Moving glass theory of driven lattices with disorder

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We study periodic structures, such as vortex lattices, moving in a random pinning potential under the action of an external driving force. As predicted in T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. **76**, 3408 (1996) the periodicity in the direction *transverse* to motion leads to a different class of driven systems: the moving glasses. We analyze using several renormalization-group techniques, the physical properties of such systems both at zero and nonzero temperature. The moving glass has the following generic properties (in $d \leq 3$ for uncorrelated disorder) (i) decay of translational long-range order, (ii) particles flow along static channels, (iii) the channel pattern is highly correlated along the direction transverse to motion through elastic compression modes, (iv) there are barriers to transverse motion. We demonstrate the existence of the *transverse critical force* at $T=0$ and study the transverse depinning. A ''static random force'' both in longitudinal and transverse directions is shown to be generated by motion. Displacements are found to grow logarithmically at large scale in $d=3$ and as a power law in $d=2$. The persistence of quasi-long-range translational order in $d=3$ at weak disorder, or large velocity leads to the prediction of the topologically ordered *moving Bragg glass.* This dynamical phase which is a continuation of the static Bragg glass studied previously, is shown to be stable to a nonzero temperature. At finite but low temperature, the channels broaden and survive and strong nonlinear effects still exist in the transverse response, though the asymptotic behavior is found to be linear. In $d=2$, or in $d=3$ at intermediate disorder, another moving glass state exists, which retains smectic order in the transverse direction: the *moving transverse glass.* It is described by the moving glass equation introduced in our previous work. The existence of channels allows us to naturally describe the transition towards plastic flow. We propose a phase diagram in temperature, force, and disorder for the static and moving structures. For correlated disorder we predict a ''moving Bose glass'' state with anisotropic transverse Meissner effect, localization, and transverse pinning. We discuss the effect of additional linear and nonlinear terms generated at large scale in the equation of motion. Generalizations of the moving glass equation to a larger class of nonpotential glassy systems described by zero temperature and nonzero temperature disordered fixed points (dissipative glasses) are proposed. We discuss experimental consequences for several systems, such as the anomalous Hall effect in the Wigner crystal, transverse critical current in the vortex lattice, and solid friction.
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I. INTRODUCTION

Interacting systems which tend to form spontaneously periodic structures can exhibit a remarkable variety of complex phenomena when they are driven by an external force over a disordered substrate. Many of these phenomena, which arise from the interplay between elasticity, periodicity, quenched disorder, nonlinearities and driving, are still poorly understood or even unexplored. For numerous such experimental systems, transport experiments are usually a convenient way to probe the physics (and sometimes the only way when more direct methods, e.g., imaging are not available). It is thus an important and challenging problem to obtain a quantitative description of their driven dynamics. Vortex lattices in type-II superconductors are a prominent example of such systems.[1] The motion of the lattice under the action of the Lorentz force (associated to a transport supercurrent) in the presence of pinning impurities has been studied in many recent experiments.[2–9] There are other examples of well-studied driven systems where quenched disorder is known to be important, such as the two-dimensional electron gas in a magnetic field which forms a Wigner crystal[10–12] moving under an applied voltage, lattices of magnetic bubbles[13,14] moving under an applied magnetic-field gradient, charge-density waves[15] (CDW) or colloids[16] submitted to an electric field, driven Josephson-junction arrays[17–19] etc. This problem may also be important in understanding tribology and solid friction phenomena,[20–22] surface growth of crystals with quenched bulk or substrate disorder,[23] domain walls in incommensurate solids.[24] One striking property exhibited by all these systems is pinning, i.e., the fact that at low temperature there is no macroscopic motion unless the applied force f is larger than a threshold critical force f_c. Dynamic properties have thus been studied for some time, quite extensively near the depinning threshold[25–27] but mostly in the context of CDW (Refs. 28–30) or for models based on driven manifolds[31,32] and their relation to growth processes[33] described by the Kardar-Parisi-Zhang (KPZ) equation.[34,31] They are, however, far from being fully understood. In addition, the full problem of a periodic *lattice* (with additional periodicity transverse to the direction of motion) was not scrutinized until very recently (see, e.g., Ref. 35 for a re-

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view). A crucial question in both the dynamics and the statics is whether topological defects in the periodic structure are generated by disorder, temperature and the driving force or their combined effect. Another important issue is to characterize the degree of order (e.g., translational order, or temporal order) in the structure in presence of quenched disorder. In the absence of topological defects it is sufficient in the statics to consider only elastic deformations. In the dynamics this leads to *elastic flow*. On the other hand, if these defects exist (e.g., unbound dislocation loops) the *internal* periodicity of the structure is lost and one must consider also plastic deformations. In the dynamics the flow is then not elastic but turn into *plastic flow* with a radically different behavior.

The *statics* of lattices with impurity disorder has been much investigated recently, especially in the context of type-II superconductors. It was generally agreed that disorder leads to a *glass phase* (often called[36,37] a *vortex glass*) with many metastable states, diverging barriers between these states,[38,1] pinning and loss of translational order. Indeed, even if for the pure elastic theory various proposals existed for the translational order,[39–43] general arguments,[37,39] unchallenged until recently, tended to show that disorder would always favor the presence of dislocations destroying the Abrikosov lattice beyond some length scale. In a series of recent works,[44–47] we have obtained a different picture of the *statics* of disordered lattices (including vortex lattices) and predicted the existence of a new thermodynamic phase, the *Bragg glass*. The Bragg glass has the following properties: (i) relative displacements grow only logarithmically at large scale, (ii) translational order decays at most algebraically and there are divergent Bragg peaks in the structure function in $d=3$ (i.e., quasi-long-range order survives) (iii) it is *topologically ordered*, (iv) it is nevertheless a true static glass phase with diverging barriers. There has been several analytical[48–50] and numerical studies[51,52] confirming this theory. The predicted consequences for the phase diagram of superconductors compare well with the most recent experiments.[47]

While some progress towards a consistent theoretical treatment has been made in the statics, it is still further removed in the dynamics. Determining the various phases of driven systems is still a widely open question. Evidence based mostly on experiments, numerical simulations and qualitative arguments indicates that quite generally one expects plastic motion for either strong disorder situations, high temperature, or near the depinning threshold in low dimensions (for CDW see, e.g., Ref. 53). Indeed there has been a large number of studies on plastic (defective) flow.[54–56] In the context of superconductors a H-T phase diagram with regions of elastic flow and regions of plastic flow was observed.[7,8] Several experimental effects have been attributed to plastic flow, such as the peak effect,[57,8,58,59] unusual broadband noise[60] and fingerprint phenomena in the I-V curve.[61,62,9] Steps in the I-V curve were also observed in Y-Ba-Cu-O near melting.[5] Close to the threshold and in strong disorder situations the depinning is observed to proceed through what can be called ''plastic channels'' [63,64] between pinned regions. This type of filamentary flow has been found[65] in simulations of two-dimensional (2D) (strong disorder) thin-film geometry (with $c_{11} \gg c_{66}$). Depinning then proceeds via filamentary channels which become increasingly denser. Filamentary flow was proposed as an explanation for the observed sharp dynamical transition observed in MoGe films[62,9] characterized by abrupt steps in the differential resistance. Interesting effects of synchronization of the flow in different channels were also observed.[65] Despite the number of experimental and numerical data[55,56] a detailed theoretical understanding of plastic motion remains quite a challenge.[66]

As in the statics, one is in a better position to describe the elastic flow regime, which is still an extremely difficult problem. This is the situation on which we focus in this paper. Though elastic flow in some cases extends to all velocities, a natural idea is to start from the large velocity region and carry perturbation theory in $1/v$. At large velocity one may think at first that since the sliding system averages enough over disorder one recovers a simple behavior, in fact much simpler than in the statics. Indeed it was observed experimentally, some time ago in neutron-diffraction experiments,[67] and in more details recently,[68] that at large velocity the vortex lattice is more translationally ordered than at low velocity. This tendency to *dynamical reordering* has also been seen in numerical simulations.[54,55,69] The $1/v$ expansion has been fruitful to compute the corrections to the velocity itself in Refs. 70, 71, and 29. Recently it was extended by Koshelev and Vinokur in Ref. 72 to compute the vortex displacements u induced by disorder and led to a description in term of an additional effective shaking temperature induced by motion. This description suggests bounded displacements in the solid and thus a perfect moving crystal at large velocity. Recently we have investigated[73] the effects of the *periodicity* of the moving lattice in the direction transverse to motion, in the same spirit which led to the prediction of the Bragg glass in the statics. It was still an open problem how much of the glassy properties remain once the lattice is set in motion. We found that, contrary to the naive expectation, some modes of the disorder are not affected by the motion even at large velocity. Thus, the large v expansion of Ref. 72 breaks down and the effects of nonlinear static disorder persists at all velocities, leading to different physics. As a result the moving lattice is *not* a perfect crystal but a *moving glass*.

The aim of this paper is to provide a detailed description of the moving glass state predicted in Ref. 73 and to present our approach to the general problem of moving lattices. A brief account of some of the new results contained here (e.g., the $T=0$ renormalization-group equations (RG) and fixed points and random forces) has already appeared in Refs. 74, 35. We use several RG approaches at zero and at nonzero temperature. Since several sections of this paper are rather technical we have chosen to expose all the results about the physics of the moving glass in Secs. II and III in a self-contained manner, avoiding all technicalities. The reader can find there the results for the existence of static channels (Sec. III A) the transverse I-V curves at $T=0$ and the dynamical Larkin length (Sec. III B), the random force and the correlation functions (Sec. III C) the various crossover lengths and the finite-temperature results (Sec. III F). Decoupling scenarios for the channels, which distinguish between two different moving glass phases: the moving Bragg glass and the moving transverse glass (Sec. III D) as well as predictions for the dynamical phase diagrams are given in (Sec. III G). Finally we discuss how the moving glass theory stands presently compared to numerical simulations (Sec. III H) and experiments (Sec. III I) and present some suggestions of further observables which would be interesting to measure.

The following sections contain the analytical derivation of the results discussed in Secs. II and III and more generally aim at making progress towards an analytic description of the moving state of interacting particles in a random potential. Since this is a vastly difficult problem, it is potentially dangerous (and unfruitful) to try to attack this problem by treating all the effects at the same time (dislocations, nonlinearities, thermal effects, etc.). Already within the simplifying assumption of an elastic flow two main types of phenomena are missed in a naive large v approach. The first one is a direct consequence of previous works on driven dynamics of CDW and elastic manifolds.[34,31] It is expected on symmetry grounds[75] that nonlinear KPZ terms $(\nabla u)^2$ are generated by motion, an effect which was studied in the driven liquid.[76] Another important effect, studied so far only within the physics of CDW, is the generation of a static *random force* convincingly argued by Krug[77] and explored in Ref. 78. If both effects are assumed to occur simultaneously, they may lead to interesting interplays which have been explored only recently and only in simple CDW models.[79] However there is still no explicit RG derivation of those terms even in CDW models. In the context of driven lattices, they have not even been discussed yet. Our aim in this paper is to remedy this situation. We derive these terms explicitly and show that other linear terms, *a priori* even more relevant are generated. Though these additional linear, nonlinear and random force terms certainly complicate seriously the problem, focusing exclusively on these terms only obscurs the physics of the present problem. Indeed the second and as we show here, most important effect in the moving structure is the crucial role of transverse periodicity to describe the dynamics. A rigorous study of the problem of moving interacting particles would be to first study the fully elastic flow of a lattice. Once the main elastic physics is understood a second step is then to allow for topological excitations (vacancies, interstitials, dislocations). In principle the results obtained within the elastic only approach can, as in the statics,[45] be used to check self-consistently the stability of the elastic flow itself. Clearly understanding the elastic flow first is a necessary step before going further. Here we carry most of the first step and propose an effective description of the second.

Even the purely elastic model turns out to be difficult to treat when all sources of anisotropies, nonlinear elasticity, and cutoff effects are included. There are no analogous terms in the statics and thus in that sense the dynamics is more difficult. Our strategy has thus been to simplify the problem in several stages and resort to simplified models. The simplified models of moving glasses that we have obtained turn out to exhibit some new physics and become interesting in their own. They call for interesting generalizations to other models exhibiting dissipative glassy behavior, as we propose. We call model I the full model of an elastic flow of a lattice containing all the above-mentioned relevant linear and nonlinear terms. Such models can also be written for other elastic structures with related kind of order (such as liquid crystal order). This model is discussed in Sec. VIII B. However its complete study goes beyond the present paper. Fortunately, a useful and further simplified model can be constructed

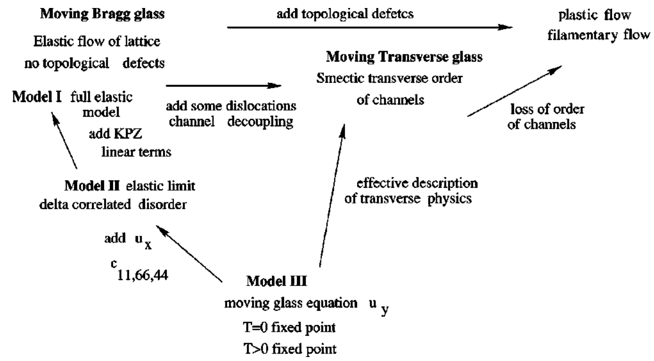

FIG. 1. Various models studied here to describe with various levels of approximation the (i) fully elastic flow of a lattice, (ii) intermediate phase with ordering transverse to motion. (iii) plastic flow.

(model II). It corresponds to considering the above full elastic model in the continuum limit. It certainly gives a very good approximation of the full model at least up to some very large scale. This model was discussed in Ref. 73 and is studied in detail here. It has both longitudinal degrees of freedom (along the direction of motion) and transverse ones. Though it is quite difficult, it can be handled by perturbative renormalization-group studies, as we show here. It has nontrivial fixed point which gives a detailed description of the moving Bragg glass phase. It turns out that most of the physics of the moving glass is contained in a further simplification of model II which retains only the transverse degrees of freedom (displacements). This model, which here we call model III, was introduced in Ref. 73 and is described by the equation of motion:

$$\eta\partial_t u+\eta v\,\partial_x u=c\nabla^2 u+F^{\rm stat}(r,u(r,t))+\zeta(r,t), \qquad (1)$$

which we call *the moving glass equation*. $F^{\rm stat}$ is a nonlinear static pinning force and we have denoted x as the direction of motion, y as the transverse direction(s), and $r=(x,y)$. The model retains only the transverse displaceemnt $u\equiv u_y$. Equation (1) was obtained simply by considering the density modes of the moving structure which are *uniform in the direction of motion*. Indeed, the key point of Ref. 73 is that the transverse physics is to a large extent independent of the details of the behavior of the structure along the direction of motion. This is because the transverse density modes, which can be termed smectic modes, see an almost static disorder and thus are the most important ones to describe the physics of moving structures with a periodicity in the direction transverse to motion. Let us emphasize that this is explicitly confirmed here by the detailed RG analysis of the properties of model II. Note that to obtain model III one sets *formally* $u_x=0$.[80] The hierarchy of models introduced here is represented in Fig. 1.

The outline of the paper is as follows. After Secs. II and III where we give a nontechnical discussion of the physical results, we start in Sec. IV by deriving an equation of motion and, carefully examining its symmetries, we introduce models I, II, III and explain the approximations leading to them. In Sec. V we perform perturbation theory on the full dynamical problem, focusing on model II. In Sec. VI we use the functional RG to study model III and thus the transverse

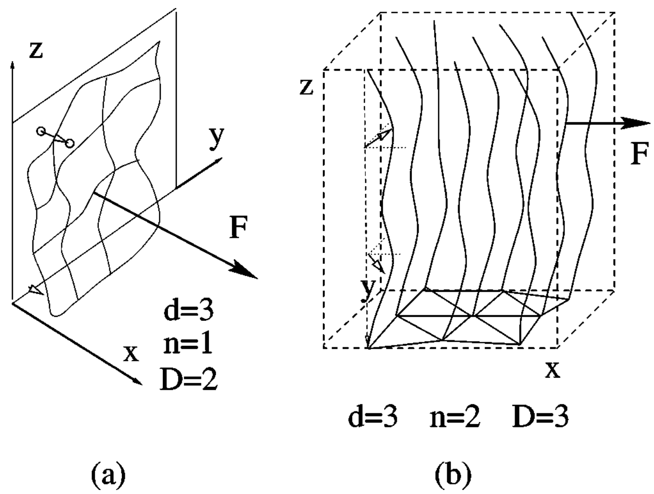

FIG. 2. Two cases of a driven structure. (a) An interface $D=2$, $n=1$, $d=3$, driven orthogonal to its internal space. (b) A triangular line lattice $D=3$, $n=2$, $d=3$ driven within its internal space.

physics in $d=3$ and $d=3-\epsilon$. We study $T=0$ and $T>0$. In Sec. VII we study a two-dimensional version of the moving glass equation model III. This allows us to obtain results in $d=2$ at $T>0$ and in $d=2+\epsilon$. Having obtained a good understanding of the transverse physics in the sections devoted to model III, we are now in good position to tackle the full problem. We treat in Sec. VIII A the RG of model II and in Sec. VIII B we examine the full model I, show that linear terms and KPZ terms are generated at large scales and discuss some consequences. Conclusions can be found in Sec. IX and many technical details have been hidden in the six Appendixes of the paper.

II. MOVING STRUCTURES AND MOVING GLASSES

A. Moving structures: General considerations

All the structures we consider share the same basic features. The static system in the absence of quenched substrate disorder consists of a network of interacting objects at equilibrium positions R_i^0, forming either a perfect lattice (periodic case) or elastic manifolds (nonperiodic case). Depending on the system the objects can be either pointlike (e.g., electrons in a Wigner crystal) or lines (vortex lines in superconductors). Deformations away from equilibrium positions are described by displacements u_i or in a coarse-grained description $u(r,t)$ where r is the internal coordinate. A complete characterization of the structure in motion uses three parameters (i) the internal dimension D, (ii) the number of components n of the displacement field u_α, and (iii) the embedding space dimension d. Two examples are shown in Fig. 2 and more details are given in Appendix E. Since we are mostly interested here in periodic structures (though not exclusively) we can set $D=d$. We consider motion along one direction called x, and we parametrize throughout all this paper the space variable r as $r=(x,y,z)$ where x is one dimension, y has *a priori* $n-1$ dimensions, and z has $d_z=d-n$ dimensions, and the displacements along motion as u_x and transverse to motion as u_y. Three-dimensional triangular flux-line lattices driven along a lattice direction thus have $d=3$, $n=2$, $r=(x,y,z)$, $u=(u_x,u_y)$, where z denotes the direction of the magnetic field. Two-dimensional triangular lattices of point vortices have $d=2$, $n=2$, $r=(x,y)$.

At finite temperatures or in the presence of quenched substrate disorder the structure is deformed. An important issue is then to characterize the degree of order. This can be expressed in terms of displacements correlation functions. The simplest one measures the relative displacements of two points (e.g., two vortices) separated by a distance r

$$\tilde{B}(r)=\frac{1}{n}\,\overline{\langle[u(r)-u(0)]^2\rangle}, \tag{2}$$

where $\langle\,\rangle$ denotes an average over thermal fluctuations and—is an average over disorder. The growth of $\tilde{B}(r)$ with distance is a measure of how fast the lattice is distorted. For thermal fluctuations alone in $d>2$, $\tilde{B}(r)$ saturates at finite values, indicating that the lattice is preserved. Intuitively it is obvious that in the presence of disorder $\tilde{B}(r)$ grows faster and can become unbounded. $\tilde{B}(r)$ can directly be extracted from direct imaging of the lattice, such as performed in decoration experiments of flux lattices. Related to $\tilde{B}(r)$ is the structure factor of the lattice, obtained by computing the Fourier transform of the density of objects $\rho(r)=\Sigma_i\delta^d(r-R_i^0-u_i)$. The square of the modulus $|\rho_k|^2$ of the Fourier transform of the density is measured directly in diffraction (neutrons, x rays) experiments. For a perfect lattice the diffraction pattern consists of δ-function Bragg peaks at the reciprocal vectors. The shape and width of any single peak around K can be Fourier transformed to obtain the translational order correlation function given by

$$C_K(r)=\overline{\langle e^{iK\cdot u(r)}e^{-iK\cdot u(0)}\rangle}. \tag{3}$$

For simple Gaussian fluctuations (and isotropic displacements) $C_K(r)=e^{-(K^2/2)\tilde{B}(r)}$ but such a relation holds only qualitatively in general (as a lower bound). $C_K(r)$ is therefore a direct measure of the degree of translational order that remains in the system. Three cases are possible (see Fig. 1 in Ref. 35): (a) for thermal fluctuations alone $C_K(r)\to$Cste, one keeps the perfect δ-function Bragg peaks, albeit with a reduced weight (b) $C_K(r)$ decays exponentially fast. The structure factor has no divergent peak, translational order is destroyed beyond length R_a, although some degree of order persists at short distance (c) $C_K(r)$ decays as a power law. The structure factor still has divergent peaks but not δ functions. One retains quasi-long-range translational order. This is the case, e.g., in $d=2$ at low temperature (Kosterlitz-Thouless) or in the Bragg glass. Depending on how much crystalline order remains in the system the structure factor has extremely different behaviors as depicted in Fig. 3.

Quite surprisingly, if one takes into account correctly the *periodicity* of the lattice, a thermodynamic phase *without dislocations* was predicted to exist in $d=3$ at weak disorder.[44,45] This phase, named the Bragg glass, has quasi-long-range order with Bragg peaks diverging at least as $q^{-(3-A_3)}$ (with $A_3\approx 1$), similar to dashed lines in Fig. 3. At the same time displacements $B(r)$ grow logarithmically at large scale. Similar predictions hold for other elastic models such as random-field XY systems, and *a priori* also for liquid

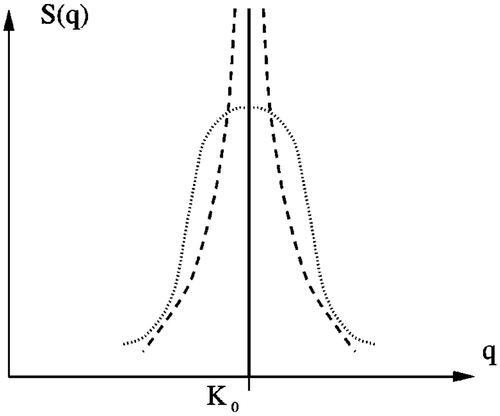

FIG. 3. Depending on the translational order remaining in the lattice the structure factor has different shapes. The thick line is the δ-function Bragg peak of a perfect lattice (including thermal fluctuations). The dashed line is the divergent Bragg peak of the Bragg glass (which retains quasi-long-range order and has no topological defects), the dotted line is the Lorentzian-like shape of a system losing its translational order exponentially fast.

crystals. The Bragg glass theory has by now received considerable numerical[51,52] and analytical confirmations.[48–50] If disorder is increased above a threshold it is predicted that there is a transition at which topological defects proliferate. They destroy the translational long-range order exponentially fast beyond a length R_D leading to finite height diffractions peaks. The height of the peak increases with the scale at which translational order is destroyed. This transition is thus characterized by the loss of the divergence in the Bragg peaks. In type-II superconductors it implies that there is a transition, upon increasing the magnetic field, predicted in Ref. 45 (see also Ref. 48), from the Bragg glass (at low fields) to another phase. The high-field phase is either the putative vortex glass[36,37] or is simply continuously related to the high-temperature phase. These predictions for the phase diagrams of superconductors has received experimental support (see Ref. 47 for a review).

What happens when an external force is applied to such a structure? One obviously important quantity to determine is the curve of velocity v versus the applied force f. Through this v-f characteristic, three main regimes can be distinguished and are shown in Fig. 4.

Far below the depinning threshold f_c the system moves through thermal activation. This is the so-called creep regime. Since the motion is extremely slow in this regime, it has been analyzed based on the properties of the *static* system.[38,1,40] The resulting v-f curve crucially depends on whether the static system is in a glass state (such as the Bragg glass) where the barriers $U(f)$ become very large when $f \to 0$, or a liquid where barriers remain finite at small f, resulting in a linear resistivity. The general form expected in the creep regime is

$$v \sim \rho_0 f e^{-U(f)/T}. \tag{4}$$

Let us emphasize that this "longitudinal" v-f characteristic has mainly been used to determine whether the *static* system (i.e., the limit $f=v=0$) is or is not in a glass state. It may not be enough though, if one wants to probe glassiness of the moving system itself, as discussed below. The second regime, near the depinning transition $f \approx f_c$, has been intensely investigated in similarity with usual critical phenomena (see, e.g., Refs. 25, 27, 26) where the velocity plays the role of an order parameter. A particularly important question in that regime is to determine whether plastic rather than elastic motion occurs.[53] Close to the threshold in low dimensions and in strong disorder situations the depinning is observed to proceed through "plastic channels" [63,64] between pinned regions as depicted in Fig. 5. This type of filamentary flow has been found[65] in simulations of 2D (strong disorder) thin-film geometry (with $c_{11} \gg c_{66}$) where depinning proceeds via filamentary channels which become increasingly denser.

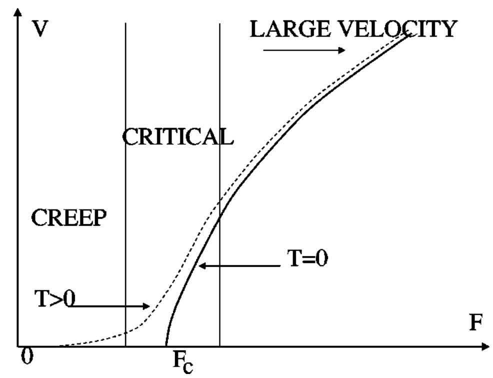

FIG. 4. A typical v-f characteristic at $T=0$ (full line) and finite temperatures (dashed line). Three main regimes can be distinguished: the creep regime for forces well below threshold, the critical regime around the threshold, and the large velocity regime well above threshold.

The third regime is above the depinning threshold $f > f_c$. This is the situation on which we focus in this paper (though some of our considerations have consequences in the other regimes as well). An important phenomenon in this regime is that of *dynamical reordering*. Indeed, it was observed experimentally, some time ago in neutron-diffraction experiments,[67] and in more detail recently,[68] that at *large velocity* the vortex lattice is more translationally ordered than at low velocity. Intuitively the idea is that at large velocity v, the pinning force on each vortex varies rapidly and disorder should produce little effect. This phenomenon was also

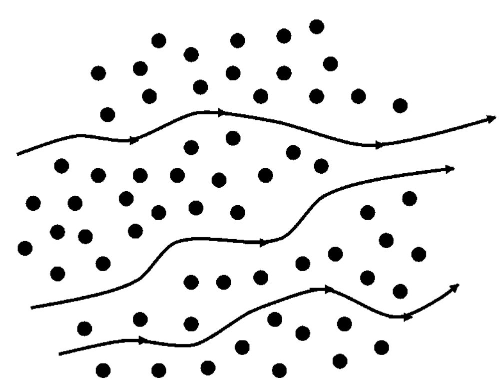

FIG. 5. Plastic flow of a network of objects submitted to an external force F, shown for simplicity in two dimensions. The motion occurs through plastic channels around pinned regions. Plastic flow might occur close to the depining threshold whereas at large velocities one expects to recover elastic flow where the whole lattice moves coherently.

known in the context of CDW.[25] The tendency to reorder has also been seen in numerical simulations.[54,55,69] Since the effects of disorder were expected to vanish at high velocity perturbation theory in $1/v$ was developed mainly to compute the v-f characteristics.[70,71,29] Recently it was extended by Koshelev and Vinokur[72] to compute the vortex displacements u induced by disorder in the moving lattice and in the moving liquid. The effect of disorder on the moving liquid was found to be equivalent to *heating* to an effective temperature $T' = T + T_{sh}$ with $T_{sh} \sim 1/v$. Thus the moving liquid was argued to survive at temperatures lower than the melting temperature $T < T_m$, and a *dynamical melting transition* to occur below T_m from a moving liquid to a moving solid upon increase of the velocity,[72] when $T' = T_m$. These arguments were then extended to describe the moving solid itself, and it was argued that there the effect of pinning could also be described[72] by some effective shaking temperature $T_{sh} \sim 1/v^2$ defined by the relation $\langle |u(q)|^2 \rangle = T_{sh}/c_{66}q^2$. This suggests bounded displacements in the solid and that at low T and above a certain velocity the moving lattice is *a perfect crystal*. As is discussed in the remainder of this paper, the picture of the moving lattices emerging from the above bold qualitative arguments[72] goes wrong in several ways.

There are several other important questions to be answered in addition to the v-f characteristics. The first one is the question of the effect of the motion on the spatial correlations and in particular whether translational order exists in a moving system. This is related to the question of plastic versus elastic flow. If plastic flow occurs, the structure factor should signal some destruction of lattice. However because a moving system is inherently anisotropic different effects appear and the decay of the structure factor is not as isotropic as in the static system (the Lorentzian in Fig. 3). This question thus remains to be investigated. A possibility, suggested by the idea of a shaking temperature,[72] would be that at large velocity one should observe δ-function Bragg peaks characteristic of a crystal at finite temperature. Such questions are discussed in detail in Sec. III. Finally determining how motion affects the phase diagram of the statics has to be investigated and depends of course on the above issues. In particular what remains of the glassy properties of the systems when in motion (slow relaxation, history dependence, nonlinear behaviors) needs to be addressed. For moving periodic systems, an equivalent question can be asked also about ''temporal order'' and its associated effects such as noise spectrum. In particular if one looks at a signal at a fixed position in space but as a function of time, one expects a periodic signal with a periodicity of a/v, having δ peaks in frequency at the multiples of the washboard frequency $\omega_0 = 2\pi v/a$. If the lattice becomes imperfect one could naively expect the Fourier peaks in frequency to broaden in a way that reflects the loss of translational order. Quite surprisingly this is not so. Indeed it can be shown for a single-component displacement field[81] (CDW) that the perfect periodicity in time remains (in the absence of topological defects). However this result is not readily applicable to a moving lattice, and it is thus crucial to determine whether this remarkable property holds in that case.

B. The moving glass

To tackle the physics of a structure with a displacement field with *more than one component* $(n > 1)$, such as a triangular lattice (by contrast with a single Q CDW), two routes seem to be possible. The commonly followed one[72,78,82] is to simply borrow from, or extend, the physics of single-component CDW,[28–30] or of elastic manifolds driven *perpendicularly* to their internal direction.[31] In this case emphasis is put on the displacements *along* the direction of motion u_x and on the proper way to model its dynamics. Such a problem has turned out to be already quite complicated in particular due to the generation of KPZ-type nonlinearities in the equation of motion. Even if degrees of freedom transverse to motion u_y exist as in the cases depicted in Fig. 6 they constitute an extension[32] of this ''longitudinal'' physics. Thus in this ''CDW paradigm'' it would seem necessary to understand first completely the physics of longitudinal modes u_x and then incorporate u_y as an extra complication. Indeed there were a few attempts to describes the physics of driven vortex lattices along those lines.[72,78]

The second approach is based on the realization that the physics of periodic structures driven along one of their internal direction is radically different[73] from the above descriptions. This stems from the fact that due to the periodicity in the transverse direction u_y a *static nonlinear pinning force* F^{stat} persists even in a fast moving system. We want to stress that this is a very general property of *any* moving structure which contains uniform density modes $K_x = 0$ in the direction of motion (as can be seen on the Fourier decomposition

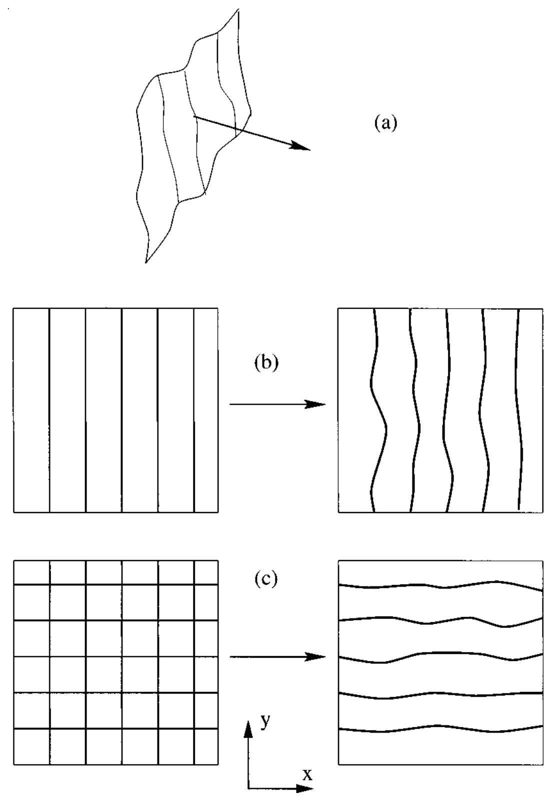

FIG. 6. Three types of dynamical systems. (a) A manifold driven perpendicular to itself. (b) A single Q CDW system. Only displacements in the direction of motion exist, but periodicity *along* the direction of motion can play a role. (c) A periodic system with transverse degrees of freedom driven along one of its symmetry directions. The correct description of this last class of systems is the moving glass fixed point where the *transverse* degrees of freedom are the important one as represented here.

of the density[73]). As illustrated in Fig. 6 the substructure formed by these modes can deform elastically in the u_y direction and sees essentially a static disorder. As is obvious from Fig. 6(c), this substructure has generically a liquid-crystal type of (topological) order and can be termed a ''smectic'' (though when $d_z=0$, e.g., for $d=3$ and $n=3$, it is rather a ''line crystal,'' see below). In all cases the basic starting point thus involves the *transverse* degrees of freedom as shown in Fig. 6, and is quite different from the ''CDW description.'' The equation which captures the main ingredients of such moving systems was derived in Ref. 73. It leads to an interesting model for transverse components $u \equiv u_y$, which has the general form in the laboratory frame:

$$\eta \partial_t u + \eta v \partial_x u = c \nabla^2 u + F^{\text{stat}}(r, u(r,t)) + \zeta(r,t). \tag{5}$$

Since this equation (model III) captures glassy features of moving systems we call it *the moving glass equation*. Although it looks like a standard pinning equation the *convection term* $\eta v \partial_x u$ dissipates even in the static limit (a reminder that we are looking at a moving system) and does *not* derive from a potential. Thus we consider this problem and its generalizations as a prototype for a new class of physical phenomena which are glassy and do not derive from a potential (or from a Hamiltonian). The first example is to choose $F^{\text{stat}}(r,u)$ periodic in the u direction:

$$F^{\text{stat}}(r, u_y) = V(r) \rho_0 \sum_{K_y \neq 0} K_y \sin K_y (u_y - y) \tag{6}$$

and corresponds to lattices (or to liquid crystals) driven in a random potential with a short-range correlator $\overline{V(r)V(r')} = g(r-r')$ of range r_f. The study of this case in Ref. 73 gave the first hint that nonpotential dynamics can indeed exhibit glassy properties and lead to *dissipative glasses*. This is a rather delicate notion because the constant dissipation rate in the system would naturally tend to generate or increase the effective temperature and kill the glassy properties. However this type of competition between glassy behavior and dissipation arises in other systems which are a generalization of the above equation. Let us briefly indicate some of the generalizations that we are proposing which are being studied here or in related works. An interesting generalization is the case of a periodic manifold with *correlated disorder*.[83] This is relevant to describe the *moving Bose glass* state of driven vortex lattices in the presence of correlated disorder. Another generalization is to extend Eq. (5) to a N-component vector u_α. It is easy to see in that case that a nonpotential nonlinear disorder is generated if $v>0$ (which reduces to the ''static random force'' for $N=1$). Thus in that model it is natural to look at a generic nonpotential disorder $F_\alpha^{\text{stat}}(r,u)$ from the start. The mean-field dynamical equations for large N and the functional renormalization group (FRG) equations at any N for a large class of such models are derived in Sec. VI and in Appendix F. A subclass of these models is nonperiodic (manifold). They are relevant to describe the random manifold crossover regime in the moving glass (see Sec. III and Fig. 12). A further subclass is then obtained by setting $v=0$. Interestingly the resulting model describes polymers (and manifolds) in random flows and can be studied both in the large-N limit[84] and using RG (Ref. 85) for any N.

