Phase diagram for large two-dimensional bipolarons in a magnetic field

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We perform a path-integral calculation of the ground state energy of a bipolaron confined to two dimensions and which is placed in a perpendicular magnetic field. The present calculation is valid for arbitrary magnetic field strength, arbitrary strength of the repulsion between the electrons, and arbitrary electron-phonon coupling constant. We find that the *bipolaron* exhibits (1) a discontinuous transition from the polaronic state to the bipolaronic state and (2) a transition from the dressed (bi)polaron state to the (bi)polaron stripped state. These three transitions depend on the magnetic field and the strength of the repulsion between the electrons.
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I. INTRODUCTION

There are increasing experimental[1,2] indications that polarons and bipolarons, which can exist under conditions of strong electron-phonon interaction, may play an important role in explaining many characteristics of high-T_c cuprates. Recently, several theoretical studies of the properties of small and large polarons and bipolarons have appeared[3–14] which go beyond the weak coupling approximation.

In the present paper we study the ground state of a large bipolaron moving in two dimensions (2D) in the case where an external perpendicular magnetic field is present. Previous work on this problem was concentrated on (1) the study of the 3D problem[11,12] where it was found that a magnetic field increased the stability region of the bipolaron. In fact in the strong magnetic field limit it was shown[12] that the 3D bipolaron problem reduces to a 1D bipolaron with an effective electron-phonon coupling constant which increases with magnetic field. (2) The strong-coupling 2D bipolaron[9] where it was found that the magnetic field reduces the stability of the bipolaron slightly. The aim of the present work is to investigate if this magnetic field induced reduction in the 2D bipolaron stability is also present at intermediate electron-phonon coupling. In order to do so we generalize the Feynman polaron theory to the bipolaron case, which is believed to be valid for arbitrary electron-phonon coupling strength.[15] Furthermore, we also address the magnetic field stripping transition, which was first studied in Ref. 16 for the polaron problem, but which was overlooked in Refs. 9,11–13 in the case of a bipolaron in a magnetic field.

In Refs. 16,17 the influence of a magnetic field on a single polaron was examined: the ground state properties of a three-dimensional and two-dimensional polaron were studied and it was found that there exists a transition of the Feynman polaron from a dressed polaron state to a stripped polaron state with increasing magnetic field strength. The physical idea behind such an effect is the following: if the electron moves too quickly (i.e., the electron frequency is larger than the optical phonon frequency) through the crystal its polarization cloud will no longer be able to follow the electron, and an electron with the band mass, instead of the dressed mass will be observed. With other words, the polaron is stripped of his polarization cloud. Such a bare electron moves in a potential well created by the phonons.

Using the path integration technique, the ground state properties of a system of two electrons interacting with each other by the Coulomb force and indirectly through the optical phonons is investigated in the limit of zero temperature. The state of the system is determined by the strength of the electron-phonon coupling constant α, the strength $U(\vec{r}_1-\vec{r}_2)$ of the Coulomb repulsion, and the strength of the magnetic field. We found four different states: (i) two stripped polarons infinitely separated, (ii) two dressed polarons infinitely separated, (iii) the bipolaron state where two electrons are dressed around the same position; and (iv) the bipolaron state in which the two undressed electrons move within a common potential well formed by the phonons.

The outline of the present paper is as follows. In Sec. II we present the Fröhlich Hamiltonian and a generalization of the Feynman trial action and show that it can be obtained from the Hamiltonian of a Feynman bipolaron model. In Sec. III we use the Feynman variational principle to derive an upper bound to the exact ground state energy of the bipolaron. Finally, in the last section our numerical results and our concluding remarks are presented.

II. FEYNMAN BIPOLARON

In this section, we present the Hamiltonian which describes two electrons interacting with the vibrational modes of a crystal and pave the way to a Feynman-type approach of the bipolaron problem. The Hamiltonian describing the 2D bipolaron in a magnetic field is given by

$$H=\frac{1}{2m}\sum_{j=1,2}\left(\vec{p}_j+\frac{e}{c}\vec{A}_j\right)^2+\sum_{\vec{k}}\hbar\omega_{\vec{k}}\left(a_{\vec{k}}^{\dagger}a_{\vec{k}}+\frac{1}{2}\right)+H_I+U(\vec{r}_1-\vec{r}_2), \tag{1}$$

with the electron-phonon interaction

$$H_I=\sum_{j=1,2}\sum_{\vec{k}}\left(V_{\vec{k}}a_{\vec{k}}e^{i\vec{k}\cdot\vec{r}_j}+V_{\vec{k}}^{*}a_{\vec{k}}^{\dagger}e^{-i\vec{k}\cdot\vec{r}_j}\right), \tag{2}$$

