Generalized CP^1 model from the t_1-t_2-J model

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A long-wavelength, low-frequency, effective theory is obtained from the t_1-t_2-J model. The action is written in terms of two-component bose spinor fields (CP^1 fields) and two spinless Fermi fields. The generalized CP^1 model is invariant under U(1) gauge transformations. The bose fields and one of the Fermi fields have charge $+1$ while the other Fermi field has charge -1 with respect to these transformations. A simple mean-field theory of a gauge-symmetry breaking, based on a four-fermion interaction, is discussed. An effective theory of frustrated antiferromagnetism is obtained integrating out the Fermi fields around the mean fields. Another option is used to parametrize the long-distance fluctuations in the t_1-t_2-J model, with the help of gauge-invariant fields. It is argued that the resulting Fermi quasiparticles of the t_1-t_2-J model have both charge and spin. The effective action is rewritten in terms of the spin $\frac{1}{2}$ Fermi spinor, which has the charge of the holes, and unit vector. [S0163-1829(98)02317-0]

I. INTRODUCTION

High-T_c superconductivity has given theorists a strong motivation to work on correlated electrons. Among the many electronic models that are being currently studied, the two-dimensional t-J model is the simplest one that captures the essential physics of strongly correlated electronic systems. Anderson[1] first applied this model to high-T_c oxides. Zhang and Rice[2] and others[3] showed that the one-band t-J model is an effective model describing the physics of the three-band Cu-O model.[4]

The underlying problem is to create an adequate field theory. Usually, the bosonic and fermionic raising and lowering operators are used to realize spin-fermion algebra. Several approximate techniques have evolved so far to deal with the t-J model. A representation for the electron operators acting on states with no double occupancy has been proposed[5] in terms of spin-fermion operators and spinless bosons that keep track of empty sites. Mean-field theory of high-T_c superconductivity based on this representation has been developed.[6] The slave-boson technique is widely applied to the study of various properties of the model,[7] and to a large range of problems: Hubbard model,[8] Kondo lattice model,[5,9] and the Anderson Hamiltonian.[10]

Mean-field theory based on the alternative Schwinger bosons slave-fermion representation has been worked out.[11] A similar mean-field approach has been used to investigate the phase diagram of the t_1-t_2-J model.[12]

It is important to stress the fact that these theories start with one and the same Hamiltonian, and use equivalent representations of the spin-fermion algebra. But these representations allow different appropriate methods of approximate calculations that may arrive at completely different description of the properties of the model. The mean-field approximations are self-consistent, but it is difficult to judge how close to the true properties of the model the results are.

Numerical calculations have been done using a large number of techniques.[13] The results of these calculations can contribute to the acceptance or rejection of mean-field-based theories, and can also indicate directions in which new aproaches should be developed.

The reduction of the three-band model to the one-band t-J model is still controversial. Many authors have argued that the resulting quasiparticles of the three-band model have both charge and spin. The effective spin-fermion model is characterized by a Kondo-like coupling of the O holes to localized Cu spins and a Heisenberg antiferromagnetic interaction among Cu spins.[14] The long-wavelength limit of the model has been written in terms of fermionic spinors and a unit vector.[15] The dynamics of the order parameter of the spin background is given by the 0(3) nonlinear σ model, and the interaction of the mobile holes with the order parameter is a current-current type of interaction.

The three-band Cu-O model contains strong interactions, and the perturbative calculation of bubble and ladder diagrams are questionable. An analytical method, that seems to be able to handle strong correlations, has been proposed.[16] The photoemission and inverse photoemission spectra of holes calculated by means of the projection technique reproduce the numerical results.

It is widely accepted that the undoped oxides superconductors can be modeled rather well by a nearest-neighbor $s=\frac{1}{2}$ antiferromagnetic Heisenberg Hamiltonian on a square lattice. It has been argued that the long-wavelength, low-temperature, behavior of the model can be described by a quantum nonlinear σ model.[17] The low-temperature behavior of the correlation length and the static and dynamic spin-correlation functions have been calculated using the renormalization-group method.[18] The results are in good agreement with the experimental data.

The present work is motivated by the successful application of the quantum-mechanical nonlinear σ model to high-T_c oxides. A long-wavelength, low-frequency, effective theory is obtained from the t_1-t_2-J model. Under the assumption that the antiferromagnetic correlations are important, I introduce two sublattices and divide the spin vector into slow mode, described by unit vector, and fast mode, described by vector orthogonal to the unit one. Then the two fermions and the unit vector are considered on an equal footing as smooth

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fields and the fast mode is treated perturbatively. The action is expanded in powers of the fast mode and the first three terms in the respective Taylor expansion are taken into account. Integrating out the fast mode one obtains the effective action written in terms of two-component bose spinor fields (CP^1 fields) and two spinless Fermi fields. The generalized CP^1 model is invariant under U(1) gauge transformation. The bose fields and one of the Fermi fields have charge $+1$ while the other Fermi field has charge -1 with respect to these transformations. A simple mean-field theory of the gauge-symmerty breaking, based on the four-fermion interaction, is discussed. An effective theory of frustrated antiferromagnetism is obtained integrating out the Fermi fields around the mean fields.

Another option is used to parametrize the long-distance fluctuations in the t_1-t_2-J model, with the help of gauge-invariant fields. It is argued that the resulting Fermi quasiparticles of the t_1-t_2-J model have both charge and spin. The effective action is rewritten in terms of the spin-$\frac{1}{2}$ Fermi spinor, which has the charge of the holes, and unit vector.

The paper is organized as follows. Section II is devoted to the derivation of the generalized CP^1 model from the t_1-t_2-J one. In Sec. III a dynamical breakdown of the gauge symmetry is discussed. The effective action is rewritten in terms of gauge-invariant fields in Sec. IV. The charge and the spin of the Fermi quasiparticles are discussed. Finally, I comment on the relations to the other effective models.

II. GENERALIZED CP^1 MODEL

The t_1-t_2-J model is defined by the Hamiltonian

$$h=t_1\sum_{\langle i,j\rangle}[c^{\dagger}_{i\sigma}c_{j\sigma}+\text{H.c.}]+t_2\sum_{\langle\langle i,j\rangle\rangle}[c^{\dagger}_{i\sigma}c_{j\sigma}+\text{H.c.}]$$
$$+J\sum_{\langle i,j\rangle}(\boldsymbol{S}_i\cdot\boldsymbol{S}_j-\tfrac{1}{4}n_in_j)-\mu\sum_i n_i. \tag{1}$$

The electron operators $c_{i\sigma}$ $(c^{\dagger}_{i\sigma})$, the spin operators $\boldsymbol{S}_i$, and the number operator n_i act on a restricted Hilbert space where the doubly-ocupied state is excluded. The sums are over all sites of a two-dimensional square lattice, $\langle i,j\rangle$ denotes the sum over the nearest neighbors, and $\langle\langle i,j\rangle\rangle$ denotes the sum over the next to nearest neighbors.

Let us represent the eight operators by means of Schwinger bosons $\varphi_{i\sigma}$ $(\bar{\varphi}_{i\sigma})$, $\sigma=1,2$ and slave-fermions ψ_i $(\bar{\psi}_i)$

$$c_{i\sigma}=\bar{\psi}_i\varphi_{i\sigma},\quad \boldsymbol{S}_i=\tfrac{1}{2}\bar{\varphi}_i\boldsymbol{\sigma}\varphi_i,$$
$$\bar{c}_{i\sigma}=\psi_i\bar{\varphi}_{i\sigma},\quad n_i=1-\bar{\psi}_i\psi_i, \tag{2}$$

where $\boldsymbol{\sigma}$ are Pauli matrices. The finite-dimensional space of representation is a subspace of the Hilbert space of bosons and fermions defined by the operator consraint

$$\bar{\varphi}_{i\sigma}\varphi_{i\sigma}+\bar{\psi}_i\psi_i=1. \tag{3}$$

The partition function can be written as a path integral over the complex functions of the Matsubara time τ $\varphi_{i\sigma}(\tau)[\bar{\varphi}_{i\sigma}(\tau)]$ and Grassmann functions $\psi_i(\tau)[\bar{\psi}_i(\tau)]$

$$\mathcal{Z}(\beta)=\int d\mu(\bar{\varphi},\varphi,\bar{\psi},\psi)e^{-S}. \tag{4}$$

