

# Linear birefringence behavior of $A_2BX_4$ -type ferroic crystals in the normal-incommensurate phase transition

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Critical behavior of linear birefringence was studied in the normal-incommensurate phase transition of various  $A_2BX_4$ -type crystals. We considered the fluctuation effect in the scaling analysis so that we could obtain a precise value of the critical exponent  $\beta$  from linear birefringence measurements. The numerical values of  $2\beta$  obtained from our experiment were  $0.72\sim 0.75$  ( $\pm 0.02$ ). These results are in agreement with the previous theoretical prediction which suggests that the normal-incommensurate phase transition of  $A_2BX_4$ -type crystals belongs to the universality class of the three-dimensional (3D)  $XY$  model. Beyond the critical region, we observed crossover from 3D  $XY$ -like behavior into classical Landau-like behavior,  $2\beta = 1$ , in  $Rb_2ZnCl_4$  and  $K_2ZnCl_4$ . [S0163-1829(98)03318-9]

## I. INTRODUCTION

Physical properties of  $A_2BX_4$ -type crystals have been widely studied by various experimental method because of the interesting characteristics of their incommensurate phase.<sup>1</sup> ( $A = Rb, K, NH_4, N(CH_3)_4$ ;  $B = Zn, Mn, Cu, Co$ ; and  $X = Cl, Br$ .) The  $A_2BX_4$ -type crystals have the same phase transition sequence as the normal (N)-incommensurate (IC)-commensurate (C) phase with descending temperature.<sup>1,2</sup> In the N phase, the  $A_2BX_4$ -type crystals have  $\beta$ - $K_2SO_4$ -type  $Pm\bar{c}n$  structure.<sup>3,4</sup> In the IC phase, modulated polarization turns up with a period irrational to the period of crystal lattice. Some characteristics of the IC phase of the  $A_2BX_4$ -type crystals are given in Table I. In the C phase, some  $A_2BX_4$ -type crystals have ferroelectricity or ferroelasticity.

The N-IC phase transition was theoretically studied by Cowley and Bruce, who predicted that the N-IC phase transition belongs to the universality class of the three-dimensional (3D)  $XY$  model.<sup>5</sup> Their theoretical study was followed by many experimental investigations, some of which agree with the expectation of the 3D  $XY$  model.<sup>6-10</sup> However, there also exist other experimental results which

agree with the 3D Ising model rather than with the 3D  $XY$  model.<sup>11-13</sup> Up to now, it is not yet conclusive experimentally whether or not the N-IC phase transition of  $A_2BX_4$ -type crystals belongs to the universality class of the 3D  $XY$  model.<sup>13</sup>

For the investigation of critical phenomena, linear birefringence (LB) is a very useful tool because it has a direct relation to the order parameter of the N-IC phase transition.<sup>14-18</sup> Therefore, the critical exponent  $\beta$  for the order parameter can be obtained from LB measurement and scaling analysis. In our previous studies, we have reported that the LB behavior of some  $A_2BX_4$ -type crystals were 3D- $XY$ -like near the N-IC phase transition temperature ( $T_i$ ).<sup>19-21</sup>

In this paper, we report the LB behavior near the N-IC phase transition of various  $A_2BX_4$ -type crystals. The critical LB behavior has been investigated systematically by considering the fluctuation effect and crossover behavior.

## II. EXPERIMENTAL PROCEDURE

In this report, we have measured the LB behavior of  $Rb_2ZnCl_4$  (RZC),  $K_2ZnCl_4$  (KZC),  $(NH_4)_2ZnCl_4$  (AZC),  $[N(CH_3)_4]_2ZnCl_4$  (TMAZC),  $[N(CH_3)_4]_2CuCl_4$

TABLE I. Characteristics of the IC phase of  $A_2BX_4$ -type crystals studied in this paper.  $T_i$  and  $T_c$  are the N-IC and the IC-C phase transition temperature, respectively. The ferro-axis means the axis along which spontaneous polarization or spontaneous strain appears. All of the LB behaviors in this study were measured along these ferro-axes. The crystals have ferroelectricity or ferroelasticity in the temperature region given in the parentheses.