where $\vec{r}_j(\vec{p}_j)$ are the position (momentum) operators of the jth electron, m is the electron band mass, $a^{\dagger}_{\vec{k}}(a_{\vec{k}})$ are the creation (annihilation) operators for phonons with wave vector $\vec{k}$, and frequency $\omega_{\vec{k}}$, $U(\vec{r}_1-\vec{r}_2)=e^2/\epsilon_\infty|\vec{r}_1-\vec{r}_2|$ is the Coulomb potential between the two electrons. The two electrons interact with an external magnetic field $\vec{B}=\mathrm{rot}\vec{A}$, which is taken along the z axis, and the vector potential is written in the symmetrical Coulomb gauge $\vec{A}_j=(B/2)(-y_j,x_j,0)$. For longitudinal optical phonons we take dispersionless phonons $\omega_{\vec{k}}=\omega_{\mathrm{LO}}$, the Fourier transform of the electron-phonon interaction, takes the form $V_{\vec{k}}=\hbar\omega_{\mathrm{LO}}\sqrt{\pi\alpha/Ak}(2\hbar/m\omega_{\mathrm{LO}})^{1/4}$ in two dimensions, where A is the area of the crystal and $\alpha=e^2/\hbar\omega_{\mathrm{LO}}(1/\epsilon_\infty-1/\epsilon_0)\sqrt{m\omega_{\mathrm{LO}}/2\hbar}$ is the dimensionless electron-phonon coupling constant, which depends on the static (ϵ_0) and high-frequency (ϵ_∞) dielectric constants.

In the well-known Feynman path-integral representation of the partition function the phonon variables can be eliminated exactly. After this elimination each electron path contributes $e^{-S[\vec{r}_1(t),\vec{r}_2(t)]}$ to the path integral. The action $S[\vec{r}_1(t),\vec{r}_2(t)]$ is defined as the time integral over the Lagrangian of this dynamical system.[15,18] For our purpose we want to calculate the partition function and therefore we introduce imaginary times $\tau=-it=\beta$, with $\beta^{-1}=Tk_B$, where T is the temperature of the system and k_B the Boltzmann constant. After eliminating the field variables,[15] we obtain the action $S[\vec{r}_1(t),\vec{r}_2(t)]$ (see also, Ref. 6)

$$S[\vec{r}_1(t),\vec{r}_2(t)]=S_e+S_{I,c}+S_{I,\mathrm{ph}}, \tag{3}$$

where

$$S_e=-\frac{1}{2}\sum_{i=1}^{2}\int_0^\beta dt\{\dot{\vec{r}}_i(t)^2+i\omega_c[x_i(t)\dot{y}_i(t)-y_i(t)\dot{x}_i(t)]\}, \tag{4}$$

is the action of two free electrons in a magnetic field with $\omega_c=eB/mc$ the cyclotron frequency. The interaction part of the action consists of the direct Coulomb repulsion

$$S_{I,c}=-\int_0^\beta dtU[\vec{r}_1(t)-\vec{r}_2(t)], \tag{5}$$

and the action which contains a memory effect as a consequence of the elimination of the phonons

$$S_{I,\mathrm{ph}}=\sum_{j,l=1,2}\sum_{\vec{k}}|V_k|^2\int_0^\beta dt\int_0^\beta dsG_{\omega_k}(t-s)e^{i\vec{k}\cdot[\vec{r}_j(t)-\vec{r}_l(s)]}, \tag{6}$$

with

$$G_\omega(u)=\frac{1}{2}n(\omega)(e^{\omega|u|}+e^{\omega(\beta-|u|)}), \tag{7}$$

the phonon Green's function, where $n(\omega)=1/(e^{\beta\hbar\omega}-1)$ is the occupation number of phonons with frequency ω.

In Feynman's polaron model one replaces the virtual phonon cloud surrounding the electron by a fictitious particle which is bound to the electron through a spring. In a bipolaron system we have two electrons, each with their own phonon cloud, and consequently the Feynman bipolaron model consists of four particles, described by the following Hamiltonian:

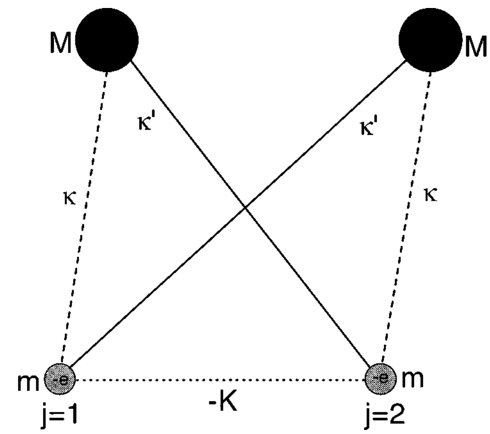

FIG. 1. Graphical representation of the Feynman bipolaron model.