The action is given by the expression

$$S=\int_0^{\beta}d\tau\Bigg\{\sum_i[\bar{\varphi}_{i\sigma}(\tau)\dot{\varphi}_{i\sigma}(\tau)+\bar{\psi}_i(\tau)\dot{\psi}_i(\tau)]$$
$$+h(\bar{\varphi},\varphi,\bar{\psi},\psi)\Bigg\}, \tag{5}$$

where β is the inverse temperature and the Hamiltonian is obtained from Eq. (1) replacing the operators with the functions. In terms of Schwinger bosons and slave fermions, the theory is U(1) gauge invariant, and the measure includes δ functions that enforce the constraint Eq. (3) and the gauge-fixing condition

$$d\mu(\bar{\varphi},\varphi,\bar{\psi},\psi)=\prod_{i,\tau,\sigma}\frac{d\bar{\varphi}_{i\sigma}(\tau)d\varphi_{i\sigma}(\tau)}{2\pi i}\prod_{i\tau}d\bar{\psi}_i(\tau)d\psi_i(\tau)$$
$$\times\prod_{i\tau}\delta(gf)\prod_{i\tau}\delta(\bar{\varphi}_{i\sigma}(\tau)\varphi_{i\sigma}(\tau)$$
$$+\bar{\psi}_i(\tau)\psi_i(\tau)-1). \tag{6}$$

I make a change of variables, introducing new Bose fields $f_{i\sigma}(\tau)[\bar{f}_{i\sigma}(\tau)]$ (Ref. 19)

$$\varphi_{i\sigma}(\tau)=f_{i\sigma}(\tau)\sqrt{1-\bar{\psi}_i(\tau)\psi_i(\tau)}=f_{i\sigma}(\tau)[1-\tfrac{1}{2}\bar{\psi}_i(\tau)\psi_i(\tau)],$$
$$\bar{\varphi}_{i\sigma}(\tau)=\bar{f}_{i\sigma}(\tau)\sqrt{1-\bar{\psi}_i(\tau)\psi_i(\tau)}$$
$$=\bar{f}_{i\sigma}(\tau)[1-\tfrac{1}{2}\bar{\psi}_i(\tau)\psi_i(\tau)]. \tag{7}$$

The inverse relations are

$$f_{i\sigma}(\tau)=\varphi_{i\sigma}(\tau)[1+\tfrac{1}{2}\bar{\psi}_i(\tau)\psi_i(\tau)],$$
$$\bar{f}_{i\sigma}(\tau)=\bar{\varphi}_{i\sigma}(\tau)[1+\tfrac{1}{2}\bar{\psi}_i(\tau)\psi_i(\tau)], \tag{8}$$

and it is easy to see that the new fields satisfy the constraint

$$\bar{f}_{i\sigma}(\tau)f_{i\sigma}(\tau)=1. \tag{9}$$

Inserting Eq. (7) into Eqs. (1)–(6), one obtains for the action

$$S=\int_0^{\beta}d\tau\Bigg(\sum_i\{\bar{f}_{i\sigma}(\tau)\dot{f}_{i\sigma}(\tau)[1-\bar{\psi}_i(\tau)\psi_i(\tau)]$$
$$+\bar{\psi}_i(\tau)\dot{\psi}_i(\tau)\}+h(\bar{f},f,\bar{\psi},\psi)\Bigg), \tag{10}$$

where the Hamiltonian is

$$h=-t_1\sum_{\langle i,j\rangle}[\bar{\psi}_i(\tau)\psi_j(\tau)f_{i\sigma}(\tau)\bar{f}_{j\sigma}(\tau)+\bar{\psi}_j(\tau)\psi_i(\tau)f_{j\sigma}(\tau)\bar{f}_{i\sigma}(\tau)]-t_2\sum_{\langle\langle i,j\rangle\rangle}[\bar{\psi}_i(\tau)\psi_j(\tau)f_{i\sigma}(\tau)\bar{f}_{j\sigma}(\tau)$$
$$+\bar{\psi}_j(\tau)\psi_i(\tau)f_{j\sigma}(\tau)\bar{f}_{i\sigma}(\tau)]-\frac{J}{4}\sum_{\langle i,j\rangle}[1-\bar{f}_i(\tau)\boldsymbol{\sigma}f_i(\tau)\bar{f}_j(\tau)\boldsymbol{\sigma}f_j(\tau)][1-\bar{\psi}_i(\tau)\psi_i(\tau)][1-\bar{\psi}_j(\tau)\psi_j(\tau)]$$
$$-\mu\sum_i[1-\bar{\psi}_i(\tau)\psi_i(\tau)]. \tag{11}$$

The partition function can be written as an integral over the fields $f_{i\sigma}(\tau),\bar{f}_{i\sigma}(\tau),\psi_i(\tau),\bar{\psi}_i(\tau)$ and the measure is given by the equality

$$d\mu(\bar{f},f,\bar{\psi},\psi)=\prod_{i,\tau,\sigma}\frac{d\bar{f}_{i\sigma}(\tau)df_{i\sigma}(\tau)}{2\pi i}\prod_{i\tau}d\bar{\psi}_i(\tau)d\psi_i(\tau)\prod_{i\tau}\delta(gf)\prod_{i\tau}\delta[\bar{f}_{i\sigma}(\tau)f_{i\sigma}(\tau)-1]\prod_{i\tau}e^{-\bar{\psi}_i(\tau)\psi_i(\tau)}, \tag{12}$$

where the last multiplier just redefines the chemical potential.

I consider two sublattices A and B, and impose the gauge-fixing conditions in the form

$$\arg f_{i1}(\tau)=0 \quad \text{if} \quad i\in A,$$
$$\arg f_{j2}(\tau)=0 \quad \text{if} \quad j\in B. \tag{13}$$

Then one can use the components of the unit vector $\boldsymbol{n}_i$ to parametrize the solution of the constraints Eq. (9)

$$i\in A:\quad f_{i1}=\bar{f}_{i1}=\frac{1}{\sqrt{2}}(1+n_{i3})^{1/2},$$
$$f_{i2}=\frac{1}{\sqrt{2}}\frac{n_i^+}{(1+n_{i3})^{1/2}},\quad \bar{f}_{i2}=\frac{1}{\sqrt{2}}\frac{n_i^-}{(1+n_{i3})^{1/2}},$$
$$j\in B:\quad f_{j1}=\frac{1}{\sqrt{2}}\frac{n_j^-}{(1-n_{j3})^{1/2}},\quad \bar{f}_{j1}=\frac{1}{\sqrt{2}}\frac{n_j^+}{(1-n_{j3})^{1/2}},$$
$$f_{j2}=\bar{f}_{j2}=\frac{1}{\sqrt{2}}(1-n_{j3})^{1/2}, \tag{14}$$

where $n_r^{\pm}=n_{r1}\pm in_{r2}$.

Now I am going to the derivation of the long-wavelength limit of the model. To that purpose one introduces a new unit vector $\boldsymbol{m}_i$ $(\boldsymbol{m}_i^2=1)$ and a vector $\boldsymbol{L}_i$

$$n_i=\sqrt{1-a^2\boldsymbol{L}_i^2}\boldsymbol{m}_i+a\boldsymbol{L}_i,\quad \text{if}\quad i\in A,$$
$$n_j=-\sqrt{1-a^2\boldsymbol{L}_j^2}\boldsymbol{m}_j+a\boldsymbol{L}_j,\quad \text{if}\quad j\in B. \tag{15}$$

The new spin-vector $\boldsymbol{m}_i$ is a smooth field on the lattice and a is the lattice spacing. The constraint $\boldsymbol{n}_i^2=1$ and the requirement that the new vector $\boldsymbol{m}_i$ should obey the same constraint $\boldsymbol{m}_i^2=1$ demand $\boldsymbol{m}_i$ and $\boldsymbol{L}_i$ to be orthogonal,

$$\boldsymbol{m}_i\cdot\boldsymbol{L}_i=0. \tag{16}$$

The next step is to substitute Eq. (15) into the fields $f_{i\sigma}(\tau)[\bar{f}_{i\sigma}(\tau)]$ Eq. (14), and then to insert them into the action. This yields an action, which depends on the vectors $\boldsymbol{m}_i,\boldsymbol{L}_i$ and the fermionic fields. I expand the action in powers of the vector $\boldsymbol{L}_i$, keeping only the first three terms in the expansion. Integrating out the vector $\boldsymbol{L}_i$, one obtains the effective action (see Appendix)

$$S_{\text{eff}}=S_{CP^1}+S_F, \tag{17}$$

where S_{CP^1} is the action of the CP^1 model (σ model) and S_F is the part of the effective action that depends on the vector $\boldsymbol{m}_i$ (complex fields $z_{i\sigma},\bar{z}_{i\sigma}$) and the fermionic fields