	modulation wave vector	$T_i$	$T_c$	ferro-axis	
$Rb_2ZnCl_4$	$\mathbf{q} = (1/3 - \delta)\mathbf{c}^*$	302 K	195 K	$a$ axis	ferroelectricity ( $\leq 195$ K)
$K_2ZnCl_4$	$\mathbf{q} = (1/3 - \delta)\mathbf{c}^*$	553 K	403 K	$a$ axis	ferroelectricity ( $\leq 403$ K)
$(NH_4)_2ZnCl_4$	$\mathbf{q} = (1/4 + \delta)\mathbf{c}^*$	406 K	364 K	$a$ axis	ferroelectricity ( $\leq 268$ K)
$[N(CH_3)_4]_2ZnCl_4$	$\mathbf{q} = (2/5 \pm \delta)\mathbf{a}^*$	297 K	280 K	$b$ axis	ferroelectricity ( $\leq 280$ K)
$[N(CH_3)_4]_2CuCl_4$	$\mathbf{q} = (1/3 - \delta)\mathbf{a}^*$	296 K	293 K	$b$ axis	ferroelasticity ( $\leq 293$ K)
$[N(CH_3)_4]_2MnCl_4$	$\mathbf{q} = (1/2 - \delta)\mathbf{a}^*$	293 K	292 K	$b$ axis	ferroelasticity ( $\leq 292$ K)

(TMACC), and  $[N(CH_3)_4]_2MnCl_4$  (TMAMC). Single crystals of  $A_2BX_4$ -type materials were grown at 310 K by the slow evaporation method from an aqueous solution of a stoichiometric mixture of  $2ACl$  and  $BCl_2$  with 10% excess of  $BCl_2$ . We obtained transparent and colorless single crystals. (TMACC and TMAMC have exceptionally light brown and red colors, respectively.)

The crystals were cut perpendicularly to the ferroelectric (or ferroelastic) axes using a wet string and polished carefully. The ferroelectric (or ferroelastic) axes for each crystal are given in Table I.

The temperature dependence of LB was measured at 632.8 nm wavelength of a He-Ne laser by the Senarmont method with an accuracy of at least  $10^{-7}$ . The LB behavior was measured in the heating run. The temperature heating rate was 0.04 K/min near  $T_i$ , and about 0.4 K/min in the temperature region far from  $T_i$ . Only for KZC, the temperature heating rate was 0.4 K/min in all of the temperature range.

### III. CRITICAL BEHAVIOR OF BIREFRINGENCE

The N-IC phase transition of dielectric materials was studied theoretically by Cowley and Bruce.<sup>5</sup> They proposed the Landau-Ginzburg-Wilson (LGW) Hamiltonian of the N-IC phase transition using the renormalization technique.  $A_2BX_4$ -type crystals belong to the case where the star of the soft mode consists of only two wave vectors  $\mathbf{q}_s$  and  $-\mathbf{q}_s$ . When  $\mathbf{q}_s$  is incommensurate, the obtained LGW Hamiltonian was of the effective Hamiltonian of the 3D XY model.<sup>5</sup> (The XY model means the isotropic  $n=2$  Heisenberg ferromagnet.)

The critical behavior of specific heat ( $C_p$ ), order parameter ( $Q$ ), and dielectric susceptibility ( $\epsilon$ ) can be expressed as

$$C_p \sim |t|^\alpha, \quad Q \sim |t|^\beta, \quad \epsilon \sim |t|^\gamma, \quad (1)$$

with the reduced temperature  $t \equiv [(T - T_i)/T_i]$  and the critical exponents  $\alpha$ ,  $\beta$ , and  $\gamma$ . The critical exponent  $\beta$  of the 3D XY model was predicted by Majkrzak *et al.* as  $2\beta = 0.70 \pm 0.04$ .<sup>9</sup> If we accept their prediction, the order parameter behavior of  $A_2BX_4$ -type crystals can be expected to be  $Q^2 \sim |t|^{2\beta} \sim |t|^{0.70 \pm 0.04}$  in the critical region of the N-IC phase transition.

