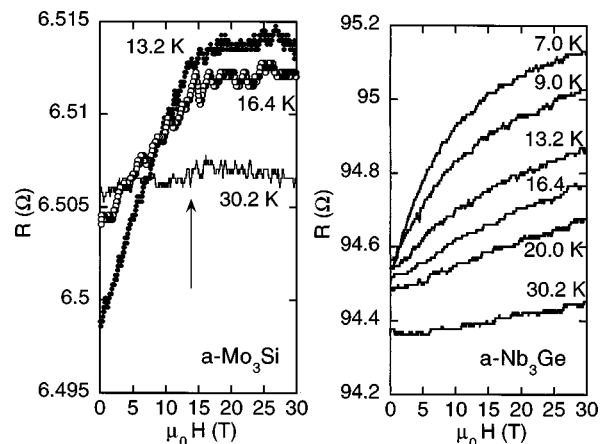


FIG. 5. R vs H at high temperatures for $a\text{-Mo}_3\text{Si}$ (left panel) and $a\text{-Nb}_3\text{Ge}$ (right panel). The arrow on the left panel marks the field $\mu_0 H_c=13.8$ T. Below this field, the resistivity increases with increasing field, and above this field, the resistivity is almost field independent.

Mo_3Si , the resistivity of $a\text{-Nb}_3\text{Ge}$ appears to increase with field up to the highest value (30 T) of our experiment.

IV. LOCALIZATION AND INTERACTION EFFECTS ON THE CONDUCTIVITY

Next, we consider the physical significance of the data. At low temperatures, the decrease of the resistivity with increasing field above H_{c2} can be well described in terms of theory of localization (see Ref. 1 for review). In this context, a negative correction to the classical Boltzmann conductivity σ_B arises from the interference of two electron paths which are on the same closed loop and are moving in two opposite directions.¹ The existence of such loops results in a localization of electrons,¹ provided that the phase coherence of the electron wave functions along these two paths can be maintained. Hence, a localization of electrons may occur if the phase coherence is not broken by inelastic scattering processes or by a magnetic field.

The effect of a magnetic field enters the localization problem via the characteristic length $L_H = [2\pi\hbar/(2e\mu_0H)]^{1/2}$, and that of temperature enters through $L_\phi(T)$: The phase coherence associated with the occurrence of localization is destroyed if the loop size is greater than L_H or $L_\phi(T)$. Consequently, the conductivity increases with increasing H or T . Thus, both the negative field coefficient for $H > H_{c2}(T)$ and the negative temperature coefficient of ρ can be explained by the destructive influence of the field and temperature, respectively, on the interference effects.

In the following, we consider various correction terms to the electrical conductivity of 3D conductors. At zero temperature, the quantum-corrected conductivity of a 3D disordered metal is^{12,18}

$$\sigma_0 = \sigma_B \left[1 - \frac{3}{(k_F l)^2} \right], \quad (1)$$

where k_F is the Fermi wave vector, and σ_B is the conductivity in the classical limit. Next, we consider the temperature-dependent localization correction to the conductivity of a 3D sample in zero field, which is given by^{1,12}

$$\Delta_T \sigma_{\text{loc}} = \frac{e^2}{\pi^2 \hbar} \frac{1}{L_\phi}. \quad (2)$$

Assuming a power law in the temperature dependence of the inelastic scattering time $\tau_{\text{in}} \sim T^{-p}$, with an exponent $p > 1$ depending on the scattering mechanism, we have $L_\phi \sim T^{-p/2}$ and $\Delta_T \sigma_{\text{loc}} \sim T^{p/2}$.

The magnetic field effect on the electron localization has been discussed by Kawabata,² which results in a correction term to the conductivity:

$$\Delta_H \sigma_{\text{loc}} = \frac{e^2}{2\pi^2 \hbar} \sqrt{\frac{e(\mu_0 H)}{\hbar}} f(x), \quad (3)$$

where $x = \hbar/[4e(\mu_0 H)L_\phi^2]$, and the asymptotic forms for $f(x)$ are $f(x) = 0.605$ for $x \ll 1$ and $f(x) = (x^{-3/2}/48)$ for $x \gg 1$. In the limit of small x , which corresponds to either large fields or weak inelastic scattering, the magnetoconductivity ($\Delta_H \sigma_{\text{loc}}$) is temperature independent:

$$\Delta_H \sigma_{\text{loc}} \approx 2.90(\mu_0 H)^{1/2}, \quad (4)$$

where σ is in $\Omega^{-1} \text{cm}^{-1}$ and $\mu_0 H$ in T.