$$
\begin{aligned}
S_F=\int_0^\beta d\tau\Bigg(&\sum_{i\in A}{}'\,\overline{\psi}_i^A(\tau)[\partial_\tau-\overline{z}_{i\sigma}\dot{z}_{i\sigma}]\psi_i^A(\tau)+\sum_{j\in B}{}'\,\overline{\psi}_j^B(\tau)[\partial_\tau+\overline{z}_{j\sigma}\dot{z}_{j\sigma}]\psi_j^B(\tau)+\frac{J}{2}\sum_{i\in A,\mu}{}'[\overline{\psi}_i^A(\tau)\psi_i^A(\tau)+\overline{\psi}_{i+a_\mu}^B(\tau)\psi_{i+a_\mu}^B(\tau)]\\
&+\frac{J}{2}\sum_{j\in B,\mu}{}'[\overline{\psi}_j^B(\tau)\psi_j^B(\tau)+\overline{\psi}_{j+a_\mu}^A(\tau)\psi_{j+a_\mu}^A(\tau)]-t_2\sum_{i\in A,\lambda}{}'[\overline{\psi}_i^A(\tau)\psi_{i+e_\lambda}^A(\tau)+\text{H.c.}]-t_2\sum_{j\in B,\lambda}{}'[\overline{\psi}_j^B(\tau)\psi_{j+e_\lambda}^B(\tau)+\text{H.c.}]\\
&+i\frac{t_1}{J}\sum_{i\in A,\mu}{}'(\boldsymbol{m}_i\times\dot{\boldsymbol{m}}_i)\cdot[\overline{\boldsymbol{e}}_{i,i+a_\mu}\overline{\psi}_{i+a_\mu}^B\psi_i^A+\boldsymbol{e}_{i,i+a_\mu}\overline{\psi}_i^A\psi_{i+a_\mu}^B]+i\frac{t_1}{J}\sum_{j\in B,\mu}{}'(\boldsymbol{m}_j\times\dot{\boldsymbol{m}}_j)\cdot[\overline{\boldsymbol{e}}_{j,j+a_\mu}\overline{\psi}_j^B\psi_{j+a_\mu}^A+\boldsymbol{e}_{j,j+a_\mu}\overline{\psi}_{j+a_\mu}^A\psi_j^B]\\
&-t_1\sum_{i\in A,\mu}{}'\{\overline{\psi}_i^A(\tau)\psi_{i+a_\mu}^B(\tau)[-z_{i1}(\tau)z_{i+a_\mu 2}(\tau)+z_{i2}(\tau)z_{i+a_\mu 1}(\tau)]+\text{H.c.}\}-t_1\sum_{j\in B,\mu}{}'\{\overline{\psi}_j^B(\tau)\psi_{j+a_\mu}^A(\tau)[-\overline{z}_{j2}(\tau)\overline{z}_{j+a_\mu 1}(\tau)\\
&+\overline{z}_{j1}(\tau)\overline{z}_{j+a_\mu 2}(\tau)]+\text{H.c.}\}-t_2\sum_{i\in A,\lambda}{}'\{\overline{\psi}_i^A(\tau)\psi_{i+e_\lambda}^A(\tau)z_{i\sigma}(\tau)[\overline{z}_{i+e_\lambda\sigma}(\tau)-\overline{z}_{i\sigma}(\tau)]+\text{H.c.}\}-t_2\sum_{j\in B,\lambda}{}'\{\overline{\psi}_j^B(\tau)\psi_{j+e_\lambda}^B(\tau)\overline{z}_{j\sigma}(\tau)\\
&\times[z_{j+e_\lambda\sigma}(\tau)-z_{j\sigma}(\tau)]+\text{H.c.}\}+\sum_{i\in A,\mu}{}'\left\{\frac{1}{32J}\dot{\boldsymbol{m}}_i\cdot\dot{\boldsymbol{m}}_i-\frac{J}{8}[\boldsymbol{m}_{i+a_\mu}(\tau)-\boldsymbol{m}_i(\tau)]^2\right\}[\overline{\psi}_i^A(\tau)\psi_i^A(\tau)+\overline{\psi}_{i+a_\mu}^B(\tau)\psi_{i+a_\mu}^B(\tau)]\\
&+\sum_{j\in B,\mu}{}'\left\{\frac{1}{32J}\dot{\boldsymbol{m}}_j\cdot\dot{\boldsymbol{m}}_j-\frac{J}{8}[\boldsymbol{m}_{j+a_\mu}(\tau)-\boldsymbol{m}_j(\tau)]^2\right\}[\overline{\psi}_j^B(\tau)\psi_j^B(\tau)+\overline{\psi}_{j+a_\mu}^A(\tau)\psi_{j+a_\mu}^A(\tau)]\\
&+\frac{\lambda}{4}\sum_{i\in A,\mu}{}'\,\overline{\psi}_i^A(\tau)\psi_i^A(\tau)\overline{\psi}_{i+a_\mu}^B(\tau)\psi_{i+a_\mu}^B(\tau)+\frac{\lambda}{4}\sum_{j\in B,\mu}{}'\,\overline{\psi}_j^B(\tau)\psi_j^B(\tau)\overline{\psi}_{j+a_\mu}^A(\tau)\psi_{j+a_\mu}^A(\tau)-\mu\sum_{i\in A}{}'[1-\overline{\psi}_i^A(\tau)\psi_i^A(\tau)]\\
&-\mu\sum_{j\in B}{}'[1-\overline{\psi}_j^B(\tau)\psi_j^B(\tau)]\Bigg)+S_{\text{add}},
\end{aligned}
\tag{18}
$$

where

$$\frac{\lambda}{4}=\frac{t_1^2}{J}-\frac{J}{2}. \tag{19}$$

I have replaced in Eq. (18) $\boldsymbol{e}_r$ and $\overline{\boldsymbol{e}}_r$ Eq. (A11) with $\boldsymbol{e}_{r,r'}$ and $\overline{\boldsymbol{e}}_{r,r'}$ where

$$
\begin{aligned}
&\mathrm{e}_{rr'1}=\frac{1}{2}(z_{r1}z_{r'1}-z_{r2}z_{r'2}),\quad \overline{\mathrm{e}}_{rr'1}=\frac{1}{2}(\overline{z}_{r1}\overline{z}_{r'1}-\overline{z}_{r2}\overline{z}_{r'2}),\\
&\mathrm{e}_{rr'2}=\frac{i}{2}(z_{r1}z_{r'1}+z_{r2}z_{r'2}),\quad \overline{\mathrm{e}}_{rr'2}=\frac{1}{2i}(\overline{z}_{r1}\overline{z}_{r'1}+\overline{z}_{r2}\overline{z}_{r'2}),\\
&\mathrm{e}_{rr'3}=-\frac{1}{2}(z_{r1}z_{r'2}+z_{r2}z_{r'1}),\\
&\overline{\mathrm{e}}_{rr'3}=-\frac{1}{2}(\overline{z}_{r1}\overline{z}_{r'2}+\overline{z}_{r2}\overline{z}_{r'1}).
\end{aligned}
\tag{20}
$$

The difference is of order a and it does not affect the long-wavelength physics.

The additional action S_{add} contains all terms in higher order of derivatives and fields. They do not contribute to the long-wavelength physics, and hereafter I shall not consider them.

Until now the fields $z_{i\sigma}$ ($\overline{z}_{i\sigma}$) have been viewed as defined by Eqs. (A5). Now, I consider $z_{i\sigma}$ ($\overline{z}_{i\sigma}$) as independent Bose fields that satisfy the constraint $\overline{z}_{i\sigma}z_{i\sigma}=1$ and the spin vector $\boldsymbol{m}_i$ as defined by the equality $\boldsymbol{m}_i=\overline{z}_i\boldsymbol{\sigma}z_i$. Then, the action (18) is invariant under the $U(1)$ gauge transformations

$$
\begin{aligned}
&z'_{r\sigma}(\tau)=e^{i\alpha_r(\tau)}z_{r\sigma}(\tau);\quad \overline{z}'_{r\sigma}(\tau)=e^{-i\alpha_r(\tau)}\overline{z}_{r\sigma}(\tau),\\
&\psi_r'^A(\tau)=e^{i\alpha_r(\tau)}\psi_r^A(\tau)\quad \text{if}\quad r\in A,\\
&\psi_r'^B(\tau)=e^{-i\alpha_r(\tau)}\psi_r^B(\tau)\quad \text{if}\quad r\in B.
\end{aligned}
\tag{21}
$$

One can restore the representation (A5) of the fields imposing the gauge-fixing condition $\arg z_{r1}=0$.

An important point in the effective model (18) is the four-fermion term. In the starting Hamiltonian (11) the four-fermion interaction is attractive. This, sometimes, leads to a speculative conjecture about superconductivity. But the sign in front of the four-fermion term in Eq. (11) is just an output of the parametrization. An additional repulsive four-fermion interaction appears in the effective theory (18) due to the interaction of the fermions with the ''fast modes'' of the spinon ($\boldsymbol{L}_i$). For the parameter range $\lambda>0$, it screens the attractive four-fermion interaction. I shall return to this term in the next section.