In the N-IC phase transition of  $A_2BX_4$ -type crystals, the order parameter  $Q$  is the amplitude of the modulated polarization. The incommensurate modulation near  $T_i$  can be described by the plane wave approximation<sup>1,17</sup>

$$P_k(x) = 2Q \cos \mathbf{q}x. \quad (2)$$

In Eq. (2),  $P_k$  is the spontaneous polarization along the  $k$  axis and  $\mathbf{q}$  is the modulation wave vector. The square of the order parameter  $Q^2$  can be easily obtained from the spatial average of the modulated polarization  $\langle P_k(x)^2 \rangle$  by

$$\langle P_k(x)^2 \rangle = Q^2. \quad (3)$$

A relation between the incommensurately modulated polarization and LB,  $\Delta n_{ij} \equiv n_i - n_j$ , is given by<sup>14,17</sup>

$$\Delta n_{ij} = \frac{1}{2} (n_j^3 g_{jk} - n_i^3 g_{ik}) \langle P_k(x)^2 \rangle + \Delta n_{ij0}, \quad (4)$$

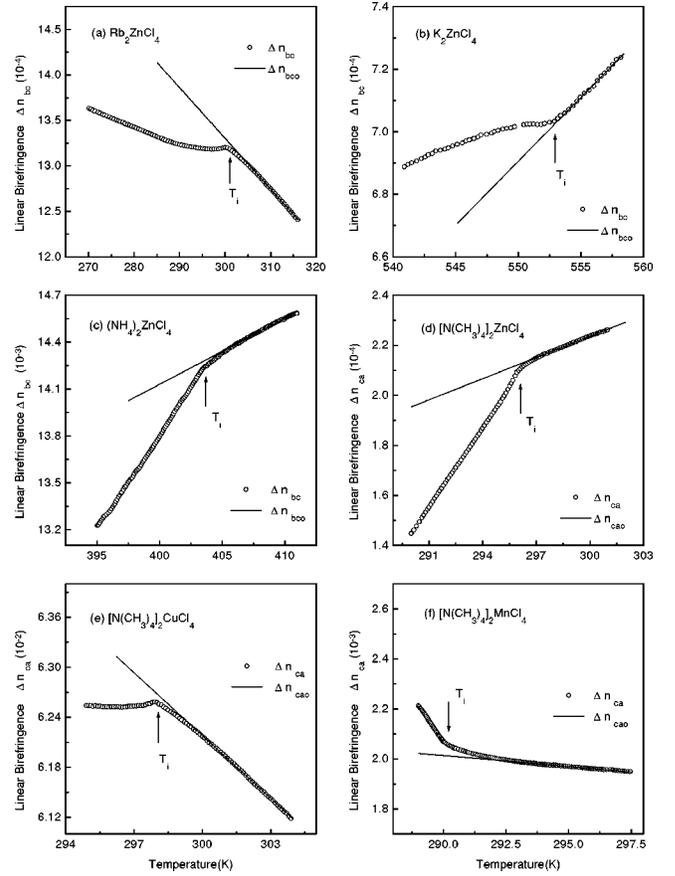


FIG. 1. The temperature dependence of  $\Delta n_{ij}$  (open circle) near  $T_i$  for (a) RZC, (b) KZC, (c) AZC, (d) TMAZC, (e) TMACC, and (f) TMAMC.  $T_i$  is indicated by an arrow. The solid line represents the base line which corresponds to  $\Delta n_{ij0}$ .

where  $i, j, k$  are crystal axes,  $n_i$  the refractive index along the  $i$  axis,  $g_{jk}$  and  $g_{ik}$  the electro-optic coefficients, and  $\Delta n_{ij0}$  the photoelastic and thermoelastic contributions.

From Eqs. (1)–(4), we can expect<sup>15</sup>

$$\Delta n \equiv |\Delta n_{ij} - \Delta n_{ij0}| \sim \langle P_k(x)^2 \rangle \sim Q^2 \sim |t|^{2\beta}. \quad (5)$$

Thus, the critical exponent  $\beta$  can be determined from the LB measurement.

### IV. RESULTS AND DISCUSSION

The results of LB measurement given in Fig. 1 shows an obvious kink at  $T_i$  which is indicated by an arrow. These results agree well with the previous LB studies of  $A_2BX_4$ -type crystals.<sup>22–27</sup>

For the investigation of the critical behavior of  $\Delta n$  and  $Q^2$ , it is important to determine correctly the base line  $\Delta n_{ij0}$  of Eq. (4). Because  $P_k(x) = 0$  in the N phase, a base line  $\Delta n_{ij0}$  is expected to be obtained from linear fitting of LB behavior in the N phase (solid line in Fig. 1). It is assumed that the base line  $|\Delta n_{ij0}|$  can be extrapolated into the IC phase.