In disordered conductors, the Coulomb interaction between electrons often has an important effect on the conductivity, because of the existence of closely spaced energy levels of electrons which experience the same disorder potential. The small energy difference ϵ of two electrons results in a long time scale \hbar/ϵ , during which the electrons are indistinguishable, and their scattering amplitudes add up due to the large number of phase-coherent paths with characteristic times smaller than \hbar/ϵ . This interaction correction to the conductivity $\Delta_T \sigma_{\text{int}}$ can be estimated by using Eq. (2), with the inelastic scattering time τ_{in} in $L_\phi (= \sqrt{D\tau_{\text{in}}})$ replaced by \hbar/ϵ . As shown by Al'tshuler and Aronov, the correction for the Coulomb interaction term in 3D samples becomes^{1,6}

$$\Delta_T \sigma_{\text{int}} = \frac{e^2}{4\pi^2 \hbar} \frac{1.3}{\sqrt{2}} \left(\frac{4}{3} - \frac{3}{2} \tilde{F} \right) \sqrt{\frac{k_B T}{\hbar D}}, \quad (5)$$

where $\frac{4}{3}$ comes from the exchange interaction among the electrons and $\frac{3}{2} \tilde{F}$ from the Hartree interaction ($0 < \tilde{F} < 1$).¹

The interaction correction in a magnetic field is given by¹

$$\Delta_H \sigma_{\text{int}} = -\frac{e^2}{4\pi^2 \hbar} \tilde{F} \sqrt{\frac{k_B T}{2\hbar D}} g(h), \quad (6)$$

where $h = g\mu_B H/k_B T$ (g is the g factor, μ_B the Bohr magneton), and the function $g(h)$ has the following asymptotic behavior: $g(h) = \sqrt{h} - 1.3$ for $h \gg 1$ and $g(h) = 0.053h^2$ for $h \ll 1$.

After considering all the above corrections, we obtain the total conductivity as follows:

$$\sigma = \sigma_0 + \Delta_H \sigma + \Delta_T \sigma, \quad (7)$$

where [see Eqs. (2)–(6)] $\Delta_H \sigma = \Delta_H \sigma_{\text{loc}} + \Delta_H \sigma_{\text{int}}$,

$$\Delta_T \sigma = \Delta_T \sigma_{\text{loc}} + \Delta_T \sigma_{\text{int}} - \frac{e^2}{\pi^2 \hbar} \frac{(k_F l)^2}{3l_{\text{in}}(T)},$$

and $l_{\text{in}} = v_F \tau_{\text{in}}$ is the inelastic electron mean free path. The last term in $\Delta_T \sigma$ is the result of thermal excitations of various inelastic processes.^{12,18}

V. DISCUSSION

A. Estimates of various correction components to the conductivity

Based on the above consideration, we find that at high fields and low temperatures, both localization and interaction terms in the magnetoconductivity [Eqs. (3) and (6), respectively] are proportional to $H^{1/2}$. In order to calculate $\Delta_H \sigma$, we first plot the total conductivity σ as a function of $H^{1/2}$ (not shown) and obtain $\sigma(H=0) = \sigma_0 + \Delta_T \sigma$ from the linear extrapolation of the σ -vs- $H^{1/2}$ dependences at low temperatures to zero field. Hence, $\Delta_H \sigma$ can be obtained by subtracting $\sigma(H=0)$ data from the total conductivity. In Fig. 6 we plot the magnetoconductivity $\Delta_H \sigma$ vs $H^{1/2}$. For $a\text{-Mo}_3\text{Si}$ and $a\text{-Nb}_3\text{Ge}$, $\sigma(H=0) \approx 7600$ (Ωcm)⁻¹ and 5200 (Ωcm)⁻¹, respectively. At low temperatures, $\sigma(H=0)$ is temperature

