Parity effect in superconducting islands with increasing lengths

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(Received 26 June 1997)

We explore island length effects in single-electron transistors with superconducting islands and normal-metal leads. It had been shown experimentally that the current versus gate-induced charge characteristics change from $2e$ periodicity to e periodicity above a temperature T^* far below T_c. According to the equilibrium model of this parity effect proposed by Tuominen *et al.*, T^* depends inversely on the logarithm of the island volume, so T^* should be depressed as the island length is increased. Our high-current-sensitivity data show that the depression of T^* with increasing island length from 2 to 10 μm agrees well with the theoretical calculation of T^*. [S0163-1829(98)00501-3]

In a typical single-electron transistor, a small Al island is weakly coupled to a bias circuit through two small-capacitance tunnel junctions and a capacitive gate as illustrated in Fig. 1. The gate charge $Q_0 = C_g V_g$ controls the most probable number of electrons on the island. When the island is superconducting, the behavior of the system depends strongly on whether the number of electrons on the island is even or odd, even though it contains $N \sim 10^9$ conduction electrons. This parity effect is due to electron pairing as described in the BCS theory of superconductivity. Recent experiments have studied these parity effects, which occur below $T^* \approx T_c/5$ whether the single-electron tunneling transistor has superconducting leads (the SSS configuration)[1–3] or normal-metal leads (the NSN configuration).[4–6] These parity effects, also known as even-odd electron number effects, have also been studied theoretically in some detail.[6–9]

In this paper, we present measurements which reveal the length dependence of the parity effects. In the NSN single-electron tunneling transistor, the current versus the gate-induced charge exhibits $2e$ periodicity at very low temperatures. According to the equilibrium model proposed by Tuominen *et al.*,[1] this $2e$ periodicity will cross over into e periodicity above a certain temperature T^*. This crossover temperature T^* depends inversely on the logarithm of the island volume, and thus its length, if other dimensions are held constant. As a result, we expect that T^* should be depressed as the island length is increased. Our experiments test for the existence of this T^* depression by studying two NSN samples with island lengths of 2 and 10 μm, respectively.

Since one might expect that deviations from the simple equilibrium model might appear in islands whose length exceeds a nonequilibrium quasiparticle diffusion length ($\sim 5-10\ \mu$m), we also studied samples with islands of up to 40 μm in length. However the amplitude of the gate modulation was so small in these longer samples that we were unable to study the transition from $2e$ to e periodicity. Our analysis indicates that this falloff in modulation amplitude stems from the classical effect of stray capacitance between the longer island and the leads, and is consistent with orthodox theory without any allowance for nonequilibrium effects. This work, which involves quite different physics from that reported here, has been submitted for publication elsewhere.[10]

Our NSN samples are fabricated using conventional electron-beam lithography followed by a double-angle thermal evaporation. The island is formed by depositing Al onto a Si substrate at a 25° angle from the substrate normal along the length of the island. The island is typically 80 nm wide and 23 nm thick. The Al is then oxidized in 50 mtorr of O_2 for 5 min. The leads are formed by depositing ≈ 50 nm of Au at a 45° angle along the leads towards the island. To ensure that the number of electrons on the island is well defined, the tunnel junction resistances must exceed the quantum resistance $R_Q = h/e^2$. In addition, the charging energy $E_c = e^2/2C_\Sigma$, where $C_\Sigma = C_1 + C_2 + C_g$, must exceed $k_B T$ so that its effects are not washed out by thermal fluctuations. In our experiment, measurements are made in a top-loading dilution refrigerator, and the current is measured with a low-noise Ithaco 1211 current preamplifier. The sample is carefully shielded to prevent photon-assisted tunneling as described by Hergenrother *et al.*[11]

Tinkham, Hergenrother, and Lu[6] had reported the temperature dependence of the current versus gate charge I-Q_0 characteristics for an NSN sample with a 2 μm long island. Its volume was estimated to be 3.2×10^{-21} m^3. A value for the energy gap $\Delta \approx 245\ \mu$eV was extracted from the current versus bias voltage I-V characteristics by using the fact that with $Q_0 = e/2$, the current should rise rapidly at $2\Delta/e$.[5] The

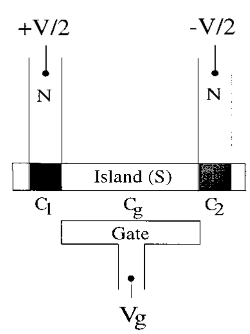

FIG. 1. Configuration of the NSN single-electron transistor. The shaded regions denote the two small tunnel junctions.