In addition, the fluctuation effect on the LB behavior is expected to be observed in the N phase. Accounting for the time-averaged static order parameter  $\langle Q \rangle_t$ , we can set  $Q = \langle Q \rangle_t + \delta Q$ , where  $\delta Q$  represents fluctuation amplitude.

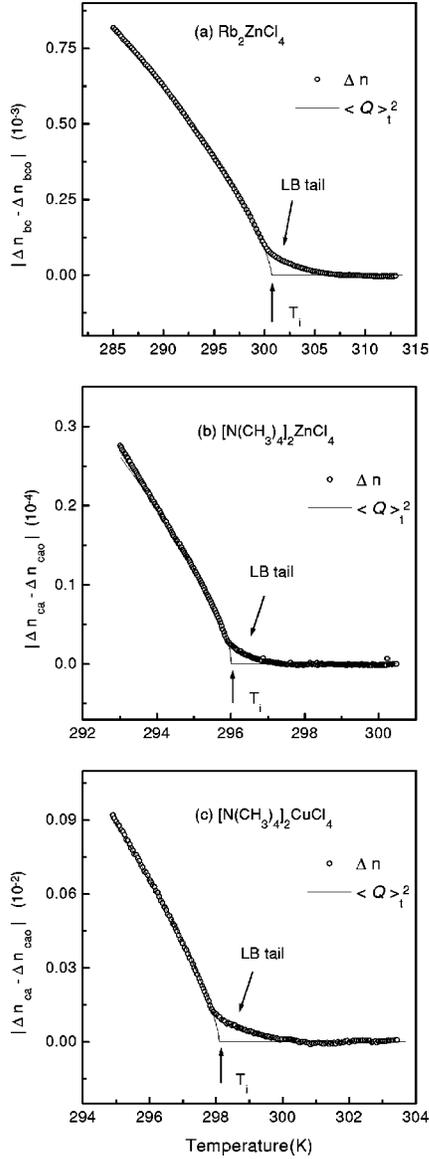


FIG. 2. The temperature dependence of  $\Delta n \equiv \Delta n_{ij} - \Delta n_{ij0}$  (open circle) for (a) RZC, (b) TMAZC, and (c) TMACC. The solid line represents the temperature dependence of the static order parameter  $\langle Q \rangle_t^2$ .

As mentioned in Eq. (4), LB is proportional to the time-averaged square of the order parameter  $\langle Q^2 \rangle_t$ ,<sup>28</sup>

$$\Delta n \sim \langle Q \rangle_t^2 + \langle \delta Q^2 \rangle_t. \quad (6)$$

TABLE II. The numerical values of the critical exponent  $2\beta$  obtained from this LB study.  $T_i$  was determined from the scaling analysis as described in the text. The  $\log_{10} t_{cr}$  is the temperature where the  $2\beta$  line and the  $2\beta^*$  line meet in Fig. 3.

	$\beta$	$T_i$	$\beta^*$	$\log_{10} t_{cr}$
Rb <sub>2</sub> ZnCl <sub>4</sub>	$2\beta = 0.72 \pm 0.02$	300.7 K	$2\beta^* = 0.99 \pm 0.02$	-0.71
K <sub>2</sub> ZnCl <sub>4</sub>	$2\beta = 0.75 \pm 0.02$	553.1 K	$2\beta^* = 1.02 \pm 0.02$	-0.92
(NH <sub>4</sub> ) <sub>2</sub> ZnCl <sub>4</sub>	$2\beta = 0.73 \pm 0.02$	403.6 K	$2\beta^* = 0.89 \pm 0.02$	-2.34
[N(CH <sub>3</sub> ) <sub>4</sub> ] <sub>2</sub> ZnCl <sub>4</sub>	$2\beta = 0.73 \pm 0.02$	296.1 K	$2\beta^* = 0.88 \pm 0.02$	-2.13
[N(CH <sub>3</sub> ) <sub>4</sub> ] <sub>2</sub> CuCl <sub>4</sub>	$2\beta = 0.75 \pm 0.02$	298.1 K		
[N(CH <sub>3</sub> ) <sub>4</sub> ] <sub>2</sub> MnCl <sub>4</sub>	$2\beta = 0.72 \pm 0.02$	290.3 K		

The term  $\langle Q \rangle_t$  must be zero in the N phase, and the effect of the order parameter fluctuation  $\Delta n \sim \langle \delta Q^2 \rangle_t$  is also expected to be observed in the LB behavior of the N phase.