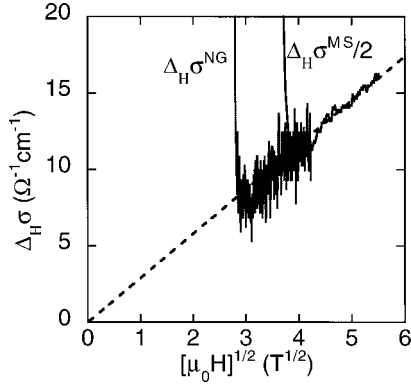


FIG. 6. $\Delta\sigma$ as a function of $H^{1/2}$ for a - Mo_3Si (MS) and a - Nb_3Ge (NG). The data for a - Nb_3Ge were taken at a lower dissipation level than those for a - Mo_3Si , and, therefore, they are noisier than the data for a - Mo_3Si .

independent, and we can associate it with the zero-temperature conductivity σ_0 which includes the quantum correction for localization [Eq. (1)].¹² The linear slope of the σ -vs- $H^{1/2}$ dependences is approximately temperature independent for the data taken at $T=35$ – 180 mK and $8T < \mu_0 H < 18$ T in the case of a - Nb_3Ge , and in the case of a - Mo_3Si for $T=0.42$ – 2 K and $15T < \mu_0 H < 30$ T. [For a - Mo_3Si , we cannot determine the slope of the σ -vs- $H^{1/2}$ dependences down to lower temperatures because of the higher H_{c2} and the limited field range for accessing the the normal-state behavior of the superconducting a - Mo_3Si films in the dilution refrigerator: $\mu_0 H_{c2}(0) \approx 13.7T < \mu_0 H < 18$ T.]

The dashed line in Fig. 6 depicts the theoretical $\Delta_H\sigma_{\text{loc}}$ curve according to Eq. (3). The agreement between the theoretical $\Delta_H\sigma_{\text{loc}}$ and experiment is good in a - Nb_3Ge , suggesting that the contribution of $\Delta_H\sigma_{\text{int}}$ is not significant. On the other hand, in a - Mo_3Si , the experimental value of $\Delta_H\sigma$ is 2 times larger than the theoretical prediction for $\Delta_H\sigma_{\text{loc}}$. Although the origin of this discrepancy is not understood, a similar trend has been observed in granular Al films by Chui *et al.*:⁷ Samples with low resistivity have $d\Delta_H\sigma/d(H^{1/2})$ up to 3 times higher than that predicted by Kawabata.² We note that neither the Coulomb interaction correction nor consideration of superconducting fluctuations can reduce the discrepancy between theory and experiment because both mechanisms result in a negative contribution to the magnetoconductivity. Therefore, significant corrections are needed to the $\Delta_H\sigma_{\text{loc}}$ term given by Kawabata.

In order to make a better comparison of the experimental $\Delta_H\sigma$ with theory, we consider the conductivity contributions due to both the superconducting fluctuation effects (σ_{fl}) and the Coulomb interaction ($\Delta_H\sigma_{\text{int}}$). The fluctuation conductivity has been calculated by Ullah and Dorsey¹⁵ in the lowest-Landau-level limit which is applicable to the high-field data. A convenient estimate for σ_{fl} , using the theory by Ullah and Dorsey,¹⁵ has been given in Ref. 14. Combining Eqs. (8) and (9) of Ref. 14, one can show that in the 3D regime, the fluctuation conductivity of isotropic superconductors in the dirty limit is

$$\sigma_{\text{fl}}^{3\text{D}} \approx 1.447\sigma_0 \frac{A_0^{3\text{D}}}{\epsilon_H^{1/2}}, \quad (8)$$

where $A_0^{3\text{D}} = 2\sqrt{2\text{Gi}}$ (Gi is the Ginzburg parameter), $\epsilon_H = t - 1 + h$, $t = T/T_c(0)$, and $h = H/H_{c2}(0)$. Using $h=2$, $T=35$ mK, and $\text{Gi} \sim 10^{-5}$ (Ref. 14), we obtain $\sigma_{\text{fl}}^{3\text{D}} \approx 0.43$ ($\Omega \text{ cm}$)⁻¹ for a - Mo_3Si and 0.8 ($\Omega \text{ cm}$)⁻¹ for a - Nb_3Ge . In view of the discussion of the dimensionality in the beginning of Sec. III, we also estimate the fluctuation conductivity in the 2D regime, $\sigma_{\text{fl}}^{2\text{D}}$, for our a - Nb_3Ge film which has a thickness $d \approx 3\xi(0)$ [see Eqs. (5)–(7) of Ref. 14]:

$$\sigma_{\text{fl}}^{2\text{D}} \approx 1.447\sigma_0 \frac{A_0^{2\text{D}}}{\epsilon_H} t, \quad (9)$$

where $A_0^{2\text{D}} = 4\sqrt{2\text{Gi}\xi(0)}/d$. Using the same values of Gi, h , and t as for the estimate of $\sigma_{\text{fl}}^{3\text{D}}$, we obtain $\sigma_{\text{fl}}^{2\text{D}} \approx 0.53$ ($\Omega \text{ cm}$)⁻¹ for a - Nb_3Ge . Comparing with the data shown in Fig. 6, we conclude that the effect of superconducting fluctuations on $\Delta_H\sigma$ at low temperatures and large magnetic fields may be neglected (see Fig. 6).

The interaction term in the magnetoconductivity ($\Delta_H\sigma_{\text{int}}$) at low temperatures and high fields [$h \gg 1$, Eq. (6)] is

$$\Delta_H\sigma_{\text{int}} = -\frac{e^2}{4\pi^2\hbar} \bar{F} \sqrt{\frac{g\mu_B H}{2\hbar D}}. \quad (10)$$

The diffusion coefficient D can be estimated from the H_{c2} data (Fig. 4) by the relation⁹

$$D = \frac{4k_B}{\pi e} \left(-\frac{dT_{c2}}{d(\mu_0 H_{c2})} \right),$$

which yields $D \approx 4 \times 10^{-5}$ m²/s for a - Mo_3Si and 2.4×10^{-5} m²/s for a - Nb_3Ge . Taking $g=2$, we obtain from Eq. (8), $\Delta_H\sigma_{\text{int}} \approx -3\bar{F}(\mu_0 H)^{1/2}$ (σ is in $\Omega^{-1} \text{ cm}^{-1}$, $\mu_0 H$ in T) for a - Mo_3Si and $-3.9\bar{F}(\mu_0 H)^{1/2}$ for a - Nb_3Ge . If we further assume $\bar{F} \approx 1$, then in order to account for the experimental value $\Delta_H\sigma \approx 5.8(\mu_0 H)^{1/2}$ in a - Mo_3Si , we have to assume that the localization term is $\Delta_H\sigma_{\text{loc}} \approx (5.8+3)(\mu_0 H)^{1/2} = 8.8(\mu_0 H)^{1/2}$, approximately 3 times larger than the theoretical prediction given by Eq. (4). Similarly, in a - Nb_3Ge , with an experimental value $\Delta_H\sigma \approx 2.9(\mu_0 H)^{1/2}$, the localization contribution would be $\Delta_H\sigma_{\text{loc}} \approx (2.9+3.9)(\mu_0 H)^{1/2} = 6.8(\mu_0 H)^{1/2}$. On the other hand, although it is difficult to obtain the dimensionless parameter \bar{F} with certainty, theory^{1,3} suggests $\bar{F} \ll 1$ rather than $\bar{F} \approx 1$. Indeed, \bar{F} depends on another parameter $X \sim (n/10^{24})^{1/3}$, where n is the carrier density in m⁻³. The Hall effect measurements on a - Mo_3Si (see below) suggest that $n \gg 10^{24}$ m⁻³. Hence, $X \gg 1$. In this limit $\bar{F} \ll 1$,^{1,3} and the interaction term $\Delta_H\sigma_{\text{int}} \sim \bar{F}$ becomes negligible.