The effective theory Eq. (18) is a theory of slow spinon modes defined on a small area around the zero vector, and fermions defined on a whole antiferromagnetic Brillouin zone. All fermionic terms are taken into account exept for those of order equal or higher than six. This permits to investigate more special phases, related to the geometry of the lattice.

To carry out the long-wavelength limit for fermions, one should know the exact Fermi surface. But for small doping, it is enough to consider the dispersion of free fermions. In the model Eq. (18) with $t_2>0$, it has minima located at zero wave vector, and the continuum limit can be achieved by means of a gradient expansion around this point. In this way one obtains the following continuum theory:

$$S_{\rm eff}=\int d^2x d\tau\Big\{\frac{2}{g^2}[\overline{D_\tau z_\sigma}D_\tau z_\sigma+c^2\overline{D_\mu z_\sigma}D_\mu z_\sigma]+\bar\psi^A D_\tau^{(A)}\psi^A$$
$$+\frac{1}{2m}\overline{D_\mu^{(A)}\psi^A}D_\mu^{(A)}\psi^A+\bar\psi^B D_\tau^{(B)}\psi^B+\frac{1}{2m}\overline{D_\mu^{(B)}\psi^B}D_\mu^{(B)}\psi^B$$
$$-\frac{2t_1}{J}\bar\psi^A\psi^B(z_1\dot z_2-z_2\dot z_1)+\frac{2t_1}{J}\bar\psi^B\psi^A(\bar z_1\dot{\bar z}_2-\bar z_2\dot{\bar z}_1)$$
$$+t_1a^2(\bar\psi^A\partial_\mu\psi^B-\partial_\mu\bar\psi^A\psi^B)(z_1\partial_\mu z_2-z_2\partial_\mu z_1)$$
$$-t_1a^2(\bar\psi^B\partial_\mu\psi^A-\partial_\mu\bar\psi^B\psi^A)(\bar z_1\partial_\mu\bar z_2-\bar z_2\partial_\mu\bar z_1)$$
$$+\frac{2}{\tilde g}[\overline{D_\tau z_\sigma}D_\tau z_\sigma+\tilde c^2\overline{D_\mu z_\sigma}D_\mu z_\sigma](\bar\psi^A\psi^A+\bar\psi^B\psi^B)$$
$$+\lambda a^2\bar\psi^A\psi^A\bar\psi^B\psi^B+\mu(\bar\psi^A\psi^A+\bar\psi^B\psi^B)\Big\}, \qquad (22)$$

where

$$D_l z_\sigma=(\partial_l-\bar z_{\sigma'}\partial_l z_{\sigma'})z_\sigma,\quad l=0,x,y,$$
$$D_l^{(A)}\psi^A=(\partial_l-\bar z_{\sigma'}\partial_l z_{\sigma'})\psi^A,\quad D_l^{(B)}\psi^B=(\partial_l+\bar z_{\sigma'}\partial_l z_{\sigma'})\psi^B, \qquad (23)$$

and the parameters are given by the equalities $g=2a\sqrt{J}$, $c=aJ$, $\tilde g=2\sqrt{J}$, $\tilde c^2=2a^2J(2t_2-J)$, and $1/2m=2t_2a^2$.

To obtain the effective action, Eq. (22), I have rescaled the Fermi fields $(1/a)\psi^R\to\psi^R$, $(R=A$ or $B)$ and have used the identities

$$(\boldsymbol m\times\partial_l\boldsymbol m)\cdot\bar{\boldsymbol e}=-i(\bar z_1\partial_l\bar z_2-\bar z_2\partial_l\bar z_1),$$
$$(\boldsymbol m\times\partial_l\boldsymbol m)\cdot\boldsymbol e=i(z_1\partial_l z_2-z_2\partial_l z_1),$$
$$\tfrac14\partial_l\boldsymbol m\cdot\partial_l\boldsymbol m=\partial_l\bar z_\sigma\partial_l z_\sigma+\tfrac14(\bar z_\sigma\partial_l z_\sigma-z_\sigma\partial_l\bar z_\sigma)^2$$
$$=(\bar z_1\partial_l\bar z_2-\bar z_2\partial_l\bar z_1)(z_1\partial_l z_2-z_2\partial_l z_1), \qquad (24)$$

where l stands for τ, x, or y and no sum over l is assumed.

III. GAUGE-SYMMETRY BREAKING

The four-fermion terms in the effective action allow an appropriate mean-field theory of gauge-symmetry breaking. To demonstrate this I arrange the Fermi fields in the form

$$S_{F^4}=\int_0^\beta d\tau\Big\{-\frac{\lambda}{4}\sum_{i\in A\mu}{}'\bar\psi_i^A\psi_{i+a_\mu}^B\bar\psi_{i+a_\mu}^B\psi_i^A$$
$$-\frac{\lambda}{4}\sum_{j\in B\mu}{}'\bar\psi_j^B\psi_{j+a_\mu}^A\bar\psi_{j+a_\mu}^A\psi_j^B\Big\}. \qquad (25)$$

Then, by means of the Hubbard-Stratanovich transformation I introduce new collective complex fields $u_{i\mu}^A(\tau),\overline{u_{i\mu}^A}(\tau),u_{j\mu}^B(\tau),\overline{u_{j\mu}^B}(\tau)$, and rewrite the exponent in the form

$$e^{-S_{F^4}}=\int\prod_{i\mu\tau}d\overline{u_{i\mu}^A}(\tau)du_{i\mu}^A(\tau)\prod_{j\mu\tau}d\overline{u_{j\mu}^B}(\tau)du_{j\mu}^B(\tau)$$
$$\times\exp\int_0^\beta d\tau\Big\{\frac{\lambda}{4}\sum_{i\in A\mu}{}'[\overline{u_{i\mu}^A}(\tau)u_{i\mu}^A(\tau)$$
$$-\overline{u_{i\mu}^A}\bar\psi_{i+a_\mu}^B(\tau)\psi_i^A(\tau)-\bar\psi_i^A(\tau)\psi_{i+a_\mu}^B(\tau)u_{i\mu}^A(\tau)]$$
$$+\frac{\lambda}{4}\sum_{j\in B\mu}{}'[\overline{u_{j\mu}^B}(\tau)u_{j\mu}^B(\tau)-\overline{u_{j\mu}^B}\bar\psi_{j+a_\mu}^A(\tau)\psi_j^B(\tau)$$
$$-\bar\psi_j^B(\tau)\psi_{j+a_\mu}^A(\tau)u_{j\mu}^B(\tau)]\Big\}. \qquad (26)$$

The mean-field approximation for the problem is just the evaluation of the path integral over the new collective fields by means of the saddle-point approximation. The stationary conditions are

$$\frac{\delta\mathcal F}{\delta u_{i\mu}^A}=0,\quad\frac{\delta\mathcal F}{\delta\overline{u_{i\mu}^A}}=0,\quad\frac{\delta\mathcal F}{\delta u_{j\mu}^B}=0,\quad\frac{\delta\mathcal F}{\delta\overline{u_{j\mu}^B}}=0, \qquad (27)$$

where

$$\mathcal F=-\frac{\lambda}{4\beta N_1}\int_0^\beta d\tau\Big[\sum_{i\in A,\mu}{}'\overline{u_{i\mu}^A}(\tau)u_{i\mu}^A(\tau)$$
$$+\sum_{j\in B,\mu}{}'\overline{u_{j\mu}^B}(\tau)u_{j\mu}^B(\tau)\Big]+\mathcal F_0, \qquad (28)$$

and $\mathcal F_0$ is the free energy of a system with Hamiltonian

$$h_{\rm m.f.}=-t_2\sum_{i\in A\lambda}{}'[\bar\psi_i^A\psi_{i+e_\lambda}^A+{\rm H.c.}]-t_2\sum_{j\in B\lambda}{}'[\bar\psi_j^B\psi_{j+e_\lambda}^B+{\rm H.c.}]$$
$$+\frac{\lambda}{4}\sum_{i\in A\mu}{}'[\overline{u_{i\mu}^A}\bar\psi_{i+a_\mu}^B(\tau)\psi_i^A(\tau)$$
$$+\bar\psi_i^A(\tau)\psi_{i+a_\mu}^B(\tau)u_{i\mu}^A(\tau)]$$
$$+\frac{\lambda}{4}\sum_{j\in B\mu}{}'[\overline{u_{j\mu}^B}\bar\psi_{j+a_\mu}^A(\tau)\psi_j^B(\tau)$$
$$+\bar\psi_j^B(\tau)\psi_{j+a_\mu}^A(\tau)u_{j\mu}^B(\tau)]+\mu\sum_{i\in A\mu}{}'\bar\psi_i^a\psi_i^A$$
$$+\mu\sum_{j\in B\mu}{}'\bar\psi_j^B\psi_j^B. \qquad (29)$$

The mean-field equations (27) have a trivial solution $u^A=\overline{u^A}=u^B=\overline{u^B}=0$, but, when $\lambda>0$ they have and nonzero solution which leads to the breaking of the gauge symmetry.