Finally, there are other simpler but interesting situations such as disorder correlated along the direction of motion or lattices moving in a periodic tin roof potential. If F^{stat} in Eq. (5) is independent of x one finds the interesting property that the steady-state measure $P[u(r)]$ is *identical* (at any $T>0$) to the one with $v=0$. This can be shown by studying the associated Fokker-Planck equation. Thus we see that the moving glass Eq. (5) hides a whole class of interesting dissipative models with glassy properties.

III. PHYSICAL RESULTS

In this section we present all the physical results on the moving glass that we have obtained in Refs. 73, 74, 35 and in the present paper. We deliberately avoid technicalities and refer to the proper sections for details.

A. Channels

One of the most striking properties of moving structures described by Eq. (5) is that the nonlinear static force F^{stat} results in the pinning of the transverse displacements $u_y(r,t)$ into preferred static configurations $u_y(r)$ in the laboratory frame. Thus the resulting flow can be described in terms of *static channels* where the particles follow each other like beads on a string. In the laboratory frame these channels are determined by the static disorder and do not fluctuate in time. They can be visualized in simulations or experiments by simply superposing images at different times. What makes the problem radically different compared to conventional systems which exhibit pinning is that despite the static nature of these channels there is constant dissipation in the steady state. This can be seen in the moving frame where each particle, being tied to a given channel (which is then moving) must wiggle along y and dissipate. In fact the existence of the channels shows in a transparent way that the wiggling of different particles in the moving frame is highly correlated in space and time, thus leading to a radically different image as the one embodied in the ''shaking temperature'' based on thermal-like incoherent motion.[72]

The channels are thus the easiest paths followed by the particles. One can see that the ''cost'' of deforming a channel along y is that dissipation is increased. Thus the channels are determined by a subtle and novel competition between elastic energy, disorder, and dissipation. As a consequence these channels are *rough*. This is a crucial difference between what would be observed for a perfect lattice as illustrated in Fig. 7.

By contrast the channels which are predicted in the moving glass are illustrated in Figs. 8 and 9. It is important to stress that the moving glass equation (5) does not assume anything about the coupling of the particle in *different* channels but only implies that the channels themselves are elastically coupled along y, and thus through compression modes.[73] Indeed on specific models such as model II one can verify explicitly that although coupling between longitudinal and transverse degrees of freedom exists *a priori*, the longitudinal degrees of freedom u_x do not feed back *at all* in the moving glass equation (see Sec. VIII A). The existence of channels naturally leads to several *a priori* possible regimes for the coupling between particles in different channels. The

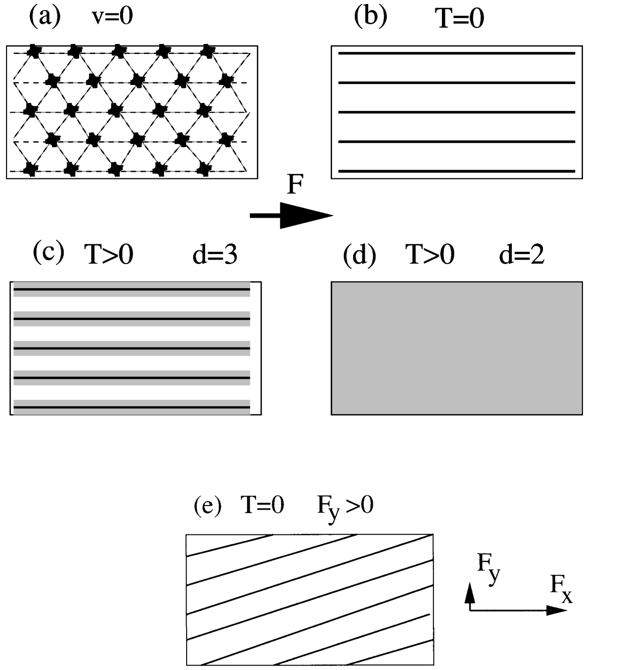

FIG. 7. (a) A snapshot of a perfect (nondisordered) lattice moving along the x direction. (b) Upon superposing images at different times one would see that at $T=0$ the particles follow perfectly straight lines. (c) At $0<T<T_m$ in $d=3$ the channels remain perfectly straight with a finite width due to the thermal fluctuation of the particles. (d) In $d=2$ since thermal fluctuations are unbounded, channels are completely blurred and cannot be defined even for $T<T_m$. (e) Even in situation (b), (c) applying an additional small force along y immediately results in tilted channels with angle F_y/F_x.

first case, represented in Fig. 8(b), is a topologically ordered moving structure corresponding to full elastic coupling between particles in different channels. Since, remarkably, this structure retains perfect topological order despite the roughness of the channels, it is reminiscent of the properties of the static Bragg glass, and thus we call it a moving Bragg glass. A second case of a moving glass corresponds to decoupling between the channels, by injections of dislocations beyond a certain length scale R_d and is called the moving transverse glass. These two regimes are discussed in more detail in Sec. III D. Finally note that in $d=3$ channels can be either ''sheets'' (for line lattices) or linear (for point lattices) as represented in Fig. 9. It is important to note that the channels in the moving glass are fundamentally different in nature from the one introduced previously[63,64] to describe slow plastic motion between pinned islands, as illustrated in Fig. 5. In the moving glass they form a manifold of almost parallel lines (or sheets for vortex lines in $d=3$), elastically coupled along y. For that reason we call them generically ''elastic channels'' (whether or not they are fully coupled or decoupled) to distinguish them from the ''plastic channels'' (even though some plastic flow may occur when elastic channels decouple). Note that in the above discussion we have concentrated on elastic channels which can *spatially decouple*. It is possible *a priori* that they may still remain *temporally coupled*, i.e. synchronized. Indeed, examples of synchronization were observed even in extremely plastic filamentary flow.[65]

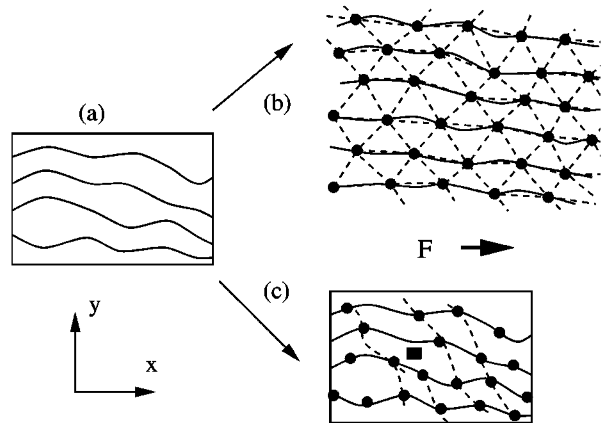

FIG. 8. (a) The motion in the moving glass occurs through rough static channels. Relative deformations grow with distance and become of order a at distances $R_a^x \sim R_a^{y2}$. Only the *channels* themselves are elastically coupled along the y direction. Depending on the dimension, velocity, and disorder strength two main cases can occur: (b) an elastic flow where the particle positions are elastically coupled between channels (in $d=3$ and weak disorder or large velocities). In this regime the lattice is topologically ordered (no dislocations) and the rows of the lattice follow the channels. This is a moving Bragg glass. (c) In $d=2$ or at stronger disorder in $d=3$ the positions of particles in different channels may decouple. Dislocations with Burgers vectors along x (indicated by the square) are then injected between some channels beyond the length R_a. This situation describes a moving transverse glass (with a smectic or a line-crystal-type topological order).

B. Dynamical Larkin length and transverse critical force

Another important property of the moving glass intimately related to the existence of stable channels is the existence of ''transverse barriers.'' Indeed it is natural physically that once the pattern of channels is established the system does not respond in the transverse direction along which it is pinned. Thus we have predicted in Ref. 73 that the response to an additional small external force F_y in the direction transverse to motion vanishes at $T=0$. A true *transverse critical force* F_c^y, such that the transverse velocity

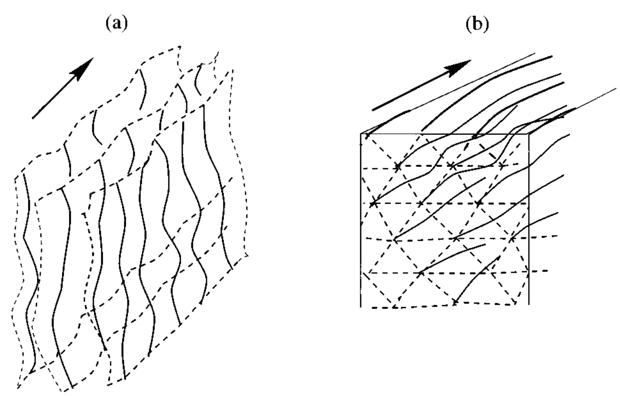

FIG. 9. Different types of topological order for the manifold of channels in $d=3$. (a) For line lattices in motion the channels are ''sheets'' and thus form an anisotropic type of smectic layering, (b) for 3d lattices of pointlike objects or for triply periodic structures (e.g., triple Q CDW) they have instead the topology of a line crystal.

$v^y=0$ for $F^y<F_c^y$, exists (and thus a transverse critical current J_c^y in superconductors) for a lattice driven along a principal lattice direction.[86]

The transverse critical force is a rather subtle effect, more so than the usual longitudinal critical force. It does *not* exist for a single particle at $T=0$ moving in a short-range-correlated random potential. By contrast even a single-particle experiences a nonzero longitudinal critical force. It does not exist either for a single driven vortex line or any manifold driven perpendicular to itself in a pointlike disordered environment. It does exist, however, even for a single particle if the disorder is sufficiently correlated *along the direction of motion* (such as a tin roof potential constant along x and periodic along y). Such disorder breaks the rotational symmetry in a drastic way. Still, in the case of a lattice driven in an uncorrelated potential a nonzero transverse critical force does seem to break the rotational symmetry of the problem. In some sense in the moving glass the transverse topological order which persists (and the elasticity of the manifold) provide the necessary correlations (through a spontaneous breaking of rotational symmetry). Thus the transverse critical force is a dynamical effect due to barriers preventing the channels to reorient.

Thus we have proposed the moving glass as a dynamical phase (a new RG fixed point) and the transverse critical force as its order parameter at $T=0$. The upper critical dimension of this phase is $d=3$ instead of $d=4$ for the static Bragg glass. Above $d=3$ weak disorder is irrelevant and the moving glass is a moving crystal. For $d\leqslant 3$ disorder is relevant in the moving crystal and leads to a breakdown of the $1/v$ expansion of Ref. 72. Divergences in perturbation theory can be treated using a RG procedure (Sec. VI). One indeed finds a different fixed point which confirms the prediction that the moving glass is a dynamical phase. Using RG and the properties of this fixed point one can compute various physical quantities (Sec. VI B 3). We find that the transverse critical force is given by

$$F_c^y \sim A\frac{c r_f}{(R_c^y)^2} \tag{7}$$

with $c=c_{11}$ in $d=2$, $c=\sqrt{c_{11}c_{44}}$ in $d=3$, and A is a nonuniversal constant. The length scale R_c^y is the *dynamical Larkin length*. It is defined as the length scale along y at which perturbation theory breaks down, nonanalyticity appears in the FRG and the (scale-dependent) mobility vanishes. Before we proceed further let us define now disorder strength parameters. For uncorrelated disorder the random potential $V(r)$ which couples to the density of the structure has short-range correlations of range r_f, $\overline{V(r,z)V(r',z)}=g(r-r')\delta^{d_z}(z-z')$ (see Sec. IV). As in Ref. 73 we denote by Δ [also denoted $\Delta(u=0)$, see Sec. IV] the bare static pinning force correlator $\Delta=\rho_0^2\Sigma_{K_y,K_x=0}K_y^2 g_K$ where ρ_0 is the average density and g_K is the Fourier transform of $g(r)$ at the reciprocal-lattice vectors. Throughout Δ_2 denotes the second derivative of the nonlinear pinning force correlator $\Delta_2=-\Delta''(0)\approx\Delta/r_f^2$ (see Sec. VI). Our result is that in $d=3$ the dynamical Larkin length is given by

$$R_c^y \sim a\exp\frac{4\pi\eta v c}{\Delta_2}, \tag{8}$$

while in $d\leqslant 2$ it reads

$$R_c^y \sim \left(a^{3-d}+\frac{4\pi\eta v c(3-d)}{\Delta_2}\right)^{1/(3-d)} \tag{9}$$

with again $c=c_{11}$ in $d=2$ and $c=\sqrt{c_{11}c_{44}}$ in $d=3$. These results are valid for large enough velocities $v\gg v_c^*$ (see below for the definition of v_c^* and results for all velocities). Note that for $v>v_c^*$ the dynamical Larkin length depends only on c_{11} (and of c_{44} in $d=3$) as it should since the physics of the moving glass is controlled by the compression modes and thus largely independent of the detailed behavior along x.[73]

Another way to estimate the dynamical Larkin length is to compute the displacements in perturbation theory of the disorder. At very short distance one can treat the pinning force in Eq. (5) to lowest order in u. This gives a model where disorder is described by a *random force* $F^{\text{stat}}(x)$ independent of u whose correlator is $\langle F^{\text{stat}}(r)F^{\text{stat}}(r')\rangle=\Delta\delta^d(r-r')$. This regime is the equivalent of the short distance Larkin regime for the statics. In the moving glass at very large velocity $v\gg v_c^*$ the displacements (2) along y grow as $B(r)=B_{\text{RF}}(r)$ (at $T=0$) with

$$B_{\text{RF}}(y)=\int\frac{dq_x dq_y d^{d_z}q_z}{(2\pi)^d}\frac{\Delta[1-\cos(q_y y)]}{(\eta v q_x)^2+(c_{11}^2q_y^2+c_{44}^2q_z^2)^2}. \tag{10}$$

The scale along y at which u_y becomes of order r_f defines the dynamical Larkin length R_c^y, i.e., $B_{\text{RF}}(y=R_c^y,x=0)\sim r_f^2$. The resulting expression coincides with the one obtained within the RG approach (up to nonuniversal prefactors). Similarly one can define a Larkin length for transverse pinning along the x direction by the condition that $u_y(x=R_c^x,y=0)-u_y(x=0,y=0)\sim r_f$. Since what determines this length is only u_y (and not u_x) it is independent of the detailed behavior along x. It is important to note that the moving glass is a very anisotropic object at large scale with a scaling $x\sim y^2$ of the internal coordinates. This implies that at large velocity ($v>v_c^*$) the Larkin length along x is very large (much larger than R_c^y), with $R_x=v(R_c^y)^2/c_{11}$ (one has also the more conventional behavior $R_c^z\sim\sqrt{c_{44}/c_{11}}R_c^y$ in $d=3$). Estimating the random force acting on a Larkin volume for the transverse displacements[73] one recovers the above estimate for F_c^y.

The resulting transverse I-V characteristics at $T=0$ is depicted in Fig. 10. The transverse depinning is studied in Sec. VI and we find the behavior near the threshold $v_y\sim|F^y-F_c^y|^\theta$ for $F^y>F_c^y$ with $\theta=1$ to lowest order in $\epsilon=3-d$. A reasonable conjecture which would be interesting to verify is that it remains $\theta=1$ to all orders.[87] Thus the transverse velocity v_y starts linearly with a slope which depends on the velocity longitudinal velocity v. It is very large for $v\ll v_c^*$ and diverges in the limit $v\to 0$.

The existence of a transverse critical force in a moving state raises interesting issues about *history dependence*. These issues are largely open and should be explored in further numerical, experimental, and theoretical work. Let us,

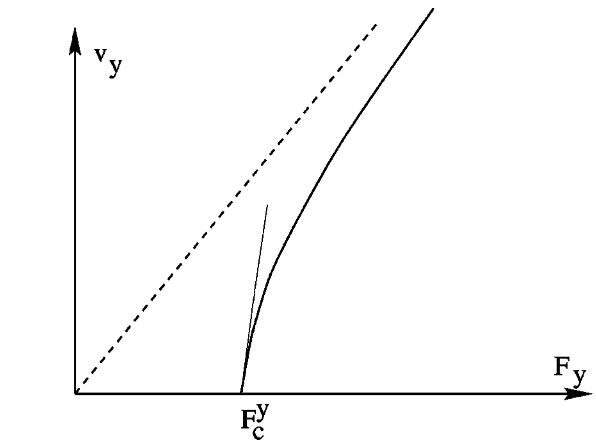

FIG. 10. Transverse v-f characteristics at $T=0$: transverse velocity v_y as a function of the applied transverse force F_y at a fixed longitudinal velocity. The behavior near threshold is found to be linear. The slope at the threshold is large for $v \leqslant v_c^*$.

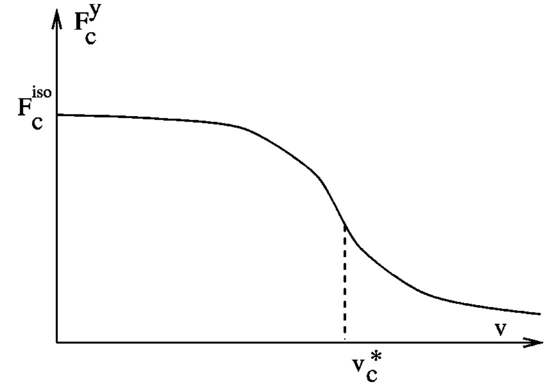

FIG. 11. Transverse critical force as a function of the longitudinal velocity. At small velocities the transverse critical force saturates to the isotropic one $F_c^{\rm iso}$. For a relation between ηv_c^* and the longitudinal critical force see the text.

for instance, consider two experiments. In the first one a force $f_x e_x + f_y e_y$ is applied to the lattice at time $t=0$ and then one waits until a steady state is reached. The velocity is then $[v_x(f),v_y(f)]$. In the second experiment one first applies a force f_x along the direction x, waits for a steady state, and then appliies f_y along y. One then measures the velocities $[v_x^w(f),v_y^w(f)]$. The question is, should one find the same result in the two experiments or not. Of course there are subtle issues which complicate the problem and needs to be further investigated [such as (i) the order of limits system size versus waiting time before a steady state is reached, (ii) whether the lattice globally rotates or breaks into crystallites, and (iii) some nonuniversality of $T=0$ dynamics] but one should still be able to find an *operational* answer. If it is found that there are such history dependence effects then that would be a strong characteristic of a glassy state (it should not happen in the liquid where one expects both answers to be the same, but in the same trivial sense as for a single particle). On the other hand, if no clear history dependence is found it has interesting consequences. We assume in the following that the global orientation of the lattice is unchanged. Then the first consequence is that there is a well-defined history-independent global v-f function. This function however is nonanalytic in a large region of the f_x,f_y plane. $v_y(f_x,f_y)$ should remain zero at least in the region $f_y < F_c^y(v_x(f_x,f_y))$ and similar regions near each of the principal symmetry axis of the crystal. This is clearly the result of the FRG calculation presented here. But then one may also guess that it may be nonanalytic too along other lattice directions (though it is possible that some of the higher symmetry directions be screened by lower ones). The transverse mobility as a function of the angle and the force should exhibit a complex (and rather strange) behavior which would be interesting to investigate further. A second interesting consequence would be that if in the above described first experiment one chooses a $f_y > 0$ smaller than $F_c^y(v_x)$, the lattice would first glide in the direction of the applied force (as small time perturbation theory would indicate) but would soon change its velocity to lock it along a symmetry axis. It is quite possible that this locking effect exists and could be a possible explanation for the behavior ubiquitously observed in experiments, namely that lattices tend to flow along their principle axis directions. Such a behavior near depinning was observed in recent decoration experiments.[88] Note that a similar locking would happen for a particle moving in a tin roof potential (but as a more trivial effect).

Another important question for experiments is to determine the transverse critical force as a function of the longitudinal velocity v. As v decreases F_c^y increases but it is intuitively clear that F_c^y cannot become larger than the longitudinal critical current (strictly speaking in the same direction y). We neglect for now the dependence of the longitudinal critical current in the orientation with respect to the lattice (which gives a numerical factor which can be incorporated). We call $F_c^{\rm iso}$ the critical current for $v=0$. As v is decreased below v_c^* the transverse critical force saturates at $F_c^{\rm iso}$. This is depicted in Fig. 11 [the large-v behavior was given in Eqs. (7)–(9)]. There is thus a crossover towards the static isotropic behavior (e.g., in the Bragg glass), assuming no dynamical phase transition as v decreases which would complicate the analysis.

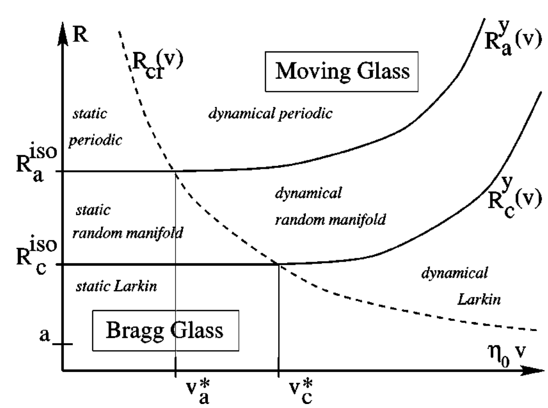

FIG. 12. Crossover as a function of the length scale R and longitudinal velocity v from the static Bragg glass behavior (at small v) to the moving glass behavior (at large v). The dashed line represents the crossover between these two regimes $R=R_{\rm cross}=c/v$. The dynamical Larkin length $R_c^y(v)$ as a function of v and the transverse translational order length $R_a^y(v)$ are indicated as plain curves. This is valid in the collective-pinning regime $R_c^{\rm iso} > a$ where $R_c^{\rm iso}$ is the static Larkin length.

This crossover can be explicitly estimated using the FRG in Secs. VI, VIII A, and physical arguments. It is convenient to discuss it using Fig. 12 (also useful for studying the crossover in the correlations—see the next section). Let us first discuss it for simplicity with isotropic elasticity $c_{11}=c_{66}=c_{44}=c$. There is a crossover length scale $R_{\rm cr}=c/v$ below which the moving glass looks very isotropic and very similar to the Bragg glass. This length scale is represented in Fig. 12 as a dashed line. Increasing the length scale R starting from a, at fixed v, one is first controlled by the static behavior until reaching that line ($R<R_{\rm cr}$) and then one is controlled by the dynamical moving glass regime for $R>R_{\rm cr}$. Similarly one can also represent the Larkin length at $v=0$ $R_c^{\rm iso}=(c^2r_f^2/\Delta)^{1/(4-d)}$ (in $d<4$). The crossover velocity v_c^* corresponds to the velocity at which $R_c^y=R_{\rm cr}$ when one has also $R_c^y=R_c^{\rm iso}$. One finds

$$v_c^*=(\Delta/cr_f^2)\ln(c^2r_f^2/a\Delta) \quad (d=3),$$

$$v_c^*=c(\Delta/c^2r_f^2)^{1/(4-d)} \quad (d\leqslant 3). \tag{11}$$

These results are valid when ($R_c^{\rm iso}>a$), i.e., in the collective-pinning regime for the statics. We denote $r_f=\min(r_f,a)$, i.e., if $r_f\sim a$ one can simply replace r_f by a in all the above formulas. Thus for $v<v_c^*$ the transverse critical current becomes of order the longitudinal one $F_c^{\rm iso}=cr_f/(R_c^{\rm iso})^2$. It is useful for the purpose of comparison with experiments to compare ηv_c^* with the longitudinal critical force $F_c^{\rm iso}$. One finds the general relation

$$\frac{\eta v_c^*}{F_c^{\rm iso}}=\frac{R_c^{\rm iso}}{r_f} \tag{12}$$

with logarithmic corrections in $d=3$, $\eta v_c^*=F_c^{\rm iso}(R_c^{\rm iso}/r_f)\ln(R_c^{\rm iso}/a)$. This result is remarkable: since for weak disorder one has usually that $R_c^{\rm iso}\gg r_f$ it shows that for a system with isotropic elasticity, the transverse critical force should remain of the order of the longitudinal one up until very far above the longitudinal threshold ($F_x\gg F_c$) (very high up in the v_x-F_x curve in Fig. 4).

The situation is different when $c_{66}\ll c_{11}$. In that case two distinct crossover velocities exist which we denote by v_{66} and v_{11}. This is because the pinning properties of the large velocity moving glass are determined by the compression c_{11}, while the pinning of the static Bragg glass are determined by shear c_{66}. The critical current in the statics is then $F_c^{\rm iso}=c_{66}r_f/(R_c^{\rm iso})^2$ with $R_c^{\rm iso}=r_f^2c_{66}^{3/2}c_{44}^{1/2}/\Delta$ in $d=3$ and $R_c^{\rm iso}=r_fc_{66}/(\Delta)^{1/2}$ in $d=2$ (we have neglected the contribution of compression modes). This critical current is much larger than the one which would be inferred from compression modes alone $F_{c,11}$. Thus when $v<v_{66}$ the transverse critical force is of the order of the isotropic static one $F_c^{\rm iso}$. For $v_{66}<v<v_{11}$, the transverse critical force decreases as v increases from $F_c^{\rm iso}$ to $F_{c,11}$ where $F_{c,11}$ is given by the same formulas as $F_c^{\rm iso}$ but with $c_{66}\to c_{11}$ and is thus much smaller (with the corresponding $R_{c,11}$ also obtained from $R_c^{\rm iso}$ by substituting $c_{66}\to c_{11}$). For $v>v_{11}$ the transverse critical force is given by the large velocity result [see Eqs. (7)–(9)]. While v_{11} is still very large, v_{66} can be within experimentally accessible order of magnitude:

$$\frac{\eta v_{66}}{F_c^{\rm iso}}=\frac{(R_c^{\rm iso})^2}{r_fR_{c,11}}. \tag{13}$$

One finds that the ratio of the two crossover velocities is $v_{11}/v_{66}=c_{11}/c_{66}$ (up to logarithms in $d=3$). One sees from Eq. (13) that a measure of the transverse critical current may lead to interesting information about the elasticity of the lattice. To estimate the transverse critical force in all regimes one could compute the dynamical Larkin length as $u_y\sim r_f$ using the complete formula (45) which contains both c_{66} and c_{11} contributions.

Finally note that one can make a simple minded argument showing directly in Eq. (5) that the convective term should not change pinning much at small v. Indeed starting from the case $v=0$, where one has a pinned state $u_{\rm stat}^{v=0}(r)$ and treating the convection term as a perturbation (which should be valid at small scales), one sees that this terms acts on the $v=0$ pinned state as an additional quenched random force. Since there is a critical force $f_c(v=0)$ in that case, it is intuitively clear that this term does not destroy completely the state $u_{\rm stat}^{v=0}(r)$ until $vr_f/R_c\sim f_c$. This argument gives back the correct value for v_c^*.

C. Displacements and correlation functions

Due to the presence of the static disorder one expects unbounded growth of displacements in the moving glass. The relative displacements induced by disorder in the moving system can be first computed in naive perturbation theory using Eq. (10). One finds

$$B(x,y)\sim\Delta\frac{y^{3-d}}{c\,\eta v}H\left(\frac{cx}{\eta v y^2}\right) \tag{14}$$

where $H(0)=\text{cst}$ and $H(z)\sim z^{(3-d)/2}$ at large z. Thus x scales as y^2 and the displacements are very anisotropic. The above formula, if taken seriously, leads to displacements growing unboundedly for $d\leqslant 3$. This is similar to the Larkin calculation for the static problem. As in the statics it indicates that the crystal is unstable to weak disorder in $d\leqslant 3$ and that perfect transverse long-range order is destroyed. Note that due to motion the upper critical dimension is now $d=3$ instead of $d=4$ for the statics. As in the statics, the above formula and perturbation theory breaks down above R_c^y and an RG approach is absolutely necessary to compute the displacements. Using an RG calculation one finds that the behavior of displacements is indeed controlled by a fixed point characteristic of the moving glass phase. One finds that the correlation function of displacements *averaged over disorder* can be rigorously separated into two parts $B(r)=\overline{\langle[u(r)-u(0)]^2\rangle}=B_{\rm RF}(r)+B_{\rm NL}(r)$ where $B_{\rm NL}(r)$ comes from the *nonlinear part* of the pinning force. While this part (computed in Ref. 45) is dominant in the Bragg glass in the moving glass this contribution is subdominant at large scales (although it can be dominant at intermediate scales) and we neglect it for now. The main contribution comes from the *static random force* which is generated both along y and x direction. The generation of such a random force, forbidden in a static system, occurs here because of the nonpotentiality induced by the motion. The complete expression of the generated random force is given in Sec. VIII A (see also Sec.

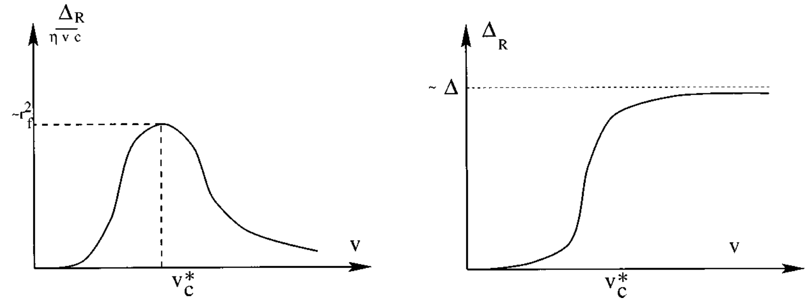

FIG. 13. Renormalized random force strength as a function of the velocity (right) and resulting amplitude in displacement correlations (left).

VI). This random force gives a contribution to the displacement which at large scale has the same spatial dependence than the one naively extrapolated from Larkin regime formula (10) and thus Eq. (14). One thus finds

$$B_{\rm RF}(r)\sim\frac{\Delta_R}{4\pi c\,\eta v}\ln(y),\quad d=3,$$

$$B_{\rm RF}(r)\sim C_d\frac{\Delta_R}{4\pi c\,\eta v}y^{3-d}\sim\frac{\Delta_R}{4\pi c\,\eta v}x^{(3-d)/2},\quad d<3. \tag{15}$$

At large scales the random force contribution to $B(r)$ dominates. Although the formula resembles the perturbative one, the amplitude of the random force is given by the *renormalized* Δ_R which has been extracted from the RG analysis and is determined by the nonlinear pinning force. In general Δ_R can be different from the perturbative Δ. In particular Δ_R must vanish when $v\to 0$. Although Δ_R is a nonuniversal quantity (contrary to the behavior in the Bragg glass) one can still obtain a reliable estimate for Δ_R by studying the crossover depicted in Fig. 12. If the velocity is smaller than the crossover velocity v_c^* the random force is renormalized downwards according to the behavior in the Bragg glass phase. Thus Δ_R will be smaller than the bare Δ. This is illustrated in Fig. 13.

The amplitude of the displacements [e.g., the prefactor of the logarithmic growth in Eq. (14)] generated by the renormalized random force is maximum around the velocity v_c^*. Even at this velocity the displacements can be estimated as $B_{\rm RF}(r)\sim r_f^2\ln(r/R_c^y)$ in $d=3$ and $B_{\rm RF}(r)\sim r_f^2(y/R_c^y)$ in $d=2$. At all other velocities the amplitude is much smaller. Given the form of the displacement correlation function the moving glass has quasi-long-range translational order in $d=3$. One finds for transverse translational order correlations:

$$C_{K_y,K_x=0}(0,y,z)\sim\left(\frac{R_a^y}{\sqrt{y^2+z^2(c_{11}/c_{44})}}\right)^{A_K},\quad A_K=\frac{K_y^2\Delta_R}{8\pi v c\,\eta}. \tag{16}$$

In particular $A_{K_0}=\pi a^2\Delta_R/(2vc\,\eta)$. The dependence in the coordinate x is $C_{K_y,K_x=0}(x,0,0)\sim(R_a^x/x)^{A_K/2}$ and thus one finds an anistropic divergence of the Bragg peaks corresponding to $K_x=0$ of the form

$$S(q)\sim\frac{1}{(c_{11}q_y^2+c_{44}q_z^2)^{2-A_K/2}}\sim\frac{1}{q_x^{2-A_K/2}}. \tag{17}$$

The question of the divergences of peaks associated with $K_x>0$ is discussed in the next section. In $d=2$ algebraic growth of displacements imply a stretched exponential decay of $C_K(r)$ and thus that the peaks in the structure factor are rounded (as the dotted line in Fig. 3).

The roughness of the channels define an additional length scale at which the wandering becomes similar to the lattice spacing. As in the statics (Bragg glass) it is possible to estimate these lengths. At large velocity these lengths are large and at this scale the system is very anisotropic. A simple argument a la Fukuyama-Lee, similar to the one in Ref. 45 gives

$$R_a^y\sim(a^2vc/\Delta)^{1/(3-d)},\quad R_a^x=v(R_a^y)^2/c. \tag{18}$$

At large v one can also obtain these lengths by looking at the displacements generated by the random force. For small $v<v_a^*$ there is a long crossover since at small scales the system looks more like the Bragg glass. As a consequence the estimates for R_a change. This illustrated in Fig. 12. v_a^* is determined roughly by $R_{\rm cr}=R_a^y$.

Let us summarize the main regimes as a function of the velocity of the moving glass, as can be seen in Fig. 12. At large velocity $v>v_c^*$ the system is already anisotropic at the scale R_c and pinning and correlations are determined directly by the asymptotic moving glass behavior. For $v_a^*<v<v_c^*$ the system is isotropic at the Larkin length and pinning is similar to the static, but the system is still very anisotropic at scales R_a^y. Finally for $v<v_a^*$ the system is almost staticlike up to R_a^y and isotropic. The random force is enormously reduced, and transverse barriers are very large. Finally note that in each configuration of the disorder the random force along y and the transverse critical force compete. The physics of the moving glass is determined by this competition.

D. Decoupling of channels and dislocations

Most of the properties of a moving structure discussed in the previous section were obtained from the moving glass equation (5), which contains only the transverse displacements u_y. They thus rest only on the channel structure itself and not on the precise motion of the individual particles along these channels. Let us now discuss the problem of the coupling of particles between different channels which is important for the issues of topological, translational order, and structure factor.

An outstanding problem in the statics is whether or not topological defects are generated by disorder in an elastic structure. Using energy arguments it was predicted that due to the periodicity a lattice is stable to dislocations at weak disorder in $d=3$ giving rise to the Bragg glass.[45] The similar question of whether disorder generates dislocations arises also for moving structures. At first sight the situation looks even more complicated to tackle analytically and furthermore precise energy arguments cannot be used because the system is out of equilibrium. However, as is becoming clear from the discussion in the previous section the issue of the creation of dislocations can now be discussed here in terms of decoupling of channels. Even in the presence of dislocations our picture of pinned channels should remain valid as long as periodicity along y is maintained. Before the channel structure was identified in Ref. 73 it was unclear how dislocations could affect a moving structure. The existence of channels then *naturally* suggests a scenario by which dislocations appear. In fact the results of Ref. 73 naturally suggests that transitions from elastic to plastic flow may now be studied as *ordering transitions* in the structure of channels.

