Tunneling between a bipolaron superconductor and a normal metal

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We show in this paper that the bipolaron theory is able to describe the main features of tunnel experiments. This theory explains the experimental value of a gap, a form of peaks at low bias voltage, and a different value of an asymmetry in different tunnel experiments. The temperature variation of the gap width was obtained as well. [S0163-1829(98)06513-8]

I. INTRODUCTION

The investigation of the tunneling through a superconductor–normal-metal junction provides important information on the excitation spectra of superconductors. In the low-temperature superconductors accurate experiments unambiguously confirmed the BCS theory. Experiments on high-T_c superconductors are much more complicated and explanations for them are far from obvious. There is still no general consensus on such important tunneling features, as the number of gaps in superconductors (one or two[1–4]); the existance of a smaller gap; the ratio $2\Delta/kT_c$ that varies from 0.7 to 2–12 in different experiments.

The main features of high-T_c junction are the presence of a large gap and the asymmetry of the tunneling current. A lot of experimental papers report on asymmetry, but its value varies from very small[3,4,5] to rather large.[5,6] The observed temperature dependence of the gap is weak (if any).[3,4] The authors of Ref. 7 presented experimental data on the tunneling current with the asymmetry depending on the density of charge carriers. There are several papers that represent symmetrical conductivity curves[3,5] as well.

The features mentioned above are not completely explained. Several theoretical models were suggested. Calculations on the basis of the local pair model[8] or bipolarons[9] led to a strongly asymmetrical tunnel conductance at low temperatures. To improve the situation and explain the asymmetry, the authors of Ref. 8 supposed a smaller coupling interaction near the junction than in the bulk material. In Ref. 10 authors supposed the existence of local centers in high-T_c materials and obtained symmetrical curves, but they did not obtain any peaks in the dI/dV-V characteristic. The authors of Ref. 11 applied the idea of a local level and considered a potential barrier in the junction that contained localized electron states. In the framework of this model the low-energy gaplike structure and the experimentally observed two-gap behavior of the conductivity at low voltages was obtained. In Ref. 9 the tunnel current was studied in the framework of the bipolaron model. The existence of both large and small gaps and the origin of the asymmetry were explained there, but the theoretical tunneling conductance was too asymmetrical compared with the experimental one.

In this paper we propose a one-particle tunneling mechanism in the bipolaronic superconductors and explain several experimental features of the tunnel conductivity curves. We discuss here the simplest form of the model and neglect the tunneling of pairs because of the smallness of the two-particle tunnel probability compared with that of the one particle. As it was predicted in Ref. 9 and will be shown below, the accounting of the polaron band leads to a good agreement with the experiment. We considered, as an example, the $YBa_2Cu_3O_{7-\delta}$ superconductor.

We investigate the tunneling through the superconductor–normal-metal (SN) contact on the basis of the bipolaron theory of high-T_c superconductors.[9,12] We explain the asymmetry of the tunneling current, the mechanism of variation of its value from very small to an arbitrary large, and weak temperature dependence of the experimental gap. In Sec. II we present the one-particle tunneling mechanism on the basis of bipolaron theory. Numerical calculations are done in Sec. III. Section IV is devoted to the discussion of the phonon contribution into the tunneling current.

II. THE TUNNELING MECHANISM

As mentioned above, we chose the simplest form of tunneling mechanism in the framework of the bipolaron model for high-T_c superconductors.[12] There are two different mechanisms for tunneling of charge carriers: two-particle tunneling, discussed in Ref. 9 and one-particle tunneling. In this paper we consider the second one assuming that the first mechanism is exponentially suppressed. At positive voltage V [Fig. 1(a)] a process of tunneling consists of a bipolaron decay into an electron in a normal metal and a polaron in a polaron band of the superconductor. Evidently, this transition is allowed only at $V>\Delta/2$ ($T=0$, $e=k_B=\hbar=1$), where $\Delta/2$ is the gap between the bipolaron band and the polaron one. At the negative voltage $V<-\Delta/2$ the usual tunneling of the electrons into the polaron band takes place [Fig. 1(b)]. As one can see from Fig. 1, the tunneling mechanism is different at $V>0$ and $V<0$, which explains the tunnel current asymmetry. The effect of the polaron tunneling into a normal metal at positive voltage is negligible at low temperature when the polaron band is practically empty. Effects of upper bipolaron bands, such as triplet and d-wave[13] bands are considered to be suppressed for the same reason. Two-particle tunneling from the upper band contributes only to a fine structure of the gap at a very small voltage. So, we suppose that different mechanisms are responsible for tunneling at different signs of voltage. To study these transitions we use the standard tunneling Hamiltonian method[14] and assume the Hamiltonian in the form