In Fig. 2, the existence of the LB tail can be easily observed just above  $T_i$  in the N phase. The relation between the LB tail and the fluctuation was discussed by Courtens and Kleemann, Schäfer, and Nouet who explained it in terms of anisotropic fluctuation.<sup>29,30</sup> The LB tail of  $A_2BX_4$ -type crystals can be similarly explained by the fluctuation effect. Microscopic origin of the anisotropic fluctuation in  $A_2BX_4$ -type crystals can be deduced from previous x-ray diffraction and neutron scattering experiments, the results of which indicate that the fluctuation consists of atomic displacements of the  $BX_4$  tetrahedra.<sup>1</sup>

The LB tail must be correctly taken into account to determine the base line  $\Delta n_{ij0}$ . Because the LB tail is not a part of  $\Delta n_{ij0}$  but caused by the fluctuation effect, for obtaining a correct  $\Delta n_{ij0}$ , it was excluded from the linear fitting of the LB behavior in the N phase. In many previous LB studies on  $A_2BX_4$ -type crystals, the existence of the LB tail is overlooked in the linear fitting for  $\Delta n_{ij0}$ .<sup>21</sup> It is thought to be the reason why our LB studies are different than the previous ones.

An initial value of  $T_i$  was determined at which we can find the kink by a direct inspection of the LB behavior. Then, with a small variation of the initial value of  $T_i$ , we were able to get a precise  $T_i$  where the best linear fitting of a  $\log_{10} \Delta n - \log_{10} t$  plot was obtained. The  $T_i$ 's determined in this way are given in Table II, and were treated as a fixed parameter afterwards.

Figure 3 shows the results of the scaling analysis, i.e., the  $\log_{10} \Delta n - \log_{10} t$  plot of LB behavior. The exponent  $2\beta$  was obtained from the inclination of the log-log plot in a proper temperature region. The obtained value of  $2\beta$  is also presented for each crystal in Table II. The  $2\beta$ 's estimated from LB measurement are distributed in the range  $2\beta = 0.72 - 0.75$  ( $\pm 0.02$ ). Within experimental error, these results are in good agreement with the theoretical prediction of the 3D XY model,  $2\beta = 0.70 \pm 0.04$ , and with some previous LB studies.<sup>18,31</sup> As mentioned in Sec. I, it has not yet been resolved whether the N-IC phase transition is XY-like or Ising-like.<sup>13</sup> It can be thought that our LB study strongly supports the 3D XY model. For the exponent  $\alpha$  and  $\gamma$ , similar results supporting the 3D XY model were also obtained by recent specific heat and nuclear magnetic resonance (NMR) studies.<sup>6,10</sup>