Knowing σ_0 , we can compute, for a given magnetic field, the correction to the conductivity $\Delta\sigma = \sigma - \sigma_0$ as a function of temperature (Fig. 7, shown for $\mu_0 H = 15$ T). At low temperatures ($T < 0.3$ K), $\Delta\sigma$ is nearly temperature independent, so that $\Delta_T\sigma \approx 0$ and $\Delta\sigma \approx \Delta_H\sigma$ at $\mu_0 H = 15$ T. In the temperature interval 0.3–30 K, the behavior of $\Delta\sigma$ is determined by the interplay of several factors: $\Delta_T\sigma_{\text{loc}}$ augments with increasing T [Eq. (2)] because of the decreasing L_ϕ , $\Delta_H\sigma$ ($\approx \Delta_H\sigma_{\text{loc}}$, because $\Delta_H\sigma_{\text{int}} \approx 0$) decreases when $x = \hbar/[4e(\mu_0 H)L_\phi^2]$ becomes comparable to unity [Eq. (3)], the Aslamazov-Larkin superconducting fluctuation conduc-

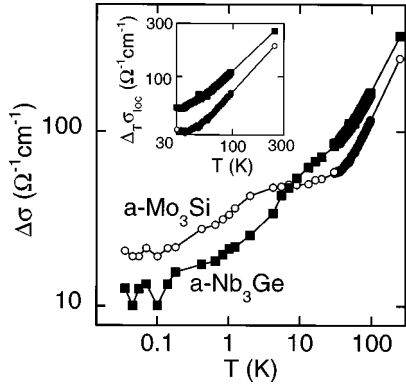


FIG. 7. The temperature dependence of $\Delta\sigma$. Inset: $\Delta_T\sigma_{\text{loc}}$ vs T (open circles for $a\text{-Mo}_3\text{Si}$, lower curve, and solid squares for $a\text{-Nb}_3\text{Ge}$, upper curve).

tivity vanishes with increasing temperature, and the interaction term $\Delta_T\sigma_{\text{int}}$ (see below) is small in the temperature region 0.3–30 K. Hence, we conclude that the increase in $\Delta\sigma$ with increasing temperature (for 0.3 K $< T < 30$ K) is largely associated with the temperature dependence of the localization correction $\Delta_T\sigma_{\text{loc}}$ [Eq. (2)].

Above $T = 30$ K the magnetoresistance becomes insignificant (Fig. 5). Therefore, the behavior of $\Delta\sigma(T)$ for $T > 30$ K is determined predominantly by the zero-field corrections to the conductivity, i.e., $\Delta\sigma(T > 30 \text{ K}) \approx \Delta_T\sigma(T > 30 \text{ K})$. We may estimate $\Delta_T\sigma_{\text{int}}$ by using Eq. (5) and by taking $\bar{F} \ll 1$. We obtain $\Delta_T\sigma_{\text{int}} \approx 4.6\sqrt{T}$ in $a\text{-Mo}_3\text{Si}$ and $5.9\sqrt{T}$ in $a\text{-Nb}_3\text{Ge}$ (σ is in $\Omega^{-1} \text{ cm}^{-1}$, T in K). These values account for approximately 25%–35% of $\Delta\sigma$ (Fig. 7) at $T = 30$ –300 K. Subtracting these values of $\Delta_T\sigma_{\text{int}}$ from the experimental data of Fig. 7 at high temperatures, we can obtain $\Delta_T\sigma_{\text{loc}}$ (Fig. 7, inset). Comparing $\Delta_T\sigma_{\text{loc}}$ with Eq. (2) gives $L_\phi \approx 3000/T$ (in \AA) for $a\text{-Mo}_3\text{Si}$ and $2300/T$ (in \AA) for $a\text{-Nb}_3\text{Ge}$. The temperature dependence of L_ϕ for $T > 30$ K is consistent with our earlier conjecture that $L_\phi \sim T^{-p/2}$ and $p > 1$.

In Fig. 8, we show the Hall effect data measured on another $a\text{-Mo}_3\text{Si}$ thin film whose resistance has been measured at fields below 15.5 T and at temperatures above 1.4 K and, within this range of the experimental parameters, shows properties consistent with those of the $a\text{-Mo}_3\text{Si}$ film described earlier. In the mixed state, there is a sign change in the Hall resistivity ρ_{xy} , which has been observed in many type-II superconductors, including high- T_c (Ref. 19) and