In the phase with broken gauge symmetry the normal Green functions read

$$S_k^{AA}(\tau-\tau')=S_k^{BB}(\tau-\tau')=\frac{1}{\beta}\sum_{\omega_n} e^{i\omega_n(\tau-\tau')}\frac{i\omega_n+\varepsilon_k}{(i\omega_n+\varepsilon_k)^2-|\gamma_k|^2} \quad (30)$$

and for the anomalous Green functions one obtains

$$S_k^{BA}(\tau-\tau')=\frac{1}{\beta}\sum_{\omega_n} e^{i\omega_n(\tau-\tau')}\frac{\gamma_k}{(i\omega_n+\varepsilon_k)^2-|\gamma_k|^2},$$

$$S_k^{AB}(\tau-\tau')=\frac{1}{\beta}\sum_{\omega_n} e^{i\omega_n(\tau-\tau')}\frac{\bar{\gamma}_k}{(i\omega_n+\varepsilon_k)^2-|\gamma_k|^2}. \quad (31)$$

The sum is over the frequencies $\omega_n=(2n+1)\pi/\beta$, $\varepsilon=\mu-4t_2\cos k_x\cos k_y$, and $\gamma_k=\overline{u_\mu^A}e^{-iak_\mu}+u_\mu^B e^{iak_\mu}$ where the order parameters u_μ^A and u_μ^B are choosen to be homogeneous.

Integrating over the fermions around the mean-field values of the order parameters one obtains an effective action that is not gauge invariant,

$$\begin{aligned}S'_{\rm eff}=\int d^2x\,d\tau\Big\{&\frac{2}{g_r^2}[\overline{D_\tau z_\sigma}D_\tau z_\sigma+c_r^2\overline{D_\mu z_\sigma}D_\mu z_\sigma]\\&+\bar{W}_l(z_1\partial_l z_2-z_2\partial_l z_1)+W_l(\bar{z}_1\partial_l\bar{z}_2-\bar{z}_2\partial_l\bar{z}_1)\\&+Z_l[(z_1\partial_\mu z_2-z_2\partial_\mu z_1)^2+(\bar{z}_1\partial_\mu\bar{z}_2-\bar{z}_2\partial_\mu\bar{z}_1)^2]\\&+M_l(\bar{z}_\sigma\partial_l z_\sigma-z_\sigma\partial_l\bar{z}_\sigma)^2\Big\},\end{aligned} \quad (32)$$

where l stands for τ,x,y.

The coefficients in front of the terms that break the gauge symmetry (W_l,Z_l,M_l) are zero if the order parameter is zero. The constants $W_l(\bar{W}_l)$ are proportional to t_1 and result from the tadpole diagrams with anomalous Green functions, and special values of the order parameters. The constants Z_l and M_l are obtained calculating the one-loop diagrams with two anomalous Green functions. Z_l are proportional to t_1^2 and M_l are proportional to t_2^2. One can get the renormalized parameter g_r of the CP^1 model and the renormalized spin-wave velocity c_r calculating the one-loop diagram with two normal Green functions, and using Eqs. (24).

Generalized CP^1 models with broken gauge symmetry have been largely discussed in the literature. An additional exitation (thirth Goldstone boson) appears in the theory as a result of gauge-symmetry breaking. A generalized CP^1 model with "W" terms only has been considered as a model of the spiral phase of a doped antiferromagnet.[20,21] Due to "W" terms the minimum of the dispersion of the z_σ ($\bar{z}_\sigma$) quanta is not at the zero wave vector, which leads to incommensurate order.

The massive CP^1 model with "M" terms only, has been investigated as a model of frustrated antiferromagnet, by means of the renormalization-group technique and $1/N$ expansion.[22,23]

As was shown above, the effective model of the antiferromagnet, which is frustrated due to the doping, also contains terms with constants Z_l. These terms split the spectrum of the antiferromagnetic magnons and make the application of the large-N expansion based on SU(N) group questionable. Moreover, if one considers a theory without massive terms $(t_2=0)$, then a large-N expansion based on the Sp(2N) group is plausible.

The magnon fluctuations influence the fermions strongly, and a mean-field theory that treats fermions separately seems to be not an adequate way to solve the model. Nevertheless, the effective theory (32) gives a good intuition for investigation of the effective model, Eq. (18), of doped antiferromagnets.

IV. EFFECTIVE THEORY IN TERMS OF GAUGE-INVARIANT FIELDS

Another option is to parametrize the long-distance fluctuations with help of gauge-invariant fields. To do this I introduce two gauge-invariant Fermi fields $c_\sigma(\bar{c}_\sigma)$,

$$\begin{aligned}c_1(\tau,\mathbf{x})&=z_1(\tau,\mathbf{x})\psi^B(\tau,\mathbf{x})-\bar{z}_2(\tau,\mathbf{x})\psi^A(\tau,\mathbf{x}),\\c_2(\tau,\mathbf{x})&=\bar{z}_1(\tau,\mathbf{x})\psi^A(\tau,\mathbf{x})+z_2(\tau,\mathbf{x})\psi^B(\tau,\mathbf{x}).\end{aligned} \quad (33)$$

Under the action of the group of rotations the fields $\psi^A(\tau,\mathbf{x}),\psi^B(\tau,\mathbf{x})$ are singlets and the Bose fields $z_\sigma(\tau,\mathbf{x})[\bar{z}_\sigma(\tau,\mathbf{x})]$ are spin-$\frac{1}{2}$ spinors. One can check that the Fermi fields $c_\sigma(\tau,\mathbf{x})[\bar{c}_\sigma(\tau,\mathbf{x})]$ transform like components of spin-$\frac{1}{2}$ spinor. Then, it is not difficult to guess the inverse relations, because there are only two singlets that can be built up by means of the Fermi $c_\sigma(\tau,\mathbf{x}),\bar{c}_\sigma(\tau,\mathbf{x})$ and Bose $z_\sigma(\tau,\mathbf{x}),\bar{z}_\sigma(\tau,\mathbf{x})$ spinors

$$\begin{aligned}\psi^B(\tau,\mathbf{x})&=\bar{z}_1(\tau,\mathbf{x})c_1(\tau,\mathbf{x})+\bar{z}_2(\tau,\mathbf{x})c_2(\tau,\mathbf{x}),\\\psi^A(\tau,\mathbf{x})&=z_1(\tau,\mathbf{x})c_2(\tau,\mathbf{x})-z_2(\tau,\mathbf{x})c_1(\tau,\mathbf{x}).\end{aligned} \quad (34)$$

Equations (33) can be regarded as a SU(2) transformation; $c_\sigma=U_{\sigma,\sigma'}\psi_{\sigma'}$ $(\psi_1=\psi^B,\psi_2=\psi^A)$ where $U_{11}=z_1$; $U_{12}=-\bar{z}_2$; $U_{21}=z_2$; $U_{22}=\bar{z}_1$. Then, it follows that the Fermi measure is invariant under the change of variables (33) and that the following equalities hold:

$$\begin{aligned}&\bar{\psi}^A(\tau,\mathbf{x})\psi^A(\tau,\mathbf{x})+\bar{\psi}^B(\tau,\mathbf{x})\psi^B(\tau,\mathbf{x})=\bar{c}_\sigma(\tau,\mathbf{x})c_\sigma(\tau,\mathbf{x}),\\&\bar{\psi}^A(\tau,\mathbf{x})\psi^A(\tau,\mathbf{x})\bar{\psi}^B(\tau,\mathbf{x})\psi^B(\tau,\mathbf{x})\\&\quad=\bar{c}_1(\tau,\mathbf{x})c_1(\tau,\mathbf{x})\bar{c}_2(\tau,\mathbf{x})c_2(\tau,\mathbf{x}).\end{aligned} \quad (35)$$