Let us examine first whether dislocations appear in the moving Bragg glass in $d=3$. The relative deformations due to disorder grow only logarithmically with distance, resulting in quasi-long-range order. At weak disorder or large velocity (since the relevant parameter is Δ/v) the prefactor of the logarithmic displacements is very small. This suggests, by analogy with the statics, that dislocations *do not appear*, leading to a stable moving Bragg glass at weak disorder or large velocity. In that case the structure factor exhibits Bragg glass-type peaks (at all the small reciprocal-lattice vectors). Note however that due to the anisotropy inherent to the motion the *shape* of each peak is highly anisotropic the length R_a^x being much larger than R_a^y. Upon increase of disorder the first likely transition corresponds to a decoupling of the channels, while the periodicity along y is maintained. This corresponds to the loss of divergent Bragg peaks at reciprocal-lattice vectors with nonzero components along the direction of motion. The peaks at reciprocal-lattice vectors along y still exhibit divergences (computed in the last section). This particular case of a moving glass was observed numerically in $d=2$ and called the moving transverse glass[89] (see next section). This phase has also a smectic type of order. One question is whether particles can hop between the channels in this phase. This however seems unlikely at zero temperature provided the channels are well defined. In the absence of such hops this decoupled phase can still be described by Eq. (5) and has nonzero transverse critical current. Increasing further the disorder should destroy even the channel structure leading to a fully plastic flow.

An estimate of the locus of the transition between the moving Bragg glass and the moving transverse glass is given by a Lindemann criterion. For the statics such a criterion has been shown to accurately predict the positional decoupling in a layered structure.[48,49] Here we extend this criterion to the dynamics

$$\overline{\langle[u_x(y=a)-u_x(0)]^2\rangle}=c_L^2a^2. \tag{19}$$

Decoupling of channels comes from displacements along the direction of motion. This scenario makes sense since displacements along x are likely to dominate (see the discussion below and Sec. V B). This is consistent with edge dislocations appearing first.

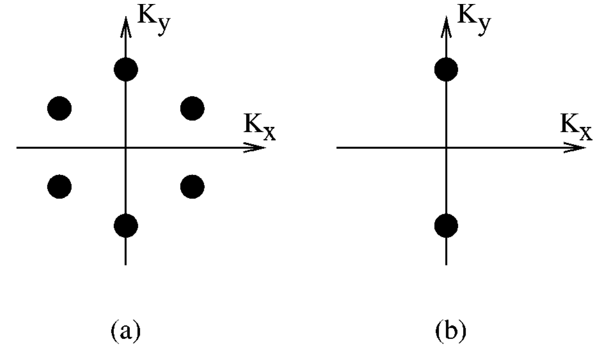

FIG. 14. Bragg peaks for the two realizations of the moving glass: (a) moving Bragg glass with quasi-long-range order and perfect topological order. (b) Moving transverse glass where channels have decoupled and quasi-long-range order in the y direction is maintained.

In $d=2$ displacements grow algebraically [see Eq. (15)]. Thus, even at weak disorder or large velocities, it is much more likely that dislocations appear at large scale. Presumably this scale corresponds to the displacements being of order a. One then easily sees that dislocations appear between the layers. Indeed

$$\overline{\langle[u_x(L_a^y)-u_x(0)]^2\rangle}\sim a^2 \tag{20}$$

is controlled by the random force along x and by c_{66} (the displacements u_y are down by a factor R_a^y/R_a^x—see below and Sec. V B). In this regime blocks of channels of variable transverse size L_a^y (depending on the strength of the disorder) are separated by dislocations.

The peaks at vectors with a nonzero K_x thus allow us to distinguish between the moving Bragg glass and the moving transverse glass. This is illustrated in Fig. 14.

In systems with a small ratio c_{66}/c_{11} and stronger disorder the peaks with $K_x>0$ have a tendency to be smaller (and decoupling becomes easier as u_x becomes larger than u_y). Indeed the displacements at large scales (and thus the decay of translational long-range order) are controlled by the random forces along y, Δ_{yy} and along x, Δ_{xx} (they remain statistically uncorrelated—see Sec. VIII A). These random forces act differently on u_y and u_x. Indeed, only P_{xx}^T (shear, c_{66}) and P_{yy}^L (compression c_{11}) lead to unbounded displacements (e.g., in $d=3$). The y random force thus acts mainly on u_y via compression, and the x random force mainly on u_x via shear. Though generally one has $\Delta_{xx}<\Delta_{yy}$ at weak disorder, if the ratio c_{66}/c_{11} is small this could strongly favor the weakening of the $K_x>0$ peaks and channel decoupling. A estimation of Δ_{xx} is performed in Sec. VIII A.

The problem of the behavior of dislocations in the moving glass system is of course still open,[90.91] and constitutes as for the statics one of the most important issues to understand. It is noteworthy, however, that although these issues are of course important to obtain the structure factors and as such, they *do not* affect the main physics of the moving glass.

E. Moving Bose glass

Similar arguments also apply in the presence of correlated (columnar $d_z=1$) disorder along the z direction. We predict in that case the existence of a ''moving Bose glass'' in $d=3$,[92] whose upper critical dimension is $d=4$. Indeed the same calculation as above,

$$B_{\mathrm{RF}}(y)=\Delta\int \frac{dq_x dq_y dq_z}{(2\pi)^2} \times \delta(q_z)\,\frac{[-\cos(q_y y)]}{(\eta v q_x)^2+(c_{11}^2 q_y^2+c_{44}^2 q_z^2)^2}, \quad (21)$$

now yields a fast growing displacement. Thus the disorder effects are stronger and one can expect thermal effects to be weaker for correlated disorder. The situation resembles the $d=2$ case at $T=0$. One can still predict a transverse critical current. Full topological order is unlikely so one should rather have a moving transverse glass type of order along y with a localization effect of the layers and thus a transverse Meissner effect along y. A detailed study of this moving Bose glass phase will be given elsewhere.[83] In a similar way, the effect of correlated disorder on another dissipative glass system (nonpotential) was found to be quite strong.[85]

F. Moving glass at finite temperature

Thus the moving glass, in its different forms, described by Eq. (5) is a new disordered fixed point at $T=0$. An important question is to understand what is the effect of thermal fluctuations. Indeed in moving systems, as can be seen by perturbation theory, the fluctuation dissipation theorem is violated and a generation of temperature by motion occurs. This corresponds to the physical effect of heating by motion. Note however that at $T=0$ a system in the absence of thermal fluctuations retains perfect time order which implies that no temperature can be generated. Thus the heating effect is not well described by a ''shaking temperature'' as introduced in Ref. 72 which would be nonzero even at $T=0$. Although the temperature grows due to motion, this effect competes with the fact that naive power counting in glassy systems suggests that the temperature is an irrelevant variable flowing to zero. The competition between these two effects is highly nontrivial and leads to physics which need to be investigated for the whole class of moving glasses of Sec. II B. Remarkably, for this class of systems, different finite temperature fixed points exists. In the case of driven lattices, we find a fixed point at finite temperature in a $d=3-\epsilon$ expansion and $d=2+\epsilon$ expansion. Similarly for randomly driven polymers very similar fixed points are obtained.[85] Thus a large class of dissipative glasses exists at nonzero temperature. In $d=3$, the fixed point is slightly peculiar since both temperature and disorder flow to zero but can be analyzed along the same lines. The properties of the finite-temperature phase are continously related to the $T=0$ one. In particular the finite-temperature moving glass exhibits the same type of rough channel structure. Channels are slightly broadened due to bounded thermal displacements around the average channel position. Thus the asymptotic behavior of the displacements and structure factor, still remains similar to that at $T=0$ discussed in the previous sections. There is, in addition, a contribution of thermal displacements. In $d=3$ they are small, and one sees that the RG methods developed here allow us to estimate more precisely the thermal heating effect, and to distinguish it clearly from the disorder effects. This is important for determining the dynamical phase diagrams. In $d=2$ the thermal displacements are large (see Sec. VII).

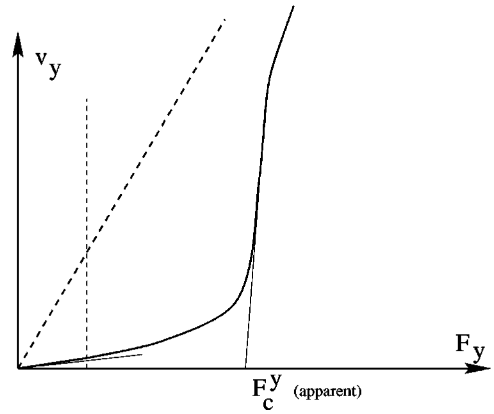

FIG. 15. Transverse critical force as a function of the velocity. v-f characteristics at finite temperature. The vertical dotted line is the crossover force f^* below which the barriers saturate (see text). Above f^* the characteristics are highly nonlinear.

The main effect of temperature is to modify the v-f characteristics. One finds (Sec. VI) that the asymptotic mobility μ_R is nonzero. However at low temperatures or at velocities not too large the v-f characteristics remain highly nonlinear. There is still an ''effective'' (or apparent) transverse critical force $F_c^y(T)$ as shown in Fig. 15. At low temperature the mobility μ_R is very small. If the velocity is not too large $v<v_c^*$ there are several regions in the v-f curve. Below the transverse pinning force slow motion due to effective barriers exist. They are a growing function of $1/f$ until one reaches the finite-temperature moving glass fixed point. Indeed reducing the transverse force probes larger and larger length scales. As depicted in Fig. 12 one is dominated until the scale R_{cr} by the Bragg glass fixed point for which temperature is strongly renormalized downwards. In this regime the v-f curve is nearly similar to the one in the static Bragg glass and thus highly nonlinear. This corresponds to a creep regime. For smaller forces (i.e., when probing scales larger than R_{cr}) one crosses over to the moving Bragg glass fixed point. At that point the FRG calculation shows that the barriers saturate. Thus below the scale f^* one recovers a linear v-f charactersitic, with an extremely small mobility. Note that the scale f^* which corresponds to the crossover scale R_{cr} can be much smaller than the critical transverse pinning force if $v<v_c^*$.

In a realistic system where $c_{66}\ll c_{11}$, v_c^* splits into two different crossover velocities as discussed in Sec. III B. When $v<v_{11}$ barriers are very high as in the static problem and there is a noticeable transverse creep regime as shown in Fig. 15. Since v_{11} is very large in practice this regime applies to a large range of longitudinal velocities. When $v<v_{66}$ one enters a regime where transverse creep becomes identical to the isotropic one.

These properties show that even at finite temperature the moving Bragg glass remains different from a perfect crystal. The definition of what is ''glassiness'' in a moving structure is a concept which has to be defined. In that respect too close analogies with the statics can be misleading. A first obvious glassy characteristic is the loss of translational order, contrary to the crystal. Note however that a similar effect could be obtained by adding a random force by hand to a perfect crystal. However the response of such a structure to an externally applied force would be *identical* to the one of a perfect crystal. Thus the glassy properties of the moving Bragg glass are necessarily stronger than such a state. The same question of history dependence as discussed above at $T=0$ can be asked. If these effects exist the question of the finiteness of the barriers might not be as important an issue as in the static case. Note however that in some other examples of nonpotential dynamics barriers can indeed be infinite.[85] Since the finite-temperature moving Bragg glass is described by a new fixed point which still contains nonlinear disorder $\Delta(u)$ the system remains obviously in a glassy regime. Some correlation functions of the system, such as four-point correlation functions, necessarily depend on the existence of this finite $\Delta(u)$ and exhibit infrared divergences leading to anomalous behavior. Others, such as the two-point response functions, have these divergences cut by the finite velocity. This is reminiscent of what happens in the statics where the order parameter of the glassy phase is the fluctuation of the susceptibility[93] whereas the averaged susceptibility itself remain inocuous. A detailed investigation of higher order nonlinear response clearly deserves further studies. Note finally that for driven lattice (and for experimental purposes), the predicted existence of high (even if asymptotically finite) barriers in a large regime of velocities is a totally unanticipated property of disordered moving systems.

G. Phase diagrams

Having established the existence of the moving Bragg glass in $d=3$ and of the moving transverse glass in $d=3$ and $d=2$ and having discussed their properties we now indicate in which region of the phase diagram these phases are expected to exist. We study the phase diagram as a function of disorder, temperature, and applied force (or velocity).

Let us first discuss the case $d=3$. We have represented in Fig. 16 a schematic expected phase diagram as a function of temperature, disorder, and applied force. For clarity we have not represented intermediate phases (or the various forms of the moving glass). Let us now discuss the main features of Fig. 16. At zero external force $F=0$, one recovers the static phase diagram.[45,47] There is a transition at finite disorder strength between the Bragg glass to an amorphous glass where dislocations proliferate. Upon applying a force the Bragg glass phase becomes the moving Bragg glass in the low velocity regime (creep regime) and continuously extend to the moving Bragg glass at higher drives. At weak enough disorder the continuity between the two phases suggests that depinning should be elastic without an intermediate plastic region. Upon raising the temperature the moving Bragg glass melts to a liquid, presumably through a first-order dynamical melting transition. The $T=0$ plane contains a pinned region for $F<F_c(\Delta)$ and it is natural to expect the Bragg glass to still exist even for a finite force $F<F_c$ until the depinning transition. At higher disorder dislocations appear and the Bragg glass is replaced by an amorphous glass. The nature of this amorphous glass is still unclear (see, e.g., Ref. 47), but it is sure to contain topological defects. Thus the depinning of this amorphous glass should be via a highly disordered filamentary plastic flow. Upon increasing the force and thus the velocity, the system should reverse back to the moving Bragg glass (MBG), since the effective disorder Δ/v decreases. At strong disorder and finite drive the liquid extends to zero temperature.

These different behaviors are also represented along each plane in Figs. 17–19. In these figures we also have indicated intermediate phases such as the Moving transverse glass. Determining the exact shape of the various boundaries is still an open and challenging problem, in particular in the square region in Fig. 16.

One of the strong features that emerges from these phase diagrams is the fact that the Bragg glass is able to survive

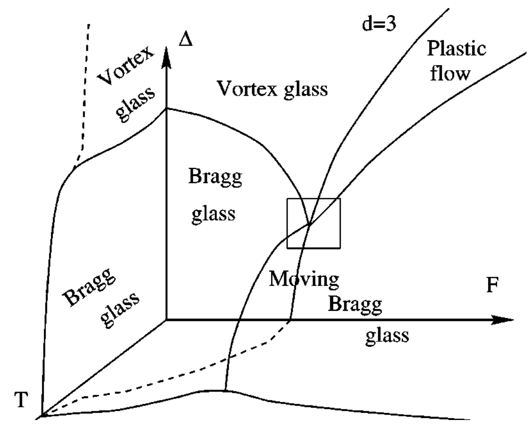

FIG. 16. Schematic phase diagram in the temperature T, disorder Δ, applied force f variables. In $d=3$ for very weak disorder, since the moving glass is likely to be topologically ordered, the possibility of a depinning without a plastic regime exists. Note that the moving Bragg glass then should extend all the way down to small f. We have not represented intermediate phases (hexatic, moving transverse glass) for clarity. Note also that the lower plane corresponds to a small but nonzero Δ. The connections between the various phase boundaries (inside the square region) is schematic.

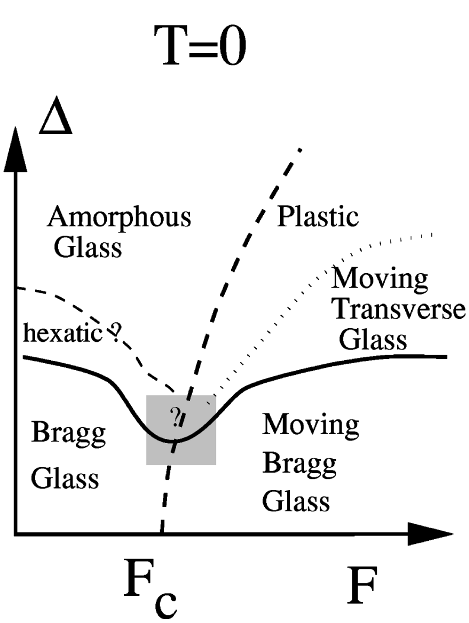

FIG. 17. Schematic phase diagram in the force f, disorder Δ in $d=3$ at $T=0$. The behavior in the square region is unclear. An interesting possibility would be a direct depinning of a hexatic into the moving transverse glass, but other scenarios are possible.

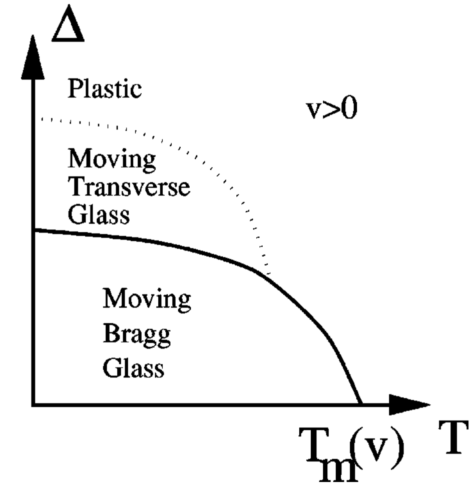

FIG. 18. Schematic phase diagram in the temperature T, disorder Δ in $d=3$ for a fixed velocity (not too small). This phase diagram is the dynamical version to the static one (containing the Bragg glass, the vortex glass, and the field-driven transition). The MBG can either thermally melt at T_m (via a first-order transition) or decouple because of disorder.

motion by turning into the moving Bragg glass. On the other hand other, more disordered phases such as the amorphous glass (vortex glass) are likely to be immediately destroyed at finite drive (and finite temperature) and to be continuously related to the liquid.

In $d=2$, the static phase diagram is still unresolved. A reasonable assumption is that there is no topologically ordered phase[45,52,1,94] although this is far from being firmly established. Accepting this as a starting point for the static phase diagram, we can now extend it to the dynamic case. Most of the transitions then reduce to simple crossovers. At $F=0$ and finite disorder dislocations are expected to be present. The resulting phase should thus be continuously connected to the liquid, although it can retain good short distance translational order. At $T=0$ there is a pinned phase until F_c, which should depin by a plastic flow. At larger drive disorder effects become smaller and one expects the system to revert to a moving glass state. As discussed earlier, due to the presence of disorder induced dislocations, this

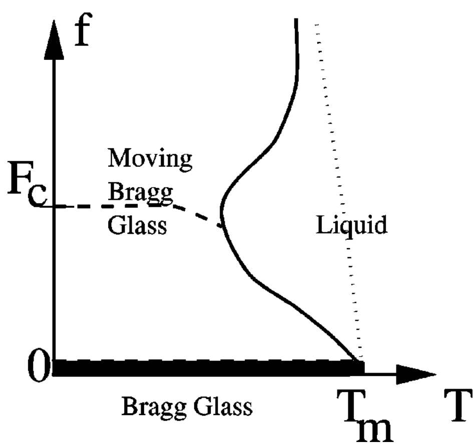

FIG. 19. Schematic phase diagram as a function of force f and temperature T in $d=3$. T_m is the melting temperature of the static system. The Bragg glass phase also exists at $T=0$ for $f<F_c$.

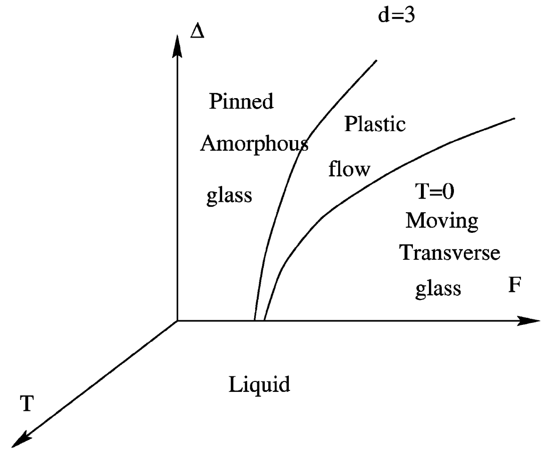

FIG. 20. Schematic phase diagram as a function of force f, temperature T, and disorder Δ in $d=2$. There is a very long crossover not represented here.

state is a moving transverse glass (if $d=2$ is above its lower critical dimension). At any finite temperature, one can use the RG flow of Sec. VII. Since the temperature renormalizes *above* the melting temperature and disorder flows to zero the resulting phase should be a driven liquid. (See Fig. 20.)

H. Comparison with numerical simulations

Some of the predictions of the moving glass theory contained in the short account of our work[73] have been later verified in several numerical simulations in $d=2$ and $d=3$. The static channels were clearly observed[89] at $T=0$ in $d=2$, and then in Ref. 95. Both Ref. 89 and Ref. 96 showed clear evidence of the transverse critical force at $T=0$ (see also Ref. 95). The transverse critical force was found to be a fraction of the longitudinal critical force which is a reasonable order of magnitude. The effect of a nonzero temperature $T>0$ weakened the effect of transverse barriers[95] in $d=2$. Some nonlinear effects still persisted for low enough transverse force and temperature.[96] Such an observation is in agreement with the discussion of Sec. III F and can be interpreted as a long crossover.

Sharp Bragg peaks were observed in the direction transverse to motion[89] at $T=0$. However the order along the x direction was found to have fast decay. This is consistent with a decoupling of the channels, and the resulting state was termed the ''moving transverse glass.''[89] Such a decoupling is in agreement with the expectations from the theory presented here [Eq. (20)], as illustrated on the phase diagram. This observed phase in the simulations is presumably the $T=0$ moving transverse glass fixed point analyzed in Sec. III D which does have a nonzero transverse critical force. This is in fact confirmed by the observation of Ref. 96 of a smectic type of order where well separated dislocations exist between the channels[90,91] consistent with the expectations discussed in Secs. III A and III D and summarized in Fig. 8. The absence of long-range order was also observed in Ref. 97 in a stronger disorder situation.

In $d=3$, a simulation of a driven discrete superconducting XY gauge model[98] finds not perfect but still well defined Bragg peaks at $T>0$ (near the melting), a result which indicates that the driven lattice is in a quasiordered moving

Bragg glass state. The melting is found to occur at lower temperature than in the pure system and the transition to remain first order up to higher fields. This is consistent with the discussion in Sec. III G. In another study on the simpler $d=3$ driven XY model at $T=0$,[99] it was found that indeed there is a phase without topological defects at large enough drive. If it carries to the lattice problem it would indicate that indeed there is a $d=3$ moving Bragg glass state.

Finally note that there are also very recent simulations of a lattice with a periodic substrate.[100,56] This is a simpler case where a transverse critical current exists (it does exist for a single particle). It would be worthwhile to also investigate this case in all details.

All the above numerically observed effects seem to be in qualitative agreement with the predictions in Ref. 73. However, it would be very useful to be able to make a more *quantitative* comparison. This should now be possible, as we give here more detailed predictions than the short account (Ref. 73). Among the various interesting topics to check are the algebraic decay of translational order in $d=3$, a detailed study of the dependence of the transverse critical force on the velocity, the exponent θ of the transverse depinning, a measure of the barriers at low temperatures, a characterization of the history dependence, and zero and at low temperature.

I. Comparison with experiments

The moving glass picture has also been confronted with experiments. Since these experiments need the characterization of a moving structure they are challenging. The transverse critical current can, in principle, be observed in transport experiments (and may show up as an hysteretic effect). These are difficult though because of dissipation in the longitudinal direction.

Decoration experiments on the *moving* vortex lattice have been performed recently by Marchevsky *et al.*[101] In these experiments the external field is slowly varied and vortices are decorated while they move. The decoration particles thus accumulate on the regions where vortices are flowing preferentially. The lattice is observed to move in the symmetry axis direction and relatively large regions of highly correlated static channels are observed. These channels do not look like ''plastic channels'' but much more like the elastic channels predicted in Ref. 73. Note however that some dislocations along y appear (defects in the layered structure). This may be due to strong disorder effects or to the geometry of the experiment (since the advancing front geometry is in the shape of a droplet some dislocations are unavoidable). Another set of experiments in $NBSe_2$, also exhibiting channels, was reported.[102] Note that there has also been several decoration experiments performed *just after* the current is turned off.[103] These can, in principle, probe the defect structure of the flowing lattice (though one may worry about transient effects) but cannot show the channel structure. Finally a recent experiment[104] on superconducting multilayers found that the flux-flow resistivity exhibit quasiperiodic oscillations as a function of the field. This was interpreted[104] in terms of dynamics matching of the moving vortex lattice with the periodic substrate. This is compatible with the presence of a quasiordered structure in motion.

Other effects of the moving glass and transverse critical force can be found in systems other than vortex lattices. Indeed, as suggested in Ref. 73, the transverse barriers may explain the anomalies recently observed in the Hall effect in a Wigner crystal in a constant magnetic field.[12,11] The qualitative analysis suggested by the moving glass theory is as follows. An electric field E_x is applied in along the x direction. The Wigner crystal starts moving along x when the applied field is larger than a ''longitudinal'' threshold $E_x > E_c$. It produces a current along x, $I_x = qv_x$ which is directly measured. Below the longitudinal threshold a highly nonlinear regime is observed where activated motion dominates. Since it is moving in a high magnetic field, the moving Wigner crystal is submitted to a transverse Lorentz force $F_y^L = qv_xB$. The geometry of the experiment is such, however, that no transverse motion is permitted in the stationary state (because of zero current boundary conditions), and thus $v_y=0$. Thus the transverse Lorentz force must be balanced by a transverse electric field, which is thus generated, and is measured as the Hall voltage V_y. In the absence of transverse pinning the Hall voltage is $V_y = LBI_x$. Remarkably, it is found in the experiment that the actual measured Hall voltage is indeed $V_y = LBI_x$ for small I_x, then experiences a plateau, and finally starts again growing linearly with a slope $dV_y/dI_x \approx LB$. We have interpreted the different behaviors upon increase of I_x as follows. For small I_x one is near the longitudinal depinning and it is probably a plastic flow regime with little transverse barriers. Then upon motion, a transition to the moving glass occurs (see Fig. 11). The existence of a nonzero transverse critical force $F_y^c > 0$ then immediately implies that there are sliding states with $v_y=0$ as long as $F_y^L < F_y^c$ and no Hall voltage is necessary.

Obviously more experiments are needed, to investigate in detail the properties of the moving glass phase. It would be interesting to probe further the channel structure by direct imaging techniques. In particular one may investigate the degree of reproducibility of the channel pattern. In particular upon sudden reversal of the velocity the channels should be *different*. The question of order and quasiorder can be probed in experiments such as neutron scattering, flux-lattice imaging magnetic noise experiments, NMR experiments, and more indirectly in transport measurements. Other imaging techniques such as μ-SR NMR electron holography,[105] can also be used. Finally it would be interesting to check for similar effects in the presence of columnar defects since, as discussed in this paper we predict the formation of a moving Bose glass.

IV. THE MODELS AND PHYSICAL CONTENT

A. Derivation of the equations of motion

Let us first derive the equation of motion for a lattice submitted to external force f. We work in the *laboratory frame*. This offers several advantages that will become obvious later. We denote by $R_i(t)$ the true position of an individual vortex in the laboratory frame. The lattice as a whole moves with a velocity v. We thus introduce the displacements $R_i(t) = R_i^0 + vt + u_i(t)$ where the R_i^0 denote the equilibrium positions in the perfect lattice with no disorder. u_i represent the displacements compared to a moving perfect lattice (and corresponds to the position of the ith particle in the moving frame). The definition of v imposes $\Sigma_i \dot{u}_i(t) = 0$

at all times. We furthermore assume that the motion is overdamped. The exact equation of motion can then be obtained from the Hamiltonian H by

$$\eta \frac{du_i(t)}{dt} = -\frac{\delta H}{\delta u_i} + f - \eta v + \zeta_i(t), \tag{22}$$

where η is the friction coefficient and the thermal noise satisfies $\overline{\zeta_i(t)\zeta_j(t')} = 2T\eta\delta_{ij}\delta(t-t')$. The Hamiltonian is the standard Hamiltonian for periodic structure in a random potential $H = H_{\rm el} + H_{\rm dis}$. $H_{\rm el}$ is the standard elastic Hamiltonian, and $H_{\rm dis}$ describes the interaction with the random potential

$$H_{\rm dis} = \int_r V(r)\rho(r) = \sum_i \int_r V(r)\delta(r - [R_i^0 + vt + u_i(t)]), \tag{23}$$

where the random potential has correlations $\overline{V(r)V(r')} = g(r-r')$ of range r_f.

In order to use the standard field description of the displacement u instead of focusing on the equation for one particle, one rewrites Eq. (22) as

$$\eta \frac{du_i(t)}{dt} = -\frac{\delta H_{\rm el}}{\delta u_i} + \int_r \partial V(r)\delta(r - [R_i^0 + vt + u_i(t)]) + f - \eta v + \zeta(R_i(t),t). \tag{24}$$

In doing so one would get the same thermal noise for two particles being at the same place at the same time, instead of the two independent noises of Eq. (22). Since such a configuration cannot happen, going from Eq. (22) to Eq. (24) is essentially exact.

As for the static case[44,45] the difficulty is to take the continuum limit of Eq. (24) since the disorder can vary at a much shorter scale than the lattice spacing a. To proceed one follows the same steps than for the static case, suitably modified to take into account the time dependence of the displacements. One first introduces a smooth interpolating displacement field $u(r,t)$ such that $u(R_i^0 + vt, t) = u_i(t)$ [see formula (A2) of Ref. 45]. The field $u(r,t)$ is the smoothest field interpolating between the actual positions $u_i(t)$. All coordinates r are expressed in the laboratory frame. The field $u(r,t)$, whose components we denote by $u_\alpha(r,t)$ thus expresses the displacement in the moving frame, as a function of the coordinates of the laboratory frame. As for the static, if one assumes the absence of dislocations at all times the particles can be labeled in a unique way. One then introduces the continuous labeling field $\phi(r,t) = r - vt - u[\phi(r,t) + vt, t]$. Thus $\phi(R_i(t),t) = \phi(R_i^0 + vt + u_i(t),t) = R_i^0$ by definition, and ϕ numbers the particles by their initial positions. In the absence of dislocations the field $\phi(r,t)$ can be single valued. To obtain the continuum limit of Eq. (24) one first performs the continuum limit in the Hamiltonian as in Ref. 45, to obtain for the disorder term

$$H_{\rm pin} = \int dr V(r)\rho(r) = -\rho_0 \int dr V(r)\partial_\alpha u_\alpha + \rho_0 \int dr \sum_{K \neq 0} V(r) e^{iK\cdot[r - vt - u(r,t)]}, \tag{25}$$

where K spans the reciprocal lattice and ρ_0 is the average density. In Eq. (25) we have made the approximation $u[\phi(r,t) + vt, t] \sim u(r,t)$. Such an approximation is exact up to higher powers of ∂u, negligible in the elastic limit, as for the static case.[45] However the dynamic case is more subtle since such terms could generate relevant terms when combined with a nonzero velocity. This is the case for example of the so-called KPZ terms generated through cutoff effects. Since it is hopeless to try to tackle from first principles all such additional terms the only safe procedure is to assume that every term allowed by symetry is generated, and has to be examined. We proceed with such a program in Sec. VIII B. For the moment we only retain the dominant terms of Eq. (25). If one then takes the derivative with respect to the smooth field $u(r,t)$ one obtains for the equation of motion in the laboratory frame

$$\eta \partial_t u_{rt}^\alpha + \eta v \cdot \nabla u_{rt}^\alpha = -\int_{r'} \Phi_{\alpha\beta}(r-r') u_{r't}^\beta + F_{\rm pin}^\alpha(r,t) + f_\alpha - \eta v_\alpha + \zeta_\alpha, \tag{26}$$

where $\Phi_{\alpha\beta}(r-r')$ is the elastic matrix. The term $\eta v \cdot \nabla u_\alpha$ comes from the standard Euler representation when expressing the displacement field in the laboratory frame. $-\eta v_\alpha$ is the average friction and in the continuum v is determined by the condition that the average of u is zero. The thermal noise satisfies in the continuum limit $\overline{\zeta_\alpha(r,t)\zeta_\beta(r',t')} = 2T\eta\delta_{\alpha\beta}\delta^d(r-r')\delta(t-t')$ and

$$F_\alpha^{\rm pin}(r,t) = -\delta H_{\rm pin}/\delta u_\alpha(r,t) = V(r)\rho_0 \sum_K iK_\alpha \times \exp(iK\cdot[r - vt - u(r,t)]) - \rho_0 \nabla_\alpha V(r) \tag{27}$$

is the pinning force. Note the difference between our Eq. (26) and the one derived in Ref. 70, which does not contain the convective term. This difference comes simply from a different definition of the displacement fields. They consider displacement fields labeled by the original position of the particle (i.e., the actual position of the particle is $r+u$) whereas for us r denotes the actual position of the vortex considered [i.e., in the presence of an external potential V the potential acting on the vortex at point r is $V(r)$ instead of $V(r+u)$ for Ref. 70]. In Ref. 106 we give a more general derivation of Eq. (27) valid even for cases where the equation of motion is *not* the derivative of a potential.

B. Models and symmetries

Before we even attempt to solve Eq. (26), let us examine the various symmetries of the problem and define several models which approximate the physical problem at various levels. The physical symmetry of the original equation of motion (22) is the global inversion symmetry $(r \to -r, u \to$

$-u, v \to -v, f \to -f$). When the force (and thus v) is along a principal lattice direction, one has then two independent inversion symmetries $I_x = (x \to -x, u_x \to -u_x, v \to -v, f \to -f)$ and $I_y = (y \to -y, u_y \to -u_y)$. These symmetries are exact and hold in all cases. They are the only symmetries of the original model (22). The proper continuum limit of Eq. (22) must thus include all terms which are relevant and consistent with these exact symmetries. We define such a model as model I, which is studied in more detail in (Sec. VIII B). The additional terms can originate from, e.g., anharmonic elasticity, cutoff effects or higher-order terms in ∇u combined with disorder, as is dicussed in Sec. VIII B.