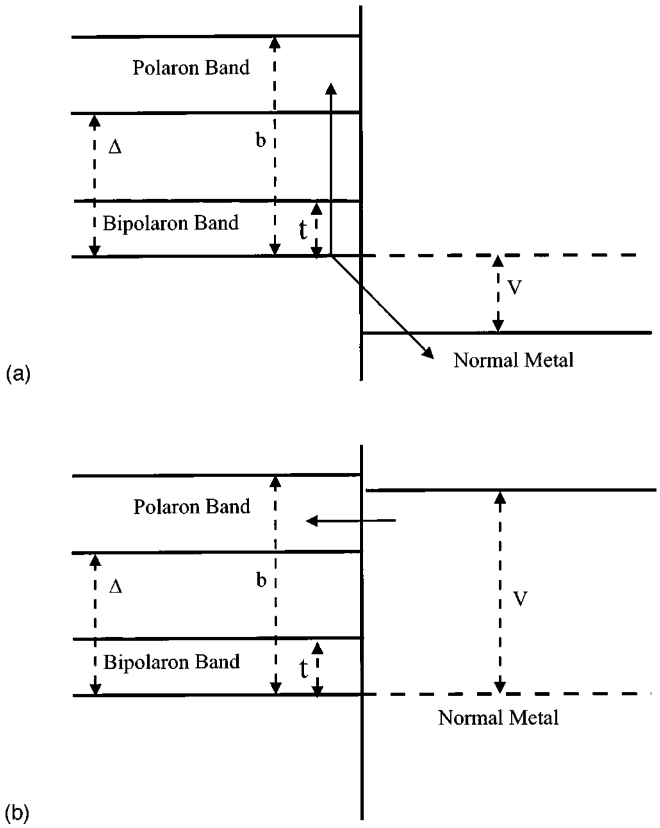

FIG. 1. Tunneling from a superconductor (S) to normal metal (N). (a) Bipolaron decay to a polaron in the polaronic band and an electron in the normal metal. (b) Electron transition from a normal metal to bipolaronic superconductor.

$$H = H_S + H_N + H_T + V\sum_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{\mathbf{k}}, \tag{1}$$

where H_T is the Hamiltonian of the bipolaronic superconductor with a strong electron coupling, and H_N is the Hamiltonian of a normal metal.

The normal metal (N) is described by the Hamiltonian

$$H_N = \sum_{\mathbf{k}} \xi_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{\mathbf{k}}, \tag{2}$$

where $a^\dagger_{\mathbf{k}}$ and $a_{\mathbf{k}}$ are the creation and annihilation operators for electrons in the normal metal, and $\xi_{\mathbf{k}}$ is an electron energy. All energies are measured from a chemical potential level. The superconductor (S) is assumed to be the material with bipolarons described by the bipolaronic Hamiltonian[12] as

$$H_S = \sum_{\mathbf{p}} E_{\mathbf{p}} b^\dagger_{\mathbf{p}} b_{\mathbf{p}} + \sum_{\mathbf{q}} \varepsilon_{\mathbf{q}} C^\dagger_{\mathbf{q}} C_{\mathbf{q}}, \tag{3}$$

$b^\dagger_{\mathbf{p}}$ and $b_{\mathbf{p}}$ are creation and annihilation operators of bipolarons, $C^\dagger_{\mathbf{q}}$, $C_{\mathbf{q}}$ are polaron operators, $E_{\mathbf{p}}$, $\varepsilon_{\mathbf{q}}$, are energy spectra of bosons and polarons, respectively. As follows from the previous discussion, the tunneling Hamiltonian may be written in the form