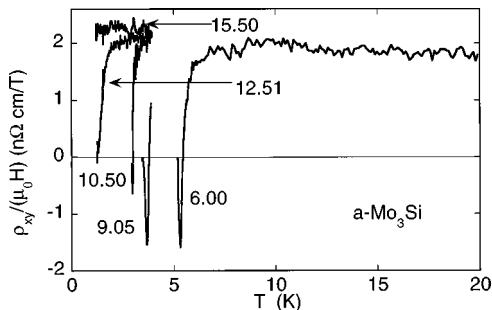


FIG. 8. The temperature dependence of $\rho_{xy}/(\mu_0 H)$ for $a\text{-Mo}_3\text{Si}$. The applied field values are shown near the curves, in units of tesla.

amorphous²⁰ superconductors. We shall not concern ourselves with the mixed-state Hall effect in this paper. In the normal state, the Hall coefficient $R_H = \rho_{xy}/(\mu_0 H)$ is positive and appears to increase on cooling from $T = 20$ K to 1.4 K by approximately 20%. The behavior of the Hall coefficient in disordered conductors is an interesting issue in its own right. However, small signal-to-noise ratio (≈ 20) in our Hall effect data does not allow us to quantify the temperature dependence of R_H .

Assuming $\rho_{xy}/(\mu_0 H) = 1/(ne)$ and taking $R_H = \rho_{xy}/(\mu_0 H) \approx 2 \text{ n}\Omega \text{ cm/T}$ (Fig. 8), we obtain for the hole density $n = 3 \times 10^{29} \text{ m}^{-3}$. From the Hall angle $\rho_{xy}/\rho = \omega_c \tau$, where $\omega_c = e(\mu_0 H)/m^*$ is the cyclotron frequency, we estimate $\tau \approx 10^{-16} m^*/m \text{ s}$ (m is the free electron mass, and m^* is the effective electron mass). From n we obtain the Fermi wave number $k_F = (3\pi^2 n)^{1/3} \approx 2 \text{ \AA}^{-1}$, and the Fermi velocity $v_F = (\hbar/m^*)k_F \approx (2 \times 10^6) m/m^* \text{ m/s}$. Thus, the mean free path is $l = v_F \tau \approx 2 \text{ \AA}$ and the parameter $k_F l \approx 4$. Compared with the diffusion coefficient obtained from H_{c2} data, the result $D = \frac{1}{3} v_F l \approx 1.3 \times 10^{-4} m/m^* \text{ m}^2/\text{s}$ is suggestive of $m^* \approx 0.3m$. However, we note that the value of n exceeds those in normal metals (like Cu, Al, Au, etc.) and seems to be somewhat overestimated. Furthermore, the assumption of a well-defined Fermi vector k_F in amorphous conductors is, strictly speaking, not accurate. So the estimates of k_F and D from the carrier density n and the Hall angle should be considered as an order-of-magnitude approximation only. Nonetheless, we may still estimate the inelastic mean free path in $a\text{-Mo}_3\text{Si}$ at high temperatures by the relation $l_{\text{in}} = 3L_\phi^2/l \approx 1.4 \times 10^7/T^2 [\text{\AA}]$. The T^{-2} dependence is a signature of the electron-electron scattering.

B. Upper critical field

Another interesting point for discussion is the zero-temperature value of the upper critical field (Fig. 4). Using the slope of the upper critical field at $T = T_c(0)$, we compute the parameter $h_{c2}(0) = H_{c2}(0)[-dT_c(0)/dH_{c2}]|_{T=T_c(0)}/T_c(0) = 0.72$ for $a\text{-Mo}_3\text{Si}$ and 0.65 for $a\text{-Nb}_3\text{Ge}$. These values are close to the result $h_{c2}(0) = 0.69$ for the orbital critical field $H_{c2}^{\text{orb}} = 0.69[-dH_{c2}/dT_c(0)]|_{T=T_c(0)}/T_c(0)$ in the standard Werthamer-Helfand-Hohenberg (WHH) theory²¹ of dirty superconductors. On the other hand, it is also necessary to compare the empirical upper critical field with the characteristic field (H_p) for the paramagnetic limit, where $\mu_B H_p \sim k_B T_c(0)$ or, more precisely,²² $H_p = \sqrt{2}\Delta(0)/g\mu_B$, with $\Delta(0)$ being the zero-temperature energy gap. Again assuming $g = 2$, we have $\mu_0 H_p = 14.5 \text{ T}$ and 5.3 T for $a\text{-Mo}_3\text{Si}$ and $a\text{-Nb}_3\text{Ge}$, respectively. The paramagnetic limit is supposed to reduce the upper critical field below its orbital value according to the formula $H_{c2}(0)^{-2} = H_p^{-2} + (H_{c2}^{\text{orb}})^{-2}$. In both $a\text{-Mo}_3\text{Si}$ and $a\text{-Nb}_3\text{Ge}$, the paramagnetic effect appears negligible. It is particularly remarkable in the case of $a\text{-Nb}_3\text{Ge}$, where the experimental value of $\mu_0 H_{c2}(0) = 7.5 \text{ T}$ exceeds that of $\mu_0 H_p = 5.3 \text{ T}$ substantially. One possible explanation may be related to the estimate of the paramagnetic limit in disordered superconductors: According to Spivak and Zhou,²³ the g factor in disordered superconductors may take any values because of electron localization. Therefore, it is only a question of probability for finding regions in the dis-