If one drops in model I the terms which are small in the elastic limit $\nabla u \ll 1$, one obtains another model that we call model II. (See Fig. 1.) It corresponds to the continuum limit of the equation of motion to obtain Eq. (26) i.e., Eq. (26) in the elastic limit. However this continuum limit is nontrivial and should be performed with care as the disorder can vary at scales much shorter than the lattice spacing.[45] Although model II is slightly simpler than model I, it only misses terms which are small in the bare equation but would be allowed by the above symmetries. Even if some of them are relevant, they would only be able to change the physics compared to model II at very large length scales. One thus expects model II to give in practice an extremely accurate description of the physics. Model II possesses a higher symmetry than model I. Let us examine the symmetries of the pinning force (27). Using the correlator of the random potential V, the correlator of the pinning force is

$$\Delta_{\alpha\beta} = \overline{F_\alpha^{\rm pin}(r,t,u_{rt}) F_\beta^{\rm pin}(r',t',u_{r't'})} = \rho_0^2 g(r-r') \times \sum_{K,K' \neq 0} iK_\alpha iK'_\beta e^{iK\cdot(r-vt-u_{rt}) + iK'\cdot(r'-vt'-u_{r't'})}. \tag{28}$$

Since u is a smooth field it has no rapidly oscillating components and thus in Eq. (28) the terms that are rapidly oscillating in $r+r'$ can be discarded. Setting $K' = -K$ in Eq. (28), one is left with

$$\Delta_{\alpha\beta} = \rho_0^2 \sum_{K \neq 0} K_\alpha K_\beta g(r-r') \exp(iK\cdot(r-r') - iK\cdot[u_{rt} - u_{r't'} + v(t-t')]). \tag{29}$$

The symmetries of Eq. (29) thus *a priori* depend on the precise form of the correlator $g(r)$. However in the elastic limit it is legitimate to replace $u_{r't'}$ by $u_{rt'}$ in the above expression. Integrating then over r' one obtains

$$\Delta_{\alpha\beta} = \rho_0^2 \sum_{K \neq 0} K_\alpha K_\beta g_K \exp(-iK\cdot[u_{rt} - u_{rt'} + v(t-t')]), \tag{30}$$

where g_K is the Fourier coefficient of the correlator $g(r)$. Since g_K is essentially zero for $K \gg 1/r_f$, the error made in the above approximation is itself of order ∇u and thus consistent with the elastic limit approximation. This justifies the choice of Eq. (30) as the pinning force correlator in model II. The disorder term then posseses the statistical tilt symmetry (STS) $u_{rt} \to u_{rt} + f(r)$ where $f(r)$ is an arbitrary function. In this case one can absorb any *static* change in u without affecting the correlations of the pinning force. Finally note that in the case of isotropic elasticity, the additional inversion symmetry $y \to -y$ holds.

Though we study the complete model II in Secs. V and VIII B its main physics can be understood[73] by noticing that the pinning force $F_\alpha^{\rm pin}(r,t)$ in Eq. (27) naturally splits into a *static* and a time-dependent part:

$$F_\alpha^{\rm stat}(r,u) = V(r)\rho_0 \sum_{K.v=0} iK_\alpha \exp(iK\cdot(r-u)) - \rho_0 \nabla_\alpha V(r),$$

$$F_\alpha^{\rm dyn}(r,t,u) = V(r)\rho_0 \sum_{K.v\neq 0} iK_\alpha \exp(iK\cdot(r-vt-u)). \tag{31}$$

The static part of the pinning force comes from the modes such that $K\cdot v=0$ which exist for any direction of the velocity commensurate with the lattice. The maximum effect is obtained for v parallel to one principal lattice direction, the situation we study now. This force originates *only* from the periodicity along y and the uniform density modes along x, i.e., the smecticlike modes. Since this static pinning force $F_\alpha^{\rm stat}(r,u)$ is along the y direction, it is useful to consider only the transverse part (along y) of the equation of motion (26) dropping $F_\alpha^{\rm dyn}$. This leads to introducing model III, defined by the following equation of motion in the laboratory frame:

$$\eta \partial_t u_y + \eta v \partial_x u_y = c\nabla^2 u_y + F^{\rm stat}(r, u_y(r,t)) + \zeta_y(r,t),$$

$$F^{\rm stat}(x,y,u_y) = V(x,y)\rho_0 \sum_{Ky\neq 0} K_y \sin K_y(u_y - y) - \rho_0 \partial_y V(r). \tag{32}$$

Thus model III only involves the *transverse* displacements u_y. It posesses the same symmetries as model II with the three additional independent symmetries $y \to -y$, $u_y \to -u_y$, and $(x \to -x, v \to -v)$ and is also defined in the elastic limit. It is to be emphasized that although the derivation of model III was given here systematically starting from an elastic description the only serious hypothesis behind model III is the existence of transverse periodicity.[73,107,74] As discussed in Sec. II B Eq. (32) is the correct starting point to describe *any* kind of structure having such transverse periodicity properties. Thus model III is the generic equation containing the physics necessary to describe these structures.

V. PERTURBATION THEORY FOR THE COMPLETE TIME-DEPENDENT EQUATION

Let us start by a simple perturbation analysis of the equation of motion model II. Such a large velocity or weak disorder expansion has a long history in various contexts such as vortex lattices[71,70] and charge-density waves.[29] The natural idea is that at large velocity the disorder term oscillates rapidly and averages to a small value and that $1/v$ is a good expansion parameter. As we will see such an idea is in fact incorrect, albeit useful, since previously unnoticed divergences appear in the perturbation theory.

A. Analysis to first order

We start from the initial equation (26) defining model II that we rewrite as

$$(R^{-1})^{\alpha\beta}_{rtr't'}u^{\beta}_{r't'}=f_{\alpha}-\eta_{\alpha\beta}v_{\beta}+F_{\alpha}(r,t,u_{rt}),\qquad(33)$$

where from now on we drop the pin subscript on f. The response kernel R is defined in Fourier space:

$$(R^{-1})^{\alpha\beta}_{qt,q't'}=\delta_{tt'}\delta_{q',-q}[\eta_{\alpha\beta}\partial_t+i\eta_0 v_{\gamma}q_{\gamma}\delta_{\alpha\beta}+C_T(q)P^T_{\alpha\beta}(q)+C_L(q)P^L_{\alpha\beta}(q)],\quad(34)$$

where P^T and P^L are the standard transverse and longitudinal projectors and the elastic matrix is $C_T(q)=c_{66}q^2$, $C_L(q)=c_{11}q^2$ for a two-dimensional problem and $C_T(q)=c_{66}q^2+c_{44}q_z^2$, $C_T(q)=c_{11}q^2+c_{44}q_z^2$ for a three-dimensional problem. The bare value of the friction coefficient η_0 is defined as $\eta_{\alpha\beta}=\delta_{\alpha\beta}\eta_0$. In Eq. (33) the velocity is fixed by the constraint that $\langle u^{\beta}\rangle=0$ to all orders in perturbation theory. This is equivalent to enforce that the linear term in the effective action[108] is exactly zero. Instead of working directly with the equation of motion it is more convenient to use the de Dominicis-Janssen-Martin-Siggia-Rose formalism (MSR).[109] The generic MSR functional is given by

$$Z[h,\hat{h}]=\int DuD\hat{u}e^{-S[u,\hat{u}]+\hat{h}u+ih\hat{u}}\qquad(35)$$

where $\hat{h},h$ are source fields. The MSR action corresponding to the equation of motion (33) and the disorder correlator (30) is $S[u,\hat{u}]=S_0[u,\hat{u}]+S_{int}[u,\hat{u}]$ with

$$S_0[u,\hat{u}]=\int_{rtr't'}i\hat{u}^{\alpha}_{rt}(R^{-1})^{\alpha\beta}_{rt,r't'}u^{\beta}_{r't'}-\int_{rt}i\hat{u}^{\alpha}_{rt}(f_{\alpha}-\eta_{\alpha\beta}v_{\beta})-\eta T\int_{r,t}(i\hat{u}^{\alpha}_{rt})(i\hat{u}^{\alpha}_{rt}),\qquad(36)$$

$$S_{\rm int}[u,\hat{u}]=-\frac{1}{2}\int_{rtt'}(i\hat{u}^{\alpha}_{rt})(i\hat{u}^{\beta}_{rt'})\times\Delta^{\alpha\beta}[u_{rt}-u_{rt'}+v(t-t')].\qquad(37)$$

Note that Eq. (37) corresponds to the action derived in Ref. 106.

The fundamental functions to compute are the disorder-averaged displacements correlation function $C^{\alpha,\beta}_{rt,r't'}=\overline{\langle u^{\alpha}_{rt}u^{\beta}_{r't'}\rangle}$ and the response function $R^{\alpha,\beta}_{rt,r't'}=\delta\overline{\langle u^{\alpha}_{rt}\rangle}/\delta h^{\beta}_{r't'}$ which measures the linear response to a perturbation applied at a previous time. They are obtained from the above functional as $C^{\alpha\beta}_{rt,r't'}=\langle u^{\alpha}_{rt}u^{\beta}_{r't'}\rangle_S$ and $R^{\alpha\beta}_{rt,r't'}=\langle u^{\alpha}_{rt}i\hat{u}^{\beta}_{r't'}\rangle_S$, respectively. Causality imposes that $R_{rt,r't'}=0$ for $t'>t$ and we use the Ito prescription for time discretization which imposes that $R_{rt,r't}=0$. We assume here time and space translational invariance (for disorder averaged quantities) and denote indifferently $C_{rt,r't'}=C_{r-r',t-t'}$ and $R_{rt,r't'}=R_{r-r',t-t'}$ by the same symbol, as well as their Fourier transforms when no confusion is possible. Note that in this problem $C_{-r,t}\neq C_{r,t}$ when v is nonzero. In the absence of disorder the action is simply quadratic $S=S_0$. The response and correlation function in the absence of disorder are thus (for $t>0$) and introducing the mobility $\mu=1/\eta$:

$$R^{\alpha\beta}_{q,t}=P^L_{\alpha\beta}(q)\mu e^{-[c_L(q)+ivq_x]\mu t}\theta(t)+P^T_{\alpha\beta}(q)\mu e^{-[c_T(q)+ivq_x]\mu t}\theta(t),$$

$$C^{\alpha\beta}_{q,t}=P^L_{\alpha\beta}(q)\frac{T}{c_L(q)}e^{-[c_L(q)\mu|t|+ivq_x\mu t]}+P^T_{\alpha\beta}(q)\frac{T}{c_T(q)}e^{-[c_T(q)\mu|t|+ivq_x\mu t]}.\qquad(38)$$

Note that the fluctuation dissipation theorem (FDT) $TR^{\alpha\beta}_{r,t}=-\theta(t)\partial_t C^{\alpha\beta}_{r,t}$ does *not* hold here (it holds only for $v=0$ or in the absence of disorder), since we are studying a moving system which does not derive from a Hamiltonian. It is easy to show that the disorder does not produce any correction to the part $i\widehat{u_t}(cq^2+ivq_x)u_t$ of the action, and thus that the parameters c (c_{11} and c_{66}) and $\eta_0 v$ are not renormalized (we consider here for simplicity the isotropic version $c=c_{11}=c_{66}$ but this property holds in general). Thus here and in the following we often denote $\eta_0 v$ simply by v. This is similar to the property of nonrenormalization of connected correlations in the statics[93] (for $v=0$) due to the statistical tilt symmetry (STS). Here the exact relation $\ln Z[h,\hat{h}_t]=\int dt\hat{h}^q_t(cq^2+ivq_x)^{-1}h_{-q}+\ln Z[0,\hat{h}_t]$ where h is an arbitrary *static* field, holds because of the STS discussed in Sec. IV B (as can be seen from a simple change of variable). It implies that the static response function $\int dt' R_{q,t,t'}=(cq^2+ivq_x)^{-1}$ is not renormalized.

Let us now study the perturbation theory in the disorder and compute the effective action $\Gamma[u,\hat{u}]$ to lowest order in the interacting part $S_{\rm int}$, using a standard cumulant expansion

$$\Gamma[u,\hat{u}]=S_0[u,\hat{u}]+\langle S_{\rm int}[u+\delta u,\hat{u}+\delta\hat{u}]\rangle_{\delta u,\delta\hat{u}},\qquad(39)$$

where the averages in Eq. (39) over $\delta u,\delta\hat{u}$ are taken with respect to S_0. The calculations are performed in Appendix A. One finds that the effective action has the same form as the bare action, up to irrelevant higher-order derivative terms, with the following modifications. First the full nonlinear form of the correlator of the pinning force is corrected by thermal fluctuations $\Delta^{\alpha\beta}_K\to\tilde{\Delta}^{\alpha\beta}_K$. In $d>2$ it reads

$$\tilde{\Delta}^{\alpha\beta}_K=\Delta^{\alpha\beta}_K e^{-(1/2)K^2B_{\infty}}\qquad(40)$$

or equivalently $\tilde{g}_K=g_K e^{-(1/2)K^2B_{\infty}}$ where $B_{\infty}\sim\langle u^2\rangle_{\rm thermal}$. We have defined $B^{\alpha\beta}_{r,t}=2(C^{\alpha\beta}_{0,0}-C^{\alpha\beta}_{r,t})$. This amounts to a smoothing out of the disorder by thermal fluctuations. Secondly, the friction coefficient matrix is corrected by $\delta\eta_{\alpha\beta}$, and the temperature by δT. Finally, the driving force is corrected by δf (we are working at fixed velocity, enforcing order by order that $f+\delta f=\eta v$). Let us start by the corrections to the driving force. We find

$$\delta f_\alpha(v) = -\sum_K \sum_{I=L,T} \int_{q,\mathrm{BZ}} K_\alpha (K\cdot P^I(q)\cdot K) g_K \times \frac{v\cdot(K+q)}{c_I(q)^2 + [\eta_0 v\cdot(K+q)]^2}. \quad (41)$$

This formula gives the lowest correction to the driving force at fixed velocity or, equivalently to the velocity at fixed driving force. It is identical to the formula (22) of Schmidt and Hauger.[71] There are small differences, unimportant in the elastic limit, which come from the different definitions of the continuum limit of the model (see discussion in Sec. IV A). A salient feature of the above formula was noticed by Schmidt and Hauger, i.e., the velocity and the force are not, in general, aligned. They are aligned however when the velocity is along one of the principal lattice directions, i.e., $K_0\cdot v = 0$, where K_0 is one of the principal reciprocal-lattice vectors (note that this is also the case for the median direction $\pi/6$). Such a feature is reasonable on physical grounds and can be confirmed by higher-order analysis of the perturbation theory (see Sec. VI). Furthermore, using the approximation $v(K+q)\sim vK$ Schmidt and Hauger found that, in $d=2$, the transverse pinning force versus the angle α between the velocity and one principal direction of the lattice has a discontinuity at $\alpha=0$. One could naively think that such a discontinuity could be interpreted as the existence of a transverse critical force. Indeed a natural interpretation of Fig. 1 of Ref. 71 would be that one needs to apply a finite force to the lattice (opposite to the transverse pinning force) to tilt sightly its velocity from the principal axis direction. Notable confusion on this subject existed in the literature.[71,70] Such an interpretation is in fact incorrect. First, as Schmidt and Hauger correctly pointed out such a discontinuity is an *artefact* of the approximation $v(K+q)\sim vK$, and disappears if the correct expression (41) is used. Furthermore it is easy to check that even with the above approximation the discontinuity exists only in $d=2$ and the function is continuous for $d>2$. Thus the first-order perturbation does not exhibit any divergence and *does not* give rise to a transverse critical current. In order to have divergences in the perturbation theory (and the associated effects) it is thus necessary to examine the perturbation theory to *second order*. We perform such a calculation in Sec. VI.

Before we do so it is interesting to examine the first-order corrections to the friction coefficient and the temperature. At $T=0$ and using the bare form of the disorder one finds

$$\delta\eta_{\alpha\beta} = \sum_K \sum_{I=L,T} \times \int_{q,\mathrm{BZ}} K_\alpha K_\beta (K\cdot P^I(q)\cdot K) g_K \times \frac{1}{[c_I(q) + i\eta_0 v\cdot(K+q)]^2}. \quad (42)$$

More general expressions are given in Appendix A, Eq. (A30). When the velocity is along a principal lattice direction one finds that $\delta\eta_{xy}=0$ and thus the friction matrix remains diagonal. However the corrections to the friction is clearly not the same along x and y. Next we give the corrections to temperature:

$$\delta(\eta T)_{\alpha\beta} = \frac{1}{2}\sum_K \int_t \Delta_K^{\alpha\delta} e^{-iK\cdot vt} \times (e^{-(1/2)K\cdot B_{0,t}\cdot K} - e^{-(1/2)K\cdot B_\infty\cdot K}). \quad (43)$$

Contrary to the velocity corrections, corrections to the temperature (43) exhibit divergences for any v already at the *first* order in the disorder. However these divergences are well hidden and can *only* appear if one looks at the *nonzero temperature* perturbation theory, which was *not done* in Refs. 70, 71. At $T=0$ one finds trivially that $\delta T=0$, showing that disorder alone cannot generate a finite temperature. Such corrections are thus nontrivial and there is in general no simple relation between $\delta(\eta T)_{\alpha\beta}$ and $\delta(\eta)_{\alpha\beta}$, due to the absence of FDT theorem. Only in the particular case where $v=0$ and of potential random forces $\Delta_K^{\alpha\beta} = K_\alpha K_\beta g_K$ the FDT theorem enforces $\delta T=0$ (see, e.g., Ref. 94 and below). The way to treat these divergences is examined together with the second order in perturbation in Sec. VI.

B. Correlation functions

The last physical information that can be extracted from the perturbation theory is about the correlation functions. The calculation is performed in Appendix A and the result at $T=0$ is given in Eq. (A33) and at $T>0$ in Eq. (A33) (at $T>0$). The static component is

$$\overline{\langle u^\alpha_{-q,t} u^\beta_{q,t}\rangle} = \sum_{K,K\cdot v=0} \sum_{I=L,T,I'=L,T} \times \int_{q,BZ} g_K \frac{P^I_{\alpha\gamma}(q)}{c_I(q) + i\eta_0 v\cdot q} \times \frac{P^{I'}_{\beta\delta}(q)}{c'_I(q) - i\eta_0 v\cdot q} D_{\gamma\delta}, \quad (44)$$

where the disorder is smoothed by the temperature as: $g_K D_{\gamma\delta} = g_K K_\gamma K_\delta e^{-(1/2)K\cdot B_{0,\infty}\cdot K}$. We now focus on the $T=0$ limit of Eq. (44). To lowest order in perturbation theory the pinning force is only along y, and corresponds to a random force of strength Δ_{yy}. We now compute the mean-squared displacements along x and y produced by this random force:

$$B_{yy} \approx \Delta_{yy} \int_{q,\mathrm{BZ}} [1-\cos(qr)] \times \left[\frac{q_x^4}{q_\perp^4\{v^2q_x^2 + [c_{66}(q_x^2+q_y^2) + c_{44}q_z^2]^2\}} + \frac{q_y^4}{q_\perp^4\{v^2q_x^2 + [c_{11}(q_x^2+q_y^2) + c_{44}q_z^2]^2\}} \right],$$

$$B_{xx}\approx\Delta_{yy}\int_{q,\mathrm{BZ}}[1-\cos(qr)]\times\left[\frac{q_x^2q_y^2}{q_\perp^4\{v^2q_x^2+[c_{66}(q_x^2+q_y^2)+c_{44}q_z^2]^2\}}+\frac{q_x^2q_y^2}{q_\perp^4\{v^2q_x^2+[c_{11}(q_x^2+q_y^2)+c_{44}q_z^2]^2\}}\right], \tag{45}$$

where $q_\perp^2=q_x^2+q_y^2$. Since $q_x\sim q_y^2$ one immediately sees from Eq. (45) that the compression modes are the ones responsible for displacements growing unboundedly in $d\leqslant 3$. The expression for B_{yy} allows one to estimate the dynamical Larkin length for transverse pinning, as discussed in Sec. III B.

On the other hand, in order to obtain a decoupling of the channels one can use a simple Lindemann criterion (19). We use Eq. (45) for B_{xx} where we neglect all terms containing c_{11} (i.e., the compression modes), assuming as is reasonable for most systems that c_{11} is very large. The decoupling between the channels is thus controlled by c_{66} whereas the roughness of the channels and the characteristics length scales of the moving glass directly depends on the compression mode c_{11} (at large velocities). Estimating the integral one finds in $d=3$:

$$B_{xx}(y=a)\sim\frac{\Delta_{yy}}{a^2c_{44}^{1/2}}\min\left[\left(\frac{a^2}{c_{66}}\right)^{3/2},(a/v)^{3/2}\right]. \tag{46}$$

This gives back the Bragg glass estimate (in the simpler case $a=r_f$). The effect of thermal fluctuation can also be added as in Ref. 47. The above perturbation expansion for the Lindemann criterion implicitly supposes that the random force is directed along the y direction. In fact, under renormalization a random force along x is of strength Δ_{xx} is also generated as discussed in Sec. III. The resulting expression for B_{xx} is identical to the one of B_{yy} in Eq. (45) with $\Delta_{yy}\to\Delta_{xx}$ and c_{11} and c_{66} interchanged. Thus that quantity is determined mostly by the shear modes. If one uses again the Lindemann criterion with a random force along x it would contribute, provided c_{66}/c_{11} is small enough to compensate for the fact that $\Delta_{xx}\ll\Delta_{yy}$. This can be quantified using the estimates given in Sec. VIII A for Δ_{xx}.

VI. RENORMALIZATION-GROUP STUDY OF THE TRANSVERSE PHYSICS (MOVING GLASS)

Up to now, we have studied the perturbation expansion of the full continuous model II, keeping both the x and y directions. Doing the second-order perturbation on that full model is tedious. Since one knows on physical grounds that the singularities in the perturbation theory comes from the *static components* of the disorder,[73] which as was discussed in Sec. II B originates from the transverse degrees of freedom, we now study the perturbation theory of the simplified *transverse equation of motion*, model III. If, as we indeed find, this perturbation theory is singular, this implies divergences in the full model II as well. We thus study it here and come back to the full model II in Sec. VIII A.

A. Zero-temperature perturbation theory to second order

To avoid cumbersome expressions, and since in this whole section we only discuss transverse degrees of freedom we skip the index y for u_y. We also discuss here for simplicity an $n=1$ component model for the transverse displacement u (which is appropriate for flux lines in $d=3$ and point vortices in $d=2$). Generalizations to $n>1$ are briefly mentioned in Appendix B.

We thus study the dynamical equation for model III.[73] Since we are dealing with an anisotropic fixed point it is useful to distinguish c_x and c_y:

$$\eta\partial_t u_{rt}=(c_x\nabla_x^2+c_y\nabla_y^2-v\,\partial_x)u_{rt}+F(r,u_{rt})+\zeta(r,t). \tag{47}$$

The bare value of the friction coefficient along y is denoted by η_0, and for simplicity we denote by v the quantity $\eta_0 v$ (which remains uncorrected to all orders in this model). The correlator of the static transverse pinning force (32) is $\overline{F(r,u)F(r',u')}=\Delta(u-u')\,\delta^d(r-r')$. Averages over solutions of Eq. (47) can be performed using the Martin-Siggia-Rose (MSR) action (35) with

$$S_0[u,\hat{u}]=\int_{rt}i\hat{u}_{rt}(\eta\partial_t+v\,\partial_x-c_x\nabla_x^2-c_y\nabla_y^2)u_{rt}-\eta T(i\hat{u}_{rt})(i\hat{u}_{rt}),$$

$$S_{\mathrm{int}}=-\frac{1}{2}\int_{rtt'}(i\hat{u}_{rt})(i\hat{u}_{rt'})\Delta(u_{rt}-u_{rt'}). \tag{48}$$

In this section we restrict ourselves to $T=0$. The corrections coming from correlation functions $\langle\delta u_t\delta u_{t'}\rangle\sim T$ then vanish, which simplify the analysis. This can be used to show that to all orders the temperature remains zero. The first-order corrections where computed in Sec. V and Appendix A. At $T=0$ there is no correction to the disorder term (to this order). There is a nontrivial correction to the kinetic term, which gives the following correction to the friction coefficient η, $\delta\eta=-\Delta''(0)\int_q\int_0^{+\infty}dttR(q,t)$. This leads to

$$\frac{\delta\eta}{\eta}=-\Delta''(0)\int\frac{d^{d-1}q_y}{(2\pi)^{d-1}}\frac{2c_x}{(4c_xc_yq_y^2+v^2)^{3/2}}. \tag{49}$$

This is *not* a divergent integral, except when $v=0$. To find divergences in the perturbation theory at $T=0$ one has to go to second order.

The second-order corrections to the effective MSR action, and thus to the coarse-grained equation of motion, are computed in Appendix B. To second order a correction to the full nonlinear disorder correlator appears and reads

$$\delta\Delta(u)=\Delta''(u)[\Delta(0)-\Delta(u)]\int_r G(r)G(r)-\Delta'(u)^2\int_r G(r)G(-r), \tag{50}$$

where $G(r)$ is the static response function

$$G(r)=\int_0^\infty dt R(r,t), \quad G(q)=\frac{1}{c_x q_x^2+c_y q_y^2+ivq_x}. \tag{51}$$

At zero velocity both terms in Eq. (50) are infrared divergent for $d\leqslant 4$, as is well known leading to the glassy effects in the statics. The key novelty with respect to the problem at $v=0$ is that due to the assymetry introduced by motion, $G(r)$ is different from $G(-r)$. As a consequence the second term in Eq. (50) is now *convergent* for $v>0$. Indeed the integral

$$\int_r G(r)G(-r)=\int_q G(q)^2=\int \frac{d^{d-1}q_y}{(2\pi)^{d-1}}\frac{2c_x}{(4c_xc_yq_y^2+v^2)^{3/2}} \tag{52}$$

is *convergent* in all dimensions for $v>0$. On the other hand, one divergence remains from the first term:

$$\int_r G(r)G(r)=\int_q G(q)G(-q)$$
$$=\int \frac{d^{d-1}q_y}{(2\pi)^{d-1}}\frac{1}{2c_yq_y^2\sqrt{4c_xc_yq_y^2+v^2}}$$
$$\sim\int \frac{d^{d-1}q_y}{(2\pi)^{d-1}}\frac{1}{2vc_yq_y^2}. \tag{53}$$

The integral (53) is divergent for $d\leqslant 3$, even for $v>0$. Thus, contrary to general belief[70–72] originating mainly from the study of the first-order perturbation in $1/v$, analysis to second-order confirms the surprising conclusion that even at *large* velocity infrared divergences occur in the perturbation theory.[73,110] Such divergences indicate the instability of the zero disorder fixed point and the breakdown of the large v expansion. They lead the system to a fixed point where the disorder plays a crucial role. The above divergence is the key to the physics of the moving glass.

B. Renormalization-group study at zero temperature

In order to handle these new divergences, and to find the fixed point which describes the large scale physics, we use a dynamical functional renormalization-group (DFRG) procedure on the effective action using a Wilson scheme. This allows us to keep track of the full function $\Delta(u)$, which is necessary since the full function is marginal at the upper critical dimension. This is equivalent to decomposing the fields into fast and slow components $u\to u+\delta u$ and $\hat{u}\to\hat{u}+\delta\hat{u}$ and to integrate the fast fields δu and $\delta\hat{u}$ over a momentum shell. This method is very similar to the method introduced in Refs. 26, 27 for the $v=0$ case, though it differs in details. An alternative RG method of mode elimination by hand yielding the same results is given in Ref. 106. It shows in a direct way how nonpotential disorder forces are generated.

1. Derivation of the RG equations

The first task is to perform a dimensional analysis of the MSR action and to determine the appropriate rescaling transformation. Since we want to describe both the $v=0$ and $v>0$ fixed points, we perform the following redefinition of space, time, and the fields (keeping also an arbitrary T) $y=y'e^l$ $x=x'e^{\sigma l}$, $t=t'e^{zl}$, $\hat{u}=\hat{u}'e^{\alpha l}$, $u=u'e^{\zeta l}$. We now impose that the action S in Eq. (48) is unchanged, which yields redefinitions of the coefficients. Since $c_y=c_y'$ since this quantity remains uncorrected to all orders, this fixes $\alpha=3-d-\sigma-z-\zeta$. One finds the rescaling:

$$\eta\to\eta'=\eta e^{(2-z)l}, \quad v\to v'=ve^{(2-\sigma)l},$$
$$c_x'=c_xe^{(2-2\sigma)l}, \quad T\to T'=Te^{(3-d-\sigma-2\zeta)l},$$
$$\Delta\to\Delta'=\Delta e^{(5-d-\sigma-2\zeta)l}. \tag{54}$$

In the case $v=0$ the natural choice is $\sigma=1$, which yields $\Delta'=\Delta e^{(4-d-2\zeta)l}$. Power counting at the Gaussian fixed point $(z=2,\ \zeta=0)$ yields the upper critical dimension $d_{\rm uc}=4$ below which disorder is relevant (and a $d=4-\epsilon$ expansion can be performed).[26,27] For $v>0$ since v is uncorrected to all orders a natural choice is $\sigma=2$. Power counting near the Gaussian fixed point $(z=2,\ \zeta=0)$ indicates that now the upper critical dimension is thus $d_{\rm uc}=3$ (with $\Delta\to\Delta'=\Delta e^{(3-d-2\zeta)l}$). As a consequence disorder terms are relevant for dimensions $d\leqslant 3$, whereas the temperature appears to be formally irrelevant (see however Sec. VI C). The elasticity term along x (c_x) now corresponds to an irrelevant operator at the anisotropic fixed point. Note that the above rescaling (54) indicates that the proper dimensionless disorder parameter is $\Delta/v\Lambda^{d-3}$, where Λ is the momentum cutoff. Here we are mostly interested in periodic systems for which, as in the statics,[45] one must set $\zeta=0$. For completeness we give however the equations for nonperiodic systems $(\zeta\geqslant 0)$.

The standard RG method consists of two steps. First, one integrates the modes between $a<y<ae^l$ or equivalently $\Lambda_0>q_y>\Lambda_0 e^{-l}$ with $\Lambda_0\sim\pi/a$, which yields corrections to the bare quantities. The cutoff procedure we choose here for convenience is to integrate over the following momentum shell:

$$\int_{sh}dq=\int_{\Lambda e^{-l}}^{\Lambda}\frac{d^{d-1}q_y}{(2\pi)^{d-1}}\int_{-\infty}^{+\infty}\frac{dq_x}{2\pi}. \tag{55}$$

This results in the same theory but with a different cutoff and corrected parameter. Second, one performs the length, time, and field rescaling (54), as well as the corresponding change of quantities (54), so as to leave the effective action invariant. The cutoff has thus been brought back to its original value. Scale-invariant theories thus correspond to fixed points of this combined transformation. Using the above, the RG equation for the disorder can be established. The shell contribution of the integral (53) is asymptotically:

$$\int_r \delta G(r)\delta G(r)\sim\int_{sh}dq_y\frac{1}{2vc_yq_y^2}\sim\frac{1}{2vc_y}A_{d-1}\Lambda^{d-3}, \tag{56}$$

where $A_d=S_d/(2\pi)^d$ and S_d is the surface of the d-dimensional sphere. In $d=3$ one has $A_2=1/(2\pi)$. Using Eqs. (50), (52), (56) we obtain after rescaling the following FRG equation for the disorder correlator:

$$\frac{d\Delta(u)}{dl}=(3-d-2\zeta)\Delta(u)+\zeta u\Delta'(u)-\Delta'(u)^2\frac{1}{2\pi}\frac{2c_x(l)\Lambda_0^2}{[4c_x(l)c_y\Lambda_0^2+v^2]^{3/2}}+\frac{1}{4\pi v c_y\sqrt{1+4c_x(l)c_y\Lambda_0^2/v^2}}\times\Delta''(u)[\Delta(0)-\Delta(u)] \quad (57)$$

where $c_x(l)=c_x e^{-2l}$. In the large scale limit it reduces to[74,35]

$$\frac{d\Delta(u)}{dl}=(3-d-2\zeta)\Delta(u)+\zeta u\Delta'(u)+\frac{1}{4\pi v c_y}\Delta''(u)\times[\Delta(0)-\Delta(u)]. \quad (58)$$

Equation (57) allows us, in principle, to examine the intermediate scales crossover when v is not very large. Indeed there is a characteristic crossover length scale $L_{\rm cross}=2\sqrt{c_x c_y}/(\eta_0 v)$ such that Eq. (58) becomes valid for $e^l \gg L_{\rm cross}/a$. Note that setting $v=0$ in the above equation (57) leads back the FRG equation for the usual manifold depinning[26,27] (up to numerical factors originating from choice of short distance cutoff, and different choices for rescalings). The RG equation for the friction coefficient [e.g., for a periodic problem ($\zeta=0$)] can be obtained. Using Eq. (49) and taking into account that $\Lambda=\Lambda_0 e^{-l}$ and $c_x(l)=c_x e^{-2l}$, and $\Delta_l''(0)=\Delta''(0)e^{(3-d)l}$ one finds the RG equation, after rescaling:

$$\frac{d\eta}{\eta dl}=2-z-\Delta_l''(0)A_{d-1}\frac{2c_x(l)\Lambda_0^{d-1}}{[4c_x(l)c_y\Lambda_0^2+v^2]^{3/2}} \quad (59)$$

thus except for $v=0$, η is corrected only by a finite amount as long as $\Delta_l''(0)$ is finite (see below).

2. Study of the FRG equation

We now study the FRG equation (58) for the periodic problem ($\zeta=0$). Thus we impose $\Delta(u)$ to be periodic of period 1 and study the interval [0,1]. One can easily restore the period a in the solution. Let us look for a perturbative fixed point in $d=3-\epsilon$. Absorbing the factor $1/4\pi v c_y \epsilon$ in $\Delta(u)$ and redefining temporarily $\epsilon l \to l$, the FRG equation reads[111]

$$\frac{d\Delta(u)}{dl}=\Delta(u)+\Delta''(u)[\Delta(0)-\Delta(u)]. \quad (60)$$

No continuous solutions such that $d\Delta(u)/dl=0$ exist.[112] This is due to the fact that the average value of $\Delta(u)$ on the interval [0,1] must increase unboundedly. Indeed integrating inside the interval one finds $(d/dl)\int\Delta(u)=\int\Delta(u)+\int\Delta'(u)^2$. It is thus natural to define $\overline{\Delta}(u)=\Delta(0)-\Delta(u)$ which satisfies

$$\frac{\overline{\Delta}(u)}{dl}=\overline{\Delta}(u)[1+\overline{\Delta}''(u)]. \quad (61)$$

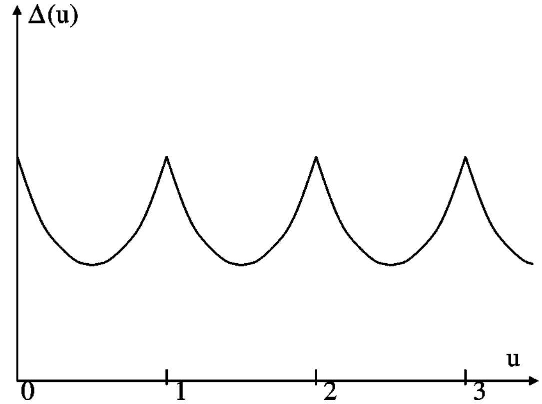

FIG. 21. Solution of the FRG equation. Note the nonanalyticity at all integers.

Note that physically one expects $\overline{\Delta}(u)\geq 0$. This equation has a fixed point $\overline{\Delta}^*(u)=u(1-u)/2$. It is shown in Appendix D that this fixed point is stable (locally attractive). Equation (60) shows that $\Delta(0)(l)$ grows unboundedly as $\Delta(0)(l)=\Delta(0)e^{\epsilon l}$ (restoring the ϵ factor). Thus the full fixed-point solution in a $d=3-\epsilon$ expansion is[74,35]

$$\Delta_l(u)=\Delta^*(u)-\Delta^*(0)+Ce^{\epsilon l},\quad \Delta^*(u)''=1, \quad (62)$$

where C is an arbitrary constant and

$$\Delta^*(u)=C^*+(\epsilon 4\pi v c_y)\left(\frac{1}{2}u^2-\frac{1}{2}u\right)\quad (0\leq u\leq 1) \quad (63)$$

and the solution repeats periodically as shown in Fig. 21. We have restored the factor $1/(4\pi v c_y)$ and $\epsilon=3-d$. In K space the fixed-point solution can be written $\Delta_K=1/K^2$ for $K\neq 0$ ($K=2\pi k$ with k integers) and $\Delta_{K=0}(l)=\Delta_l(u=0)+(1/12)=\Delta_0(u=0)e^l+(1/12)$.