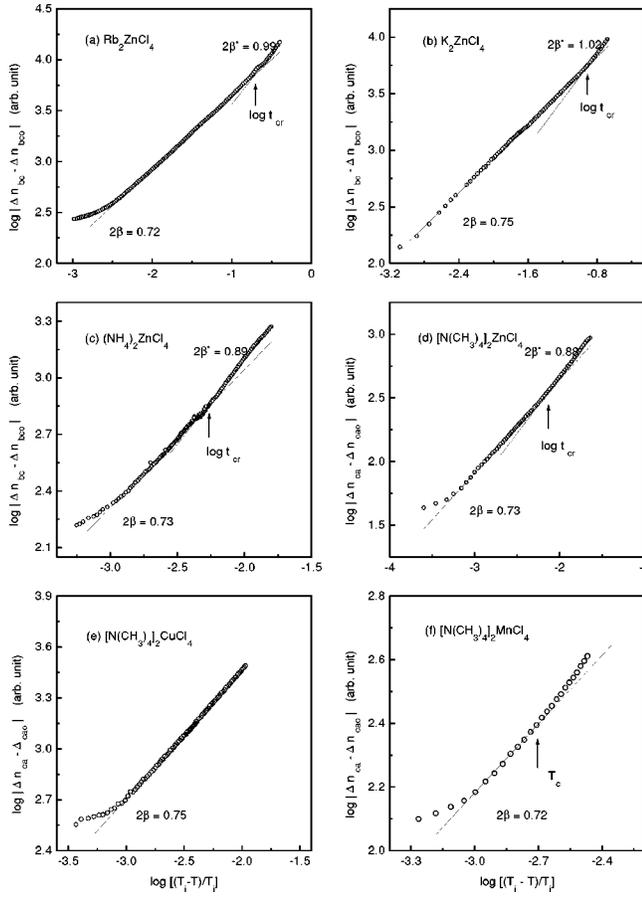


FIG. 3. The result of scaling analysis (open circle) of LB behavior for (a) RZC, (b) KZC, (c) AZC, (d) TMAZC, (e) TMACC, and (f) TMAMC. The solid line represents the slope of the exponent  $2\beta$ .

There have been some reports that the exponent obtained from the LB behavior is not  $2\beta$  but another exponent. Régis *et al.* reported that LB behaved as  $\Delta n \sim |t|^{0.87 \pm 0.02}$  for TMAZC.<sup>32</sup> In order to explain the result, they introduced  $C_2$  symmetry breaking in the N-IC phase transition of  $A_2BX_4$ -type crystals. The LB would not behave as  $\Delta n \sim |t|^{2\beta} \sim |t|^{0.70}$ , but as  $\Delta n \sim |t|^{2\beta} \sim |t|^{0.845}$  under the assumption of symmetry breaking.<sup>15</sup> However, their result is inconsistent with our results. In addition, the  $C_2$  symmetry breaking was not observed in x-ray diffraction studies in the N-IC phase transition.  $A_2BX_4$ -type crystals have  $Pm\bar{c}n$  structure in the N phase, and still preserve the  $Pm\bar{c}n_{ss\bar{1}}$  or  $Pm\bar{c}n_{1s\bar{1}}$  structure in the IC phase.<sup>3,4</sup>

On the basis of a critical review of existing LB theory ( $\Delta n \sim |t|^{2\beta}$ ), Ivanov *et al.* asserted that the existing LB theory was wrong and LB had to behave as  $\Delta n \sim |t|^{1-\alpha}$ .<sup>33</sup> If we accept their assertion, our results  $\Delta n \sim |t|^{0.72 \sim 0.75}$  must result in  $\alpha = 0.25 \sim 0.28$  ( $\pm 0.02$ ), which does not agree with any  $\alpha$  of specific heat studies on  $A_2BX_4$ -type crystals.<sup>6,11</sup>

Because of the inconsistency between these previous LB experiments, Levanyuk even suggested that  $2\beta$  of  $A_2BX_4$ -type crystals can vary in the range  $\frac{1}{2} < 2\beta < 1$  in the vicinity of the tricritical point.<sup>1</sup> But, the  $2\beta$ 's obtained from this LB study are the same for various  $A_2BX_4$ -type crystals within experimental error. (This means that  $2\beta$  does not depend on each  $A_2BX_4$ -type crystals but has a regular value.)

It is inferred that the origins of the inconsistency were due

to overlooking the LB tail and taking an inappropriate base line. The exponent obtained from the LB study is  $2\beta$  as described in Eq. (5).

In Figs. 3(a) and 3(b),  $\log_{10} \Delta n$  follows the  $2\beta = 0.72 \sim 0.75$  line in a wide temperature region  $\log_{10} t \leq -1$ . This means that the critical region of RZC and KZC is on the order of tens of a degree, and very wide compared to  $\text{KH}_2\text{PO}_4$  and Ising-type ferroelectric materials.<sup>34</sup> The temperature region where the classical Landau theory is valid can be determined by the Ginzburg criterion<sup>34,35</sup>

$$\frac{k_B}{16\pi e \Delta C} \xi(0)^{-3} \ll |t|^{1/2}. \quad (7)$$

In Eq. (7),  $k_B$  is the Boltzmann constant,  $\Delta C$  the jump in heat capacity per unit volume at  $T_i$ , and  $\xi(0)$  the zero temperature correlation length. In the view point of the Ginzburg criterion, the range of the critical region is mainly dependent on  $\xi(0)^{-3}$  of Eq. (7).<sup>35</sup> The wide critical region of RZC and KZC indicates that  $\xi(0)$  is very short compared to Ising-type ferroelectric materials. It is inferred that the characteristics of the N-IC phase transition of  $A_2BX_4$ -type crystals are dominated by short-range interactions.