$$H_T = \frac{1}{\sqrt{N_1}} \sum_{\mathbf{p},\mathbf{q},\mathbf{k}} (D_{\mathbf{p},\mathbf{q},\mathbf{k}} b^\dagger_{\mathbf{p}} C_{\mathbf{q}} a_{\mathbf{k}} + D^*_{\mathbf{p},\mathbf{q},\mathbf{k}} b_{\mathbf{p}} a^\dagger_{\mathbf{k}} C^\dagger_{\mathbf{q}})$$
$$+ \sum_{\mathbf{q},\mathbf{k}} (\mathcal{M}_{\mathbf{q},\mathbf{k}} C^\dagger_{\mathbf{q}} a_{\mathbf{k}} + \mathcal{M}^*_{\mathbf{q},\mathbf{k}} a^\dagger_{\mathbf{k}} C_{\mathbf{q}}), \tag{4}$$

where $D_{\mathbf{p},\mathbf{q},\mathbf{k}}$ is the matrix element of the bipolaron decay into a polaron (in the polaron band) and an electron (in the normal metal); $\mathcal{M}_{\mathbf{q},\mathbf{k}}$ is the matrix element of the electron transition into a polaron, N_1 is the number of cells in the superconductor. The determination of the tunneling matrix element is not a simple problem, but as it will be pointed out in the next section, the main features of tunneling curves are independent of the form of the matrix element and we consider a simplest case, $D_{\mathbf{p},\mathbf{q},\mathbf{k}} = \text{const}$ and $\mathcal{M}_{\mathbf{q},\mathbf{k}} = \text{const}$.

III. THE TUNNELING CURRENT

Our starting point is the calculation of the tunneling current using the standard method of the tunneling Hamiltonian:[14]

$$\langle J \rangle = \left\langle \frac{dN}{dt} \right\rangle = -\text{Im}\langle [H_T, N] \rangle, \tag{5}$$

where $\langle J \rangle$ is the tunneling current and $\langle \ldots \rangle$ means the equilibrium state average. N is the operator of number of particles in the normal metal. We choose the cubic cell units for the sake of simplicity. The calculation of the commutator (5) gives

$$\langle J \rangle = \frac{1}{\sqrt{N_1}} \sum_{\mathbf{p},\mathbf{q},\mathbf{k}} [D_{\mathbf{p},\mathbf{q},\mathbf{k}} \langle b^\dagger_{\mathbf{p}} C_{\mathbf{q}} a_{\mathbf{k}} \rangle - D^*_{\mathbf{p},\mathbf{q},\mathbf{k}} \langle b_{\mathbf{p}} a^\dagger_{\mathbf{k}} C^\dagger_{\mathbf{q}} \rangle]$$
$$- \sum_{\mathbf{q},\mathbf{k}} [\mathcal{M}_{\mathbf{q},\mathbf{k}} \langle C^\dagger_{\mathbf{q}} a_{\mathbf{k}} \rangle - \mathcal{M}^*_{\mathbf{q},\mathbf{k}} \langle a^\dagger_{\mathbf{k}} C_{\mathbf{q}} \rangle]. \tag{6}$$

This expression may be written in the form

$$\langle J \rangle = 2\,\text{Im}\left[\frac{1}{\sqrt{N_1}} \sum_{\mathbf{p},\mathbf{q},\mathbf{k}} D_{\mathbf{p},\mathbf{q},\mathbf{k}} \langle b^\dagger_{\mathbf{p}} C_{\mathbf{q}} a_{\mathbf{k}} \rangle - \sum_{\mathbf{q},\mathbf{k}} \mathcal{M}_{\mathbf{q},\mathbf{k}} \langle C^\dagger_{\mathbf{q}} a_{\mathbf{k}} \rangle \right]. \tag{7}$$

The averages $\langle b^\dagger_{\mathbf{p}} C_{\mathbf{q}} a_{\mathbf{k}} \rangle$ and $\langle C^\dagger_{\mathbf{q}} a_{\mathbf{k}} \rangle$ are found from the following equations:

$$\frac{d}{dt} \langle b^\dagger_{\mathbf{p}} C_{\mathbf{q}} a_{\mathbf{k}} \rangle = -\langle [H, b^\dagger_{\mathbf{p}} C_{\mathbf{q}} a_{\mathbf{k}}] \rangle,$$

$$\frac{d}{dt} \langle C^\dagger_{\mathbf{q}} a_{\mathbf{k}} \rangle = -\langle [H, C^\dagger_{\mathbf{q}} a_{\mathbf{k}}] \rangle,$$