To get the effective action in terms of the fields $c_\sigma(\bar{c}_\sigma)$ and the unit vector $\mathbf{m}=\bar{z}\boldsymbol{\sigma}z$, one has to use the relations

$$\begin{aligned}\bar{c}_\sigma\partial_\tau c_\sigma=\bar{\psi}^A D_\tau^{(A)}\psi^A+\bar{\psi}^B D_\tau^{(B)}\psi^B+\bar{\psi}^A\psi^B(z_1\partial_\tau z_2-z_2\partial_\tau z_1)\\-\bar{\psi}^B\psi^A(\bar{z}_1\partial_\tau\bar{z}_2-\bar{z}_2\partial_\tau\bar{z}_1),\end{aligned} \quad (36)$$

$$\begin{aligned}\frac{1}{2i}\bar{c}\boldsymbol{\sigma}c\cdot(\mathbf{m}\times\partial_\tau\mathbf{m})=\bar{\psi}^A\psi^B(z_1\partial_\tau z_2-z_2\partial_\tau z_1)\\-\bar{\psi}^B\psi^A(\bar{z}_1\partial_\tau\bar{z}_2-\bar{z}_2\partial_\tau\bar{z}_1),\end{aligned} \quad (37)$$

$$\partial_\mu \overline{c}_\sigma \partial_\mu c_\sigma = \overline{D_\mu^{(A)} \psi^A} D_\mu^{(A)} \psi^A + \overline{D_\mu^{(B)} \psi^B} D_\mu^{(B)} \psi^B + \overline{D_\mu z_\sigma} D_\mu z_\sigma (\overline{\psi}^A \psi^A + \overline{\psi}^B \psi^B) + (\overline{\psi}^B \partial_\mu \psi^A - \partial_\mu \overline{\psi}^B \psi^A)(\overline{z}_1 \partial_\mu \overline{z}_2 - \overline{z}_2 \partial_\mu \overline{z}_1) - (\overline{\psi}^A \partial_\mu \psi^B - \partial_\mu \overline{\psi}^A \psi^B)(z_1 \partial_\mu z_2 - z_2 \partial_\mu z_1), \quad (38)$$

$$(\overline{c}\boldsymbol{\sigma}\partial_\mu c - \partial_\mu \overline{c}\boldsymbol{\sigma} c)\cdot(\boldsymbol{m}\times\partial_\mu \boldsymbol{m}) = -4i\overline{D_\mu z_\sigma} D_\mu z_\sigma (\overline{\psi}^A \psi^A + \overline{\psi}^B \psi^B) - 2i(\overline{\psi}^B \partial_\mu \psi^A - \partial_\mu \overline{\psi}^B \psi^A)(\overline{z}_1 \partial_\mu \overline{z}_2 - \overline{z}_2 \partial_\mu \overline{z}_1) + 2i(\overline{\psi}^A \partial_\mu \psi^B - \partial_\mu \overline{\psi}^A \psi^B)(z_1 \partial_\mu z_2 - z_2 \partial_\mu z_1), \quad (39)$$

where μ stands for x or y. Taking into account the above equalities and Eqs. (24) one obtains

$$S_{\rm eff} = \int d\tau d^2 x \left\{ \frac{1}{2g^2}(\partial_\tau \boldsymbol{m}\cdot\partial_\tau \boldsymbol{m} + c^2 \partial_\mu \boldsymbol{m}\cdot\partial_\mu \boldsymbol{m}) + \overline{c}_\sigma \partial_\tau c_\sigma + \frac{1}{2m}\partial_\mu \overline{c}_\sigma \partial_\mu c_\sigma + i\gamma_\tau \overline{c}\boldsymbol{\sigma} c\cdot(\boldsymbol{m}\times\partial_\tau \boldsymbol{m}) - i\gamma_r(\overline{c}\boldsymbol{\sigma}\partial_\mu c - \partial_\mu \overline{c}\boldsymbol{\sigma} c)\cdot(\boldsymbol{m}\times\partial_\mu \boldsymbol{m}) + \lambda_0 \overline{c}_1 c_1 \overline{c}_2 c_2 + \frac{1}{2g_0^2}(\partial_\tau \boldsymbol{m}\cdot\partial_\tau \boldsymbol{m} + c_0^2 \partial_\mu \boldsymbol{m}\cdot\partial_\mu \boldsymbol{m})\overline{c}_\sigma c_\sigma + \mu \overline{c}_\sigma c_\sigma \right\}, \quad (40)$$

where

$$g = 2a\sqrt{J}; \quad c = aJ, \quad g_0 = 2\sqrt{J};$$

$$c_0 = 4a^2 J\left(t_1 + 2t_2 - \frac{J}{2}\right);$$

$$\frac{1}{2m} = 2a^2 t_2; \quad \lambda_0 = a^2\left(\frac{4t_1^2}{J} - 2J\right); \quad \gamma_\tau = \frac{t_1}{J} + \frac{1}{2};$$

$$\gamma_r = a^2\left(\frac{t_1}{2} + t_2\right). \quad (41)$$

It follows from the effective theory Eq. (40) that the resulting Fermi quasiparticles $c_\sigma(\overline{c}_\sigma)$ of the t_1-t_2-J model have both charge and spin. Let us trace the origin of the result. In the presence of the next to nearest-neighbor hopping the dispersion of the charge carriers (holons) has a twofold degenerate minimum. One can introduce two sublattices, and then the charged spinless particles are two, ψ^A and ψ^B. An unexpected result is that in the long-wavelength, low-frequency limit these fields can be mapped [Eqs. (33) and (34)] onto the spin-$\frac{1}{2}$ spinor with the same charge.

Without the four-fermion term the effective action coincides with the effective action proposed in Ref. 15. The special point is that the effective model in Ref. 15 is obtained from spin-fermion one that results from a strong-coupling expansion of the three band Cu-O model. The Fermi quasiparticles have a transparent physical interpretation, which is not so in the case of model (40).

V. CONCLUSIONS

In this paper a long-wavelength, low-frequency, effective theory of the t_1-t_2-J model was explicitly derived. The effective action was written as a generalized CP^1 model [Eqs. (18) and (22)] in terms of bose spinor fields and two spinless Fermi fields. A mean-field theory of gauge-symmetry breaking, based on a four-fermion interaction was discussed in Sec. III. The breakdown of the gauge symmetry leads to a frustration of the antiferromagnetically ordered system, and the ground state is a long-range spiral state.

Now, let us consider the opposite limit, when the four-fermion interaction is weak and one can drop it. It is convenient to introduce the composite U(1) gauge field $A_\mu = -i\overline{\varphi}_\sigma \partial_\mu \varphi_\sigma$. Integrating out the bosons and fermions one can perform a large-N expansion, where N is the number of bosonic as well as the fermionic fields. In the leading order of $1/N$ a mass of the φ_σ bosons is generated dynamically. Therefore, within a large-N expansion, a doping induced quantum phase transition from an antiferromagnetically ordered state at zero temperature to a quantum disordered spin-liquid state takes place.[24] The CP^1 representation seems to be preferable in this case because there are reasons to believe that the low-energy excitations in the disordered phase of quantum spin systems are spin one-half deconfined spinons.[25]

The two quite different pictures demonstrate that the true ground state of the model Eq. (22) should be looked for within an approach that treats the four-fermion interaction and the fermion-boson interaction on an equal footing.

In Sec. IV it was demonstrated that the effective model can be rewritten in terms of unit vector that denotes the antiferromagnetic order parameter for the spin-background and spin-$\frac{1}{2}$ fermion Eq. (40). In the case of weak current-current and four-fermion interactions, one obtains that the Fermi quasiparticles of the model have the charge of the holes and spin $\frac{1}{2}$. This means that if we consider the low-lying Fermi states of the t_1-t_2-J model and those of spin-$\frac{1}{2}$ quasiparticles, with appropriate Landau parameters, there is one-to-one correspondence between them. This result was supported by numerical calculations.[26] The authors use an exact diagonalization technique on small clusters to study the momentum distribution function of the lightly doped t-J model. They explicitly perform the above-mentioned mapping.

In the physically relevant case, most edequate technique should be applied. The σ-model part of the action can be treated in the same way as in Ref. 18. To deal with the four-fermion term by means of the renormalization group, one has to use techniques described in Ref. 27. The nontrivial point is the current-current interaction that strongly influences both the spinon spectrum and the long-wavelength behavior of the fermions.

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APPENDIX

Let us substitute the representation, Eq. (15), for the spin vector into the fields $f_{i\sigma}(\tau)[\overline{f}_{i\sigma}(\tau)]$ Eq. (14), and then to insert them into the action. I shall expand the action in powers of the vector $\boldsymbol{L}_i$, keeping only the first three terms in the expansion.