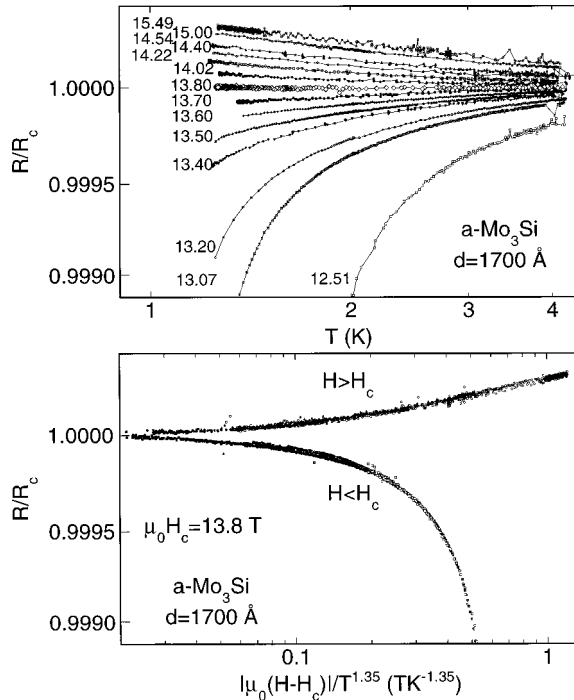


FIG. 9. R/R_c vs T for $a\text{-Mo}_3\text{Si}$ at magnetic fields near 13.8 T (top). Scaling of the data from the top panel according to Eq. (9) is shown in the bottom panel.

ordered sample where the condition $\mu_B H_p \gg k_B T_c(0)$ may be satisfied. Thus, the paramagnetic limit H_p in disordered superconductors may be very large, which explains our observations and earlier reports²⁴ of large H_{c2} in amorphous materials. However, the upward curvature in the $H_{c2}(T)$ line as predicted by Spivak and Zhou²³ is not observed in our samples down to $T = 35$ mK. This issue requires further theoretical investigation.

C. Comparison with 2D amorphous superconductors

In ultrathin amorphous superconducting films (with typical thickness of a few nm), the so-called “zero-temperature magnetic-field-induced superconductor-to-insulator transition”²⁵ has been observed.^{8,26} The experimental signature of this transition is a scaling relation for the sheet resistance:²⁵

$$R(H, T) = R_c F(|H - H_c|/T^{1/z\nu}), \quad (11)$$

where R_c and H_c are the critical resistance and critical field, respectively [$R(H = H_c) = R_c$], $F(H, T)$ the scaling function, and z and ν the critical exponents. In the work by Hebard and Paalanen on $a\text{-InO}_x$,⁸ the value of $R_c \approx 5$ k Ω is quite close to the theoretical estimate (R_Q) by Fisher,²⁵ where $R_Q = h/4e^2 \approx 6.4$ k Ω is the quantum resistance, and $z\nu \approx 1.3$. In a later work by Yazdani and Kapitulnik on $a\text{-Mo}_x\text{Ge}$,²⁶ the critical resistance has been shown to be nonuniversal, ranging from 600 Ω to 2 k Ω , with a comparable value of $z\nu \approx 1.36$.