$$H_F=\sum_{j=1,2}\left[\frac{1}{2m}\left(\vec{p}_j+\frac{e}{c}\vec{A}_j\right)^2+\frac{\vec{P}_j^2}{2M}+\frac{\kappa}{2}(\vec{r}_j-\vec{R}_j)^2\right]+\frac{\kappa'}{2}[(\vec{r}_1-\vec{R}_2)^2+(\vec{R}_1-\vec{r}_2)^2]-\frac{K}{2}(\vec{r}_1-\vec{r}_2)^2, \tag{8}$$

where $(\vec{r}_j,\vec{p}_j)$ are the electron coordinates with mass m and which interact with a second particle, called the fictitious particle, with coordinates $(\vec{R}_j,\vec{P}_j)$ of mass M. κ, κ' are the oscillator strengths characterizing the interaction of the electrons with the fictitious particles. The Coulomb repulsion between the electrons is approximated by a quadratic repulsion with strength K. The resulting bipolaron model is illustrated in Fig. 1. Note that the model is determined by the four parameters M, κ, κ', and K. The action corresponding to the Hamiltonian (8), in which we have eliminated the coordinates of the fictitious particle, is given by

$$S_t[\vec{r}_1(t),\vec{r}_2(t)]=-\int_0^\beta dt\left[\sum_{j=1}^{2}\frac{m}{2}\dot{\vec{r}}_j(t)^2-\frac{K}{2}[\vec{r}_1(t)-\vec{r}_2(t)]^2\right]-\int_0^\beta dt\int_0^\beta dsG_w(t-s)\times\left[\frac{\hbar(\kappa^2+\kappa'^2)}{4Mw}\sum_{j=1,2}[\vec{r}_j(t)-\vec{r}_j(s)]^2+\frac{\hbar\kappa\kappa'}{Mw}[\vec{r}_1(t)-\vec{r}_2(s)]^2\right]. \tag{9}$$

Notice that the self-interaction and the repulsive Coulomb interaction in the original action [Eq. (3)] is in this trial action replaced by quadratic functions. This is similar to what was done by Feynman in its original work.[15] It is also well known that the Coulomb potential can be reduced exactly to that of an harmonic oscillator in four dimensions.[19]

Recently, the present authors[10] obtained the ''exact'' eigenfrequencies of the Feynman bipolaron model (8) in the presence of a magnetic field. Such a magnetic field couples the polaron motion in the two directions perpendicular to the magnetic field which results in seven nonzero eigenfrequencies for the diagonalized bipolaron model. The diagonalized Hamiltonian (8) was found to be

$$H_F=\sum_{i=1}^{7} s_i\left(c_{\vec{k}}^{\dagger}c_{\vec{k}}+\frac{1}{2}\right), \tag{10}$$

with the eigenfrequencies s_i which are found as the positive real solutions of the algebraic equation (for details we refer to Ref. 10)

$$s^2\{-s^2(s^2-v^2)^2+\omega_c^2(s^2-w^2)^2\}\{[s^4+(2\gamma_3-v^2)s^2+\varrho^4-2\gamma_3 w^2]^2-\omega_c^2 s^2(s^2-w^2)^2\}=0, \tag{11}$$

and the trivial solution $s_8=0$. In Eq. (11) we defined $v^2=(\kappa+\kappa')/\mu$, $w^2=(\kappa+\kappa')/M$ is the square of the frequency of the oscillator (analogous to the Feynman parameter w in the single-polaron problem), $\varrho^4=4\kappa\kappa'/M$, and $\gamma_3=K/m$, where $\mu^{-1}=m^{-1}+M^{-1}$.

The polaron limit is obtained by decoupling the two electrons from each other, i.e., $\kappa'=K=0$ in Eq. (11). This results in the equation

$$s^2(s^2-v^2)^2-\omega_c^2(s^2-w^2)^2=0, \tag{12}$$

for the eigenfrequencies which was first obtained in Ref. 16. Next we consider the zero magnetic field limit of Eq. (11) and find

$$s^2[-s^2(s^2-w^2)^2][s^4+(2\gamma_3-v^2)s^2+\varrho^4-2\gamma_3 w^2]^2=0, \tag{13}$$

which results in the four eigenfrequencies $s_4=0$,

$$s_1^2=\frac{M+m}{mM}(\kappa+\kappa')=v^2, \tag{14a}$$

and

$$s_{2,3}^2=\frac{1}{2}\left\{v^2-\frac{2K}{m}\pm\left[\left(\frac{M-m}{mM}(\kappa+\kappa')-\frac{2K}{m}\right)^2+\frac{4}{mM}(\kappa-\kappa')^2\right]^{1/2}\right\}, \tag{14b}$$

as obtained in Refs. 4 and 6.