and for the tunnel current we obtain

$$\langle J \rangle = 2\,\text{Im}\left(\sum_{\mathbf{p},\mathbf{q},\mathbf{k}} |D_{\mathbf{p},\mathbf{q},\mathbf{k}}|^2 \frac{\varphi_{\mathbf{p}}[1 - f^{(N)}_{\mathbf{k}} - f^{(\text{pol})}_{\mathbf{q}}] - f^{(N)}_{\mathbf{k}} f^{(\text{pol})}_{\mathbf{q}}}{-E_{\mathbf{p}} + \varepsilon_{\mathbf{q}} + \xi_{\mathbf{k}} + V - i\delta} \right.$$
$$\left. - \sum_{\mathbf{q},\mathbf{k}} |\mathcal{M}_{\mathbf{q},\mathbf{k}}|^2 \frac{f^{(N)}_{\mathbf{k}} - f^{(\text{pol})}_{\mathbf{q}}}{\varepsilon_{\mathbf{q}} - \xi_{\mathbf{k}} - V - i\delta} \right)_{\delta \to 0}, \tag{8}$$

$f^{(N)}_{\mathbf{k}} = f^{(N)}(\varepsilon_{\mathbf{k}})$ and $f^{(\text{pol})}_{\mathbf{q}} = f^{(\text{pol})}(\varepsilon_{\mathbf{q}})$ are Fermi distributions of electrons and polarons, $\varphi_{\mathbf{p}} = \varphi(E_{\mathbf{p}})$ is the Bose distribution of bipolarons, $\varphi_p = n_0 \delta_{\mathbf{p},\mathbf{p_0}} + \varphi'_{\mathbf{p}}$, n_0 is the number of particles in the condensate at temperatures $T < T_c$. The spectra of particles are chosen in the forms $E_{\mathbf{p}} = \mathbf{p}^2/(2m_b)$, $\varepsilon_{\mathbf{q}} = \mathbf{q}^2/(2m_p) + \Delta/2$, $\xi_{\mathbf{k}} = \mathbf{k}^2/(2m_e) - E_F + \xi_0$, and ξ_0 is the additional energy of the electrons in the normal metal. This energy shift usually appears when one connects two pieces of different crystals to make their chemical potentials equal. We accounted for this value using a condition $\langle J(V=0) = 0 \rangle$.

The second term in Eq. (8) describes the electron tunneling from the normal metal to the polaronic band and vice versa.

The first term describes the bipolaron decay. One of two electrons constituting a pair tunnels to the normal metal and the second goes to the polaron band simultaneously. At $T < T_c$ the charge conservation is provided partly by the bipolaron condensate motion with a momentum $\mathbf{p}_0 \neq 0$. The estimation of this momentum using the known values of the current leads to a small value of the boson energy $E_{\mathbf{p_0}} \leqslant 10^{-2}$ meV and may be neglected in the calculations. We assume $D_{\mathbf{p},\mathbf{q},\mathbf{k}} = D_0$, $\mathcal{M}_{\mathbf{q},\mathbf{k}} = \mathcal{M}_0$ for both matrix elements to simplify our calculations. The final expression for the tunnel current is

$$\begin{aligned}\langle J \rangle = A_0 \Bigg\{ & \frac{\sqrt{2}\pi^2}{m_b^{3/2}} T^{3/2} \int_{\Delta/2T}^{b/T} dx \sqrt{x - \Delta/2T} \\ & \times [f^{(N)}(x - V/T - \xi_0/T) - f^{(\mathrm{pol})}(x)] \\ & - \frac{\sqrt{2}\pi^2}{m_b^{3/2}} T^{3/2} \gamma \rho_0 \int_{\Delta/2T}^{b/T} dx \sqrt{x - \Delta/2T} \\ & \times [f^{(N)}(x + V/T + \xi_0/T) - f^{(\mathrm{pol})}(x)] \\ & - T^3 \gamma \int_0^{t/T} dx \sqrt{x} \int_{\Delta/2T}^{b/T} dy \sqrt{y - \Delta/2T} \{ \varphi'(x) \\ & \times [f^{(N)}(x + y + V/T + \xi_0/T) - f^{(\mathrm{pol})}(y)] \\ & - f^{(\mathrm{pol})}(y) f^{(N)}(-x - y - V/T - \xi_0/T) \} \Bigg\}, \end{aligned} \tag{9}$$

$$A_0 = \frac{2\sqrt{2} m_b^{3/2} m_p^{3/2} m_e^{3/2} |\mathcal{M}_0|^2 \sqrt{E_F} N_1 N_2}{\pi^6},$$

where m_b, m_p, m_e are the masses of bipolarons, polarons, and electrons in the normal metal, respectively, and N_2 is a number of cells in the normal metal. $\gamma \equiv |\mathcal{D}_0/\mathcal{M}_0|^2$, and ρ_0 is the number of particles per cell in the condensate. The first term in the expression (9) represents the one-particle transitions between the polaron band and normal metal. The second term describes the decay and the transition of bipolarons in the condensate. The third term is the same one for supracondensate bosons.