Thus there is an ever growing average to the correlator. Remarkably, this *does not spoil* the above fixed point, since one can always separate $\Delta(0)$ and $\Delta(u)-\Delta(0)$ in the starting MSR action. In perturbation theory one sees that $\Delta(0)$ has no feedback at all into the nonlinear part. It simply means that there is an unrenormalized random force which simply adds to a nonlinear pinning force, which is described by $\Delta^*(u)$. Note that this solution has cusp nonanalyticity at all integers u. At the initial stages of the RG $\Delta''(0)$ is negative [since $\Delta(u)$ is an analytic function with a maximum at $u=0$]. However one easily sees that $\Delta_l''(0)$ becomes infinite at a finite length scale (interpreted as the dynamical Larkin length) see Sec. VI B 3, the function becomes nonanalytic and $\Delta''(0+)$ becomes positive.

Once the solution is known in $d=3-\epsilon$ it is straightforward to obtain it in the physically relevant dimension $d=3$. In $d=3$ defining $\Delta_l(u)=(1/l)\hat{\Delta}_l(u)$ and introducing $l'=\ln l$ one finds that $\hat{\Delta}_{l'}(u)$ satisfies again Eq. (60). Thus the physics is controlled by the slow decrease to zero of disorder at large scale with the following stable fixed-point behavior:

$$\Delta_l(u)\sim\Delta(0)+\frac{4\pi v c_y}{2l}(u^2-u). \quad (64)$$

The random force term does not grow by rescaling in $d=3$.

Finally, let us point out that Eq. (58) presents several differences and some remarkable similarities with the one describing the statics FRG and the dynamical FRG for $v=0$ in a $d=4-\epsilon$ expansion. Let us call $R(u)$ the correlator of the random potential. The statics FRG equation and its periodic fixed point was given in Ref. 45 (see Eqs. (5.2) and (5.5) with Δ denoting R). The dynamic FRG equation for $(v=0)$ is

$$\frac{d\Delta(u)}{dl}=\Delta(u)+\Delta''(u)[\Delta(0)-\Delta(u)]-\Delta'(u)^2. \quad (65)$$

Since one has $\Delta(u)=-R''(u)$ it yields the solution periodic in $[0,1]$ (Ref. 45) $\Delta^*(u)=\frac{1}{36}(1-6u+6u^2)$. Remarkably both the solution for $v=0$ and for $v>0$ are nonanalytic at integer u, though the detailed form of these solutions is different. Since this nonanalyticity is related to glassiness and pinning one can expect a certain continuity of properties between the moving and nonmoving case. The main difference however is that in Eq. (65) $\Delta(0)=\Delta(0^+)$ starts growing at the initial RG stages as for $v>0$ but is stopped at its fixed-point value $\frac{1}{36}$ beyond the scale at which a nonanalycity develops (Larkin length). This effect is due to the term $-\Delta'(0+)^2$ and physically means that in the case $v=0$ displacements grow much more slowly at larger scales. The system remembers that it is a potential system and thus $\int\Delta(u)$ remains zero if it is zero at the start (at least formally, see however Ref. 27). Thus no random force can be generated. By contrast for the moving system one has asymptotically $\Delta_l(0)\sim\Delta_\infty e^{\epsilon l}$. As is discussed later this corresponds to the generation of a random force (which cannot exist in the statics).

Note that for v not very large one can see in Eq. (57) that there is a long crossover during which the term $-\Delta'(0+)^2$ acts. This is the static random manifold regime as depicted in Fig. 12. Thus the actual value of Δ_∞ should be decreased compared to the value naively suggested by perturbation theory, an effect studied in the next section.

3. Physical results at $T=0$

We now extract some of the physics of the moving glass from the FRG analysis. From the equation for the second derivative of the force correlator:

$$\frac{d\Delta''(0)}{dl}=(3-d)\Delta''(0)-C(l)\Delta''(0)^2 \quad (66)$$

with $1/C(l)=4\pi v c_y\sqrt{1+4c_x e^{-2l}c_y\Lambda_0^2/v^2}$ it is possible to extract the length scale R_c^y at which $\Delta''(0)$ becomes infinite. We first estimate it in the large velocity regime $L_{\rm cross}\ll a$ where one can set $C(l)=1/(4\pi v c_y)$. In $d=3$ one has $\Delta_l''(0)=-\Delta_2/(1-\Delta_2 l/4\pi v c_y)$. where $\Delta_l''(0)=-\Delta_2$ is the bare value. Thus $R_c^y=ae^{4\pi\eta_0 v c_y/\Delta_2}$. This length scale, introduced in Refs. 73, 35, and discussed in Sec. III B is analogous of the Larkin length for the statics. Indeed R_c^y coincides with the scale at which the scale-dependent mobility $\mu(L)$ vanishes as depicted in Fig. 22. This can be seen from Eq. (59). The divergence of $\Delta_l''(0)$ at $L=e^l=R_c^y$ drives $\mu(L)=1/\eta(L)$ to zero for all larger scales. Beyond that scale pinning starts to play a role.

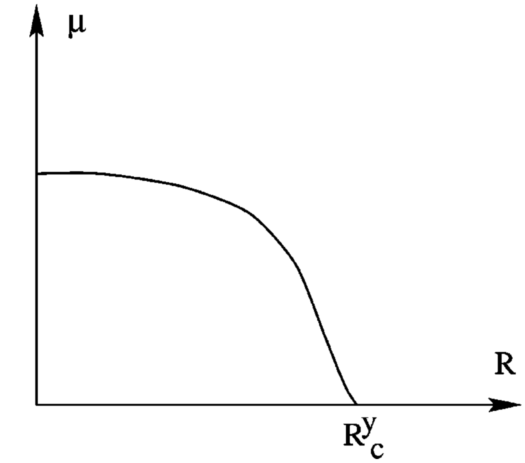

FIG. 22. Scale-dependent mobility. It vanishes beyond the dynamical Larkin length (at $T=0$).

In $d<3$ one has $\Delta_l''(0)=-\Delta_2 e^{\epsilon l}/(1-\Delta_2(e^{\epsilon l}-1)/4\pi v c_y\epsilon)$. Thus one obtains the dynamical Larkin length as given in Eq. (9) where we have restored the proper a and η dependence (with $c\equiv c_y$). Note that it is the second derivative Δ_2 of the force correlator which appears in the Larkin length. For a realistic disorder with a correlation length r_f one has $\Delta_2\sim\Delta(0)/r_f^2$. Using this relation, one checks that Eq. (9) is the one obtained[73] by estimating the length scale at which $u_{\rm dis}\sim r_f$.

One can also determine the dynamical Larkin length when the velocity is not very large. Restoring the proper dependence of $C(l)$ in l in Eq. (66) gives that $1/\Delta_2=\int_0^{l_c}e^{\epsilon l}C(l)$ with $l_c=\ln(R_c^y/a)$. This yields after some algebra in $d=3$:

$$R_c^y=ae^{4\pi\eta_0 v c_y/\Delta_2}\frac{1}{2}\left(1+e^{-8\pi\eta_0 v c_y/\Delta_2}+(1-e^{-8\pi\eta_0 v c_y/\Delta_2})\sqrt{1+\frac{4c_x c_y\Lambda_0^2}{(\eta_0 v)^2}}\right) \quad (67)$$

and in $d=2$ one finds

$$(R_c^y)^2=a^2+\left(\frac{4\pi\eta_0 v c_y}{\Delta_2}\right)^2+a\frac{8\pi\eta_0 v c_y}{\Delta_2}\sqrt{1+\frac{4c_x c_y\Lambda_0^2}{(\eta_0 v)^2}}, \quad (68)$$

where we recall $\Lambda_0\sim\pi/a$. These formulas interpolate smoothly between the Bragg glass and moving glass results. Finally, note that since the above equations are exact (within the $3-\epsilon$ FRG approach) the calculations of the Larkin lengths are independent of whether there is an intermediate random manifold regime, i.e., it holds both for $r_f\ll a$ and $r_f\sim a$. Nonuniversal irrelevant operators of course change the numerical values of the prefactors but the above expressions should be correct when all the Larkin lengths are large.

One of the remarkable properties of the moving state is the existence of transverse pinning.[73] This is demonstrated from the FRG fixed point, due to the *nonanalyticity* of the fixed point Eq. (63). Adding an external force f_y along y

[i.e., in the left hand side of Eq. (47)] generates a velocity v_y. The naive perturbation theory results, formula (A30), for $\delta f_y(v_y)$ (the correction to the applied force at fixed v_y) reads $\delta f(v_y) = \int_t R_{r=0,t}\Delta'(v_y t)$. In the limit of vanishingly small v_y one gets a nonzero limit $\delta f_y(0^+) = -F_c^y$, i.e., a transverse critical force only if the function is nonanalytic with $\Delta'(0^+) < 0$. The critical force is thus given by summing up the contributions of all the successive shells

$$F_c \approx -\int_{\ln R_c^y}^{+\infty} dl \Delta_l'(0^+) A_{d-1} \frac{\Lambda_0^{d-1} e^{-(d-1)l}}{\sqrt{4c_x(l)c_y\Lambda_0^2 + v^2}} e^{-(3-d)l}, \tag{69}$$

where quite logically only scales beyond the Larkin length give a nonvanishing contribution. Using the asymptotic value for $\Delta'(0^+)^* = \epsilon 4\pi \eta_0 v c_y$ one finds

$$F_c^y \approx C c_y a (R_c^y)^{-2} \tag{70}$$

and in $d=3$:

$$F_c^y \approx C' \frac{c_y a}{(R_c^y)^2 \ln(R_c^y/a)}. \tag{71}$$

In Eqs. (70) and (71) the prefactors C and C' are not universal (and in $d=3$ the additional logarithm correction will also be affected by higher orders in perturbation theory). In the above formulas we have assumed a direct passage from the Larkin scale regime to the asymptotic periodic regime, thus $r_f \sim a$. If $r_f \ll a$ an intermediate random manifold regime is first reached where the typical value for $\Delta_l'(0^+)$ is rather a/r_f. This yields to the replacement of a by r_f in the numerators of Eqs. (70), (71). Remarkably this coincides with the estimate obtained[73] by balancing forces. Note that this result can be obtained here *without any reference* to a Larkin length along the x direction. This illustrates that the physics of the moving glass depends only on the periodicity along the y direction. Note that the full v-f characteristics can also be computed from the FRG using Eq. (A30) (see Ref. 106).

It is interesting also to study the behavior near the transverse depinning threshold. Because of the absence of IR divergence in the integral for η Eq. (59) the exponent at the threshold remains uncorrected, i.e., $v_y \sim |f_y - F_c^y|^\theta$ with $\theta = 1$ (to first order in ϵ). The slope can easily be estimated from $\Delta_l''(0)$ and the above results and becomes large at small velocities as shown in Fig. 10.

We now study the displacement correlations. The growth of the average of $\int_0^1 \Delta(u)$ implies that there is a static random force generated. However, unlike in the $v=0$ case, the critical force does not kill the random force in the FRG equation. In fact the moving glass is dominated by the *competition* between the random force and the critical force. Although the existence of such a random force has no effect on pinning it affects strongly the positional correlation functions. In particular the relative displacements correlation function (2) becomes, in the presence of the random force at large scale, identical to Eqs. (10) and (14) with a renormalized disorder $\Delta_{\text{ren}}(0)$ (as discussed in Sec. III C):

$$\Delta_{\text{ren}}(0) = \Delta(0) - \int_0^{+\infty} dl e^{-\epsilon l} \Delta_l'(0^+)^2 \times \frac{1}{2\pi} \frac{2c_x e^{-2l}\Lambda_0^2}{(4c_x e^{-2l} c_y \Lambda_0^2 + v^2)^{3/2}}, \tag{72}$$

and one can simply set $\epsilon = 0$ to get the result in $d=3$.

C. RG study at finite temperature $T>0$

We now extend the analysis to finite temperature. In principle the FRG equations can also be written for any temperature. We study both the case $v=0$ and $v>0$ (since no such derivation exist in the litterature). In the $v=0$ case the temperature is formally irrelevant. In fact it is dangerously so (see below) as it cuts off the properties of the fixed point (the nonanalyticity) and thus modifies some observables leading to barrier determination. Here as we see the temperature for $v>0$ is even more so, very dangerously irrelevant by power counting. The dimensional rescaling Eq. (54) yields $T \to T' = T e^{(1-d-2\zeta)l}$ and thus that T is irrelevant. This turns out to be incorrect: if one adds a small $T>0$ onto the $T=0$ moving glass fixed point, FRG indicates that it flows upwards very fast (while if $T=0$ to start with, it remains so).

1. Derivation of the RG equations

Letting $T>0$ leads to several important modifications in the perturbation theory. The general idea is that the disorder is changed everywhere roughly as $\Delta_K e^{-(1/2)K \cdot B_{0,t} \cdot K}$ (with $t \to \infty$). Of course this has to be checked carefully which is done in detail in Appendix B and we give here only the main results. Near the upper critical dimension ($d_u = 4$ for $v=0$ and $d_u = 3$ for $v>0$) thermal displacements are bounded:

$$\lim_{t\to\infty} B_{0,t} = B_\infty = 2T \int_q \frac{1}{cq^2} = \frac{2T}{c} A_d \frac{\Lambda^{d-2}}{d-2} \quad (v=0)$$

$$= 2T \int_q \frac{1}{c_x q_x^2 + c_y q_y^2 + i v q_x} \quad (v>0). \tag{73}$$

We rescale T by a nonuniversal quantity and define $\tilde{T}$ the "dimensionless temperature" $\frac{1}{2} B_\infty = \tilde{T}$. Remarkably it is possible to replace everywhere Δ_K by the smoothed disorder:

$$\tilde{\Delta}_K = \Delta_K e^{-\tilde{T} K^2}, \quad \tilde{\Delta}(u) = \sum_K \Delta_K e^{iKu} e^{-\tilde{T} K^2}. \tag{74}$$

The divergent part of the correction to the friction coefficient is now Eq. (A17), thus the same as before, except one must use the smoothed disorder.

Let us now compute the renormalization of the temperature by disorder (for a related calculation see also Ref. 85). As was mentionned earlier there is a nontrivial divergence in the correction to the temperature. Using Eq. (43) one finds

$$\eta \delta T = \sum_K \int_{t>0} \Delta_K (e^{-(1/2)K \cdot B_{0,t} \cdot K} - e^{-(1/2)K \cdot B_\infty \cdot K} - K^2 t T R_{r=0,t} e^{-(1/2)K \cdot B_{0,t} \cdot K}). \tag{75}$$

When $v=0$ this integral can be simplified using the FDT relation $2TR_{r=0,t}=\theta(t)(d/dt)B_{0,t}$ which gives $\delta T=0$. This yields the RG equation, after rescaling:

$$\frac{dT}{dl}=(2-d-2\zeta)T \quad (v=0). \tag{76}$$

For $v>0$ the second part does not diverge anymore, and the divergence of the first one can be extracted as follows:

$$\eta\delta T \sim \sum_K \int_{t>0} \widetilde{\Delta}_K(e^{-(1/2)K\cdot(B_{0,t}-B_\infty)\cdot K}-1) \sim \frac{1}{2}\sum_K \int_0^{+\infty} dt \widetilde{\Delta}_K K^2(B_\infty - B_{0,t}). \tag{77}$$

Since we are also interested in the crossover from $v=0$ to $v>0$ we use Eqs. (77), (38), and (49) to estimate the large time behavior of Eq. (75) and obtain

$$\frac{\delta T}{T}\sim \sum_K K^2\widetilde{\Delta}_K \int_{q_y} \frac{v^2}{2c_y q_y^2(4c_xc_yq_y^2+v^2)^{3/2}}. \tag{78}$$

Note that it vanishes, as it should when $v\to 0$. This yields the following RG equation for the temperature $\widetilde{T}$:

$$\frac{d\widetilde{T}}{\widetilde{T}dl}=2-d-2\zeta-\frac{\widetilde{\Delta}''(0)}{(4\pi c_y v)(1+4c_xc_y\Lambda_0^2e^{-2l}/v^2)^{3/2}}. \tag{79}$$

using rescaling (54) and (73). We now look at the corrections to disorder. The calculation is done in Appendix B. The divergent contributions to the disorder correlator are, adding first and second order in perturbation

$$\Delta_P^R=e^{-\widetilde{T}P^2}\Bigg[\Delta_P+e^{\widetilde{T}P^2}\sum_{K,K'=P-K}\Delta_K e^{-\widetilde{T}K^2}\Delta_{K'}e^{-\widetilde{T}K'^2}\left(K^2\int_r G(r)G(r)+K'K\int_r G(r)G(-r)\right) - P^2\Delta_P\sum_{K'}\Delta_{K'}e^{-\widetilde{T}K'^2}\int_r G(r)G(r)\Bigg]. \tag{80}$$

The key point is that using $K\cdot K'=(P^2-K^2-K'^2)/2$ all exponential factors rearrange and at the end everything can be written only using the smoothed function $\widetilde{\Delta}_K$. This yields the RG equation for the disorder:

$$\frac{d\widetilde{\Delta}(u)}{dl}=\widetilde{T}\widetilde{\Delta}''(u)+(3-d-2\zeta)\widetilde{\Delta}(u)+\zeta u\widetilde{\Delta}'(u) + f_1(l)\widetilde{\Delta}''(u)[\widetilde{\Delta}(0)-\widetilde{\Delta}(u)]-f_2(l)\widetilde{\Delta}'(u)^2, \tag{81}$$

where f_1 and f_2 are the same coefficients as in Eq. (57). We have used that the smoothed function $\widetilde{\Delta}(u)$ itself has an explicit cutoff dependence. Note that this equation is correct for any T and to second order in the disorder. Although it can also be obtained by a small-T expansion (expanding the first-order correction in $\langle S_{\rm int}\rangle$ both in Δ and T, see Appendix A) such an expansion is potentially dangerous since it works only if $T\Delta''\leqslant\Delta$. This happens to be the case here because at the fixed point $T\Delta''\sim\epsilon\Delta$. However it may not be true for other problems and does not allow us to treat larger temperatures. In addition to Eq. (81) the RG equation for the friction coefficient is identical to Eq. (59) with $\Delta\to\widetilde{\Delta}$ (and also yields $z=2$ for $v>0$).

2. Analysis of FRG equations at T>0

Let us now analyze the FRG equations at $T>0$ for the periodic case in an $\epsilon=3-d$ expansion and in $d=3$. We write Δ instead of $\widetilde{\Delta}$ and T instead of $\widetilde{T}$ everywhere for convenience. One has

$$\frac{d\Delta(u)}{dl}=\epsilon\Delta(u)+T\Delta''(u)+\Delta''(u)[\Delta(0)-\Delta(u)], \tag{82}$$

$$\frac{dT}{Tdl}=-1+\epsilon-\Delta''(0). \tag{83}$$

We have absorbed the factor $1/4\pi v c_y$ in $\Delta(u)$. Let us first search for a fixed point. We thus assume that $dT/dl=0$ with $T=T^*$, which implies that $\Delta''(0)=-1+\epsilon$. Using T^* we now search for a fixed point for $\Delta(u)-\Delta(0)$ as we did for the $T=0$ fixed point. Let us set $\Delta(u)=\Delta(0)-T^*g(u)$ with $g(u)>0$ and periodic. One gets

$$g''=-\frac{\epsilon}{T^*}+\frac{\epsilon-\Delta''(0)}{T^*}\frac{1}{1+g}=-V'(g). \tag{84}$$

Equation (84) is the classical equation of motion of a particle in the potential $V(g)$. It always has a periodic solution starting from $g=0$. Thus the solution is

$$u=\int_0^g\frac{dg}{\sqrt{-2V(g)}}, \quad V(g)=\frac{\epsilon}{T^*}g-\frac{\epsilon-\Delta''(0)}{T^*}\ln(1+g). \tag{85}$$

This yield a condition since we have fixed the period to be $u=1$: $\frac{1}{2}=\int_0^{g_{\max}}[dg/\sqrt{-2V(g)}]$ where $V(g_{\max})=0$, the other condition being $\Delta''(0)=-1+\epsilon$. Both conditions determine T^* and $\Delta''(0)$. From this we get the fixed-point temperature:

$$T^{*}=\frac{\epsilon^{2}}{4(\int_{0}^{y_{\max}}dy/\sqrt{2[\ln(1+y/\epsilon)-y]})^{2}}\sim\frac{\epsilon^{2}}{8\ln(1/\epsilon)}. \tag{86}$$

Thus we find that there is a finite-temperature fixed point. This is the moving glass $T>0$ fixed point. Though we have not investigated in detail the stability of this fixed point it is likely to be attractive. Indeed one sees clearly in Eq. (83) that at high T one expects $\Delta''(0)$ to be small (since Δ is smoothed by temperature), while at low T $\Delta''(0)$ grows very fast thereby T increases. Note that similar finite T fixed points were found for other nonpotential systems.[85] On the other hand, setting $u=0$ in Eq. (82) shows that the random force $\Delta(0)$ is still generated, though it grows slower than for $T=0$. Thus there should be a reduction of the displacements induced by the random force, due to nonzero temperature. There is an interesting crossover at low T where $\Delta''(0)$ first starts to increase violently before it finally decreases again to its fixed-point value. This crossover is discussed in the following section.

The case of $d=3$ can be studied similarly. For $T=0$ one looks again for a solution decaying as $1/l$. As we noticed before ϵ and $1/l$ plays the same role. Indeed the substitution $\Delta(u)=\hat{\Delta}(u)/l$ in $d=3$ and the substitution $\Delta(u)=\epsilon\hat{\Delta}(u)$ in $d=3-\epsilon$ leads to the same equation for the fixed point $\hat{\Delta}(u)$ in both cases. Thus the asymptotic fixed-point solution in $d=3$ can be obtained from the solution in $d=3-\epsilon$ approximately as $\overline{\Delta}(u)\sim 1/l\overline{\Delta}[u,\epsilon=1/l,T^{*}(\epsilon=1/l)]$ with the corresponding flow of temperature:

$$T(l)\sim\frac{1}{8l^{2}\ln(l)}. \tag{87}$$

This solution is not exact now since temperature flows, however one can check that the flow of temperature is slow enough so that this is a consistent approximation. Thus at large scale temperature decays back to $T=0$. Indeed the fixed-point function is very similar to the $T=0$ fixed-point function except in small layers around integer u. Near the origin the term $T\Delta''(0)$ is of same order as $\epsilon\Delta(0)$ in $d=3-\epsilon$. Thus the main effect of temperature is to round the nonanalyticity.

3. Physical results at finite temperature $T>0$

We now discuss the behavior of the mobility $\mu_R=1/\eta_R$ and of the I-V characteristics. One can compute the mobility from the RG equation by integrating the $T>0$ version of Eq. (59) over all scales:

$$\ln\left(\frac{\mu(l)}{\mu_0}\right)=-\int_0^l\Delta_l''(0)A_{d-1}\frac{2c_xe^{-2l}\Lambda_0^{d-1}}{(4c_xe^{-2l}c_y\Lambda_0^2+v^2)^{3/2}}. \tag{88}$$

The asymptotic mobility as given by $\mu_R=\mu(l=\infty)$. Since asymptotically there are only finite corrections to η as soon as $v>0$ this integral converges. Thus there is a nonzero asymptotic mobility μ_R in the $T>0$ moving glass (by contrast with what one would have in the static Bragg glass at $T>0$). However the renormalized mobility μ_R is very small (for experimental purposes) in two important cases (i) at low temperature, (ii) for velocities not very large $v\ll v_c^*$. This leads to the v-f characteristics shown in Fig. 15. A complete calculation of μ_R in all regimes can be made by examining the RG equations derived above. We give here a rough estimate of the barriers in these two cases (i) and (ii).

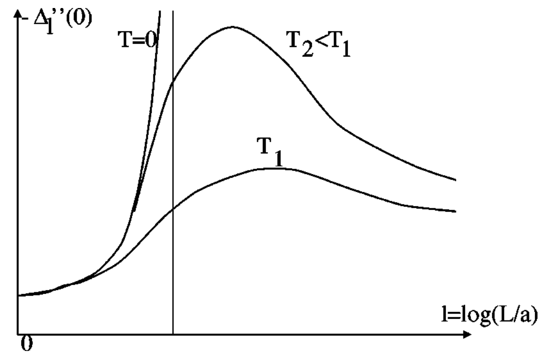

FIG. 23. Behavior of the second derivative $-\Delta''(0)$ of the disorder correlator as a function of the scale around the $T=0$ dynamical Larkin length R_c^y. At $T=0$ there is a divergence at R_c^y which is rounded at $T>0$. However $-\Delta''(0)$ still passes through very large values before eventually decaying slowly towards its fixed-point value. This results in high barriers at low temperature as discussed in the text.

Let us start by the low-temperature, high velocity behavior ($v\gg v_c$, $R_{\rm cr}<a$). A key point is that at low temperature μ_R is determined by the short scale contributions. Indeed there must be some continuity with the $T=0$ flow, where $-\Delta_l''(0)$ diverges after a finite length scale, the Larkin length R_c^y (as discussed above). Thus at low temperature $-\Delta_l''(0)$ first shoots up near the dynamic Larkin length, strongly renormalizing the mobility downwards, before the temperature catches on and reduces it back to its fixed-point value $-\Delta^{*''}(0)\sim 1$. Note that this fixed-point value corresponds to values of disorder *much larger* than the original disorder. Indeed restoring the factors (in $d=3$) gives for the original disorder dimensionless parameter $\Delta_2/(4\pi vc)\sim\ln(a/R_c^y)\ll 1$ at weak disorder, while asymptotically one has $\Delta_2^R/(4\pi vc)=1$. The global behavior with length scale is illustrated in the Fig. 23.

The small-T behavior in $d=3$ can be estimated as follows. Let us denote by T_0 the bare value. One has the exact equations:

$$\frac{d\Delta''(0)}{dl}=T\Delta''''(0)-\Delta''(0)^2, \tag{89}$$

$$\frac{d\Delta''''(0)}{dl}=-7\Delta''(0)\Delta''''(0)+T\Delta^{(6)}(0), \tag{90}$$

and Eq. (83) with $\epsilon=0$. We roughly estimate the scale $R_c(T_0)=ae^{l^*}$ at which the term $T\Delta''''(0)$ starts slowing down the growth of $\Delta''(0)$. We denote the bare values of the derivatives of the disorder correlator by $\Delta_2=-\Delta''(0)$ and $\Delta_4=\Delta''''(0)$. At $T=0$ higher and higher moments have more rapid growth: $\Delta_4(l)\sim 1/(1-\Delta_2 l)^7$ and $\Delta_6(l)\sim 1/(1-\Delta_2 l)^{16}$, etc. A natural hypothesis is that the effects of temperature is to smooth out the highest moments first. Assuming that only one length scale exists at $T>0$, this allows us to replace $\Delta_6 \to \text{Cste}\Delta_4^2/\Delta_2$ in the above equations. The equations can now be solved for Δ_4 and Δ_2 and T. The length scale l^* at which a nonzero temperature modifies the flow of Δ_2 is given by $(1-\Delta_2 l^*)^6 = T_0 e^{-l^*}\Delta_4/(\Delta_2)^2$. This length scale is very close to the Larkin length and the end result is

$$-\Delta''(0,l=l^*) = \frac{\Delta_2}{(T_0/T^*)^{1/6}}, \quad T^* = \frac{\Delta_2}{4\pi v c}\frac{R_c^y}{a}. \quad (91)$$

The renormalized mobility can then be estimated from Eq. (88). Restoring all the dimensional factors one finds

$$\mu_R \sim \mu_0 e^{-U_c}, \quad (92)$$

where in $d=3$ U_c can be estimated as

$$U_c \sim \left(\frac{R_{\text{cr}}}{R_c^y}\right)^2 \max\left[\frac{g}{[(T/T_m)(a/R_c^y)(1/g)]^{1/6}} \times \ln\left[1/\left(\frac{T}{T_m}\frac{a}{R_c^y}\frac{1}{g}\right)\right]\right]. \quad (93)$$

$R_{\text{cr}} = c/v$ has been defined in Sec. III B, $T_m = ca^d$ and $g = \Delta_2/(4\pi v c_y)$. For simplicity we have assumed here $r_f \sim a$. We stress that this estimate is a rough one, and that more work is needed to obtain better estimates by studying in more detail the solution of the FRG equation at all scales. Also, ours is probably a lower bound. This estimate indicates that for very large velocities one has to go to very low temperatures to experience significant barriers.

Barriers are much larger when the velocity is not very large $v < v_c$ the case which we study now. It must be stressed that the crossover velocity v_c which determines the barriers corresponds to c_{11} (or to c_{11} and c_{44}) and thus may be extremely large (see discussion in Sec. III B). One can use the results for barriers in the Bragg glass, which grow as $U_b = U_c(R/R_c)^\theta$, where θ is the energy exponent ($\theta = d-2$ asymptotically in the Bragg glass), $U_c = \Delta R_c^{d/2}$ the barrier at the pinning force and R_c is the isotropic Larkin length. These barriers grow until the crossover length R_{cr} is reached, as indicated in Fig. 12. Thus the asymptotic mobility can be estimated as

$$\mu_\infty \propto \mu_0 e^{-U_c(R_{\text{cr}}/R_c)^\theta/T}. \quad (94)$$

VII. MOVING GLASS EQUATION IN $D=2$ AND $D=2+\epsilon$

As stressed in Sec. I, it is important to first study the elastic theory as a function of the dimension d, before attempting to include topological defects. Up to now we have studied the moving glass equation in a $d=3-\epsilon$ expansion. This study is of course mostly relevant for the physical dimension $d=3$. To study the other physically interesting dimension $d=2$ another RG calcluation can be performed. For the *statics* $v=0$ the RG approach was constructed by Cardy and Ostlund (CO).[113] It was later extended to study equilibrium dynamics[114] and, with some additional assumptions, to study the problem in $d=2+\epsilon$. In the case $v=0$ it yields a marginal glass phase in $d=2$ for $T<T_c$ described by a line of perturbative fixed points. Extensions to models with $n>1$ components necessary to describe a lattice close[94] and far from equilibrium[115] were also studied.

In this section we first show that in $d=2$ the CO fixed line is *unstable* to a finite v on the simplest case of the $n=1$ component moving glass equation. We derive the RG equations for the case $v>0$. We stress that this is a toy model since it is clear that in $d=2$ additional instabilities to dislocations occur at the temperatures where we can control the behavior of the model. However it is instructive, and provides the first necessary step to introduce the other instabilities.

A. $d=2$

1. RG equations in $d=2$

We now study Eq. (47) in $d=2$ splitting the pinning force $F = f_1(r,u_{rt}) + f_2(x)$ where $f_1(r,u_{rt})$ is the random nonlinear pinning force with $\overline{f_1(r,u_{rt})f_1(r',u_{r't'})} = \Delta(u_{rt} - u_{r't'})\delta(r-r')$ and $f_2(x)$ is the disorder originating from long-wavelength disorder Eq. (25) with $\overline{f_2(q)f_2(-q)} = \Delta q^2 + \Delta_0$. In addition such terms are generated in perturbation theory (at least for $v=0$) and should thus be added from the start.

We use everywhere the shorthand notation $v \equiv \eta_0 v$, where η_0 is the bare value of the friction coefficient. Since we are looking at a periodic system one has $\Delta(u) = \Sigma_K \Delta_K e^{iKu}$. However in $d=2$ where temperature is *marginal* the harmonics are relevant at different temperatures. This remains true at $v>0$. It is thus enough to consider the lowest harmonic $\Delta(u-u') = g\cos(u-u')$. Perturbation theory is carried in Appendix C using the MSR formalism. Note that the random forces f_2 can be eliminated by a shift and do not feedback in the RG (see Appendix C). In addition due to the tilt symmetry (galilean invariance) c_x, c_y, and v have no corrections. One finds to first order in g the following corrections to the friction coefficient, the temperature, and the disorder:

$$\delta\eta = g\int_0^{+\infty} d\tau\,\tau R(0,\tau)e^{-(1/2)B(0,\tau)},$$

$$\delta(\eta T) = g\int_0^{+\infty} d\tau(e^{-(1/2)B(0,\tau)} - e^{-(1/2)B(0,\tau=\infty)}),$$

$$\delta g = g e^{-(1/2)B(0,\tau=\infty)}, \quad (95)$$

where $B(r,t) = \langle[(u_{rt}-u_{00}]^2\rangle_0$ and $R(r,t) = \langle\delta u_{rt}/\delta h_{00}\rangle_0$ are, respectively, the correlation and response functions in the theory without disorder. In the case $v=0$ the FDT ensures $\delta T=0$, as in the previous sections. This property does not hold any more when $v\neq 0$ and T renormalizes upward in $d=2$. Similarly as for the statics, disorder is relevant below a certain temperature T_g. To determine T_g one computes the mean-square displacements in the absence of disorder. According to whether $v=0$ or $v>0$ one finds, using a regularization discussed in the Appendix C, two different large-time behaviors:

$$B(0,t,a) = \frac{4T}{T_c}(\ln[v\mu t/a] + C/2) \quad (v>0),$$

$$B(0,t,a) = \frac{2T}{T_c}(\ln[c\mu t/a^2] + C/2) \quad (v=0), \quad (96)$$

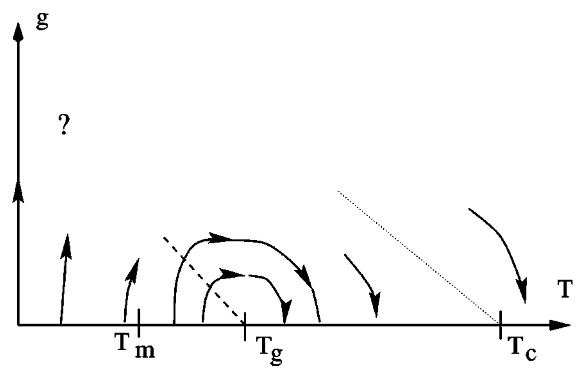

FIG. 24. RG flow diagram in $d=2$. The flow is circular around an instability temperature at $T=T_g=T_c/2$. The Cardy-Ostlund line of the fixed point of the statics (dotted line) which starts at $T=T_c$ is unstable when $v>0$. For $d=2+\epsilon$ there is a finite T moving glass fixed point (presumably attractive) on the dashed line at $g\sim\epsilon$ (the resulting spiraling flow is not shown). Continuity with the FRG result suggests that this fixed point moves upward to the $T=0$ axis as d goes from $d=2$ to $d=3$. In $d=2$ a zero-temperature moving glass fixed point is expected at infinite $g=-\Delta''(0)$ (if the lower critical dimension d_{lc} for the $T=0$ moving glass is $d_{lc}\leqslant 2$).

where $T_c=4\pi c$ is the transition temperature of the static system. Remarkably, as can be seen from Eq. (96), T_g is *half* of the Cardy-Ostlund glass temperature T_c of the statics.

Thus the CO line is unstable and both disorder and temperature are generated. To obtain the RG equations we restrict ourselves to the case when the starting cutoff is large enough $a^2v^2/(4c^2)\gg 1$ (or the velocity large enough) so that one is already in the asymptotic regime. Of course at small velocity there is a complicated crossover where the short distance properties are dominated by the static solution, but the large distance properties are again given by the present RG equations. Introducing the dimensionless coupling constant $\tilde{g}=ga/(vT_c)$ Eqs. (95) and (96) allow us to obtain the correction to ηT (see Appendix C) and the RG equations upon a change of cutoff $a'=ae^{dl}$. Note that the exponential decay of the response function at large time (as in Sec. VI) cuts all divergences in η. One finds

$$\frac{d\tilde{g}}{dl}=\left(1-\frac{2T}{T_c}\right)\tilde{g}+O(\tilde{g}^2),$$

$$\frac{dT}{Tdl}=2C_1\tilde{g}+O(\tilde{g}^2),$$

$$\frac{d\eta}{dl}=0, \tag{97}$$

where $C_1=e^{-C/2}$ is a nonuniversal constant. Equation (97) can be compared with the Cardy-Ostlund RG equation[113] for $v=0$ where the eigenvalue is $2(1-T/T_c)$.