In the process of diagonalizing Eq. (8), two canonically conjugate constants of motion enter:

$$\Pi_1=\frac{1}{4}(x_1+x_2)-\frac{1}{2\omega_c}(p_{1y}+p_{2y})-\frac{1}{2\omega_c}(P_{1y}+P_{2y}) \tag{15a}$$

and

$$\Pi_2=\frac{1}{4}(y_1+y_2)+\frac{1}{2\omega_c}(p_{1x}+p_{2x})+\frac{1}{2\omega_c}(P_{1y}+P_{2x}), \tag{15b}$$

which satisfy the commutation relation $[\Pi_1,\Pi_2]=-i/2\omega_c$. They are related to the position of the classical orbit center. The explicit time evolution of the electron position coordinates are found to be

$$x_1(t)=\Pi_1+i\sum_{j=1}^{7} d_j(c_j e^{is_j t}+c_j^{\dagger}e^{-is_j t}) \tag{16a}$$

and

$$y_1(t)=\Pi_2-i\sum_{j=1}^{7} d_j(c_j e^{is_j t}+c_j^{\dagger}e^{-is_j t}), \tag{16b}$$

where c_j ($c_j^{\dagger}$) are annihilation (creation) operators for quantized motion of the internal degrees of freedom and which satisfy $[c_j,c_l^{\dagger}]=\delta_{j,l}$. Similar expressions are obtained for the coordinates of the second electron.

The coefficients d_i are rather complicated, but in the case of $K=0$ they reduce appreciably and are given by

$$d_i^2=\frac{1}{4s_i}\frac{s_i^2-w^2}{3s_i^2+2(-1)^i\omega_c s_i-v^2},\quad i=1\cdots 3 \tag{17a}$$

and

$$d_i^2=\frac{1}{4}\frac{s_i^2-w^2}{4s_i^3-(-1)^i\omega_c(3s_i^2-w^2)-2v^2 s_i},\quad i=4\cdots 7. \tag{17b}$$

III. THE BIPOLARON GROUND STATE ENERGY

The ground state energy of the bipolaron system is calculated using the Feynman variational principle which provides an upper bound to the exact bipolaron ground state energy E_{bip}. This variational principle states that

$$F_{\mathrm{bip}}\leqslant F_t-\frac{1}{\beta}\langle S-S_t\rangle_t, \tag{18}$$

where F_t is the free energy of some trial action S_t. $\langle\cdots\rangle_t$ is a path integral average with weight e^{S_t}. This inequality is valid for real actions S and S_t but may break down when S and S_t contain imaginary terms as in the present case. This problem was studied in Refs. 20,21, where only minor deviations from Eq. (18) were found in the presence of an external magnetic field. Therefore, as a first step towards the ultimate goal of solving the bipolaron problem in a magnetic field, we assume that Eq. (18) is valid as was done in Ref. 16. In the present paper S is given by Eq. (3) and we take for the trial action S_t the expression given by Eq. (9).

From the diagonalized Hamiltonian [see Eq. (8)] one notices that the partition function of the Feynman model consists of the partition function of seven one-dimensional harmonic oscillators. Besides one has to sum over allowed values of the constants of motion Π_x and Π_y, which is equal to $(L_xL_y/2\pi)m\omega_c$ when we assume that the system is confined to move in a box with dimensions L_x and L_y. Then the partition function is given by

$$Z_F=\frac{A}{2\pi\hbar}m\omega_c\sqrt{\frac{m}{2\pi\hbar^2\beta}}\prod_{i=1}^{7}\frac{1}{2\sinh(\beta\hbar s_i/2)}, \tag{19}$$