To begin with I address the terms with time derivatives

$$S_{\mathrm{kin}}=\int_0^\beta d\tau\sum_i \{\overline{f}_{i\sigma}(\tau)\dot{f}_{i\sigma}(\tau)[1-\overline{\psi}_i(\tau)\psi_i(\tau)]+\overline{\psi}_i(\tau)\dot{\psi}_i(\tau)\}$$

$$=\int_0^\beta d\tau\left(\sum_{i\in A}{}'\left\{\frac{i}{2}\vec{\mathcal{A}}(\boldsymbol{n}_i)\cdot\dot{\boldsymbol{n}}_i(\tau)[1-\overline{\psi}_i^A(\tau)\psi_i^A(\tau)]+\overline{\psi}_i^A(\tau)\dot{\psi}_i^A(\tau)\right\}+\sum_{j\in B}{}'\left\{\frac{i}{2}\vec{\mathcal{A}}(-\boldsymbol{n}_j)\cdot\dot{\boldsymbol{n}}_j(\tau)\times[1-\overline{\psi}_j^B(\tau)\psi_j^B(\tau)]+\overline{\psi}_j^B(\tau)\dot{\psi}_j^B(\tau)\right\}\right), \tag{A1}$$

where

$$\frac{i}{2}\vec{\mathcal{A}}(\boldsymbol{n}_r)=\overline{f}_{r\sigma}(\boldsymbol{n}_r)\frac{\partial}{\partial\boldsymbol{n}_r}f_{r\sigma}(\boldsymbol{n}_r) \tag{A2}$$

is the vector potential of a Dirac magnetic monopol at the center of the unit sphere. It obeys locally

$$\boldsymbol{\partial}_{\boldsymbol{n}}\times\vec{\mathcal{A}}(\boldsymbol{n})=\boldsymbol{n}. \tag{A3}$$

Substituting Eq. (15) into Eq. (A1) and keeping the terms up to order a, one obtains

$$S_{\mathrm{kin}}=\int_0^\beta d\tau\left\{\frac{i}{2}\sum_i(-1)^{|i|}\vec{\mathcal{A}}(\boldsymbol{m})\cdot\dot{\boldsymbol{m}}_i(\tau)+\frac{i}{2}a\sum_i\left(\frac{\partial\mathcal{A}^\alpha}{\partial m_{i\beta}}L_{i\beta}(\tau)\dot{m}_{i\alpha}(\tau)+\vec{\mathcal{A}}(\boldsymbol{m})\cdot\dot{\boldsymbol{L}}_i(\tau)\right)+\sum_{i\in A}{}'\overline{\psi}_i^A(\tau)[\partial_\tau-\overline{z}_{i\sigma}(\tau)\dot{z}_{i\sigma}(\tau)]\psi_i^A(\tau)+\sum_{j\in B}{}'\overline{\psi}_j^B(\tau)\times[\partial_\tau+\overline{z}_{j\sigma}(\tau)\dot{z}_{j\sigma}(\tau)]\psi_j^B(\tau)-\frac{i}{2}a\sum_{i\in A}{}'\overline{\psi}_i^A(\tau)\times\left[\frac{\partial\mathcal{A}^\alpha(\boldsymbol{m})}{\partial m_{i\beta}}L_{i\beta}(\tau)\dot{m}_{i\alpha}(\tau)+\vec{\mathcal{A}}(\boldsymbol{m})\cdot\dot{\boldsymbol{L}}_i(\tau)\right]\psi_i^A(\tau)-\frac{i}{2}a\sum_{j\in B}{}'\overline{\psi}_j^B(\tau)\left[\frac{\partial\mathcal{A}^\alpha(\boldsymbol{m})}{\partial m_{j\beta}}L_{j\beta}(\tau)\dot{m}_{j\alpha}(\tau)+\vec{\mathcal{A}}(\boldsymbol{m})\cdot\dot{\boldsymbol{L}}_j(\tau)\right]\psi_j^B(\tau)\right\}, \tag{A4}$$

where I have introduced two complex fields $z_{i\sigma}(\tau)[\overline{z}_{i\sigma}(\tau)]$,

$$z_{r1}=\overline{z}_{r1}=\frac{1}{\sqrt{2}}(1+m_{r3})^{1/2},$$

$$z_{r2}=\frac{1}{\sqrt{2}}\frac{m_r^+}{(1+m_{r3})^{1/2}},\quad \overline{z}_{r2}=\frac{1}{\sqrt{2}}\frac{m_r^-}{(1+m_{r3})^{1/2}}, \tag{A5}$$

which satisfy $\overline{z}_{r\sigma}z_{r\sigma}=1$ and $\boldsymbol{m}_r=\overline{z}_r\boldsymbol{\sigma}z_r$.

The first term in Eq. (A4) is not important in the two-dimensional case and I ignore it. The second term, after integration by parts, can be written in the form

$$\frac{i}{2}a\int_0^\beta\sum_i\left(\frac{\partial\mathcal{A}^\alpha}{\partial m_{i\beta}}L_{i\beta}(\tau)\dot{m}_{i\alpha}(\tau)+\vec{\mathcal{A}}(\boldsymbol{m})\cdot\dot{\boldsymbol{L}}_i(\tau)\right)=\frac{i}{2}a\int_0^\beta\sum_i\left(\frac{\partial\mathcal{A}^\alpha}{\partial m_{i\beta}}-\frac{\partial\mathcal{A}^\beta}{\partial m_{i\alpha}}\right)L_{i\beta}(\tau)\dot{m}_{i\alpha}(\tau)=\frac{i}{2}a\int_0^\beta\sum_i\epsilon_{\beta\gamma\alpha}(\boldsymbol{\partial}_{\boldsymbol{m}}\times\vec{\mathcal{A}})_\gamma L_{i\beta}(\tau)\dot{m}_{i\alpha}(\tau)=\frac{i}{2}a\int_0^\beta\sum_i(\boldsymbol{m}_i\times\dot{\boldsymbol{m}}_i)\cdot\boldsymbol{L}_i, \tag{A6}$$

where Eq. (A3) is used.

The last two terms in Eq. (A4) can be canceled by the transformation

$$\psi_r^R(\tau)\to e^{i(a/2)\Delta_r(\tau)}\psi_r^R(\tau), \tag{A7}$$

where

$$\dot{\Delta}_r(\tau)=\frac{\partial\mathcal{A}^\alpha(\boldsymbol{m})}{\partial m_{r\beta}}L_{r\beta}(\tau)\dot{m}_{r\alpha}(\tau)+\vec{\mathcal{A}}(\boldsymbol{m}_r)\cdot\dot{\boldsymbol{L}}_r(\tau) \tag{A8}$$

and R stands for A or B. After this transformation phases appear only in the hopping terms in the form of $\exp\{(i/2)a(\Delta_r-\Delta_{r'})\}$. In the continuum limit, $\Delta_r-\Delta_{r'}$ is of the order of a. Hence, the phases give no contribution to the effective action. This means, that in the long-wavelength, low-frequency limit one can ignore the last two terms in Eq. (A4).

Dealing with the hopping terms it is convenient to represent the vector $\boldsymbol{L}_i$ in the form

$$\boldsymbol{L}_i=\overline{\kappa}_i\mathbf{e}_i+\kappa_i\overline{\mathbf{e}}_i, \tag{A9}$$

where the complex vectors $\mathbf{e}_i$ and the conjugated vector $\overline{\mathbf{e}}_i$ are orthogonal to the vector $\boldsymbol{m}_i$ and satisfy

$$\mathbf{e}_i^2=\overline{\mathbf{e}}_i^2=0,\quad \overline{\mathbf{e}}_i\cdot\mathbf{e}_i=\tfrac{1}{2}. \tag{A10}$$

The explicit expressions for the vectors are

$$\mathrm{e}_{i1}=\frac{1}{2}(z_{i1}z_{i1}-z_{i2}z_{i2}),\quad \overline{\mathrm{e}}_{i1}=\frac{1}{2}(\overline{z}_{i1}\overline{z}_{i1}-\overline{z}_{i2}\overline{z}_{i2}),$$

$$\mathrm{e}_{i2}=\frac{i}{2}(z_{i1}z_{i1}+z_{i2}z_{i2}),\quad \overline{\mathrm{e}}_{i2}=\frac{1}{2i}(\overline{z}_{i1}\overline{z}_{i1}+\overline{z}_{i2}\overline{z}_{i2}),$$

$$\mathrm{e}_{i3}=-z_{i1}z_{i2},\quad \overline{\mathrm{e}}_{i3}=-\overline{z}_{i1}\overline{z}_{i2}. \tag{A11}$$