B. Analysis of RG equations in $d=2$

Let us analyze the RG flows. We introduce the reduced temperature $\tau=(2T-T_c)/T_c$ and $\overline{g}=2C_1\tilde{g}$. The trajectories are the arches of parabolaes represented in Fig. 24 centered around $T_c/2$, of equation $\overline{g}-\overline{g_0}=\frac{1}{2}(\tau_0^2-\tau^2)$ (note that close to $T_g=T_c/2$ these trajectories are not modified by the higher-order terms).

As can be seen from Fig. 24 if one starts at small disorder with temperature $T_c/2-\Delta T$, both disorder and temperature first increase pushing the system in a region where the disorder is irrelevant, ending up with a disorder free system at about $T_c/2+\Delta T$. This has several physical consequences.

(i) At finite velocity the effect of disorder is weaker than in the statics, which manifests itself in the RG equation since weak disorder becomes irrelevant for $T>T_c/2$, a region which is already deep in the glass phase in the statics. This effect is analogous to the dimensional shift from $d_{uc}=4$ in the statics to $d_{uc}=3$.

(ii) However we still find a transition at $T=T_c/2$ below which disorder is relevant and grows under RG. That such a region where disorder is relevant exists in $d=2$ is compatible with the FRG findings in $d=3-\epsilon$ and clearly shows that even in motion one still has to consider the effect of the random potential. However due to the importance of the thermal effects in $d=2$, at large enough scales the disorder stops being relevant since the temperature also increases. The length scale ξ at which disorder becomes again negligible can be estimated from the RG and reads at small $g\ll\tau_0^2$, $\xi\sim(\tau_0^2/g)^{1/(4\tau_0)}$. ξ becomes extremely large when the disorder is weak or when one starts at low enough temperatures.

(iii) Finally, we find that disorder generates an additional temperature. This renormalization of temperature is physically very different from the "shaking temperature" of Ref. 72. In particular the value of the generated temperature in our case does not depend on the strength of the disorder but on the temperature itself and the distance to T_g. In particular, and in a similar way as for the FRG, if one had started at $T=0$ no temperature is generated, as can be seen from Eq. (97).

Using the RG flow one can compute the displacements. For the connected correlations one finds

$$\overline{\langle[u(x)-u(0)]^2\rangle-\langle u(x)-u(0)\rangle^2}\sim T_R(\tau,g)\ln x. \tag{98}$$

We have used the exponent at the fixed point, which is a correct procedure because the fixed point is approached fast enough as $\tau_0-\tau(l)\sim\tau_0 e^{-\tau_0 l}$. These correlations are nonmonotonic as a function of T with an almost cusp (rounded by g) at $T_c/2$, and increase below $T_c/2$. T_R is given by the above trajectory equation setting $\tau_0=(2T-T_c)/T_c$ and $\tau=(2T_R-T_c)/T_c$.

In $d=2$ thermal RG effects are obviously important. This raises the issue of whether the moving glass phase exists at finite T. Since at low-temperatures the disorder is renormalized to nonperturbative values, one cannot rule out from the above calculations that a low-temperature moving glass phase exists. In that case an additional fixed point which controls the transition is necessary. In the absence of such a fixed point the moving glass would always be unstable at finite temperature in $d=2$ due to temperature renormalization. This last scenario is supported by the FRG results in $d=3-\epsilon$ and by the calculations in $d=2+\epsilon$ of Sec. VII C. Of course this is separated from the issue of the existence of a $T=0$ moving glass phase in $d=2$ which is likely from the zero-temperature FRG study of Sec. VI.

C. Moving glass equation in $d=2+\epsilon$

We now follow Goldschmidt and Schaub[114] and continue the above RG equations to $d=2+\epsilon$. The ϵ simply shifts the dimensions of the operators. We stress that this is based on the assumption that the RG functions are well behaved around $d=2$. This procedure has been used in other cases such as the $O(n)$ model.[116] Taking into account the dimensions, one readily obtains using the same reduced variables, to lowest order:

$$\frac{d\overline{g}}{dl}=-(\epsilon+\tau)\overline{g}-b\overline{g}^2,$$

$$\frac{d\tau}{dl}=-2\epsilon+\overline{g} \tag{99}$$

with $b=B/(4C_1^2)$ being a universal (regularization independent) number (by analogy with CO).[94] Finally one has also $d\eta/dl=0$. These equations now have a fixed point at $\overline{g}=2\epsilon$ and $\tau=-(1+2b)\epsilon$. To lowest order the eigenvalues are $\lambda_{\pm}=-b\epsilon\pm i\sqrt{2\epsilon}$. Thus without needing to compute the coefficient b we know that there is a fixed point, and we know the leading behavior of the eigenvalues $\lambda_{\pm}\sim\pm i\sqrt{\epsilon}$. Such spiraling fixed points have been obtained in other problems (e.g., Ref. 117). However to know whether the fixed point is attractive or repulsive, one needs to know the real part, which is controlled by the $O(g^2,\tau^2,g\tau)$ terms in the RG equation. For instance, the RG equation contains at least:

$$\frac{d\overline{g}}{dl}=-(\epsilon+\tau)\overline{g}-b\overline{g}^2,$$

$$\frac{d\tau}{dl}=(-2\epsilon+\overline{g}+c\overline{g}^2)(1+\tau). \tag{100}$$

Inspection shows that b actually controls the leading behavior of the real part. So this is the only nonlinear term we need to compute. Results from FRG (see Appendix B) and static CO lead us to expect that $b>0$, but to settle the question and obtain the universal value of b an actual calculation along the lines of Ref. 94 is needed. It is tempting to associate this fixed point to a finite temperature moving glass fixed point (analogous to the finite T fixed points found in recent manifold studies[85]). Finally we note that at this fixed point one has $z=2$ since we find that $d\eta/dl=0$ (thus there are large but finite barriers).

VIII. TOWARDS A COMPLETE DESCRIPTION OF ELASTIC FLOWS

In this section we go beyond the transverse description of the moving glass and study models II and I.

A. Study of the complete dynamical equation in elastic limit (model II)

In this section we come back to model II which contains both degrees of freedom transverse u_y and along motion u_x and the full time dependence of the pinning force. If one describes the elastic flow of a solid, i.e., in a regime, or a range of scales where there are no topological defects, this model improves on model III (see Sec. I for a discussion of the regimes where it is useful). As discussed in Sec. IV it is still an approximation, but a rather good one, of the full (but intractable) model of the elastic flow (model I).

Since model II is still quite difficult, our aim in this section is more limited than in Sec. VI. We show that the main features of the transverse physics of the moving glass embedded in model III are also present when the degrees of freedom u_x along the motion are added. In fact we show explicitly that in model II the RG equations for quantities involving u_y remain *identical* to the one of model III. The two important issues we discuss are the ones of the existence or not of an extra temperature generated by motion, and of the generation of a static ‘‘random force.’’ We perform perturbation theory up to second order in the disorder and examine the terms generated as well as the divergences. We develop an approach which allows to treat *all harmonics* $\Delta_K^{\alpha\beta}$ of the disorder correlator. First, we find that a static ‘‘random force’’ is generated in the direction of motion. This may seem surprising at first, because first-order perturbation theory gives a pinning force along x which rapidly oscillates. However, as our calculation shows, to second order the various washboard frequency harmonics interfere to produce a static random force. Second, we identify the divergences in perturbation theory and follow the evolution of the full disorder correlator under renormalization. We show that up to some details the resulting picture is close, if not identical, to the one given by model III. We work at $T=0$ but the approach can be extended to $T>0$ along the lines of Sec. VI.

1. General properties

Here we study Eqs. (33), (34) keeping the time dependence. More specifically we are interested in the model for a triangular lattice ($n=2$ component model) with the force applied along a symmetry direction. The equation of motion of model II reads for lines in $d=3$:

$$\begin{aligned}\eta_{xx}\partial_t u_x+\eta_0 v\,\partial_x u_x+(c_{66}\nabla^2+c_{44}\partial_z^2)u_x+(c_{11}-c_{66})\\ \times\partial_x(\partial_x u_x+\partial_y u_y)\\ =f-\eta_{xx}v+F_x^{\mathrm{pin}}(r,t,u),\end{aligned} \tag{101}$$

$$\begin{aligned}\eta_{yy}\partial_t u_y+\eta_0 v\,\partial_x u_y+(c_{66}\nabla^2+c_{44}\partial_z^2)u_y+(c_{11}-c_{66})\\ \times\partial_y(\partial_x u_x+\partial_y u_y)\\ =F_y^{\mathrm{pin}}(r,t,u),\end{aligned} \tag{102}$$

and setting $c_{44}=0$ to describe lattices of points in $d=2$. Note that $\eta_{xy}=0$ from the symmetry ($u_y\to-u_y$, $y\to-y$). We have allowed for different η_{xx} and η_{yy} since, even if they start identical (and equal to η_0) they do not remain so under renormalization. The statistical tilt symmetry ensures that the elastic coefficients and $\eta_0 v$ remains uncorrected. Note that in later calculations it is convenient to rescale the z direction, setting $z=\sqrt{c_{44}/c_{66}}z'$. The correlator of the pinning force can be written as

$$\overline{F_\alpha(r,t,u)F_\beta(r',t',u')}=\delta^d(r-r')\Delta^{\alpha\beta}[u-u'+v(t-t')]$$
$$=\delta^d(r-r')\sum_K \Delta_K^{\alpha\beta}$$
$$\times e^{-iK\cdot[u-u'+v(t-t')]}. \tag{103}$$

It contains all lattice harmonics K. Due to the modes with $K_x\neq 0$ it is an explicit *periodic function of time* with frequencies all integer multiples of the washboard frequency $2\pi\omega_0=v/a$. In addition it contains nonlinear *static* components $K_x=0$, $K_y\neq 0$ (which lead to model III treated in Sec. VI). Finally it contains a static u-*independent* component, $\Delta_{K=0}$ which, as we discuss below, is the static random force. This random force is strictly zero in the bare model $\Delta_{K=0}(l=0)=0$, but is generated in perturbation theory, as we show below.

The idea behind the method presented here is that at $T=0$ all the time dependence *remains strictly periodic* to all orders in perturbation theory in the disorder (plus a static part). Only frequencies multiple of ω_0 can be generated. Indeed to lowest order $F_y^{\rm pin}(x,y,t,u=0)$ is periodic in time, and yields a u periodic. Iterating perturbation theory thus leads only to periodic u and $F_y^{\rm pin}(x,y,t,u)$ with integer multiples of ω_0. This property allows us to construct a closed RG scheme of the above model with a renormalized disorder which remains of the form (103).

An immediate consequence is that *no temperature is generated* when $T=0$ at the start. Temperature is defined as the zero-frequency limit of the incoherent noise [or using the MSR formalism the vertex function $\eta T=\Gamma_{\hat{u}\hat{u}}(q=0,\omega\to 0^+)$]. Thus here one has

$$\eta\delta T=\int d\tau\sum_{K\neq 0}(\Delta_{\rm ren})_K^{\alpha\beta}e^{-iK\cdot v\tau}=0. \tag{104}$$

The static random force $K=0$ mode leads to a $\delta(\omega)$ part in u and thus is distinct from the temperature. The fact that no temperature is generated at $T=0$ is a rather strong property of the elastic flow. In physical terms in the $T=0$ elastic laminar periodic flow all particles strictly replace each other after time $\tau_0=a/v$, i.e., $R_{i_x,i_y}(t+\tau)=R_{i_x+1,i_y}(t+\tau)$ where $i=i_x,i_y$ are integer labels for the particles. Although this laminar periodic flow may become *unstable* to chaotic motion, we proceed here assuming that such instabilities to chaos happen only at finite large enough disorder, or at large enough scale. We reserve the study of the stability of this flow (chaos, nonperturbative effects, etc.) to future study. Finally note that this periodic flow is allowed by the assumed absence of topological defects in the system. Dislocations, if present, may ruin periodicity and introduce a small additional temperature (though this is far from established).

We start by establishing the possible form for the disorder correlator (at any order in perturbation theory) based on the symmetries of the problem (model II). This is necessary here because the bare disorder is *potential* $\Delta_K^{\alpha\beta}(0)=g_K K_\alpha K_\beta$, but it does not remain of this form in perturbation theory. We are interested here in the case when the velocity is along the lattice direction. Our analysis here is very general, and we will specify when we apply it to the case of a triangular lattice. More details are contained in Appendix B using the more rigorous MSR formalism.

The symmetries are as follows. First one can exchange t with t' and u with u' in Eq. (103) and relabel the disorder term. This gives $\Delta_{-K}^{\alpha\beta}(v)=\Delta_K^{\beta\alpha}(v)$. This is by construction and from the specific dependence in t and t' of the disorder. Second, the action must be real and thus $\Delta_{-K}^{\alpha\beta}(v)=\Delta_K^{\alpha\beta}(v)^*$. Third, the symmetry T_y ($u_y\to -u_y$, $y\to -y$, $\hat{u}_y\to -\hat{u}_y$) yields that $\Delta^{yy}_{K_x,-K_y}(v)=\Delta^{yy}_{K_x,K_y}(v)$, $\Delta^{xx}_{K_x,-K_y}(v)=\Delta^{xx}_{K_x,K_y}(v)$, $\Delta^{xy}_{K_x,-K_y}(v)=-\Delta^{xy}_{K_x,K_y}(v)$, $\Delta^{yx}_{K_x,-K_y}(v)=-\Delta^{yx}_{K_x,K_y}(v)$. Similarly, because of T_x one finds $\Delta^{yy}_{-K_x,K_y}(-v)=\Delta^{yy}_{K_x,K_y}(v)$, $\Delta^{xx}_{-K_x,K_y}(-v)=\Delta^{xx}_{K_x,K_y}(v)$, $\Delta^{xy}_{-K_x,K_y}(-v)=-\Delta^{xy}_{K_x,K_y}(v)$, $\Delta^{yx}_{-K_x,K_y}(-v)=-\Delta^{yx}_{K_x,K_y}(v)$. Note that the global symmetry T_xT_y implies that $\Delta_{-K}^{\alpha\beta}(v)=\Delta_K^{\alpha\beta}(-v)$. Thus one finds that one can split the disorder correlator into $\Delta_K^{\alpha\beta}(v)=\Delta_{S,K}^{\alpha\beta}(v)+\Delta_{A,K}^{\alpha\beta}(v)$, where $\Delta_{S,K}^{\alpha\beta}(v)$ is real, symmetric in $\alpha\beta$, even in K, and even in v and $\Delta_{A,K}^{\alpha\beta}(v)$ is imaginary, antisymmetric in $\alpha\beta$, odd in K, and odd in v. This naturally leads to the decomposition

$$\Delta_{S,K}^{\alpha\beta}(v)=\Delta_1^K v^2\delta_{\alpha\beta}+\Delta_2^K K_\alpha K_\beta+\Delta_3^K v_\alpha v_\beta, \tag{105}$$

$$\Delta_{A,K}^{\alpha\beta}(v)=i\Delta_4^K(v_\alpha K_\beta-v_\beta K_\alpha), \tag{106}$$

where all Δ_i^K are even in K and v and real. The bare disorder has only Δ_2^K nonzero and thus possesses the extra symmetry $\Delta_{-K}^{\alpha\beta}=\Delta_K^{\alpha\beta}$ or equivalently $(u\to -u, v\to -v)$. Because of the convection term $v\partial_x u$ in the equation of motion, this additional symmetry does not hold to higher orders in perturbation theory. It is natural to suppose that to any fixed order in perturbation theory the Δ_i^K are regular when $v\to 0$. Thus in the limit $v=0$ one recovers a strictly potential problem (all terms except Δ_2^K vanish).

Finally note that model III is a particular case of model II which corresponds to the following choice of bare parameters: (i) isotropic response $c_{11}=c_{66}$, (ii) $\Delta_K^{xy}=0$. Then clearly the equations along x and y decouple. Another particular case is to start from $\Delta_K^{\alpha\beta}=g_K K_\alpha K_\beta$ and use isotropic elasticity. Then the equations are only coupled through the time-dependent part of the nonlinear pinning force along y which depends on u_x (one has $\Delta_K^{xy}\neq 0$). It would be interesting to check whether this is enough to change the behavior compared to model III.

At $T=0$ the lowest-order corrections to the disorder come from second-order perturbation theory. The calculation of the effective action to second order for the most general model is performed in Appendix B. The full correction to the disorder correlator is obtained as $\delta\Delta_K^{\alpha\beta}$ given in Eq. (B9).

2. Generation of the static random force

Setting $K=0$ in Eq. (B9) one gets the general expression for the static random force correlator $\delta\Delta_0^{\alpha\beta}$, i.e., a u-independent Gaussian random term $F(r)$ in the original equation of motion. This random force has zero crossed correlations, i.e., $\delta\Delta_0^{xy}(v)=0$. Thus there are two independent random forces one along x of strength Δ_0^{xx} and one along y of strength Δ_0^{yy}. This is consistent with formula (105) which gives $\Delta_K^{xy}(v) = i\Delta_4^K K_y v$ and vanishes for $K=0$. It can also be seen explicitly on the above expression which is found to be symmetric in $\alpha\beta$ (using all the above symmetries). Since we know that $\Delta_0^{xy}(v)$ must be antisymmetric in xy (from the above $K_y \to -K_y$ transformation) it must vanish. The expression for the random force is still complicated as it does involve *all disorder harmonics*. One must carefully distinguish between:

(i) The static random force generated to lowest order in the bare disorder $O(\Delta^2)$ (i.e., at the initial stage of the RG). To this order one can use the *bare disorder* and the resulting perturbative expression of the random force (evaluated below) is found to be well behaved and without IR divergences. Of course once even a small finite random force is generated, it is relevant by power counting and must be taken into account (though it does not feedback in the RG for the nonlinear disorder).

(ii) The static random force generated to higher orders in perturbation theory (or at the next stages of the RG). There we find IR divergences. This means that nontrivial corrections originating from the nonlinear disorder have to be also taken into account to estimate the random force generated. This is done in the next subsection.

Thus we start by giving the expression of the random force generated from the bare disorder (i). Setting $\Delta_K^{\alpha\beta} = g_K K_\alpha K_\beta$ in Eq. (B9) one finds

$$\delta\Delta_0^{\alpha\beta} = \sum_K \int_q K_\alpha K_\beta g_K^2 \{[K\cdot R(-ivK,-q)\cdot K] \times [K\cdot R(ivK,q)\cdot K] - [K\cdot R(ivK,q)\cdot K]^2\}. \tag{107}$$

Note that it does vanish for $v=0$ as it should since the problem becomes potential in that limit. One can first specify this formula for the case $c_{11}=c_{66}$. We also set $\eta_{xx}=\eta_{xy}$ (which is consistent since we are just looking at the lowest-order contribution in perturbation theory). This yields[35]

$$\delta\Delta_0^{\alpha\beta} = \sum_K \int_{q,\mathrm{BZ}} K^4 K_\alpha K_\beta g_K^2 \frac{2(\eta_0 v)^2 (K_x+q_x)^2}{[c^2 q^4 + (\eta_0 v)^2 (K_x+q_x)^2]^2}. \tag{108}$$

A random force is indeed generated along both the x and y directions i.e., there is a positive Δ_0^{xx} and Δ_0^{yy}. The cross terms vanish $\Delta_0^{xy}=0$. The integral for Δ_0^{yy} is infrared divergent, as discussed above (which is natural from the analysis in Sec. VI) and is examined in the next section.

Let us estimate the magnitude of the static random force generated along x. From the above expression (109) one finds in the case $d_z=0$ relevant for point lattices (in $d=1,2,3$):

$$\Delta_0^{xx} \approx \frac{C_d}{(\eta_0 v)^2} \sum_{K,|K_x|\geq K_m} K^4 g_K^2 + \frac{C_d'}{(\eta_0 v)^{2-d/2} c^{d/2}} \sum_{K,|K_x|<K_m} K^4 |K_x|^{d/2} g_K^2, \tag{109}$$

where $K_m = |K_0| \max(1, v_{\mathrm{cr}}/v)$. There is a crossover velocity $\eta_0 v_{\mathrm{cr}} \sim c\pi/a$. One has defined $C_d = 2S_d/[d(2\pi)^d]$ and $C_d' = S_d(4-d)\pi/[8(2\pi)^d \sin(\pi d/4)]$ and $n=2$ for triangular lattices. This yields for $r_f \ll a$:

$$\Delta_0^{xx} \approx \frac{g^2 a^{n-d}}{(\eta_0 v)^2 r_f^{4+n}} \min\left[1, \left(\frac{va}{v_{\mathrm{cr}} r_f}\right)^{d/2}\right]. \tag{110}$$

In the case $c_{66} \ll c_{11}$ one can simply retain only the transverse mode (thus setting $c=c_{66}$ in the above formulas). In the case $d_z=1$ (relevant for line lattices) we only give the large v estimate (valid for $v \gg v_{\mathrm{cr}} = c_{66}\pi/a$). It reads

$$\Delta_0^{xx} \approx \frac{1}{\sqrt{c_{44}}(\eta_0 v)^{3/2}} \sum_K K^4 |K_x|^{1/2} g_K^2 \sim \frac{g^2 a^n}{\sqrt{c_{44}}(\eta_0 v)^{3/2} r_f^{n+4+1/2}}. \tag{111}$$

Let us stress again that we have defined the random force as $\Delta_{K=0}^{\alpha\beta}$, which is the correct definition based on the RG decomposition. Note that one must distinguish it from $\Delta(u=0)$. If one was to compute the displacements correlator $\langle uu \rangle$ to order Δ^2, in order to show that the displacements feel the random force (e.g., grow unboundedly in the x or y direction) one would need $\Delta(u=0)$ to second order (and to be consistent the response function corrected to order Δ). It would be thus tempting to attribute the random force to $\Delta(u=0)$, but such a definition would not be carried out beyond the simple perturbative approach. While it makes little difference in order of magnitude estimates for the effect of the x random force, it is drastically different along y (one quantity $\Delta_{K=0}$ is IR divergent and the other is not, see Sec. VI).

3. RG study of model II

We now look for the divergences in perturbation theory which appear in model II. We address only the case $v>0$. Let us look again at Eq. (B9). It contains infrared divergences of the same type that was discussed in Sec. VI. These divergences occur only for q-momentum integrals which are *both* (i) zero-frequency integrals ($K\cdot v=0$ terms), (ii) involve q and $-q$. These are of the form

$$D_{\gamma\rho,\delta\lambda} = \int_q G^{\gamma\rho}(-q) G^{\delta\lambda}(q), \tag{112}$$

where $G^{\alpha\beta}(q) = R^{\alpha\beta}(\omega=0,q)$ is the static response function. Here it has the form

$$G^{\alpha\beta}(q) = \sum_I \frac{P^I(q_x,q_y)}{c_I(q) + ivq_x}, \tag{113}$$

where $I=T,L$ index the transverse and longitudinal projectors and elastic eigenenergies as defined in Eq. (38). The key point (for $n=2$, e.g., triangular lattices) is that one has here $q_x \sim q_y^2 \sim q_z^2$. Thus all projector elements are subdominant for small q (i.e., kill the IR divergences) except the two elements $P_{yy}^L \sim P_{xx}^T \sim q_y^2/(q_x^2+q_y^2) \sim 1$. Thus the only IR divergent elements among Eq. (112) are $D_{yyyy}, D_{xxyy}, D_{yyxx}, D_{xxxx}$. Explicit calculation of the divergent parts gives

$$D_{yyyy}\sim\int_{q_y,q_z}\frac{1}{2v(c_{11}q_y^2+c_{44}q_z^2)},$$

$$D_{xxxx}\sim\int_{q_y,q_z}\frac{1}{2v(c_{66}q_y^2+c_{44}q_z^2)}, \tag{114}$$

$$D_{xxyy}=D_{yyxx}\sim\int_{q_y,q_z}\frac{1}{v[(c_{11}+c_{66})q_y^2+2c_{44}q_z^2]}. \tag{115}$$

Let us now analyze the consequences of these divergences. From Eq. (B9) the relevant corrections to disorder are

$$\delta\Delta_K^{\alpha\beta}=D_{\gamma\rho,\delta\lambda}\sum_{P=(0,P_y)}\Big(-K_\gamma K_\delta\Delta_K^{\alpha\beta}\Delta_P^{\rho\lambda}+(K-P)_\gamma \times(K-P)_\delta\Delta_{K-P}^{\alpha\beta}\frac{1}{2}[\Delta_P^{\rho\lambda}+\Delta_{-P}^{\rho\lambda}]\Big). \tag{116}$$

Note that there is no component $\Delta_{P=(0,P_y)}^{xx}$ in the bare action but that it is generated in perturbation theory. This expression simplifies because of the symmetries discussed above and the fact that only the terms (114) appear in Eq. (116) (the term $\Delta_{P_y}^{\rho\lambda}$ always occurs in sums symmetrized over $\rho\lambda$ and one can use that $\Delta_{P_y,P_x=0}^{xy}=-\Delta_{P_y,P_x=0}^{yx}$ to cancel all crossed D_{xxyy} terms). From what remains one finally obtains the following RG equations:

$$\frac{d\Delta_K^{\alpha\beta}}{dl}=\epsilon\Delta_K^{\alpha\beta}+\sum_{P=(0,P_y)}\{A_{11}\Delta_{P_y}^{yy}[-K_yK_y\Delta_K^{\alpha\beta}+(K-P)_y \times(K-P)_y\Delta_{K-P}^{\alpha\beta}]+A_{66}\Delta_P^{xx}K_xK_x(\Delta_{K-P}^{\alpha\beta}-\Delta_K^{\alpha\beta})\} \tag{117}$$

with $A_{11}=1/(4\pi v\sqrt{c_{11}c_{44}})$ and $A_{66}=1/(4\pi v\sqrt{c_{66}c_{44}})$ if one uses the same regularization as in Sec. VI.

The RG equations (117) are thus the generalization to model II of the RG equations of model III and contain both the physics of u_x and u_y. They show that in the moving Bragg glass, nontrivial, nonlinear effects also occur in the direction of motion. A more detailed study of these equations will be given elsewhere.[83] Here we give their salient features. The RG equations (117) exhibit remarkable features. First, the subset of these equations for $\Delta_{(0,K_y)}^{yy}$ closes onto itself. Indeed setting $K_x=0$ in Eq. (117) one recovers exactly the RG equation (58) (with $\zeta=0$) of the moving glass model III. Thus we have shown that model III describes correctly the transverse physics, as announced, even if longitudinal degrees of freedom are present (i.e., within model II).

One can also write the above RG equations as coupled differential equations for three periodic functions of two variables $\Delta^{\alpha\beta}(u_x,u_y)$ with $\alpha,\beta=xx,yy,xy$. We temporarily use the shorthand notation $u=u_y$ and $v=u_x$. We denote $\Delta_{K_x=0}^{yy}(u)$ by $\Delta_1(u)$ and absorb the factor ϵ/A_{11} in the Δ. These coupled RG equations become

$$\frac{d\Delta_{\alpha\beta}(u,v)}{dl}=\Delta_{\alpha\beta}(u,v)+[\Delta_1(0)-\Delta_1(u)]\partial_u^2\Delta_{\alpha\beta}(u,v) +\gamma[\Delta_2(0)-\Delta_2(u)]\partial_v^2\Delta_{\alpha\beta}(u,v) \tag{118}$$

with $\gamma=A_{66}/A_{11}$ and $\Delta_1(u)=\int dv\,\Delta_{yy}(u,v)$ and $\Delta_2(u)=\int dv\,\Delta_{xx}(u,v)$. The function $\Delta_1(u)$ obeys the closed RG equation (60), identical to the moving glass RG. Thus $\Delta_1(u)$ converges towards the moving glass fixed point of Sec. VI $\Delta_1^*(u)-\Delta_1^*(0)=\frac{1}{2}u(u-1)$. Let us now examine the behavior of the other components of the disorder with $K_x=0$ (and $\alpha\beta=xx,xy$). They obey the equation

$$\frac{d\Delta_{\alpha\beta}(u)}{dl}=\Delta_{\alpha\beta}(u)+\Delta_{\alpha\beta}''(u)[\Delta_1(0)-\Delta_1(u)]. \tag{119}$$

By setting $u=0$ in this equation, one easily sees that $\Delta_{\alpha\beta}(u)$ also becomes *nonanalytic* beyond the dynamical Larkin length. Indeed the divergence of $\Delta_1''(0)\to-\infty$ at the Larkin length $l_c=\ln R_c^y$ (see Sec. VI) implies that $\Delta_{\alpha\beta}''(0)$ also diverges. Thus the full $\Delta_{\alpha\beta}(u)$ does become nonanalytic. Note that the solution of Eq. (119) for $u=0$ is simply $\Delta_2(l)=(\Delta_2/\Delta_1)\Delta_1(l)$. It is then easy to show that $\Delta^{\alpha\beta}(u)=C\Delta_1^*(u)$ is a stable fixed-point solution (up to the usual growing constant). Indeed, inserting the fixed-point value $\Delta_1(u)$ in Eq. (119) at $u=0$ one finds exactly the stability operator of the original fixed point which was discussed in Appendix D. This proves the stability of the fixed point for the whole equation (119). To determine the constant C we use Eq. (119) to compute $g=\Delta_{\alpha\beta}''(0^+)$. One gets $g\sim\int_{l>l_c}dl[1-\Delta_1''(0^+,l)]$. The exponential convergence of Δ_1 towards its fixed point implies that g is a finite constant which determines C. At the fixed point one can thus replace $\Delta_2(u)=C\Delta_1(u)$ in Eq. (118).

Thus we have shown that the RG equations in a moving Bragg glass decouple completely along y, giving back one of the generic moving glasses studied in Sec. VI. A complete study of the system (117) will be given elsewhere.[83]

B. Full model for the elastic flow (model I)

We now come back to the problem of establishing the correct long-wavelength hydrodynamic description of a moving structure with some internal order described by a displacement field $u_\alpha(r,t)$. The first step is to write an equation of motion which contains all terms which are (i) allowed by symmetry and (ii) *a priori* relevant in the long-wavelength limit by power counting. We carry this step here, check that all these terms are indeed generated in perturbation theory from the original equation of motion (22), and estimate their magnitude. The second step, which is to solve the universal large distance physics of such an equation turns out here to be a formidable task, which goes beyond this paper.

As we have discussed in Sec. IV the problem of driven lattices possesses some additional ''almost exact'' symmetries which allow us to simplify the hydrodynamic description and to extract some of the physics. This has led us to study model II (Sec. VIII A) which possesses the statistical tilt symmetry forbidding many terms, and the simpler model III (Sec. VI) (the moving glass equation) which contains most of the physics of moving structures, i.e., the physics of the transverse degrees of freedom.

As discussed in Sec. IV the only exact symmetries of the problem, for motion along a symmetry direction of the moving structure (x axis), are the spatial inversions along the directions transverse to the velocity. Power counting shows that the general form for the equation of motion in $d \leqslant 3$ is (model I):

$$\eta_{\alpha\beta}\partial_t u_\beta + L^{\gamma}_{\alpha\beta}\partial_\gamma u_\beta - C^{\gamma\delta}_{\alpha\beta}\partial_\gamma\partial_\delta u_\beta - K^{\delta\epsilon}_{\alpha\beta\gamma}\partial_\delta u_\beta \partial_\epsilon u_\gamma = F^{\rm dis}_\alpha(r,u,t) + \zeta_\alpha(r,t) + f_\alpha - \eta_{\alpha\beta}v_\beta + \delta f_\alpha, \qquad (120)$$

where the velocity v is fixed by the convention that $d/dt\int_r u^{\gamma}_{rt} = 0$ and f is the applied force. The KPZ terms are allowed because u_α is dimensionless at the upper critical dimension $d_{\rm uc=3}$ in a power counting at $T=0$. This can also be seen by writing a MSG formulation of Eq. (120). The above equation of motion is not fully complete unless one specifies the relevant disorder and thermal noise correlators. The thermal noise has a Gaussian correlator $\overline{\zeta_\alpha(t)\zeta_\beta(t')} = 2(\eta T)_{\alpha\beta}\delta(t-t')\delta^d(r-r')$, in general anisotropic. The correlator of the pinning force $F^{\rm dis}_\alpha(r,u)$ (which has zero average) is Gaussian and of the form (103) for periodic structures. Note however that in general, $\Delta_{\alpha\beta}(u)$ is not a potential disorder.

Compared to model II, cutoff effects which break the exact statistical tilt symmetry allow new terms to be generated, such as linear terms which correct the original convection term $v\partial_x u$ and nonlinear KPZ-type terms. The linear terms are obviously relevant and the nonlinear KPZ terms, in presence of the disorder, are relevant for $d \leqslant 3$. Thus once they are generated, even if their bare values are very small they may grow under RG and become important at large scale [a full solution of the RG equations for Eq. (120) would be needed in order to conclude]. However one may guess that since the statistical tilt symmetry is almost exact, the scale at which these new terms are able to change the physics compared to models II and III (if they do) may be very large. Finally note that there are also small corrections to the elastic matrix.