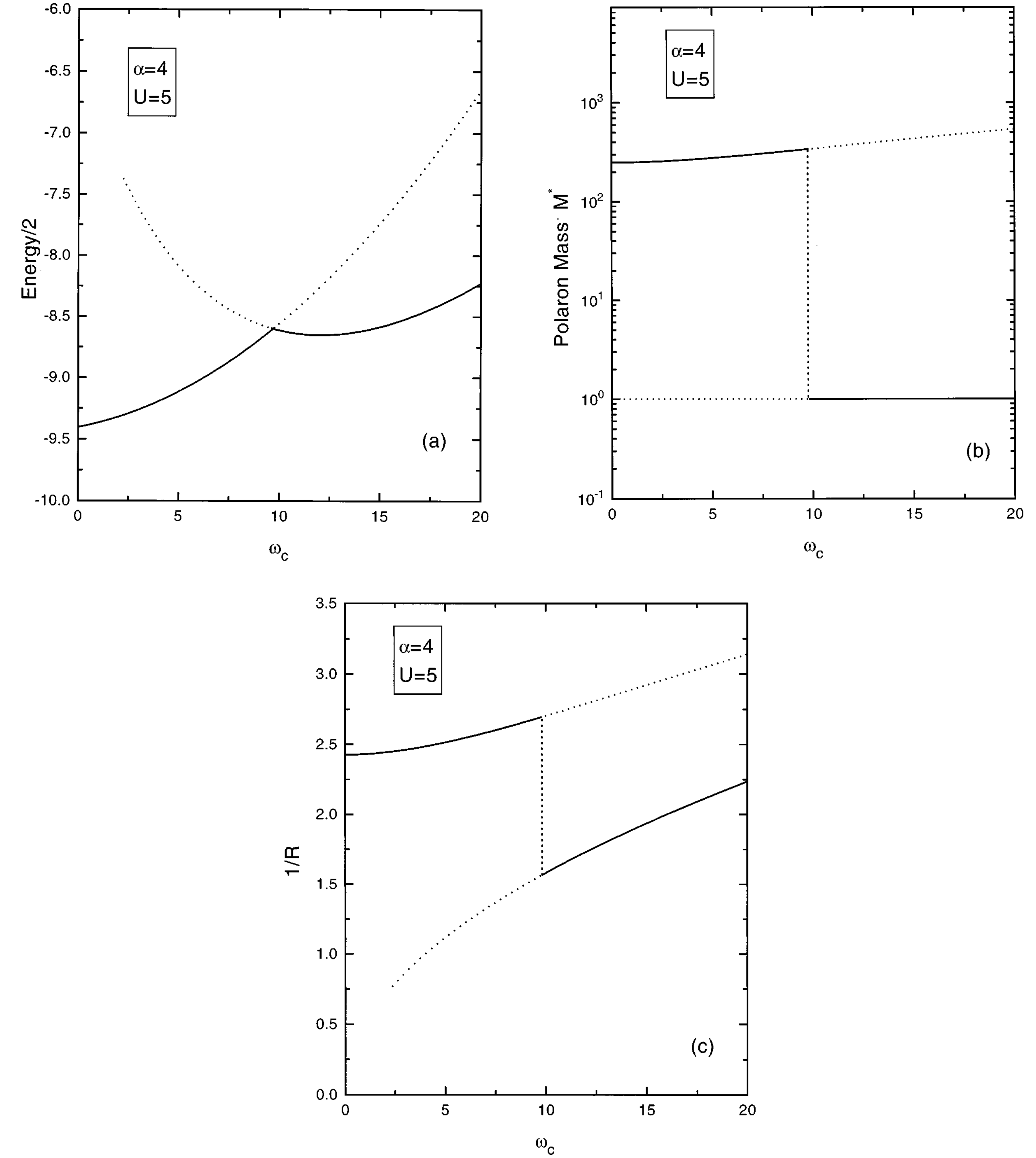

FIG. 2. Ground state energy (a), the polaron mass (b), and the inverse mean-square distance (c) as a function of the cyclotron frequency for a fixed electron-phonon coupling constant of $\alpha=4$ and Coulomb repulsion $U=5$.

from which the free energy, $F_F=-(1/\beta)\ln Z_F$, is easily calculated. At zero temperature the free energy reduces to the zero point energy of the model Hamiltonian, i.e., $\frac{1}{2}\Sigma_{i=1}{}^7 s_i$.

The bipolaron ground state energy E_{bip}, is obtained as the zero temperature limit of the free energy (18). In what follows we use dimensionless units $\hbar=\omega_{\mathrm{LO}}=m=1$, and consequently the bipolaron energy is expressed in units of $\hbar\omega_{\mathrm{LO}}$. Furthermore, we introduce the notation $\omega_1^2=v^2$ and $\omega_{2,3}^2=\frac{1}{2}[v^2\pm\frac{1}{2}\sqrt{v^4-4\varrho^4}]$, which are the squares of the frequencies given by Eq. (13) for $K=0$. Notice that they satisfy $\omega_1^2=\omega_2^2+\omega_3^2$ and $\omega_2^2\omega_3^2=\varrho^4$ and they also satisfy the following inequalities: $\omega_2\geqslant w\geqslant\omega_3\geqslant 0$. For $K=0$ and using the