Generally speaking, the factor  $(n_j^3 g_{jk} - n_i^3 g_{ik})$  of Eq. (4) is also dependent on temperature. However, for RZC and KZC, the  $\log_{10} \Delta n$  agrees very well with the  $2\beta$  line in the wide temperature range, so that the factor  $(n_j^3 g_{jk} - n_i^3 g_{ik})$  must have negligible temperature dependence in the logarithmic scale.

Because Cowley and Bruce's theoretical description is based on the renormalization technique, it is valid only in the critical region near  $T_i$ . The exponent of  $Q$  beyond the critical region  $\beta^*$  is expected to be  $2\beta^* = 1$  as described by Landau theory (mean field theory).<sup>34</sup> In Figs. 3(a) and 3(b), it is observed that  $\Delta n$  deviates from the  $2\beta = 0.72 \sim 0.75$  line, and asymptotically approaches the  $2\beta^* = 0.99 \sim 1.02$  line. This is consistent with the expectation of the crossover behavior from the 3D XY model into Landau theory. A similar behavior of  $Q$  was observed for  $\text{SrTiO}_3$  and  $\text{LaAlO}_3$ .<sup>36</sup>

Near the IC-C phase transition temperature ( $T_c$ ), the plane wave approximation of Eq. (2) is not valid and high order harmonics and discommensurations appear, so that  $\Delta n$  is not proportional to  $Q^2$  any more.<sup>1,2,17</sup> In Figs. 3(c) and 3(d), the LB behavior of AZC and TMAZC follows the  $2\beta = 0.73$  line only in the region  $\log_{10} t \leq -2$ , and approaches the  $2\beta^* = 0.88 \sim 0.89$  line in the region  $\log_{10} t \geq -2$ . As presented in Table I, the IC phase region of AZC and TMAZC is so narrow that  $T_c$  is close to  $T_i$ . Thus, the LB behavior cannot follow the  $2\beta = 0.73$  line because  $\Delta n$  is not proportional to  $Q^2$  in the region  $\log_{10} t \geq -2$ . The ambient exponent  $2\beta^* = 0.88 \sim 0.89$  is thought to be not a crossover behavior of the order parameter but an effect of the high order harmonics and discommensuration near  $T_c$ .

A new phase transition of AZC was recently reported just below  $T_i$  under uniaxial stress.<sup>37</sup> This could be an explanation for the narrow critical region of AZC. The ambient exponent of TMAZC,  $2\beta^* = 0.88$ , is close to  $2\beta = 0.87$ , obtained by Régis *et al.*, and it might be an origin of their wrong results.<sup>32</sup>

For TMACC and TMAMC, the IC phase region is very narrow, and any crossover behavior was not observed as seen

in Figs. 3(e) and 3(f). The IC phase region is only 5 K and even 0.6 K wide for TMACC and TMAMC, respectively. A more detailed description of the crossover phenomena is presented in Table II.

Whether the  $\beta^*$  of each crystal is a Landau-like one (such as RZC and KZC) or the ambient one (such as AZC and TMAZC) is thought to be dependent on the temperature range of the IC phase. The temperature range of the IC phase is mainly governed by the anisotropy of the thermodynamic potential of each crystal. In general, the stronger the anisotropy, the narrower the region of the IC phase.<sup>2</sup>

## V. SUMMARY

The critical behavior of LB was studied near the N-IC phase transition of various  $A_2BX_4$ -type crystals. The existence of the LB tail above  $T_i$  was observed and is explained

by the fluctuation effect. The values of  $2\beta$  obtained from LB are distributed in the range 0.72–0.75 ( $\pm 0.02$ ). The  $2\beta$  obtained in this study could be evidence supporting the prediction that the N-IC phase transition of  $A_2BX_4$ -type crystals belongs to the universality class of the 3D XY model. Our results also imply that LB behaves as  $\Delta n \sim |t|^{2\beta}$ , rather than  $\Delta n \sim |t|^{\tilde{\beta}}$  or  $\Delta n \sim |t|^{1-\alpha}$ . The asymptotic approach to  $2\beta^* = 1$  was observed beyond the critical region for RZC and KZC. It is thought to be the crossover from XY-like behavior into Landau-like behavior.

## ACKNOWLEDGMENTS

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