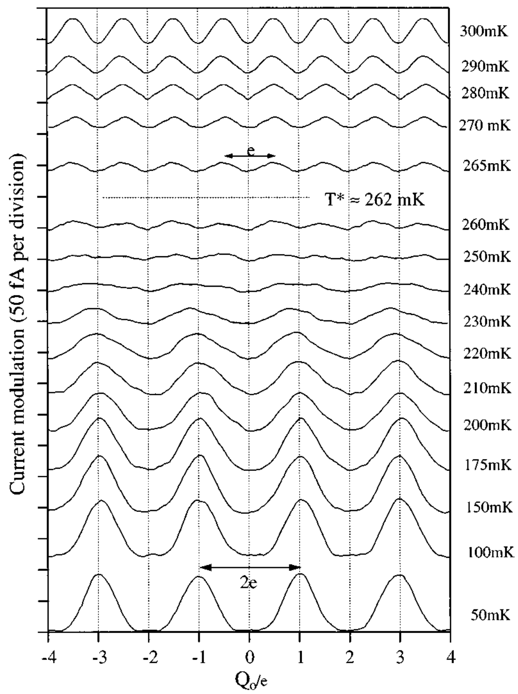

FIG. 2. Experimental I-Q_0 curves at a bias voltage $V = 140\ \mu$V and temperatures ranging from 50 to 300 mK for an NSN sample with a 10 μm long island. The curves are displaced upward successively for clarity. At low temperatures, the curves are strongly $2e$ periodic. As temperature is gradually increased, the curves evolve until they become completely e periodic above a crossover temperature of about 262 mK for this sample.

experimental crossover temperature T^* at which the current changes from being $2e$ periodic to e periodic (as described below) is about 280 mK. More recently we have done another temperature dependence measurement in an NSN sample with a 10 μm island. This sample has an island volume of 1.6×10^{-20} m^3 and $\Delta=260\ \mu$eV. A series of I-Q_0 curves at different temperatures are illustrated in Fig. 2. The evolution of the periodicity changes of this NSN sample is qualitatively similar to the 2 μm sample reported earlier. At low temperatures, the curves are strongly $2e$ periodic with two-electron tunneling (Andreev peaks) occurring at odd-integer values of Q_0/e. As the temperature is increased, the Andreev peaks decrease in size. Eventually single-electron tunneling dominates so that the current becomes e periodic and the peaks shift to half-integer values of Q_0/e. The magnitude of the current modulation grows with a further increase in the temperature. From observation, these I-Q_0 curves in Fig. 2 are $2e$ periodic at temperatures up to and including 260 mK. At 265 mK and above, only e periodicity can be distinguished in these curves at the available signal-to-noise ratio. Thus, we define the crossover temperature from $2e$ to e periodicity as $T^*\approx262$ mK, compared to 280 mK in the shorter sample. From Fig. 2, we see that the waveforms are very sensitive to the temperature. Even a 5 mK change in temperature results in noticeably different wave shapes, so T^* can be determined to a precision of $\sim2\%$.

To calculate the actual device current I-Q_0 theoretically, it would be necessary to do a kinetic calculation,[12,13] numerically solving a master equation to find the self-consistent steady-state nonequilibrium populations of all relevant states, and the resulting current. However, in the limit of low bias voltage, state populations will be near the $V=0$ equilibrium values for the same gate-induced charge Q_0. At sufficiently low bias voltages, we expect that the period (e or $2e$) of the current will be determined by the period with which the populations vary with Q_0 in equilibrium. This ''equilibrium model'' greatly simplifies the calculation of the periodicity crossover, and hence the T^* depression.[1,6,14] This model describes the system free energy of the single-electron transistor, which depends on whether there is an even or odd number of electrons on the island. It was predicted by Averin and Nazarov[7] that at $T=0$ and $H=0$, the ground-state energy[15] is higher by an amount Δ when the number of electrons on the island is odd (due to the presence of an unpaired electron which exists as a quasiparticle excitation) compared to when the number is even (all electrons paired). At finite temperatures, it is appropriate to generalize this to an even-odd *free*-energy difference F_0 which includes an entropy contribution. With such a parity-dependent free-energy difference, the system free energy is inherently $2e$ periodic in Q_0.

To make this more quantitative, at low ($eV\ll E_c$) bias voltage, the system free energy can be written in the following form:[5,14]

$$F_{\rm sys}(n,Q_0)\approx\frac{(-ne+Q_0)^2}{2C_\Sigma}+p_nF_0, \tag{1}$$

with

$$p_n=\begin{cases}0, & \text{if } n \text{ is even,}\\ 1, & \text{if } n \text{ is odd.}\end{cases}$$

Here, n is the number of excess electrons on the island.[16] The first term in Eq. (1) is the charging energy and the second term is the even-odd free energy difference which gives rise to the parity effect.