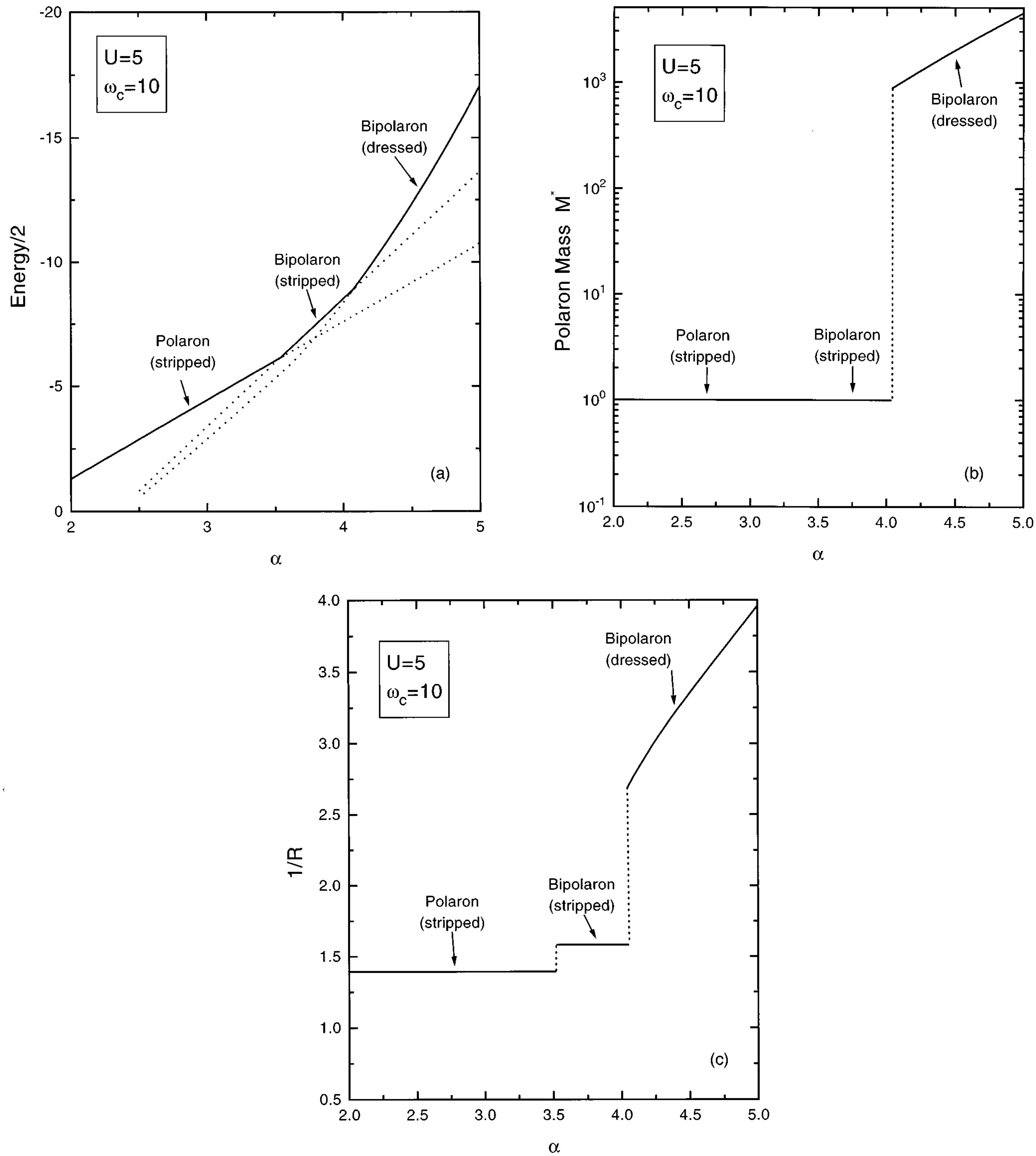

FIG. 3. The same as Fig. 2 but now as a function of the coupling constant α and for a nonzero Coulomb repulsion $U=5$ and $\omega_c=10$.

notation (11) and following the approach of Ref. 16 we derive the following estimate for the bipolaron energy:

$$E_{\mathrm{bip}}=\frac{1}{2}\sum_{i=1}^{7} s_i-2w-2\left[(v^2-w^2)\sum_{i=1}^{7}\frac{d_i^2 s_i}{s_i+w}+\frac{\varrho^4}{w}\sum_{i=4}^{7}\frac{d_i^2}{s_i+w}\right]+\frac{U}{2}\sqrt{\frac{\pi}{D_{12}(0)}}-\alpha\sqrt{\frac{\pi}{2}}\int_0^{\infty}du\,e^{-u}\left[\frac{1}{\sqrt{D_{11}(u)}}+\frac{1}{\sqrt{D_{12}(u)}}\right], \tag{20}$$