The above approach consists of writing a model independent equation (120) based on symmetry arguments. It may be useful in proving the universality of the behaviors of various structures. However, in many cases it is much more instructive to start from a given simple model without disorder, such as Eq. (22), and to estimate the bare values of the new terms to first order in a perturbation theory in disorder. Indeed, in the absence of disorder the above equation of motion reduces to

$$\eta^0_{\alpha\beta}(\partial_t u_\beta + v\partial_x u_\beta) = (C^0)^{\gamma\delta}_{\alpha\beta}\partial_\gamma\partial_\delta u_\beta + f_\alpha - \eta^0_{\alpha\beta}v_\beta + \zeta_\alpha(r,t). \qquad (121)$$

We have thus computed in Appendix A the corrections to first order in perturbation theory with respect to disorder to all terms of Eq. (120). Though the above equation (120) looks formidable many terms are zero from the exact inversion symmetry. We thus now explicitly specify the terms allowed in the equation of motion, for the case of an elastic structure described by an $n=2$ component displacement field (u_x, u_y). In $d=2$ and $d=3$ the equation along y should be odd under the inversion $(u_y \to -u_y,\ y \to -y)$ and also under $(z \to -z)$, while the equation along x must be even under these transformations. This yields in $d=3$

$$\begin{aligned}\eta_{yy}\partial_t u_y + v_1\partial_x u_y + v_2\partial_y u_x &= (c_1\partial_x^2 + c_2\partial_y^2 + c_3\partial_z^2)u_y + c_4\partial_x\partial_y u_x + (a_1\partial_x u_x \\ &\quad + a_2\partial_y u_y)\partial_x u_y + (a_3\partial_x u_x + a_4\partial_y u_y)\partial_y u_x \\ &\quad + a_5\partial_z u_x\partial_z u_y + F^{\rm dis}_y(r,u,t) + \zeta_y(r,t),\end{aligned}$$

$$\begin{aligned}\eta_{xx}\partial_t u_x + v_3\partial_x u_x + v_4\partial_y u_y &= (c_5\partial_x^2 + c_6\partial_y^2 + c_7\partial_z^2)u_x + c_8\partial_x\partial_y u_y + a_6(\partial_x u_x)^2 \\ &\quad + a_7(\partial_y u_x)^2 + a_8(\partial_x u_y)^2 + a_9(\partial_y u_y)^2 + a_{10}\partial_x u_x\partial_y u_y \\ &\quad + a_{11}\partial_x u_y\partial_y u_x + a_{12}(\partial_z u_x)^2 + a_{13}(\partial_z u_y)^2 \\ &\quad + F^{\rm pin}_x(r,u,t) + f_x - \eta_{xx}v + \delta f_x + \zeta_x(r,t), \qquad (122)\end{aligned}$$

and the same in $d=2$ with $c_3 = a_5 = c_7 = a_{12} = a_{13} = 0$. The physical interpretation of the linear terms is that now the local velocity explicitly depends on the local strain rates of the structure. The first possible effect of these terms would be to generate instabilities (see below). In the absence of such instabilities, it is unlikely that these terms alter the transverse physics. Although the full analysis of Eq. (122) goes beyond the present paper, some arguments support this picture. Indeed one can see that small additional linear terms do *not* remove the divergence in perturbation theory which was the hallmark of the moving glass. Let us write $v_1 = v + w$, $v_2 = aw$, $v_3 = v - w$, $v_4 = bw$ and consider w as small compared to v. Also we choose for simplicity isotropic elasticity $c_1 = c_2 = c_4 = c_5 = c_7 = c_3 = c$, $c_8 = c_4 = 0$. Then the eigenvalues of the $\omega = 0$ (static) response matrix are $D^{\pm}(q) = i(q_x v \pm w\sqrt{q_x^2 + abq_y^2}) + cq^2$ and eigenvectors $(\delta u_x, \delta u_y) = (bq_y, q_x \pm \sqrt{q_x^2 + abq_y^2})$. Note that one must have $ab > 0$ otherwise an instability develops. The perturbation theory result shows that indeed $ab > 0$ at least to lowest order in the disorder. One finds for instance that the integral which is the key of the FRG equation for the transverse modes Eqs. (53)–(114) becomes

$$\int_q G_{yy}(q)G_{yy}(-q) = \int_q \frac{c^2q^4 + (v-w)^2q_x^2}{[c^2q^4 + (vq_x + w\sqrt{q_x^2 + abq_y^2})^2][c^2q^4 + (vq_x - w\sqrt{q_x^2 + abq_y^2})^2]}. \qquad (123)$$

As is easily seen this integral is infrared divergent for $d \leqslant 3$, logarithmically in $d=3$ and powerlike in $d=2$ (since w is a small correction to v). The divergences occur in two hyperplanes $q_x = \pm(abw/\sqrt{v^2-w^2})\,q_y$ which are tilted symmetrically with respect of the direction of motion. Thus the main conclusions of Sec. VI are unchanged.

Let us now reexamine the moving glass equation, i.e., model III, and ask whether cutoff effects (absence of exact statistical tilt symmetry) generate relevant terms. By definition this equation involves only u_y and thus the only *a priori* relevant terms allowed by symmetry are

$$\eta_{yy}\partial_t u_y + v_1\partial_x u_y = (c_1\partial_x^2 + c_2\partial_y^2 + c_4\partial_z^2)u_y + a_2\partial_y u_y \partial_x u_y + F_y^{\text{pin}} + \zeta_y . \tag{124}$$

Now, we have shown in Sec. VI that at the moving glass fixed point $\partial_x \sim \partial_y^2$ and thus the KPZ term a_2 is irrelevant by power counting. Note that a cubic KPZ terms $(\partial_y u_y)^2\partial_x u_y$ is allowed by symmetry but again irrelevant near $d=3$. The moving glass equation model III is thus stable to cutoff effects and perfectly consistent. This lead us to claim[73] that while previous descriptions of moving systems, such as manifolds driven in periodic[118] or disordered potentials,[75,77] focused on the generation of dissipative KPZ, such terms are much less important in the moving glass equation, a problem which, because of periodicity, belongs to a universality class.

Finally one can also reexamine model II, the physics of which is presumably very similar to model III at least as far as the transverse degrees of freedom are concerned. This certainly holds below a (large) length scale max$(L_{\text{lin}}, L_{\text{KPZ}})$. Above one must worry about the new terms. Power counting at $d=3$ (where u_y and u_x are dimensionless) in the equation for the transverse degrees of freedom u_y (using that $\partial_x \sim \partial_y^2$ in the absence of the new terms) shows that the only KPZ term marginally relevant at the model II fixed point in $d=3$ (and thus the dangerous one) is the term $a_4\partial_y u_y \partial_y u_x$. Note also that the linear term $v_2\partial_y u_x$ also becomes relevant there and changes the counting. In the end it is probable that all terms in Eq. (122) have to be treated simultaneously to get the physics beyond max$(L_{\text{lin}}, L_{\text{KPZ}})$.

Finally we note that the arguments given in the previous section about the fact that no temperature is generated at $T=0$ are unspoiled by the terms generated here in the equation of motion compared to model II. That these terms may lead to other instabilities of the periodic time ordered flow resulting in chaotic motion is clearly an interesting possibility deserving further investigations.

IX. CONCLUSION

In this paper we have studied the problem of moving structures (such as vortex lattices) in a disordered medium following the physical approach developed in Ref. 73. The main emphasis in that approach is that because of degrees of freedom transverse to motion, periodic structures have a radically different physics than more conventional driven manifolds. The main consequence of our study is that the moving structures remain different from perfect structures (e.g., a perfect crystal) at all velocities (for $d \leqslant 3$ for uncorrelated disorder). In particular they still exhibit glassy behavior. The moving configurations can be generally described in terms of *static channels* which are the easiest paths in which particles follow each other in their motion. We have introduced here several degrees of approximation of the problem of moving periodic structures, embodied in several models. The simplest one, model III, introduced in Ref. 73 focuses only on the transverse degrees of freedom. A more complex one model II also contains degrees of freedom along the direction of motion. We have studied these models using several renormalization-group techniques as well as physical arguments. All our calculations and results confirm that focusing on the transverse degrees of freedom (model III) gives the main physics for this problem. Indeed we have shown explicitly that the more complete model II leads to the same transverse physics as model III.

At zero temperature we have explicitly demonstrated that the physics of the moving glass is governed by a nontrivial attractive disordered fixed point. Using the RG, we have explicitly demonstrated the existence of the transverse critical force predicted in Ref. 73, which is related to the nonanalytic behavior of the renormalized disorder correlator at the fixed point. Its actual value, computed from the RG coincides with the estimate given in Ref. 73 based on the existence of a dynamical Larkin length R_c^y. We have also found that at $T=0$ no temperature is generated because perfect time periodicity is maintained. A static random force is also generated both along and perpendicular to the directions of motion. As a consequence relative displacements in both the x and y directions grow logarithmically in $d=3$, but algebraically in $d=2$. Thus in $d=3$ at weak disorder or at large velocity, the moving glass retains quasilong range order and divergent Bragg peaks. Since the decay of translational order is very slow in $d=3$ we predict that a glassy moving structure with quasi-long-range order and perfect topological order in all directions exists: *the moving Bragg glass.* The determination of its physical properties is the main result of this paper. This phase is the natural continuation to nonzero velocities of the static Bragg glass.

We have investigated the effect of a nonzero initial temperature. We found that the moving Bragg glass survives at finite temperature as a phase distinct from a perfect crystal and with properties continuously related to the zero-temperature moving Bragg glass. At low temperature the moving Bragg glass still exhibits highly nonlinear transverse velocity-transverse force characteristics with an ''effective transverse critical current'' (in the same sense as for the longitudinal critical current). At $T>0$ the FRG calculation indicates that the asymptotic behavior is linear but with a strongly suppressed transverse mobility at low temperature.

The existence of elastic channels provides a precise way to look at the problem of generation of dislocations in moving structures. The natural transition is now a decoupling of the channels with dislocations decoupling the adjacent layers. It is indeed easier to decouple the channels via shear deformations than to destroy the channel structure altogether. This leads to expect another moving glass phase which keeps a periodicity along y, which has been termed *moving transverse glass.* Since it retains a periodicity along the direction perpendicular to motion it shares the properties of moving glasses, and in particular it exhibits a nonzero transverse critical force at $T=0$.

We have given predictions for the phase diagram of moving systems. It shows that the existence of the Bragg glass phase in the statics has profound implications on the dynamical phase diagram as well. Indeed it is natural to connect continuously the static Bragg glass (at $v=0$) to the moving Bragg glass (at $v>0$). Thus there should be a wide range of velocities (down from the creep region to the fast moving region) where effects associated with transverse periodicity (such as the transverse critical force) should be observed. We have analyzed the crossovers between the Bragg glass properties and the moving glass properties in the region where the velocity is not large.

Further experimental consequences should be investigated in details for vortex systems in motion. A direct measurement of the transverse I-V characteristics at low temperature would be of great interest. But consequences for the phase diagrams should be explored too. It was predicted[45] that the static Bragg glass should undergo a transition into an amorphous glassy state upon increase of disorder. As discussed previously[47] the field-induced transition observed in many experiments is the likely candidate for such a transition. A similar prediction can be made in the dynamics. At fixed velocity the moving Bragg glass should melt, upon increasing the temperature, into a moving liquid at a temperature smaller than the static system. Since the moving Bragg glass is topologically ordered this transition is likely to be first order. Upon an increase of the disorder (or equivalently the magnetic field for vortex systems), at fixed velocity, the moving Bragg glass should experience a transition into an amorphous moving phase. However, unless the velocity is small, since the effective disorder is smaller in moving systems this transition should occur at a higher value of disorder (or magnetic field) than for the statics. A detailed investigation of these transitions may help understand the nature of the high-field pinned phase. Indeed the nature of the transition away from the Bragg glass may change once the system moves if the moving amorphous phase is different from the static amorphous phase. If the ''vortex-glass'' phase[36,37] exists at all in the statics in $d=3$, one may expect that it would not survive as smoothly as the Bragg glass once the system is set in motion. Finally, it would be interesting to investigate whether the anomalous response to transverse forces could have an impact on the anomalous Hall angle. As we have discussed, other experimental systems, such as Wigner crystals, seem to be a promising arena to investigate the physics presented here. The effects predicted here also provide a strong motivation to reexamine other moving systems such as double Q or triple Q CDW's.

Another direct experimental consequence, in the case of correlated disorder is that one should observe a ''moving Bose glass.'' Static columnar disorder in vortex systems is strong but at large velocity one should expect that the effective disorder becomes weaker. Thus the Bose glass driven at low temperature should have interesting properties such as discussed here. The resulting moving Bose glass should exhibit a transverse critical force and retain a transverse Meissner effect in the direction perpendicular to motion.

The properties of periodic driven systems discussed in this paper also suggest many other directions of investigation. As for the statics one outstanding problem is to treat properly dislocations in the moving glass system. A controlled calculation may seem out of reach, but on the other hand, the existence of elastic channels suggests a precise way to look at the problem of generation of dislocations in moving structures and may provide a starting point.[119] Solving this issue starting from large velocity is already a formidable task, but could help us to understand what happens close to the threshold. Indeed here again only simple cases, inspired from the manifold or CDW with scalar displacements and no transverse periodicity, have been considered previously. As in the statics it is possible that the physics is modified in a quite surprising way, and certainly all the issues about critical behavior close to threshold, dynamics reordering, and elastic to plastic motion transitions, have to be reconsidered. These issues are of major theoretical concern but also of large practical importance. Finally we note that though model I remains to be tackled in order to reach a complete description of the lattice elastic flow, we have shown that the extra linear terms do not seem to change drastically the main features of perturbation theory. The KPZ terms remain to be treated, but an interesting possibility would be that again because of periodicity their effect would be weaker than expected.

Another interesting issue is to understand to which extent a moving, or more generally a nonpotential system can be glassy. This concept may seem doomed from the start since one could conclude that the constant dissipation in the system would tend to kill glassy properties. However there too the situation may be more subtle and leave room for unexpected behavior. We have proposed the moving glass as a first physical realization of a ''dissipative glass,'' i.e., a glass with a constant dissipation rate in the steady state. Other realizations of nonpotential glassy systems have been studied, such as in spin systems[120] or for elastic manifolds in random flows such as polymers.[84,85]

It is important to characterize these glassy effects in driven systems. Too close analogy with glassiness in the statics could be misleading. As we have discussed it is interesting to check whether the presence of a transverse critical force leads to history dependence effects. The role of the temperature in moving systems and its relation to entropy production remains puzzling. It is natural to expect, as in other related nonpotential systems[120,84,85] that the absence of the fluctuation dissipation theorem leads to a generation of a temperature. This is of course what happens in the RG approach presented here. This heating effect however is very different from the ''shaking temperature'' (since it disappears at $T=0$ in the moving Bragg glass) and rather is likely to be related to the entropy production. Hopefully the methods introduced here should allow us to understand this relation better. We have found within the FRG that at finite temperature the physics is controlled by a nontrivial finite-temperature moving glass fixed point. This result is strengthened by the fact that another nonzero $T>0$ fixed point has also been obtained in the problem of randomly driven polymers, a problem which does have a dissipative glassy phase.[85] The problem of understanding dissipative glassy systems is also related to the study of general *non-Hermitian* random operators. Indeed nonpotential dynamical problems (including, e.g., the moving glass) are described by a Fokker-Planck operator whose spectrum is not necessarily real (by contrast with potential problems which are purely relaxational). These Fokker-Planck problems with complex spectrum (which could be called ''dynamical non-Hermitian quantum mechanics'') are related to problems which have received a renewed interest recently (such as vortex lines with tilted columnar defects,[121,79] spin relaxation in random magnetic fields,[122] and diffusion of particles and polymers in random flows.[123,124,85] Exploring this connection, as well as the very interesting question of the classification of these glasses and the study of their physical properties is still largely open.

Finally other fascinating open questions remain. The $T=0$ moving Bragg glass fixed point, at least within model II, is a time-periodic state. The general question of the stability of periodic attractors towards chaotic motion is still very much open. It is related to problems of time coupling and decoupling in nonlinear dynamics, such as synchronization of oscillators in Josephson arrays, which has been studied extensively recently[125–130] or synchronization by disorder.[131,132] Indeed the existence of channels where particles follow each other may provide the equivalent of the no-crossing property which allowed to demonstrate the temporal order for CDW's.[81] The relation between instability to chaos and possible nonperturbative generation of a temperature is also intriguing. Indeed one important issue is whether dislocations when present do generate an additional temperature or chaos. Finally, the general question of dynamical elastic instabilities is also related to recently investigated questions about solid friction. It would be interesting to investigate in solid friction quantities analogous to the transverse response and the transverse critical force once the solid is in motion. By analogy with the effects discussed here, one may expect the existence of a transverse threshold in some regime of solid friction.

Note added in proof. Recently, G. D'anna *et al.* (unpublished) gave experimental evidence of a transverse critical current in YBCO crystals.

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APPENDIX A: FIRST-ORDER PERTURBATION THEORY

1. General analysis

In this appendix we compute the effective action $\Gamma[u,\hat{u}]$ to first order in the interacting part $S_{\rm int}$ using the standard formula (39). We remain as general as possible, in order to treat several problems and cases simultaneously, and specialize only at the end to particular cases. We thus choose the following disorder term [as it appears in the MSR action (37)]

$$S_{\rm int}=-\frac{1}{2}\int_{rr'tt'}(i\hat{u}^{\alpha}_{rt})(i\hat{u}^{\beta}_{r't'})\Delta^{\alpha\beta}[u_{rt}-u_{r't'}+v(t-t'),r-r']. \tag{A1}$$

This allows us to treat several problems. It allows us to treat short-range correlated disorder keeping the cutoff dependence which allows to generate the extra linear and KPZ terms. It also allows to treat correlated disorder. The disorder correlator is chosen as

$$\Delta^{\alpha\beta}[u_{rt}-u_{r't'}+v(t-t'),r-r'] =\sum_K \Delta^{\alpha\beta}_K(r-r')e^{-iK\cdot[u_{rt}-u_{r't'}+v(t-t')]}, \tag{A2}$$

where the symbol Σ_K denotes a discrete sum of lattice harmonics for a *periodic* problem and a continuous sum $\Sigma_K \equiv \int d^dk/(2\pi)^d$ for a *nonperiodic* manifold. For the model (29) one has $\Delta^{\alpha\beta}_K(r)=K_\alpha K_\beta g(r)e^{iKr}$. This is the *bare* starting correlator (it is itself corrected and does not remain under this form, see below). In the case of model II (30), i.e., the continuum limit of the above model, one can replace $g(r-r')e^{iK(r-r')}\to g_K\delta(r-r')$. This is because the scale at which the displacement field varies is large compared to the correlation length of the disorder (see discussion of Sec. IV B). Since we know that nonpotential terms may be generated under RG (from FDT violation) we rather study from the start the continuous model:

$$\Delta^{\alpha\beta}_K(r-r')\to\Delta^{\alpha\beta}_K\delta(r-r'). \tag{A3}$$

We work at finite T and also specify to $T=0$. With these definitions one finds from Eq. (39):

$$\Gamma[u,\hat{u}]=S_0+\int_{rt}(i\hat{u}^{\alpha}_{rt})\Sigma^{\alpha}_{rt}[u]-\frac{1}{2}\int_{rr'tt'}(i\hat{u}^{\alpha}_{rt}) \times(i\hat{u}^{\beta}_{r't'})D^{\alpha\beta}_{rt,r't'}[u] \tag{A4}$$

with

$$\Sigma^{\alpha}_{rt}[u]=-\int_{r't'}\sum_K R^{\gamma\beta}_{rt,r't'}(-iK_\gamma)\Delta^{\alpha\beta}_K(r-r') \times e^{-iK\cdot[u_{rt}-u_{r't'}+v(t-t')]}e^{-(1/2)K\cdot B_{rt,r't'}\cdot K}, \tag{A5}$$

$$D^{\alpha\beta}_{rt,r't'}[u]=\sum_K \Delta^{\alpha\beta}_K(r-r')e^{-iK\cdot[u_{rt}-u_{r't'}+v(t-t')]} \times e^{-(1/2)K\cdot B_{rr',tt'}\cdot K}. \tag{A6}$$

We have used that $\Delta^{\beta\alpha}_{-K}(r'-r)=\Delta^{\alpha\beta}_K(r-r')$ which comes from simple relabeling in Eq. (A1). One can now use time and space translational invariance and express the above under the form, e.g., $D^{\alpha\beta}_{rt,r't'}[u]=D^{\alpha\beta}(u_{rt}-u_{r't'},t-t',r-r')$. At $T=0$ it reads simply

$$\Gamma[u,\hat{u}]=S[u,\hat{u}]-\int_{rr'tt'}(i\hat{u}^{\alpha}_{rt})R^{\gamma\beta}_{rt,r't'} \times\Delta^{\alpha\beta;\gamma}[u_{rt}-u_{r't'}+v(t-t'),r-r']. \tag{A7}$$

Thus to this order the effect of temperature amounts to replace everywhere formally:

$$\Delta_K^{\alpha\delta}(r-r')\rightarrow\Delta_K^{\alpha\delta}(r-r')e^{-(1/2)K\cdot B_{r-r',t-t'}\cdot K}. \quad (A8)$$

Temperature has thus two important effects (i) it generates a time dependence and (ii) it smoothes out the disorder. One can already see that there are two important different cases. Either (high enough dimension) there is a time persistent part to the correlator $\lim_{t\to\infty} B_{r,t}=B_\infty<+\infty$, in which case the disorder is smoothed out, or $\lim_{t\to\infty} B_{r,t}=\infty$ and the disorder gets smaller at larger scales (low dimension). Another interpretation of the above result is that the corrected equation of motion includes (i) a new, nonrandom, time retarded force $\Sigma_\alpha[u]$ [see Eq. (A5)] and (ii) a corrected pinning force which has an extra time dependence:

$$(R^{-1})^{\alpha\beta}_{rtr't'}u^\beta_{r't'}=-\Sigma^\alpha_{rt}[u]+\widetilde{F}_\alpha(r,t,u_{rt})+f_\alpha-\eta_{\alpha\beta}v_\beta, \quad (A9)$$

where the new (time-dependent) pinning force correlator is $D^{\alpha\beta}(u,r,t)$. We now separate the relevant contributions in this complicated nonlinear, equation of motion. We also define:

$$\begin{aligned}\Sigma^{\alpha\beta}(u_{rt}-u_{r't'},t-t',r-r')&=\frac{\delta\Sigma^\alpha_{rt}[u]}{\delta u^\beta_{r't'}}\\&=R^{\gamma\delta}_{rt,r't'}\bigg(\langle\Delta^{\alpha\delta;\gamma\beta}(u_{rt}-u_{r't'})\rangle\\&\quad-\delta_{tt'}\int_{t''}R^{\gamma\delta}_{rt,r't''}\langle\Delta^{\alpha\delta;\gamma\beta}(u_{rt}-u_{r't''})\rangle\bigg).\end{aligned} \quad (A10)$$

On the functional expression (A1) we can identify the corrections to various terms. The first thing to do is to obtain the corrected response and correlation functions. For that, one simply has to expand Eq. (A4) in powers of u and $\hat{u}$ up to quadratic order. This yields Γ^1 and Γ^2, respectively, the linear and quadratic part. The linear term proportional to $\hat{u}$ gives the correction to the force $\delta f^\alpha=-\Sigma^\alpha[u=0]$.

The linear term in the effective action in Eqs. (36) and (37) becomes

$$-\int_{rt}(i\hat{u}^\alpha_{rt})[f_\alpha-\eta_{\alpha\beta}v_\beta+\delta f_\alpha(v)], \quad (A11)$$

where the correction to the force is given by

$$\delta f_\alpha(v)=\int_{r,t}R^{\gamma\beta}_{r,t}\sum_K(-iK_\gamma)\Delta_K^{\alpha\beta}(r)e^{-iK\cdot vt}e^{-(1/2)K\cdot B_{r,t}\cdot K}. \quad (A12)$$

At $T=0$ this can also be written as

$$\delta f_\alpha(v)=\int d\tau dr R^{\gamma\delta}(r,\tau)\Delta^{\alpha\delta;\gamma}(v\tau,r). \quad (A13)$$

The quadratic part of the effective action reads

$$\begin{aligned}\Gamma^2[u,\hat{u}]=S_0^2&+\int_{rr'tt'}(i\hat{u}^\alpha_{rt})\Sigma^{\alpha\beta}(0,t-t',r-r')u^\beta_{r't'}\\&-\frac{1}{2}\int_{rr'tt'}(i\hat{u}^\alpha_{rt})(i\hat{u}^\beta_{r't'})D^{\alpha\beta}(0,t-t',r-r').\end{aligned} \quad (A14)$$

The correction to the response function is thus

$$(\delta R^{-1})^{\alpha\beta}(q,\omega)=-\int_{rt}(1-e^{i(q\cdot r+\omega t)})R^{\gamma\delta}(r,t)K_\gamma K_\beta\Delta_K^{\alpha\delta}(r)e^{-iK\cdot vt}e^{-(1/2)K\cdot B_{r,t}\cdot K}. \quad (A15)$$

It is useful to perform the small q,ω expansion to obtain the corrections to the friction coefficient, the linear terms, and the elastic matrix. From the general equation of motion (120) one has

$$(\delta R^{-1})^{\alpha\beta}(q,\omega)=(i\omega)\delta\eta_{\alpha\beta}+(iq_\rho)\delta L^\rho_{\alpha\beta}+q_\rho q_\sigma\delta C^{\rho\sigma}_{\alpha\beta}+\text{h.o.t.} \quad (A16)$$

One finds

$$\delta\eta_{\alpha\beta}=\int_{rt}tR^{\gamma\delta}_{rt}K_\gamma K_\beta\Delta_K^{\alpha\delta}(r)e^{-iK\cdot vt}e^{-(1/2)K\cdot B_{r,t}\cdot K}, \quad (A17)$$

$$\delta L^\rho_{\alpha\beta}=\int_{rt}r^\rho R^{\gamma\delta}_{rt}K_\gamma K_\beta\Delta_K^{\alpha\delta}(r)e^{-iK\cdot vt}e^{-(1/2)K\cdot B_{r,t}\cdot K}, \quad (A18)$$

$$\delta C^{\rho\sigma}_{\alpha\beta}=\frac{1}{2}\int_{rt}r^\rho r^\sigma R^{\gamma\delta}_{rt}K_\gamma K_\beta\Delta_K^{\alpha\delta}(r)e^{-iK\cdot vt}e^{-(1/2)K\cdot B_{r,t}\cdot K}. \quad (A19)$$

This is a very general expression which we particularize to special cases below. At $T=0$ one has

$$\delta\eta^{\alpha\beta}(v)=-\int d\tau dr\,\tau R^{\gamma\delta}(r,\tau)\Delta^{\alpha\delta;\gamma\beta}(v\tau,r). \quad (A20)$$

Note that $d\delta f_\alpha(v)/dv_\beta=-\delta\eta_{\alpha\beta}(v)$. In the limit $v\to0$ one finds $\delta f_\alpha(v)\sim-\delta\eta_{\alpha\beta}v_\beta$ using an integration by part, provided the function Δ is analytic. A nonanalytic Δ yields a critical force (see Sec. VI).

The complete $\hat{u}\hat{u}$ term in the quadratic part of the effective action is $D^{\alpha\beta}(0,t-t',r-r')$. It allows us to compute the corrected correlation functions using the corrected response function:

$$\langle u^{\alpha}_{-q,-\omega}u^{\beta}_{q,\omega}\rangle=\{R^{(1)}(\omega,q)\cdot[2\eta T+D(0,\omega,q)]\cdot R^{(1)}(-\omega,-q)\}_{\alpha\beta}, \quad \text{(A21)}$$

$$R^{(1)}(\omega,q)=\frac{1}{R^{-1}(\omega,q)+\delta R^{-1}(\omega,q)}, \quad \text{(A22)}$$

where $\cdot$ denotes the matrix multiplication of indices. Examining the large time and space behaviors one finds that there are two important corrections.

(i) A correction to the temperature (from the equal-time piece):

$$\delta(\eta T)_{\alpha\beta}=\frac{1}{2}\int_{rt}[D^{\alpha\beta}(0,t,r)-D^{\alpha\beta}(0,t=+\infty,r)], \quad \text{(A23)}$$

(ii) a correction to a static random force. It is identified as the time persistent part of the disorder:

$$D^{\alpha\beta}(0)=\lim_{t\to\infty}\int_{rt}D^{\alpha\beta}(0,t=+\infty,r). \quad \text{(A24)}$$

It yields to a static part $\delta(\omega)$ in the displacement correlation. Note however that there is additional important correction to the *nonlinear* part of the disorder, which we now identify.

Finally we turn to the nonlinear terms in the effective action and the correction to the disorder and generation of KPZ terms. It is important to follow not just the random force but the complete nonlinear static part of the disorder term. It is identified as

$$\lim_{t-t'\to\infty} D^{\alpha\beta}(u_{rt}-u_{r't'},t-t',r-r') = \lim_{t-t'\to\infty}\sum_K \Delta_K^{\alpha\beta}(r-r')e^{-iK\cdot[u_{rt}-u_{r't'}+v(t-t')]}\times e^{-(1/2)K\cdot B_{r-r',t-t'}\cdot K}. \quad \text{(A25)}$$

Finally, the nonlinear KPZ terms can be easily seen to be generated already to this order. Expanding $\Sigma^{\alpha}_{rt}[u]$ to second order in the field u one finds

$$\delta K^{\rho\sigma}_{\alpha\beta\gamma}=\frac{1}{2}\int_{rt}r^{\rho}r^{\sigma}R^{\epsilon\delta}_{rt}K_{\beta}K_{\gamma}(iK_{\epsilon})\times\Delta_K^{\alpha\delta}(r)e^{-iK\cdot vt}e^{-(1/2)K\cdot B_{r,t}\cdot K}. \quad \text{(A26)}$$

2. Explicit evaluation of the corrections in specific models

(i) *Model II:* We first study the continuous model II valid in the elastic limit. It is obtained by the substitution (A3). One finds first that

$$\delta L^{\rho}_{\alpha\beta}=0, \quad \delta C^{\rho\sigma}_{\alpha\beta}=0, \quad \delta K^{\rho\sigma}_{\alpha\beta\gamma}=0, \quad \text{(A27)}$$

i.e., that no linear, KPZ terms are generated, and that there is no correction to the static part of the response function. These are consequences of the statistical tilt symmetry (see Sec. VI). The only corrections are

$$\delta f_{\alpha}(v)=\int_t R^{\gamma\beta}_{r=0,t}\sum_K(-iK_{\gamma})\Delta_K^{\alpha\beta}e^{-iK\cdot vt}e^{-(1/2)K\cdot B_{0,t}\cdot K}, \quad \text{(A28)}$$

$$\delta\eta_{\alpha\beta}=\int_{rt}tR^{\gamma\delta}_{r=0,t}K_{\gamma}K_{\beta}\Delta_K^{\alpha\delta}e^{-iK\cdot vt}e^{-(1/2)K\cdot B_{0,t}\cdot K}. \quad \text{(A29)}$$

For the bare problem one can further substitute $\Delta_K^{\alpha\delta}=K_{\alpha}K_{\beta}g_K$. Note the symmetries $\eta_{\alpha\beta}(-v)=\eta_{\alpha\beta}(v)$ and $\delta f_{\alpha}(-v)=-\delta f_{\alpha}(v)$. Let us further specify to the problem of a periodic lattice. The bare perturbation theory (i.e., starting from $\eta_{\alpha\beta}=\eta_0$) gives Eqs. (41) and (42) in the text.

One can also look at the dressed perturbation theory (i.e., adding from the start the terms which are generated). Suppose the velocity along a principal lattice direction x. Then the symmetry $y\to-y$ ensures in Eq. (A28) that to all orders $\eta_{xy}=0$. However η_{xx} and η_{yy} are in general different. Inversion of the response tensor then leads to (for $d=2$):

$$\frac{P^T(q)_{\alpha\beta}}{c_T(q)+i\eta_{yy}\omega+ivq_x}+\frac{P^L(q)_{\alpha\beta}}{c_L(q)+i\eta_{yy}\omega+ivq_x}+\frac{(\eta_{yy}-\eta_{xx})i\omega e_x^{\alpha}e_x^{\beta}}{c_T(q)+i\eta_{xx}\omega+ivq_x}. \quad \text{(A30)}$$

The correlation functions can also be computed using that here $D^{\alpha\beta}(0,t-t',r-r')=D^{\alpha\beta}(t-t')\delta(r-r')$. Starting with the bare action and expanding to lowest order in disorder yields, at $T=0$:

$$\langle u^{\alpha}_{-q,-\omega}u^{\beta}_{q,\omega}\rangle=\sum_K\sum_{I=L,T,I'=L,T}\int_{q,\mathrm{BZ}}(2\pi)\delta(\omega-K\cdot v)g_K K_{\gamma}K_{\delta}\frac{P^{I}_{\alpha\gamma}(q)}{c_I(q)+i\eta_0(\omega+v\cdot q)}\frac{P^{I'}_{\beta\delta}(q)}{c'_I(q)-i\eta_0(\omega+v\cdot q)}, \quad \text{(A31)}$$

which is a sum of oscillating functions, plus a static one. At $T>0$ the expression is more complicated. It reads

$$\delta\langle u^{\alpha}_{-q,-\omega}u^{\beta}_{q,\omega}\rangle=\sum_K \sum_{I=L,T,I'=L,T}\int_{q,\mathrm{BZ}}\frac{P^I_{\alpha\gamma}(q)}{c_I(q)+i\eta_0(\omega+v\cdot q)}\frac{P^{I'}_{\beta\delta}(q)}{c'_I(q)-i\eta_0(\omega+v\cdot q)}D_{\gamma\delta}(\omega)+\frac{P^I_{\alpha\gamma}(q)}{c_I(q)+i\eta_0(\omega+v\cdot q)}\frac{P^{I'}_{\beta\delta}(q)}{c'_I(q)-i\eta_0(\omega+v\cdot q)}2i\eta T(\omega+v\cdot q)\delta\eta_{\gamma\delta} \quad \text{(A32)}$$

with $D_{\gamma\delta}(\omega)=\int_t g_K K_\gamma K_\delta e^{-(1/2)K\cdot B_{0,t}\cdot K}e^{i\omega t-iK\cdot vt}$.

(ii) *Model I:* In the full model I one can use Eq. (A17) to compute the new, linear term which is generated. If one studies the bare disorder one may substitute $\Delta^{\alpha\beta}_K(r)\to K_\alpha K_\beta g(r)e^{iKr}$. When v is along a principal lattice direction one finds by symmetry that only $L^{yy}_x=v_1$, $L^{yx}_x=v_2$, $L^{xx}_x=v_3$, $L^{xy}_y=v_4$ are nonzero. Note that by symmetry $y\to-y$, $K_y\to-K_y$ one has $\delta\eta_{xy}=0$, $\delta\eta_{yx}=0$, $\delta f_y=0$. Using symmetries one finds

$$L^{yy}_x=v_1=\eta v-\sum_K\int_{\tau r}xK^2K_y^2R(r,\tau)g(r)e^{iK\cdot(r-v\tau)}, \quad \text{(A33)}$$

$$L^{xy}_y=v_4=L^{yx}_y=v_2=-\sum_K\int_{\tau r}yK^2K_yK_xR(r,\tau)\times g(r)e^{iK\cdot(r-v\tau)}, \quad \text{(A34)}$$

$$L^{xx}_x=v_3=\eta v-\sum_K\int_{\tau r}xK^2K_x^2R(r,\tau)g(r)e^{iK\cdot(r-v\tau)}. \quad \text{(A35)}$$

Note that $v_2=v_4$ exactly and that to lowest order in v one has $v_2=v_3$ and $\eta+\delta\eta_{xx}(v=0)=\eta+\delta\eta_{yy}(v=0)$. The corrections to η read

$$\delta\eta_{\alpha\alpha}(v)=\sum_K K^2K_\alpha^2\int d\tau dr\,\tau g(r)R(r,\tau)e^{iK\cdot(r-v\tau)} \quad \text{(A36)}$$

with $\alpha=x,y$. Note that $v\eta(v)$ has a maximum as observed in solid friction (dynamical friction smaller than static one).

APPENDIX B: DYNAMICAL EFFECTIVE ACTION TO SECOND ORDER AND ANALYSIS OF DIVERGENCES

In this appendix we obtain the perturbative expression of the effective dynamical action to second order in disorder. At each step we remain as general as possible so that our expressions can be applied to study a large class of models and situations. Then we study particular situations and identify the terms which correct the bare disorder by performing a short distance or time expansion. We focus mainly on divergences occurring near $d=4$ (for $v=0$) and $d=3$ (for $v>0$). Note that the operators are local in r but nonlocal in time, which makes the expansion more involved. We will use the fact that the calculation performed in Appendix C of Ref. 94 (for $d=2$ and $v=0$) has similarities with the present, in order to skip some details. A detailed version of the present calculation can be found in Ref. 106.

We study here *a priori* both the periodic manifold case or the nonperiodic one. The only difference is that in the periodic case one has discrete Σ_K to be replaced by $\int_K$ in the continuous case.