The density of the bosons in the condensate ρ_0 is calculated using the law of the particle conservation:

$$N = 2N_0^B + 2N_{\mathbf{p}\neq\mathbf{0}}^B + N^P,$$

where N is the number of particles, N_0^B is the number of bosons (bipolarons) in the condensate, N^P is the number of particles in the polaron band, and $N_{p\neq 0}^B$ is the number of the supracondensate bosons. A more detailed equation has the form

$$\begin{aligned}\rho_0 = \frac{\rho}{2} - \frac{\sqrt{2} m_b^{3/2}}{2\pi^2} \int_0^t dE [\sqrt{E} \varphi(E)] \\ - \frac{\sqrt{2} m_p^{3/2}}{2\pi^2} \int_{\Delta/2}^b d\varepsilon [\sqrt{\varepsilon - \Delta/2} f(\varepsilon)],\end{aligned}$$

where ρ is the number of particles per cell and ρ_0 is the number of the condensate bosons per cell.

Let us compare now the values of the matrix elements $\mathcal{M}$ and D. Both of them contain the matrix element of one-electron tunneling through a junction that can be estimated in standard form[15] $\mathcal{M} \sim \exp[-\int \sqrt{2m_e(U-E)}]dx$ and $D \sim \exp[-\int \sqrt{2m_e(U-E')}]dx$. Here U is the potential of the barrier (the same for both matrix elements) and E, E' are the energies of tunneling electrons. Keeping in mind that $E - E' \sim \Delta/2 \sim 10$ meV $\ll U \sim 1$ eV we can conclude $\mathcal{M} \approx D$ and hence $\gamma \sim 1$. Of course, this is only an estimation and we may consider γ a free parameter. As we will see below, the value $\gamma \sim 1$ does not contradict the experiment.

The main contribution into the expression (9) at $T \ll T_c$ is given by the first and the second terms, because of small number of supracondensate bosons.

At $T = 0$ the expression can be calculated analytically and $\langle J \rangle$ takes a form

$$\langle J \rangle = A_0 \frac{\sqrt{2}\pi^2}{m_b^{3/2}} \times \begin{cases} 0, & |V| < \Delta/2, \\ \frac{2}{3}(V - \Delta/2)^{3/2}, & \Delta/2 < V < b, \\ -\frac{2}{3}(|V| - \Delta/2)^{3/2} \rho_0 \gamma, & \Sigma/2 < -V < b, \\ \frac{2}{3}(b - \Delta/2)^{3/2}, & V > b, \\ -\frac{2}{3}(b - \Delta/2)^{3/2} \rho_0 \gamma, & V < -b, \end{cases}$$

and dJ/dV is

$$\frac{dJ}{dV} = A_0 \frac{\sqrt{2}\pi^2}{m_b^{3/2}} \times \begin{cases} 0, & |V| < \Delta/2, \\ (V - \Delta/2)^{1/2}, & \Delta/2 < V < b, \\ (|V| - \Delta/2)^{1/2} \gamma \rho_0, & \Delta/2 < -V < b, \\ 0, & |V| > b. \end{cases} \tag{10}$$

We calculate also the tunnel current with the other form of the matrix element, (NP) $\mathcal{M}_{qk} = \mathcal{M}_0 \sqrt{q_x}$. This momentum dependence is well known in the case of the ordinary one-particle tunneling.[14] The same result takes place for two-particle tunneling[9] as well. In this case the tunnel current has the following form at $T = 0$:

$$J = A_1 \times \begin{cases} 0, & |V| < \Delta/2, \\ -\frac{2}{3}(|V| - \Delta/2)^{3/2} \gamma \rho_0, & \Delta/2 < -V < b, \\ V^2/2 - V\Delta/2, & \Delta/2 < V < b, \\ -\frac{2}{3}(b - \Delta/2)^{3/2} \gamma \rho_0, & V < -b, \\ b^2/2 - b\Delta/2, & V > b, \end{cases} \tag{11}$$