where

$$D_{11}(u)=\sum_{i=1}^{7} d_i^2(1-e^{-s_i u}) \tag{21a}$$

and

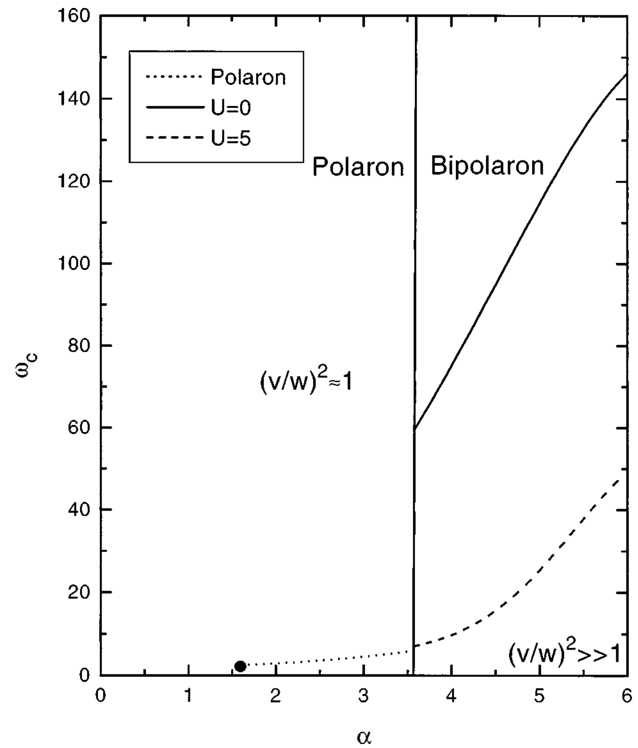

FIG. 4. Phase diagram for the bipolaron in a magnetic field.

$$D_{12}(u)=\sum_{i=1}^{3} d_i^2(1-e^{-s_i u})+\sum_{i=4}^{7} d_i^2(1+e^{-s_i u}), \quad (21b)$$

with $U=e^2/\epsilon_\infty$. Introducing the ratio of the dielectric constants $\eta=\epsilon_\infty/\epsilon_0$ we obtain the following relation between the Coulomb and electron-phonon coupling constant: $U=\sqrt{2}\alpha/(1-\eta)$. Thus in the physical allowed region we must have $U \geq \sqrt{2}\alpha$. In general the integral in Eq. (20) has to be calculated numerically and subsequently E_{bip} has to be minimized with respect to the three variational parameters v, w, and ϱ. The above expression (20) reduces to the one polaron result of Ref. 17 in the presence of a magnetic field for $U\to\infty$ and to the result of Ref. 6 in the limit of zero magnetic field. In the strong coupling limit the results of Ref. 9 are recovered.

The square of the mean-square separation R between the two electrons is given by

$$R^2=\langle[\vec{r}_1(u)-\vec{r}_2(u)]^2\rangle=4D_{12}(0). \quad (22)$$

In order to calculate the bipolaron transition we have to compare the bipolaron energy with the energy of two separate polarons which are infinitely far apart. This limit is contained in Eq. (20) and is obtained by choosing $\kappa'=K=0$ and $U=0$. Now only two variational parameters v and w are left. The resulting upper bound E_{pol} to the ground state energy[17] is given by

$$E_{\mathrm{pol}}=\frac{1}{2}\sum_{i=1}^{3} s_i-w-(v^2-w^2)\sum_{i=1}^{3}\frac{s_i d_i^2}{(w+s_i)} -\frac{1}{2}\sqrt{\frac{\pi}{2}}\alpha\int_0^\infty \frac{e^{-u}}{\sqrt{D(u)}}du, \quad (23)$$

where

$$D(u)=\sum_{i=1}^{3} d_i^2(1-e^{-s_i u}) \quad (24)$$

and

$$d_i^2=\sum_{i=1}^{3}\frac{1}{2s_i}\frac{s_i^2-w^2}{3s_i^2+2(-1)^i\omega_c s_i-v^2}, \quad (25)$$

with the eigenfrequencies s_i to be determined from the solution of the third order algebraic equation, Eq. (12), in s_i^2. The bipolaron transition occurs when the ground state energy E_{bip} of the bipolaron is equal to the ground state energy $2E_{\mathrm{pol}}$ of two separate polarons, i.e., $E_{\mathrm{bip}}=2E_{\mathrm{pol}}$.