At low temperatures, F_0 reduces to a simple form, and exhibits a nearly linear relationship with temperature:

$$F_0\approx\Delta-k_BT\ \ln N_{\rm eff}(T), \tag{2}$$

where Δ is the superconducting gap and $N_{\rm eff}(T)$ is the effective number of quasiparticle states available for thermal excitation. The corresponding entropy is $k_B\ln(N_{\rm eff})$. The low-temperature approximation gives

$$N_{\rm eff}(T)\approx V_I\rho_n(0)\sqrt{2\pi\Delta k_BT}, \tag{3}$$

where V_1 is the volume of the superconducting island, and $\rho_n(0)$ is the normal density of states (including the spin degeneracy) in the island per unit energy and per unit volume. For aluminum, $\rho_n(0)\approx1.45\times10^{47}$ m^{-3} J^{-1}.

As the temperature is raised, F_0 gradually decreases to zero, at which point the periodicity of the I-Q_0 curves changes from being $2e$ to e. The crossover temperature T_0^* is defined as the temperature at which F_0 becomes zero in the nearly linear approximation of Eq. (2),

$$T_0^*=\frac{\Delta}{k_B\ln N_{\rm eff}(T_0^*)}. \tag{4}$$

Table I shows the results of using Eq. (4) to calculate T_0^* for each sample using the appropriate experimental values of

TABLE I. Comparison of theoretical and experimental values of T^*.

Island length	Experimental T^*	Experimental Δ	Estimated $N_{\rm eff}$	Calculated $T_0^*=\Delta/k_B \ln(N_{\rm eff})$
2 μm	280 mK	245 μeV	2.7×10^4	296 mK
10 μm	262 mK	260 μeV	1.4×10^5	270 mK
$\frac{T^*_{10\ \mu\rm m}}{T^*_{2\ \mu\rm m}}$	0.93			0.91

Δ and the values of $N_{\rm eff}$ calculated for each sample using Eq. (3). These T_0^* values agree remarkably well with the measured values T^* for both samples. This good agreement may be somewhat fortuitous, however, because the experimental crossover at T^* is not sharp, so that the measured value may depend slightly on the signal-to-noise ratio. Also T_0^* is based on the extrapolated zero crossing of the linear approximation to F_0, while F_0 actually goes to zero only asymptotically. However, these small distinctions in the definitions of T^* and T_0^* should have very similar effects in both samples and hence cancel out in the ratio. Accordingly, we now examine the comparison of the ratios.

According to Eq. (4), we expect the ratio of the crossover temperatures for the two different island lengths to be

$$\frac{T^*_{2\ \mu\rm m}}{T^*_{10\ \mu\rm m}} \approx \frac{\Delta_{2\ \mu\rm m} \ln N_{\rm eff}^{10\ \mu\rm m}(T^*_{0,10\ \mu\rm m})}{\Delta_{10\ \mu\rm m} \ln N_{\rm eff}^{2\ \mu\rm m}(T^*_{0,2\ \mu\rm m})}. \tag{5}$$

As seen in Table I, when the island length is increased from 2 to 10 μm, the experimentally determined crossover temperature T^* decreases by $\sim 7\%$, while the theoretically predicted decrease in T_0^* is $\sim 9\%$. Since most of the systematic uncertainties cancel in taking the ratios, we consider the excellent agreement of the predicted and measured *ratios* of T^* for the two sample lengths to be very significant, despite the small magnitude of the logarithmic length effect.

In conclusion, this investigation of the island length effect in single-electron transistors has further supported the predictions of the theory of the parity effect by demonstrating a depression of T^* as island length increases. The good agreement of the data with these theoretical calculations shows that the measured depression of the crossover temperature T^* is consistent with the predicted proportionality of T^* to the inverse of the logarithm of the island volume.

The authors would like to thank D. C. Ralph and R. J. Fitzgerald for their assistance. This research was supported in part by NSF Grants No. DMR-92-07956 and DMR 94-00396, ONR Grants No. N00014-89-J-1565, N00014-96-1-0108, and N00014-94-1-0808, and by JSEP Grant No. N00014-89-J-1023.

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[15] This is the excess energy above the Fermi level for an electron to tunnel onto the island.

[16] We assume for simplicity that the parity of n is equal to the parity of N, the total number of conduction electrons on the island.