where $\gamma = \sqrt{2}\pi|D|^2/(\sqrt{m_p}|\mathcal{M}_0|^2)$, $A_1 = 8\sqrt{2} m_p^2 m_e^{3/2}|\mathcal{M}_0|^2 \times \sqrt{E_F} N_1 N_2/(\pi^5)$ and the conductivity has the form

$$\frac{dJ}{dV} = A_1 \times \begin{cases} 0, & |V| < \Delta/2, \\ (|V| - \Delta/2)^{1/2} \gamma \rho_0, & \Delta/2 < -V < b, \\ V - \Delta/2, & \Delta/2 < V < b, \\ 0, & |V| > b. \end{cases} \tag{12}$$

One can see by comparing the expressions (10) and (12), that the main features of the tunnel conductivity (the wide gap, the existence of two peaks and zero conductivity at high voltage) do not depend on the form of the matrix element.

Let us consider the results of numerical calculations of the tunneling current at the different parameters γ, ρ and temperature T using the formula (9). Temperature dependencies of the tunneling current are represented in Figs. 2(a) and 2(b). One can see that, in spite of different mechanisms of tunneling at $V > 0$ and $V < 0$, the temperature dependencies of the value of both peaks are similar at a wide range of temperatures. In Fig. 2 we present the asymmetry of the curves for different values of the parameter $\gamma\rho$ [Figs. 2(a) and 2(b)]. A slight variation of the charge carrier density ρ or junction structure, which is represented by the parameter γ, leads to the alternation of the asymmetry. This might be a reason for the difference in the experimental results of Refs. 3 and 4.

One can see from the inset in Fig. 2(a) that the effective gap Δ_{eff} increases with decreasing temperature. The physical explanation is obvious: the bosons leave the condensate and fill the upper levels of the boson band. It leads to the decreasing of the effective gap between the boson (bipolaron) and the fermion (polaron) bands.

Figure 3 represents the comparison of our results with the experimental data. Figure 3(a) is obtained with the parameter $\gamma\rho = 1.8$ and fits the experiment[5] at $|V/T_c| < 2$. The experimental asymmetrical tunneling current in Fig. 3(b) was found in Ref. 5 as well. It may be fitted by our theoretical curve with the parameter $\gamma\rho = 1.28$. It is assumed that the junction structure varies for different samples that leads to the small variation of the parameter $\gamma\rho$. The behavior of the experimental curve at $|V/T_c| > 2$ is explained, in our opinion, by a phonon contribution that is considered in the next section.