The effective action to second order in the interaction term is[114]

$$-2\Gamma^{(2)}[W]=\langle S_{\mathrm{int}}[W+\delta W]^2\rangle_{\delta W}-\langle S_{\mathrm{int}}[W+\delta W]\rangle^2_{\delta W}-\left\langle\frac{\delta S_{\mathrm{int}}[W+\delta W]}{\delta W}\right\rangle_{\delta W}G\left\langle\frac{\delta S_{\mathrm{int}}[W+\delta W]}{\delta W}\right\rangle_{\delta W} \quad \text{(B1)}$$

with $W=(u,\hat{u})$ and $\delta W=(\delta u,\delta\hat{u})$ and a Gaussian average over δW is performed using the bare quadratic action. The last term merely ensures that all one-particle reducible diagrams be absent. A tedious calculation then yields for the $i\hat{u}i\hat{u}$ term in the effective action, a formula identical to Eq. (C3) of Ref. 94 with the replacement $u_t-u_{t'}\to u_{rt}-u_{rt'}+v(t-t')\equiv U_{rtt'}$, since here we are dealing with a situation where $v>0$. In Eq. (C3) of Ref. 94 the symbol $\langle F[u]\rangle$ means $\langle F[u+\delta u]\rangle_{\delta u}$ and $\langle..\rangle_c$ denotes a connected average between the vertices, i.e., $\langle F[u_1]G[u_2]\rangle_c=\langle F[u_1]G[u_2]\rangle-\langle F[u_1]\rangle\langle G[u_2]\rangle$. Note that simplifications occur in the particular case $T=0$ since the connected terms then vanish identically, and one can also drop the averages $\langle F[u]\rangle=F(u)$. Using the assumption of time and space translational invariance one finds

$$\Gamma=-\frac{1}{2}\int_{rr't_1t_2}(i\hat{u}^{\alpha}_{rt_1})(i\hat{u}^{\beta}_{r+r',t_2})\delta\Delta^{\alpha\beta}_{r'} \quad \text{(B2)}$$

as a sum of four terms: $\delta\Delta=\Sigma_{i=1,4}\delta\Delta^{(i)}_{\mathrm{eff}}$:

$$\delta\Delta^{(1)}_{r'}=2R^{\delta\lambda}(\tau_2,0)R^{\gamma\rho}(\tau_1,r')\langle\Delta_{\beta\lambda;\gamma\delta}(U_{r+r',t_2,t_2-\tau_2})\times[\Delta_{\alpha\rho}(U_{r,t_1,t_2-\tau_1})-\Delta_{\alpha\rho}(U_{r,t_1,t_2-\tau_1-\tau_2})]\rangle_c,$$

$$\delta\Delta^{(2)}_{r''}=\frac{1}{2}\delta(r'')R^{\gamma\rho}(\tau,-r')R^{\delta\lambda}(\tau',-r')\langle\Delta_{\alpha\beta;\gamma\delta}(U_{r,t_1,t_2})\times[\Delta_{\rho\lambda}(U_{r+r',t_1-\tau,t_1-\tau'})+\Delta_{\rho\lambda}(U_{r+r',t_2-\tau,t_2-\tau'})-\Delta_{\rho\lambda}(U_{r+r',t_1-\tau,t_2-\tau'})-\Delta_{\rho\lambda}(U_{r+r',t_2-\tau,t_1-\tau'})]\rangle,$$

$$\delta\Delta^{(3)}_{r'}=R^{\gamma\rho}(\tau_2,r')R^{\delta\lambda}(\tau_1,-r')\times[\langle\Delta_{\alpha\rho;\delta}(U_{r,t_1,t_2-\tau_2})\Delta_{\beta\lambda;\gamma}(U_{r+r',t_2,t_1-\tau_1})\rangle-\langle\Delta_{\alpha\rho;\delta}(U_{r,t_1,t_1-\tau_1-\tau_2})\Delta_{\beta\lambda;\gamma}(U_{r+r',t_2,t_1-\tau_1})\rangle$$

$$-\Delta_{\alpha\rho;\delta}(U_{r,t_1,t_2-\tau_2})\Delta_{\beta\lambda;\gamma}(U_{r+r',t_2,t_2-\tau_1-\tau_2})\rangle],$$

$$\delta\Delta^{(4)}_{r''}=\delta(r'')R^{\gamma\rho}(\tau_2,0)R^{\delta\lambda}(\tau_1,-r')\langle\Delta_{\alpha\beta;\delta}(U_{r,t_1,t_2}) \times[\Delta_{\lambda\rho;\gamma}(U_{r+r',t_1-\tau_1,t_1-\tau_1-\tau_2}) -\Delta_{\lambda\rho;\gamma}(U_{r+r',t_2-\tau_1,t_2-\tau_1-\tau_2})]\rangle_c, \quad \text{(B3)}$$

where some terms have vanished from the Ito time discretization property $R^{\gamma\beta_1}_{t'_2t'_1}R^{\delta\beta_2}_{t'_1t'_2}=0$.

We have written these terms in that form for simplicity, but one must keep in mind that in addition they must be symmetrized under $\alpha\rightarrow\beta$ and $r\rightarrow -r$ when necessary. Up to now this is very general. Note that the expression of the second order correction to the kinetic term is given in Ref. 106. We now consider several cases.

1. Static degrees of freedom at zero temperature

In this subsection we first set $T=0$, and thus $\delta\Delta^{(1)}=\delta\Delta^{(4)}=0$ and other averages can be dropped. Here we also only study static disorder and we thus drop the $v(t-t')$ terms thus setting $U_{rtt'}=u_{rt}-u_{rt'}$ in all the above formulas. The bare response function $R^{\alpha\beta}(\tau,r)$ remains however arbitrary. In the moving lattice problem this amounts to restricting ourselves to the modes $K_x=0$, which are of interest for studying the *transverse components* $u\cdot v=0$ that see only a static disorder (assuming they can be decoupled) and v still appears in the response function. Since we keep u as a vector with arbitrary number of components, the equations that we obtain can be applied to other problems with static disorder (e.g., the usual manifold case $v=0$, periodic or not, nonpotential problems etc.). The only remaining terms are $\delta\Delta^{(2)}_{r''}$ and $\delta\Delta^{(3)}_{r'}$ in Eq. (B3). It is then easy to perform a short time, short distance operator expansion in the variables r',τ_1,τ_2. This yields, up to higher-order irrelevant gradient terms, the following total correction to the pinning force correlator:

$$\delta\Delta_{\alpha\beta}(u)=\Delta_{\alpha\beta;\gamma\delta}(u)[\Delta_{\alpha'\beta'}(0)-\Delta_{\alpha'\beta'}(u)] \times\int_r G_{\gamma\alpha'}(r)G_{\delta\beta'}(r)-\Delta_{\alpha\alpha';\delta}(u)\Delta_{\beta\beta';\gamma}(u) \times\int_r G_{\gamma\alpha'}(r)G_{\delta\beta'}(-r), \quad \text{(B4)}$$

where we have defined the static response $G_{\alpha\beta}(r)=\int_0^\infty d\tau R_{\alpha\beta}(\tau,r)$. Note that this formula is valid for a large class of models. It does *not* suppose for instance that the random force correlator is the second derivative of a random potential. It is important to note that the condition that the random force is the gradient of a potential, i.e., $\Delta_{\alpha\beta}(u)=-\partial_\alpha\partial_\beta R(u)$ where $R(u)$ is the correlator of the random potential (not to be confused with the response function) remains true only when $G(r)=G(-r)$. Indeed, in that case, assuming the symmetry that $G_{\alpha\beta}(r)=G_{\beta\alpha}(r)$, one finds

$$\delta R(u)=\left[\frac{1}{2}R_{;\gamma\delta}(u)R_{;\alpha'\beta'}(u)-R_{;\gamma\delta}(u)R_{;\alpha'\beta'}(0)\right] \times\int_r G_{\gamma\alpha'}(r)G_{\delta\beta'}(r). \quad \text{(B5)}$$

If $G(r)\neq G(-r)$ a nonpotential part is generated to the disorder. If u has only an $N=1$ component it remains a total derivative. If we study models with $N>1$ component fields and a non-FDT response function we generate nonpotential disorder. From Eq. (B4) a generalized FRG equation can be derived, which depends on the divergences contained in the response function. A special case corresponds to $v>0$ and isotropic response (the simplest N component generalization of the moving glass equation (5). Then one finds

$$\frac{d\Delta_{\alpha\beta}(u)}{dl}=\epsilon\Delta_{\alpha\beta}(u)+\zeta u_\gamma\Delta_{\alpha\beta,\gamma}(u)+C\Delta_{\alpha\beta;\gamma\delta}(u) \times[\Delta_{\gamma\delta}(0)-\Delta_{\gamma\delta}(u)], \quad \text{(B6)}$$

where C is a numerical constant. The temperature can be added. In the isotropic case it simply produces an extra term $-T\Delta_{\alpha\beta,\gamma\gamma}(u)$ in the above equation.

Higher derivative terms have been neglected. In the periodic case, the annihilation term[94] produces a gradient random force term (the so-called Cardy-Ostlund term) relevant in the statics in $d\leqslant 2$, but unimportant here.

2. Full dynamical problem at zero temperature

In this subsection we still set $T=0$ leading to the same simplifications as in the previous subsection, but we keep the $v(t-t')$ terms. So we are studying the full dynamical problem of a driven lattice (i.e., with transverse and longitudinal displacement fields). The effective action is the sum of the following two terms:

$$\Gamma_1=-\frac{1}{4}\int_{rr'tt'\tau\tau'}(i\hat{u}^\alpha_{rt})(i\hat{u}^\beta_{rt'})R^{\gamma\rho}(\tau,r')R^{\delta\lambda}(\tau',r')\Delta_{\alpha\beta;\gamma\delta}[u_{rt}-u_{rt'}+v(t-t')][\Delta_{\rho\lambda}(u_{r-r',t-\tau}-u_{r-r',t-\tau'}+v(\tau'-\tau)) +\Delta_{\rho\lambda}(u_{r-r',t'-\tau}-u_{r-r',t'-\tau'}+v(\tau'-\tau))-\Delta_{\rho\lambda}(u_{r-r',t-\tau}-u_{r-r',t'-\tau'}+v(t-t'+\tau'-\tau)) -\Delta_{\rho\lambda}(u_{r-r,t'-\tau}-u_{r-r',t-\tau'}+v(t'-t+\tau'-\tau))],$$

$$\Gamma_2=-\frac{1}{2}(i\hat{u}^\alpha_{rt})(i\hat{u}^\beta_{r+r',t'})R^{\gamma\rho}(\tau,r')R^{\delta\lambda}(\tau',-r')\{\Delta_{\alpha\rho;\delta}(u_{r,t}-u_{r,t'-\tau}+v(t-t'+\tau)) \times[\Delta_{\beta\lambda;\gamma}(u_{r+r',t'}-u_{r+r',t-\tau'}+v(t'-t+\tau'))-\Delta_{\beta\lambda;\gamma}(u_{r+r',t'}-u_{r+r',t'-\tau-\tau'}+v(\tau+\tau'))] -\Delta_{\alpha\rho;\delta}(u_{r,t}-u_{r,t-\tau-\tau'}+v(\tau+\tau'))\Delta_{\beta\lambda;\gamma}(u_{r+r',t'}-u_{r+r',t-\tau'}+v(t'-t+\tau'))\}. \quad \text{(B7)}$$

We can now perform a short distance and time expansion and compute the correction to the random force correlator. Expressed as $\Delta_{\alpha\beta}(U)$, with $U=u-u'+v(t-t')$ it reads

$$\delta\Delta_{\alpha\beta}(U)=\int_{q,\tau\tau'}\Delta_{\alpha\beta;\gamma\delta}(U)R^{\gamma\rho}(\tau,q)R^{\delta\lambda}(\tau',-q)\left[\Delta_{\rho\lambda}(v(\tau'-\tau))-\frac{1}{2}[\Delta_{\rho\lambda}(U+v(\tau'-\tau))+\Delta_{\rho\lambda}(-U+v(\tau'-\tau))]\right]$$
$$+R^{\gamma\rho}(\tau,q)R^{\delta\lambda}(\tau',q)[\Delta_{\alpha\rho;\delta}(U+v\tau)(\Delta_{\beta\lambda;\gamma}(-U+v\tau'))-\Delta_{\beta\lambda;\gamma}(v(\tau+\tau'))$$
$$-\Delta_{\alpha\rho;\delta}(v(\tau+\tau'))\Delta_{\beta\lambda;\gamma}(-U+v\tau')]. \tag{B8}$$

It is also convenient to study the Fourier transform of the correlator $\Delta_{\alpha\beta}(U)=\Sigma_K\Delta_K^{\alpha\beta}e^{iKU}$ and to compute the correction to $\Delta_K^{\alpha\beta}$. We express it using the response function in $R^{\gamma\rho}(s=i\omega,q)$ spatial Fourier transform and time Laplace transform. It is the sum of two contributions and reads

$$\delta\Delta_K^{\alpha\beta}=-K_\gamma K_\delta\Delta_K^{\alpha\beta}\sum_{K'}\int_q\Delta_{K'}^{\rho\lambda}R^{\gamma\rho}(-ivK',-q)R^{\delta\lambda}(ivK',q) \tag{B9}$$
$$+\frac{1}{2}\sum_{K'}\int_q(K-K')_\gamma(K-K')_\delta\Delta_{K-K'}^{\alpha\beta}[\Delta_{K'}^{\rho\lambda}R^{\gamma\rho}(-ivK',-q)R^{\delta\lambda}(ivK',q)$$
$$+\Delta_{-K'}^{\rho\lambda}R^{\gamma\rho}(ivK',-q)R^{\delta\lambda}(-ivK',q)] \tag{B10}$$
$$-\sum_{K'}\int_q K'_\delta(K'-K)_\gamma\Delta_{K'}^{\alpha\rho}\Delta_{K'-K}^{\beta\lambda}R^{\gamma\rho}(ivK',q)R^{\delta\lambda}(iv(K'-K),q) \tag{B11}$$
$$+K_\delta\Delta_K^{\alpha\rho}\sum_{K'}\int_q K'_\gamma\Delta_{K'}^{\beta\lambda}R^{\gamma\rho}(iv(K+K'),q)R^{\delta\lambda}(ivK',q)$$
$$-K_\gamma\Delta_{-K}^{\beta\lambda}\sum_{K'}\int_q K'_\delta\Delta_{K'}^{\alpha\rho}R^{\gamma\rho}(ivK',q)R^{\delta\lambda}(iv(K'-K),q), \tag{B12}$$

where the first half comes from Γ_1 and the second from Γ_2.

3. Study at finite temperature

In this subsection we study the case of finite $T>0$ and the static disorder case (corresponding to Sec. B 1 above). The study is rather tedious and we skip some details. Since that situation has already been analyzed (but applied to the different case of a periodic manifold in $d=2$) we refer to Ref. 94 for further details. We concentrate mostly on what is needed for the analysis near $d=d_u$ ($d_u=4$ for $v=0$ and $d_u=3$ for $v>0$). The result[94] is that the short distance expansion of the effective action up to second order in disorder produces a $i\hat{u}i\hat{u}$ term which can be written as

$$\int_{r,t_1,t_2}\delta\Delta_K^{\alpha\beta}e^{-(1/2)K\cdot B_{0,t_1-t_2}\cdot K}(i\hat{u}_{rt_1}^{\alpha})(i\hat{u}_{r,t_2}^{\beta})e^{iK(u_{rt_1}-u_{rt_2})}, \tag{B13}$$

which is thus of the same form as the first-order term and which thus corrects it. Here again, other operators (such as higher gradients) are produced, but they are irrelevant near d_u. We are using extensively the assumed symmetries $\Delta_K^{\alpha\beta}=\Delta_K^{\beta\alpha}=\Delta_{-K}^{\alpha\beta}$. We are *not* using the potential condition that $\Delta_K^{\alpha\beta}\sim K_\alpha K_\beta$, since this is wrong in general (see discussion above). We find that the corrections $\Delta_K^{\alpha\beta}$ are *a priori* the following, starting with the terms which do not give a contribution.

(i) The terms with connected averages (1) and (4) give contributions given by formulas (i) and (iii) of Eq. (C19) of Ref. 94. One can check that this term does not produce a divergence.

(ii) The last two terms of $\delta\Delta^{(3)}$ give

$$\delta\Delta_K^{\alpha\beta}=\sum_{K'}[K_\delta K'_\gamma(\Delta_K^{\alpha\rho}\Delta_{K'}^{\beta\lambda})|_{\alpha\beta}$$
$$-K_\gamma K'_\delta(\Delta_{K'}^{\alpha\rho}\Delta_K^{\beta\lambda})|_{\alpha\beta}]R_{r,\tau_2}^{\gamma\rho}R_{-r,\tau_1}^{\delta\lambda}$$
$$\times e^{-(1/2)K'\cdot B_{0,\tau_1+\tau_2}\cdot K'}e^{-K\cdot(C_{r,-\tau_1}-C_{r,\tau_2})\cdot K'}, \tag{B14}$$

where $(\cdots)_{\alpha\beta}$ means symmetrization over the indices α,β. This term was unlikely to produce a divergence for the same reason as above, but in any case it does not since it vanishes. Indeed one sees on this expression that this quantity vanishes because of the symmetry $\tau_1\to\tau_2$ which makes the summmand over K' odd under $K'\to-K'$.

(iii) Finally, the terms which produce divergences are the term $\delta\Delta^{(2)}$ and the first term of $\delta\Delta^{(3)}$. They give a total contribution:

$$\delta\Delta_P^{\alpha\beta}=\sum_{K,K'=P-K}(K_\gamma K_\delta\Delta_K^{\alpha\beta}\Delta_{K'}^{\rho\lambda}R_{-r,\tau}^{\gamma\rho}R_{-r,\tau'}^{\delta\lambda}e^{K\cdot(2C_{0,0}-C_{r,-\tau}-C_{r,-\tau'})\cdot K'}+K'_\gamma K_\delta\Delta_K^{\alpha\rho}\Delta_{K'}^{\beta\lambda}R_{r,\tau}^{\gamma\rho}R_{-r,\tau'}^{\delta\lambda}e^{K\cdot(2C_{0,0}-C_{r,\tau}-C_{r,-\tau'})\cdot K'})$$
$$-P_\gamma P_\delta\Delta_P^{\alpha\beta}\sum_{K'}\Delta_{K'}^{\rho\lambda}R_{-r,\tau}^{\gamma\rho}R_{-r,\tau'}^{\delta\lambda}e^{-(1/2)K'\cdot B_{0,\tau'-\tau}\cdot K'}\cosh P\cdot(C_{r,-\tau}-C_{r,-\tau'})\cdot K'. \quad \text{(B15)}$$

This term produces a divergence at d_u. It is simply the finite-temperature generalization of Eq. (B4) above. Since $B(r,\tau)$ is finite (and cutoff dependent) at large r, τ and since at $T=0$ the infrared divergence came from the large r, τ values, the new IR divergence is the same as the old one, with a coefficient obtained by simply by taking the large r, τ limit in the exponential factors. Near d_u the large time or space limit of $B(r,\tau)=2(C_{0,0}-C_{r,\tau})$ is proportional to the temperature:

$$\lim_{\max(r,t)\to\infty}B^{\beta\alpha}(r,\tau)=2C_{0,0}^{\beta\alpha}=B_\infty\delta_{\alpha\beta}$$
$$=2T\int_q\left(\frac{P_{\beta\alpha}^T(q)}{c_T(q)}+\frac{P_{\beta\alpha}^L(q)}{c_T(q)}\right).$$

The final divergent contribution is

$$\delta\Delta_P^{\alpha\beta}=\sum_{K,K'=P-K}(K_\gamma K_\delta\Delta_K^{\alpha\beta}\Delta_{K'}^{\rho\lambda}R_{-r,\tau}^{\gamma\rho}R_{-r,\tau'}^{\delta\lambda}$$
$$+K'_\gamma K_\delta\Delta_K^{\alpha\rho}\Delta_{K'}^{\beta\lambda}R_{r,\tau}^{\gamma\rho}R_{-r,\tau'}^{\delta\lambda})e^{K\cdot B_\infty\cdot K'}$$
$$-P_\gamma P_\delta\Delta_P^{\alpha\beta}\sum_{K'}\Delta_{K'}^{\rho\lambda}R_{-r,\tau}^{\gamma\rho}R_{-r,\tau'}^{\delta\lambda}e^{-(1/2)K'\cdot B_\infty\cdot K'}. \quad \text{(B16)}$$

APPENDIX C: ANALYSIS IN $d=2$

We start from the MSR dynamical action corresponding to the equation studied in Sec. VII. It reads $S=S_0+S_2+S_{\rm int}$ with S_0 and $S_{\rm int}$ as defined in 48 and $S_2=\frac{1}{2}\int_{q,t,t'}(i\hat{u}_{q,t})\times(i\hat{u}_{-q,t'})(\Delta q^2+\Delta_0)$. Here one studies $\Delta(u)=g\cos(u)$. In the absence of disorder the free action S_0 yields correlators as in Eq. (38), e.g., $C(q,\omega)=2T\eta/|c_xq_x^2+c_yq_y^2+ivq_x+i\eta\omega|^2$ and

$$B(r,t)=\int_q\frac{2T}{c_xq_x^2+c_yq_y^2}[1-e^{-(c_xq_x^2+c_yq_y^2)|t|/\eta}$$
$$\times\cos(qr+vq_xt/\eta)]. \quad \text{(C1)}$$

Note that for $v>0$ $C(r,t)\neq C(-r,t)$. In the presence of disorder one studies perturbation theory expanding in the interaction term $S_{\rm int}$ using the quadratic part S_0+S_2 as the bare action. The response function of S_0+S_2 is identical to the one of S_0 with the correlation function changed as: $C_{q,t}\to C_{q,t}+C_{{\rm stat}q,t}$ with $C_{{\rm stat}q,t}=(\Delta q^2+\Delta_0)/(c^2q^4+v^2q_x^2)$ which is purely static and does not appear in any diagram of perturbation theory.

As in Appendix A, we compute the effective action $\Gamma[u,\hat{u}]$ in perturbation of $S_{\rm int}$. To lowest order one gets

$$\Gamma=S_0+S_2+\int_{rtt'}(i\hat{u}_{rt})g\sin(u_{rt}-u_{rt'})R_{rtrt'}e^{-(1/2)B_{rtrt'}}$$
$$-\frac{1}{2}(i\hat{u}_{rt})(i\hat{u}_{rt'})g\cos(u_{rt}-u_{rt'})e^{-(1/2)B_{rtrt'}}, \quad \text{(C2)}$$

where $B_{rtr't'}=C_{rtrt}+C_{r't'r't'}-C_{rtr't'}-C_{r't'rt}$. This yields immediately the corrections to first order in g for the friction coefficient, the temperature, and the disorder given in the text (95). The correction to η comes from a a gradient expansion in time which yields a correction to the term $i\hat{u}_{rt}\eta\partial_tu_{rt}$. The correction to ηT comes from a correction to the $i\hat{u}_{rt}i\hat{u}_{rt}$, and the correction to the disorder comes from the long-time limit of the exponential.

To compute the RG equations from Eq. (95), we have to decide on a regularization scheme. Here we choose to take an infrared regulator by defining a large time $t_{\max}$ but no infrared regulator in momentum q. The ultraviolet cutoff is enforced via a Gaussian cutoff in momentum, i.e., we define

$$B(r,t,a)=\int_q\frac{d^2q}{(2\pi)^2}\frac{2T}{cq^2}(1-e^{-cq^2\mu|t|}e^{iqr+ivq_x\mu t})e^{-a^2q^2}, \quad \text{(C3)}$$

where the mobility $\mu=1/\eta$ has been introduced. Equation (C3) can be readily evaluated as

$$B(r,t,a)=\frac{2T}{c}\int_{a^2}^{+\infty}ds\int_q\frac{d^2q}{(2\pi)^2}[e^{-sq^2}$$
$$-e^{-(s+c\mu t)q^2}\cos(vq_x\mu t)]$$
$$=\frac{T}{2\pi c}\int_0^{c\mu t/(c\mu t+a^2)}\frac{du}{u}\left(\frac{1}{1-u}\right.$$
$$\left.-e^{-u(y^2+(x+v\mu t)^2)/4c\mu t}\right), \quad \text{(C4)}$$

where in the intermediate stage we have integrated over q and performed an intermediate change of variable $u=c\mu t/(c\mu t+s)$. Using

$$\int_0^z\frac{du}{u}(1-e^{-ru})=C+\ln(rz)-Ei[-rz])$$

$$Ei[x]=\int_{-\infty}^xe^t\frac{dt}{t},$$

$$Ei[-x]\sim_{x\ll0}C+\ln(-x)-x+\frac{x^2}{4}+O(x^3), \quad \text{(C5)}$$

one obtains for Eq. (C4)

$$B(r,t,a)=\frac{T}{2\pi c}\left(\ln\left[\frac{c\mu|t|+a^2}{a^2}\right]+C+\ln\left[\frac{y^2+(x+v\mu t)^2}{4(c\mu|t|+a^2)}\right]\right.$$
$$\left.-Ei\left[\frac{-[y^2+(x+v\mu t)^2]}{4(c\mu|t|+a^2)}\right]\right). \tag{C6}$$

This is used to obtain, for the RG corrections (see text):

$$\frac{\delta(\eta T)}{\eta T_c}=\tilde{g}e^{-(T/T_c)C}\int_{\eta a/v}^{t_{\max}}\frac{v\,dt}{\eta a}\left(\frac{vt}{\eta a}\right)^{-2T/T_c}$$
$$=\tilde{g}e^{-(T/T_c)C}dl+\tilde{g}e^{dl(1-2T/T_c)}e^{-(T/T_c)C}$$
$$\times\int_{\eta a'/v}^{t_{\max}}\frac{v\,dt}{\eta a'}\left(\frac{vt}{\eta a'}\right)^{-2T/T_c}. \tag{C7}$$

APPENDIX D: STABILITY OF THE $T=0$ FRG FIXED POINT

Here we diagonalize the RG flow around the fixed point (63) and show that it is a locally *attractive* fixed point within the space of periodic functions on [0,1]. $\overline{\Delta}(u)$ satisfies the RG equation (61) with $\overline{\Delta}(0)=\overline{\Delta}(1)=0$. We have checked numerically that analytic initial functions (i.e., with zero odd derivatives at 0) converge towards the nonanalytic fixed point $\overline{\Delta}^*(u)=u(1-u)/2$. The stability analysis is performed by writing $\overline{\Delta}(u)=\overline{\Delta}^*(u)+\delta(u)$. One then has to solve the eigenvalue problem:

$$\frac{1}{2}u(1-u)\delta''(u)=-\lambda\,\delta(u). \tag{D1}$$

The eigenfunctions are such that

$$\frac{1}{2}u(1-u)\delta_n''(u)=-\frac{1}{2}n(n-1)\delta_n(u). \tag{D2}$$

One can also define the variable $u=(1+v)/2$. Then the eigenfunctions are the Jacobi polynomials $\delta_n(u)=P_n^{-1.-1}(v)$ (see Ref. 133, p. 779) and form an orthonormal complete set. They can be written as

$$\delta_n(u)=\frac{(-1)^n}{n!}u(1-u)\frac{d^n}{du^n}[u(1-u)]^{n-1}$$
$$=\frac{(-1)^n}{2^n n!}(1-v^2)\frac{d^n}{dv^n}(1-v^2)^{n-1}. \tag{D3}$$

Because of the $u\to-u$ symmetry, which due to periodicity becomes $u\to(1-u)$ symmetry, we can restrict ourselves to n an even and nonzero positive integer. The lowest eigenmodes are thus $\delta_2(v)=(v^2-1)/4$ (eigenvalue -1), $\delta_4(v)=3(1-6v^2+5v^4)/16$ (eigenvalue -6), $\delta_6(v)=5(v^2-1)(1-14v^2+21v^4)/32$ (eigenvalue -15), etc. Note that they satisfy $\delta_n(v=-1)=\delta_n(v=1)=0$ as requested. All these eigenfunctions are nonanalytic (though by combining several one may get analytic ones). Rendering the initial function nonanalytic is presumably the role of the nonlinearity. This equation is interesting since it is the simplest case on which one can work out the full stability spectrum and it may enlighten us about the generation of nonanalyticity in these types of RG equations.

APPENDIX E: PARAMETRIZATION OF MOVING STRUCTURES

Moving structures can be generally parametrized by their internal space D, the number of components n of the displacements fields (characterizing its deformations, or the number of components of the order parameter) and the embedding space d. We denote for convenience by the same symbol the space itself and its dimension.

In the statics one can distinguish several cases. The problem of manifolds in random potentials has been studied for e.g., (i) fully oriented manifolds where D and n are orthogonal ($d=D+n$) such as directed polymers or interfaces, (ii) isotropic manifolds $n=d$ such as self-avoiding chains in random potentials, and (iii) problem of lattices where usually $d=D$ and $n\leqslant d$. Lattices with $D<d$ are possible in principle, such as flat but fluctuating tethered membranes $D=n<d$ or isotropic tethered membranes $D<d=n$ or any intermediate case (the so-called tubules).

In the driven dynamics, let us call x the direction in the embedding space along which the system is driven. One can distinguish the following cases.

(A) The structure is elastic (i.e., not liquid) in the direction where it is driven. Then there is a displacement u_x along x and x also belongs to the n space. There are two subcases:

(A1) x also belongs to the internal space D. This is the problem of manifolds driven *along one of their internal dimensions*, to which the moving glass studied here belongs. A general parametrization in that case would be

$$D=(x,y_1,z),\quad n=[u_x,u_y=(u_{y_1},u_{y_2})],\quad d=(x,y_1,y_2,z). \tag{E1}$$

It allows for manifolds with $D<d$ which do not entirely fill space (i.e., with ''height'' degrees of freedom u_{y_2}). Then a parametrization of the dimensions (and the subspaces) is

$$D=1+d_1+d_z,\quad n=1+d_1+d_2,\quad d=1+d_1+d_2+d_z, \tag{E2}$$

where d_1 and d_2 are the number of components of u_{y_1} and u_{y_2}, respectively, and d_z is the number of components of z. In this paper we have mainly studied the case $d=D$ ($d_2=0$) but with $d_1>0$. Note that a single Q CDW has $d_1=d_2=0$ ($u_y=0$) and $d=D$.

(A2) x does not belong to the internal space D. This is the problem of manifolds driven *perpendicular to their internal dimension*. The general parametrization in that case is

$$D=(y_1,z),\quad n=[u_x,u_y=(u_{y_1},u_{y_2})],$$
$$d=(x=u_x,y_1,y_2,z), \tag{E3}$$

and thus $D=d_1+d_z$, $n=1+d_1+d_2$, and $d=1+d_1+d_2+d_z$. It also contains the case of a driven order parameter u which does not couple at all to internal space (such as a vector spin order parameter). Indeed in that particular case one can define the ''embedding space'' as the sum $d=D+n$ (and thus $u_{y_1}=0$).

(B) The structure is a *liquid* in the direction where it is driven. Then x belongs to D space but not to n space. Then one sets $u_x=0$ in case (A1) above, i.e., $n=d_1+d_2$. The parametrization is thus

$$D=(x,y_1,z), \quad n=(u_{y_1},u_{y_2}), \quad d=(x,y_1,y_2,z) \quad \text{(E4)}$$

with $D=1+d_1+d_z$, $n=d_1+d_2$, $d=1+d_1+d_2+d_z$. The transverse moving glass is one example with $d_2=0$ and $(d_1=1,\ d_z=0)$ in $d=2$ and $(d_1=1,\ d_z=1)$ in $d=3$ (a moving line lattice giving a smectic sheets structure of channels) and $(d_1=2,\ d_z=0)$ in $d=3$ (a moving point lattice giving a line crystal structure of channels). Note that as for any liquid scalar density fluctuations should, in principle, be also incorporated in a complete description.

APPENDIX F: HARTREE METHOD

For completeness we give here the Hartree equations exact in the large-N limit, for model III generalized to N components. The equations at $v=0$ were derived and analyzed in Ref. 84 (see also Refs. 134 and 135). These equations will be analyzed further in a future publication. The Hartree equations are

$$\partial_t R_{ktt'} = -(k^2+ivk_x)R_{ktt'} + 4\int_0^t ds V_2''(B_{ts}) \times R_{ts}(R_{ktt'}-R_{kst'}),$$

$$\partial_t C_{ktt'} = -(k^2+ik_xv)C_{ktt'} + 2\int_0^{t'} ds V_1'(B_{ts})R_{-kt's} + 4\int_0^t ds V_2''(B_{ts})R_{ts}(C_{ktt'}-C_{ks,t'}) + 2TR_{-kt't} \quad \text{(F1)}$$

where $R_{tt'}=\int_k R_{ktt'}$, $B_{tt'}=\int_k B_{ktt'}$, and $B_{ktt'}=C_{ktt}+C_{kt't'}-C_{ktt'}-C_{-ktt'}$. Note that $C_{ktt'}=C_{-kt't}$, where V_2 contains only the potential part of disorder, while V_1 contains all disorder (see Ref. 84 for definitions). One can look for a time-translational invariant solution of these equations: $R_{ktt'}=r_k(t-t')$, $C_{ktt'}=c_k(t-t')$(Ref. 84) (in the statics this is the equivalent of a replica symmetric solution). It can be written in Fourier-transform version:

$$r_k(\omega)=\frac{1}{i\omega+k^2+ivk_x+\Sigma(0)-\Sigma(\omega)},$$

$$c_k(\omega)=\frac{2T+D(\omega)}{|i\omega+k^2+ivk_x+\Sigma(0)-\Sigma(\omega)|^2}. \quad \text{(F2)}$$

Note that $c_k(\omega)=c_{-k}(-\omega)$. We have defined

$$\Sigma(\omega)=-4\int_{-\infty}^{+\infty} dt e^{i\omega t}V_2''(b(t))r(t),$$

$$D(\omega)=4\int_{-\infty}^{+\infty} dt e^{i\omega t}V_1'(b(t)), \quad \text{(F3)}$$

where $r(t)=\int_k r(t)$ and $b(t)=\int_k b_k(t)=\int_k[2c_k(t=0)-c_k(t)-c_{-k}(t)]$. Note that $b_k(t)=\int_\omega(1-e^{i\omega t})[c_k(\omega)+c_{-k}(\omega)]$ and $b(t)=\int_\omega[1-\cos(\omega t)]c(\omega)$ where $c(\omega)=\int_k c_k(\omega)$ is an even function of ω.

A superficial analysis of the above equation indicates that this nonperiodic problem has an asymptotically linear response and is not glassy for $v>0$ for $N=\infty$ (while it has a nonlinear asymptotic response for $v=0$ both at N finite[84] and N infinite[85]). Indeed the response function appears to be massive since integrating over k_x one has

$$r(\omega)=\frac{1}{2}\int_{k_y}\frac{1}{[k_y^2+i\omega+\Sigma(0)-\Sigma(\omega)+v^2/4]^{1/2}}. \quad \text{(F4)}$$

Thus the response to an applied force being $F/V=[1-\Sigma'(0)]$ would be linear at least at the naive level (for a more detailed behavior one must add a small transverse force and follow the methods of Ref. 136). This is related to the absence of divergence for η noticed in the FRG Sec. VI. Further investigations would be necessary however before reaching a conclusion. One should makes sure that no transition occurs in the above equations (such can happen in the case $v=0$). Also, it is possible that the glassy physics found in Sec. VI, which comes from a renormalization of the disorder, is not fully captured here by the most naive large-N limit.

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