IV. PHASE DIAGRAM

In this section we present the numerical results for the polaron and bipolaron energy. First we investigate the bipolaron state as a function of the magnetic field. A typical result of the bipolaron energy per particle as a function of ω_c is shown in Fig. 2(a) for $\alpha=4$ and fixed Coulomb repulsion $U=5$. The dotted curves are the results of the metastable states. We also calculated numerically the corresponding mass of the Feynman bipolaron model $2M^*=2(v/w)^2$ and show it in Fig. 2(b). Note that for large ω_c we have $M^*\approx 1$ which is the free electron mass and consequently the bipolaron transition corresponds to the stripping transition which is analogous to a similar transition which was found for a single polaron in Ref. 16. The inverse of the mean square separation $1/R$, in units of $\sqrt{m\omega_{\mathrm{LO}}/\hbar}$, is shown for the same set of parameters in Fig. 2(c). With increasing ω_c, a discontinuous transition occurs at $\omega_c\approx 9.724$, at which point the bipolaron stripped state has a lower energy. For $\omega_c<9.724$ the stable bipolaron state consists of two heavy polarons and the energy increases with magnetic field similar to the case of a quasifree particle. Within the present Feynman type of approach the polaron dressed state is still mobile and has an effective mass which is more than two orders of magnitude larger than the bare electron mass. At the transition, the two polarons find it energetically more favorable to move within one effective potential well created by the phonons. The behavior of the ground state energy of the stripped state as a function of the magnetic field [see Fig. 2(a)] is typically the one of a particle bound in a potential well. The electrons are not dressed and move inside this well. In the present approach the composite system: electron + well is translational invariant in contrast to e.g., the approach of Ref. 9 where the electron in the strong coupling limit is bound in space. The size of the bipolaron state is larger than in the dressed state. We found that the magnetic field at which the stripping transition occurs depends very strongly on the strength of the repulsive Coulomb interaction.

Next we investigate the system as function of the electron-phonon coupling strength. In Fig. 3(a), a typical result for the energy per polaron is depicted as a function of electron-phonon coupling strength α for a fixed value of the Coulomb repulsion $U=5$ and the cyclotron frequency $\omega_c=10$. The corresponding mass and inverse radius are shown in Figs. 3(b) and 3(c), respectively. Note that we find two transition points. For small α we have two separate stripped polarons each with an effective mass which is slightly larger than 1, i.e., $M^*\approx 1$. Notice that $R<\infty$ in this state which is a consequence of the presence of the magnetic field. In the polaron state we have $R^2=\langle \vec{r}_1^{\,2}\rangle+\langle \vec{r}_2^{\,2}\rangle=2\langle \vec{r}_1^{\,2}\rangle\approx 2l_B^2=2\hbar/m\omega_c$ which is finite for $B\neq 0$. With increasing α, a transition is found at $\alpha\approx 3.53$ where the stripped bipolaron has a lower energy. When we further increase α, the bipolaron self-energy increases continuously until $\alpha\approx 4.09$ at which point the dressed bipolaron state has a lower energy and consequently is the stable state. The one-polaron mass increases with three orders of magnitude and the bipolaron radius decreases with almost a factor of 2.

Our results are summarized in the polaron-bipolaron phase diagram which is shown in Fig. 4, where the magnetic field at which the transition occurs is plotted versus the electron-phonon coupling constant for fixed values of the electron-electron Coulomb repulsion. The phase diagram exhibits four possible states for this system. First there is the polaron-bipolaron transition which occurs for $\alpha\approx 3.6$ and which is practically independent of ω_c. This transition occurs at slighly larger α values with increasing ω_c, e.g., for $\omega_c=0$ it occurs at $\alpha=3.564$, while for $\omega_c=160$ it takes place for $\alpha=3.597$. In the polaron region we have the polaron stripping transition which is given by the dotted curve and which ends in the critical point $\alpha=1.60$, $\omega_c=2.10$ where a second order transition takes place. In the bipolaron region we also have a stripping transition with increasing magnetic field. This transition is shown for $U=0$ (solid curve) and for $U=5$ (dashed curve). Notice that with increasing U smaller magnetic fields are needed in order to induce the stripping transition.

V. CONCLUSION

In the present work we used the Feynman path-integral method to study the ground state properties of a system consisting of two electrons moving in two dimensions which interact with each other by the Coulomb force and through optical phonons in the presence of the magnetic field. The ground state energy, the Feynman polaron mass, and the size of the state were studied numerically which showed a very rich behavior. Namely, for certain values of α, U, and ω_c the bipolaron undergoes phase transitions where the mass and the mean square separation $1/R$ between the two electrons exhibit a discontinuity. This discontinuity can be a consequence of the polaron-bipolaron transition or of the stripping transition. The later transition was overlooked in previous work[9,11–13] on the magnetic field dependence of the bipolaron.

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