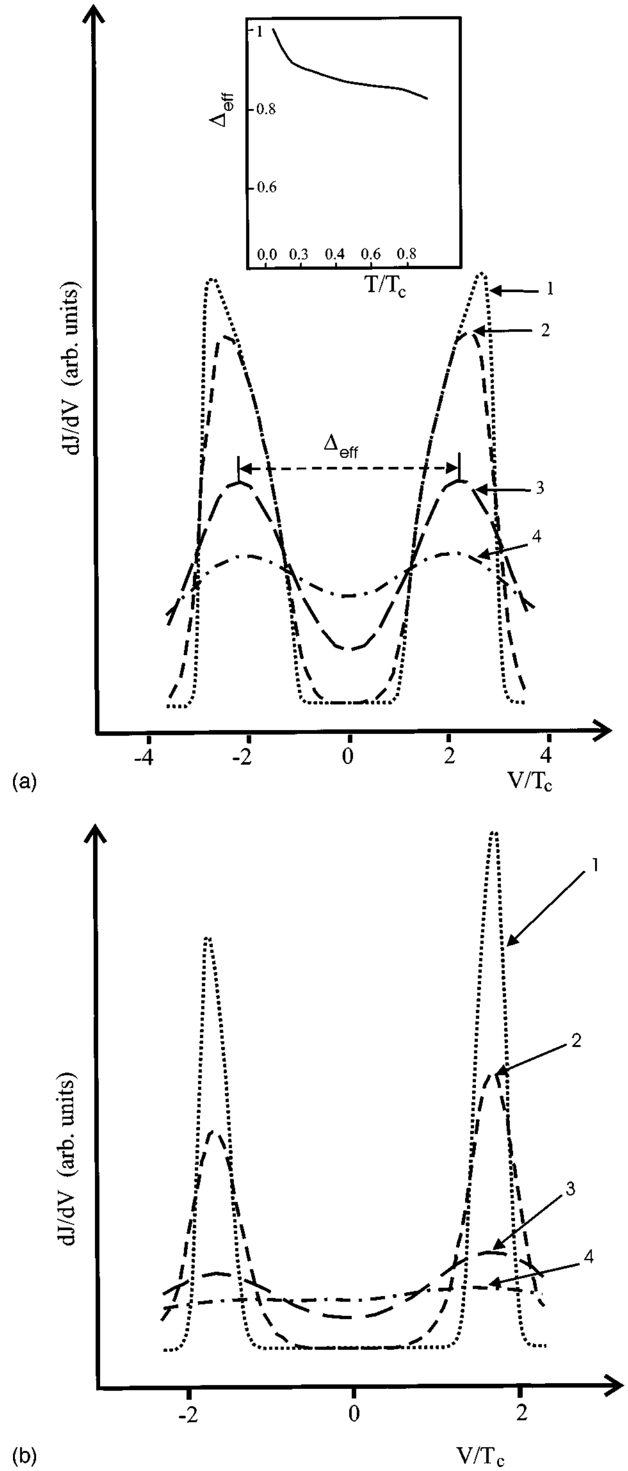

FIG. 2. dJ/dV vs V/T_c for different temperature T, (1) $T/T_c = 0.05$, (2) $T/T_c = 0.17$, (3) $T/T_c = 0.55$, $4 - T/T_c = 0.75$. $m_b = 1/t$ and $m_p = 1/(b - \Delta)$, $\Delta/(2T_c) = 1.04$, $b/T_c = 2.91$, $t/T_c = 0.47$, $T_c = 85$ K; (a) $\gamma\rho = 2$, (b) $\gamma\rho = 1.7$. Inset shows the dependence Δ_{eff} on T/T_c.

IV. THE ROLE OF PHONONS

Our theoretical results are in a good agreement with experiments at low voltage and describe the gap features quite well. But as one can see in Fig. 3 there is an essentially different behavior of the experimental and theoretical curves at high bias voltage.

We have considered above the tunneling process where absorption and emission of phonons were neglected. It will be shown here that the phonons considerably improves our fit. We keep in mind that only emission is essential at low temperatures. Here only one-phonon emission is taken into account. A momentum dependence of the phonon energy is neglected too. The calculation leads to the tunneling current of the form

$$J = J_0 + J_1. \tag{13}$$

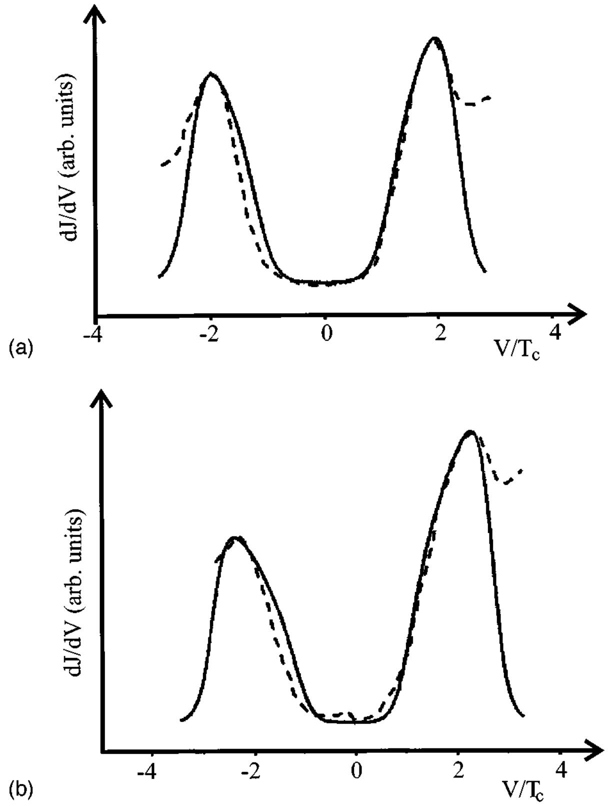

FIG. 3. dJ/dV vs V/T_c for different $\gamma\rho$ calculated within the framework of the bipolaron model with parameters $m_b=1/t$ and $m_p=1/(b-\Delta)$, $\Delta/(2T_c)=1.06$, $b/T_c=2.93$, $t/T_c=0.4$, $T_c=85$ K; (a) $\gamma\rho=1.8$, (b) $\gamma\rho=1.28$. The dotted lines are the experimental curves (Ref. 10).