For comparison, we perform detailed resistivity measurements in 3D $a\text{-Mo}_3\text{Si}$ films near the field $\mu_0 H = 13.8$ T where the resistivity is approximately temperature independent over a wide temperature range (Fig. 9, top panel). The bottom panel of Fig. 9 shows a successful attempt to scale

the data using Eq. (9). [We note that in our $a\text{-Nb}_3\text{Ge}$ film, there is no substantial field and temperature range where scaling given by Eq. (9) works.] The form of the scaling function is similar to that reported in Refs. 8 and 26. However, there are several important differences between our data and those on the ultrathin amorphous films.^{8,26} First, the effective critical resistance of our 3D $a\text{-Mo}_3\text{Si}$ films (of the order of ohms) is much smaller than the R_c value reported in Refs. 8 and 26. Second, the apparent exponent is $z\nu \approx (1.35)^{-1}$ for the $a\text{-Mo}_3\text{Si}$ films, compared with $z\nu \approx 1.3$ in ultrathin films.^{8,26} Third, at low temperatures, where the scaling given by Eq. (9) is supposed to work well, we find that the scaling relation actually breaks down when the resistance of $a\text{-Mo}_3\text{Si}$ becomes temperature independent (see Fig. 2).

The puzzling scaling behavior of our $a\text{-Mo}_3\text{Si}$ films at finite temperatures may be simply coincidental, because the theoretical prediction²⁵ is developed for 2D amorphous superconductors. We note that the key assumption of the field-tuned phase transition²⁵ is that only the phase of the order parameter is relevant for the occurrence of this phase transition at $T = 0$. The same assumption is likely to break down in the case of a 3D superconductor, because the modulation of the amplitude of the order parameter may no longer be negligible. We also caution that the mere existence of scaling behavior is not sufficient to prove a true second-order phase transition at $T = 0$. Nonetheless, the seemingly excellent scaling of the resistivity data in Fig. 9 may be suggestive of interesting physics for future investigation.

In addition to the variety of interesting phenomena associated with the magnetoconductivity, upper critical field, and scaling of the electrical resistivity of amorphous superconductors at low temperatures, we also note the possibility of quantum vortex lattice melting²⁷ at $T = 0$ and below H_{c2} , provided that the sheet resistance R is a significant fraction of the quantum resistance R_Q . However, the amorphous films presented in this work yield $R \ll R_Q$ ($R \approx 6.5$ Ω and 95 Ω for $a\text{-Mo}_3\text{Si}$ and $a\text{-Nb}_3\text{Ge}$, respectively), suggesting that the issue of quantum melting of the vortex system is difficult to resolve with certainty. This topic is beyond the scope of our current study and is better considered in Refs. 27 and 28 where the results of the non-Ohmic transport measurements are presented.

VI. SUMMARY

In summary, we have investigated the magnetoconductivity of three-dimensional amorphous films of $a\text{-Mo}_3\text{Si}$ and $a\text{-Nb}_3\text{Ge}$ in magnetic fields up to 30 T and temperatures down to 35 mK. A decrease in the resistivity with increasing field is observed above H_{c2} in both compounds at low temperatures. This decrease of field-induced resistivity agrees within a factor of 2 with the localization theory. At higher temperatures, above $T_c(0)$, in $a\text{-Mo}_3\text{Si}$ there is a significant cross-over from strong field dependence to weak field dependence of ρ at 13–14 T, near its upper critical field. This feature is not present in $a\text{-Nb}_3\text{Ge}$. The temperature dependence of the conductivity in the normal state is found to be dominated by the localization corrections, and the phase-coherence length is estimated at high temperatures in both $a\text{-Mo}_3\text{Si}$ and $a\text{-Nb}_3\text{Ge}$. The combination of the normal-state Hall effect and resistivity data in $a\text{-Mo}_3\text{Si}$ allows a determination of the carrier concentration n and mean free path l , although the

value of n seems overestimated. Various correction terms to the conductivity, including the temperature, magnetic field, and quantum fluctuation effects on the localization and Coulomb interaction, are estimated and compared quantitatively. Finally, the empirical H_{c2} values in both systems are insensitive to the conventional paramagnetic effect, which may be understood in terms of localization of the charge carriers in these disordered superconductors.

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