The fields $f_{i\sigma}(\tau)[\overline{f_{i\sigma}}(\tau)]$ depend on the vectors $\boldsymbol{m}_i(\tau)$ and the fields $\kappa_i(\tau)[\overline{\kappa_i}(\tau)]$. I expand them in powers of $\kappa_i(\tau)$ and $\overline{\kappa_i}(\tau)$ up to linear terms. This yields

$$f_{i\sigma}\overline{f}_{j\sigma}\simeq -z_{i1}z_{j2}+z_{i2}z_{j1}+a\kappa_i$$

$$f_{j\sigma}\overline{f}_{i\sigma}\simeq -\overline{z}_{i1}\overline{z}_{j2}+\overline{z}_{i2}\overline{z}_{j1}+a\overline{\kappa}_i, \tag{A12}$$

if $i\in A$ and $j=i+a_\mu$;

$$f_{j\sigma}\overline{f}_{i\sigma}\simeq -\overline{z}_{j2}\overline{z}_{i1}+\overline{z}_{j1}\overline{z}_{i2}+a\overline{\kappa}_j,$$

$$f_{i\sigma}\overline{f}_{j\sigma}\simeq -z_{j2}z_{i1}+z_{j1}z_{i2}+a\kappa_j, \tag{A13}$$

if $j\in B$ and $i=j+a_\mu$;

$$f_{i\sigma}\overline{f}_{j\sigma}\simeq 1+z_{i\sigma}(\overline{z}_{j\sigma}-\overline{z}_{i\sigma}),$$

$$f_{j\sigma}\overline{f}_{i\sigma}\simeq 1+\overline{z}_{i\sigma}(z_{j\sigma}-z_{i\sigma}), \tag{A14}$$

if $i,j\in A$ and $j=i+e_\lambda$;

$$f_{j\sigma}\overline{f}_{i\sigma}\simeq 1+\overline{z}_{j\sigma}(z_{i\sigma}-z_{j\sigma}),$$

$$f_{i\sigma}\overline{f}_{j\sigma}\simeq 1+z_{j\sigma}(\overline{z}_{i\sigma}-\overline{z}_{j\sigma}), \tag{A15}$$

if $j,i\in B$ and $i=j+e_\lambda$. The two lattice's directions $(a,0)$ and $(0,a)$ are noted by $a_\mu, \mu=x,y$, and $e_\lambda=[a_x+a_y,a_x-a_y]$. I have used again the two complex fields defined by Eq. (A5).

Collecting the results above, one can write the action in the form

$$S=S_0+S_L+S_{LL}. \tag{A16}$$

The term which does not depend on $\boldsymbol{L}$ reads

$$\begin{aligned}
S_0={}&\int_0^\beta d\tau\Bigg(\sum_{i\in A}{}'\,\overline{\psi}_i^A(\tau)[\partial_\tau-\overline{z}_{i\sigma}\dot{z}_{i\sigma}]\psi_i^A(\tau)+\sum_{j\in B}{}'\,\overline{\psi}_j^B(\tau)[\partial_\tau+\overline{z}_{j\sigma}\dot{z}_{j\sigma}]\psi_j^B(\tau)+\frac{J}{2}\sum_{i\in A,\mu}{}'[\overline{\psi}_i^A(\tau)\psi_i^A(\tau)\\
&+\overline{\psi}_{i+a_\mu}^B(\tau)\psi_{i+a_\mu}^B(\tau)]+\frac{J}{2}\sum_{j\in B,\mu}{}'[\overline{\psi}_j^B(\tau)\psi_j^B(\tau)+\overline{\psi}_{j+a_\mu}^A(\tau)\psi_{j+a_\mu}^A(\tau)]-t_2\sum_{i\in A,\lambda}{}'[\overline{\psi}_i^A(\tau)\psi_{i+e_\lambda}^A(\tau)+\text{H.c.}]\\
&-t_2\sum_{j\in B,\lambda}{}'[\overline{\psi}_j^B(\tau)\psi_{j+e_\lambda}^B(\tau)+\text{H.c.}]-t_1\sum_{i\in A,\mu}{}'\{\overline{\psi}_i^A(\tau)\psi_{i+a_\mu}^B(\tau)[-z_{i1}(\tau)z_{i+a_\mu 2}(\tau)+z_{i2}(\tau)z_{i+a_\mu 1}(\tau)]+\text{H.c.}\}\\
&-t_1\sum_{j\in B,\mu}{}'\{\overline{\psi}_j^B(\tau)\psi_{j+a_\mu}^A(\tau)[-\overline{z}_{j2}(\tau)\overline{z}_{j+a_\mu 1}(\tau)+\overline{z}_{j1}(\tau)\overline{z}_{j+a_\mu 2}(\tau)]+\text{H.c.}\}\\
&-t_2\sum_{i\in A,\lambda}{}'\{\overline{\psi}_i^A(\tau)\psi_{i+e_\lambda}^A(\tau)z_{i\sigma}(\tau)[\overline{z}_{i+e_\lambda\sigma}(\tau)-\overline{z}_{i\sigma}(\tau)]+\text{H.c.}\}\\
&-t_2\sum_{j\in B,\lambda}{}'\{\overline{\psi}_j^B(\tau)\psi_{j+e_\lambda}^B(\tau)\overline{z}_{j\sigma}(\tau)[z_{j+e_\lambda\sigma}(\tau)-z_{j\sigma}(\tau)]+\text{H.c.}\}\\
&+\frac{J}{8}\sum_{i\in A,\mu}{}'[\boldsymbol{m}_{i+a_\mu}(\tau)-\boldsymbol{m}_i(\tau)]^2[1-\overline{\psi}_i^A(\tau)\psi_i^A(\tau)][1-\overline{\psi}_{i+a_\mu}^B(\tau)\psi_{i+a_\mu}^B(\tau)]\\
&+\frac{J}{8}\sum_{j\in B,\mu}{}'[\boldsymbol{m}_{j+a_\mu}(\tau)-\boldsymbol{m}_j(\tau)]^2[1-\overline{\psi}_j^B(\tau)\psi_j^B(\tau)][1-\overline{\psi}_{j+a_\mu}^A(\tau)\psi_{j+a_\mu}^A(\tau)]-\frac{J}{2}\sum_{i\in A,\mu}{}'\,\overline{\psi}_i^A(\tau)\psi_i^A(\tau)\overline{\psi}_{i+a_\mu}^B(\tau)\psi_{i+a_\mu}^B(\tau)\\
&-\frac{J}{2}\sum_{j\in B,\mu}{}'\,\overline{\psi}_j^B(\tau)\psi_j^B(\tau)\overline{\psi}_{j+a_\mu}^A(\tau)\psi_{j+a_\mu}^A(\tau)-\mu\sum_{i\in A}{}'[1-\overline{\psi}_i^A(\tau)\psi_i^A(\tau)]-\mu\sum_{j\in B}{}'[1-\overline{\psi}_j^B(\tau)\psi_j^B(\tau)]\Bigg).
\end{aligned} \tag{A17}$$

It is convenient to write the linear term in the form

$$S_L=a\int_0^\beta d\tau\left[\sum_{i\in A}{}'(\overline{\kappa}_i\rho_i^A+\kappa_i\widetilde{\rho}_i^A)+\sum_{j\in B}{}'(\overline{\kappa}_j\rho_j^B+\kappa_j\widetilde{\rho}_j^B)\right], \tag{A18}$$

where

$$\rho_i^A=\frac{i}{2}(\boldsymbol{m}_i\times\dot{\boldsymbol{m}}_i)\cdot\mathbf{e}_i-t_1\sum_\mu\ \overline{\psi}_{i+a_\mu}^B\psi_i^A,$$

$$\widetilde{\rho}_i^A=\frac{i}{2}(\boldsymbol{m}_i\times\dot{\boldsymbol{m}}_i)\cdot\overline{\mathbf{e}}_i-t_1\sum_\mu\ \overline{\psi}_i^A\psi_{i+a_\mu}^B, \tag{A19}$$

$$\rho_j^B = \frac{i}{2}(\boldsymbol{m}_j \times \dot{\boldsymbol{m}}_j)\cdot \mathbf{e}_j - t_1 \sum_\mu \bar{\psi}_j^B \psi_{j+a_\mu}^A ,$$

$$\tilde{\rho}_j^B = \frac{i}{2}(\boldsymbol{m}_j \times \dot{\boldsymbol{m}}_j)\cdot \bar{\mathbf{e}}_j - t_1 \sum_\mu \bar{\psi}_{j+a_\mu}^A \psi_j^B .$$

Finally, the bilinear term is

$$S_{LL} = \frac{Ja^2}{2}\int_0^\beta d\tau \left[\sum_{i\in A}{}' \boldsymbol{L}_i^2 (1-\bar{\psi}_i^A \psi_i^A)(1-\bar{\psi}_{i+a_\mu}^B \psi_{i+a_\mu}^B) + \sum_{j\in B}{}' \boldsymbol{L}_j^2 (1-\bar{\psi}_j^B \psi_j^B)(1-\bar{\psi}_{j+a_\mu}^A \psi_{j+a_\mu}^A)\right], \quad \text{(A20)}$$

where $\boldsymbol{L}_i^2 = \bar{\kappa}_i \kappa_i$.

The last step is to integrate over the vector $\boldsymbol{L}_i$. The integral over $\boldsymbol{L}_i$ is defined as an integral over the independent variables $\bar{\kappa}_i(\tau)$ and $\kappa_i(\tau)$. Carrying out the integration, one obtains the action of the effective theory (17).

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