Here J_0 is the expression (9), and J_1 is the one-phonon contribution

$$\langle J_1\rangle = zA_0\Bigg(\frac{\sqrt{2}\pi^2}{m_b^{3/2}}\,T^{3/2}\int_{\Delta/2T}^{b/T}dx\,\sqrt{x-\Delta/2T}\cdot[f^{(N)}(x-V/T-\xi_0/T-\omega/T)-f^{(\mathrm{pol})}(x)]$$
$$-\frac{\sqrt{2}\pi^2}{m_b^{3/2}}\,T^{3/2}\gamma\rho_0\int_{\Delta/2T}^{b/T}dx\,\sqrt{x-\Delta/2T}[f^{(N)}(x+V/T+\xi_0/T+\omega/T)-f^{(\mathrm{pol})}(x)]$$
$$-T^3\gamma\int_0^{t/T}dx\,\sqrt{x}\int_{\Delta/2T}^{b/T}dy\,\sqrt{y-\Delta/2T}\cdot\{\varphi(x)[f^{(N)}(x+y+V/T+\xi_0/T+\omega/T)-f^{(\mathrm{pol})}(y)]-f^{(\mathrm{pol})}(y)f^{(N)}(-x-y-V/T-\xi_0/T-\omega/T)\}\Bigg), \tag{14}$$

z is the multiplier that contains the constant of the electron-phonon interaction.

The expression (13) for $T\neq 0$, calculated numerically at arbitrary temperature, is shown in Fig. 4. It becomes evident

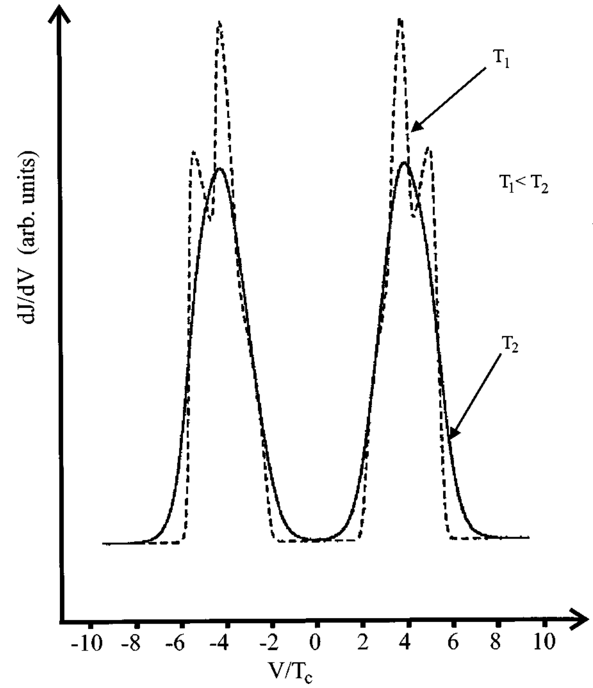

FIG. 4. dJ/dV vs V/T_c for different T calculated on the basis of the bipolaron model. $\gamma\rho=1$, $m_b=1/t$ and $m_p=1/(b-\Delta)$, $\Delta/(2T_c)=3.2$, $b/T_c=6.45$, $t/T_c=2.8$, $T_c=85$ K, $\omega_0/T_c=5$, $z=0.5$.

that the phonon radiation is responsible for the behavior of the tunneling current at high voltage. The phonon radiation increases effectively the width of the peaks and does not influence the gap width. A similar phonon structure has been observed in the experiment that is described in Ref. 16.

V. DISCUSSION

We have shown in our paper, that the bipolaron model[2] is able to explain the main experimental features of the tunneling current. The mechanism of the tunneling current consists of the bipolaron decay to the polaron in the superconductor and the electron in the metal. This model explains the experimental value of the gap, the form of the gap peak, and the different value of the asymmetry in different tunnel experiments. The temperature dependence of the gap is one of the hallmarks of the BCS theory. Our result on the temperature dependence of the gap width is shown in the inset of Fig. 2. This curve is not similar to that of the BCS theory.

Taking into account the phonon emission, even in the simplest form, improves the agreement with the experiment at high voltage.

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