PHYSICAL REVIEW B VOLUME 57, NUMBER 17 1 MAY 1998-I

Proximity-effect enhancement induced by roughness of the superconductor–normal-metal interface

R. G. Mints and I. B. Snapiro
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel
(Received 19 August 1997; revised manuscript received 5 December 1997)

Critical-temperature reduction ΔT_c is considered for a thin film of a superconductor (S) with a rough surface covered by a thick layer of a normal metal (N). The value of ΔT_c induced by the roughness of the SN interface can be much higher than ΔT_c for a film with a plain surface if the superconductor is extremely anisotropic. [S0163-1829(98)04817-6]

Proximity effects are determined by the penetration of electrons from the normal metal into the superconductor and by the penetration of Cooper pairs from the superconductor into the normal metal.[1,2] A dramatic manifestation of proximity effects is the reduction of the critical temperature ΔT_c of a thin superconducting film covered by a thick layer of a normal metal.[1] In the case of an isotropic superconductor with a plain superconductor–normal-metal (SN) interface the value of ΔT_c is given by the formula[3]

$$\frac{\Delta T_c}{T_c} = -\frac{\gamma^2\pi^2}{4}\frac{\xi^2(0)}{d^2}, \tag{1}$$

where $\gamma \approx 0.74$ is a numerical factor, $\xi(0)$ is the coherence length at zero temperature, and d is the thickness of the superconducting film $[d \gg \xi(0)]$.

Proximity effects in anisotropic high-temperature copper oxide superconductors are currently under thorough experimental and theoretical study.[5] The reduction of the transition temperature for an anisotropic superconductor depends on the orientation of the SN interface relative to the symmetry axes of the superconductor. The value of $|\Delta T_c|$ is maximal when the SN interface is perpendicular to the axis corresponding to the maximum value of the coherence length. For an anisotropic superconductor with a rough SN interface the local orientation of the film surface relative to the symmetry axes is varying along the surface. A certain average value of the squared coherence length appears then in Eq. (1). Depending on the roughness this average value can be determined by the highest of the coherence lengths even if for a plain surface with the same orientation ΔT_c is determined by the smallest of the coherence lengths. Therefore, for a strongly anisotropic superconductor the roughness of the SN interface can drastically increase ΔT_c.

In the present paper we treat the reduction of the critical temperature ΔT_c for a rough superconducting film of an anisotropic superconductor covered by a thick layer of a normal metal and located on top of a substrate from an insulator with a plane surface $(z=0)$, where the z axis is along the c direction and the xy plane is parallel to the ab planes. The average thickness of the film d is considered to be bigger than the coherence lengths at zero temperature $\xi_\mu(0)$ $(\mu = a,b,c)$, i.e., $d \gg \xi_\mu(0)$. We assume that the roughness is small and describe the SN interface as $z = z_S(x,y) = d + f(x,y)$, with $|f(x,y)| \ll d$ and a zero average value $\langle f \rangle$ of $f(x,y)$,

$$\langle f \rangle = \frac{1}{A}\int dx\,dy\, f(x,y) = 0, \tag{2}$$

where A is the area of the film. The typical length scale l of variations of the film thickness $f(x,y)$ is considered to be from the interval $\xi_\mu(0) \ll l \ll \xi_\mu(T)$.

To find the reduction of the transition temperature we treat the Ginzburg-Landau free energy

$$\mathcal{F} = \frac{H_c^2}{4\pi}\int d^3\mathbf{r}\left[-\Psi^2 + \frac{1}{2}\Psi^4 + \xi_\mu^2\left|\frac{d\Psi}{dx_\mu}\right|^2\right], \tag{3}$$

where the order parameter Ψ depends on the three coordinates x, y, and z in the case of a rough SN interface and H_c is the critical magnetic field. To minimize the free energy (3) we have to formulate the boundary conditions for $\Psi(x,y,z)$ at both surfaces of the superconducting film. The boundary condition at the plane interface with the substrate from an insulator is[2,3]

$$\left.\frac{\partial \Psi}{\partial z}\right|_{z=0} = 0. \tag{4}$$

In general, at the SN interface we have[2–4]

$$\mathbf{n}\hat{m}\nabla\Psi\Big|_{\mathrm{SN}} = \frac{1}{b}\Psi\Big|_{\mathrm{SN}}, \tag{5}$$

where $\mathbf{n}$ is a unit vector normal to the SN interface; the eigenvalues of the dimensionless tensor $\hat{m}$ in the coordinate system aligned with the principal axes are m/m_μ, and m and m_μ are the effective masses in the normal metal and in the superconductor; b is the extrapolation length $(b \ll d)$. The left side of Eq. (5) is of order Ψ/ξ_{av}, where the length scale ξ_{av} is a certain combination of the correlation lengths ξ_μ. In the close vicinity of T_c the value of the extrapolation length b is temperature independent contrary to ξ_{av}. At the superconducting-to-normal transition point ξ_{av} becomes of order d and the left side of Eq. (5) can be disregarded. The boundary condition at the SN interface then has the form[6]

$$\Psi|_{z=d+f(x,y)} = 0. \tag{6}$$

0163-1829/98/57(17)/10318(2)/$15.00

To find the value of $\mathcal{F}$ we use a trial function

$$\Psi=\Phi(x,y)\cos\frac{\pi z}{2[d+f(x,y)]}. \qquad (7)$$

This function satisfies the standard Ginzburg-Landau boundary conditions at the film surfaces,

$$\Psi|_{z=d+f(x,y)}=0, \quad \left.\frac{\partial\Psi}{\partial z}\right|_{z=0}=0, \qquad (8)$$

and to find $\Phi(x,y)$ we have to minimize the free energy.

Substituting Eq. (7) into the functional (3) and assuming that $|f|\ll d$ we obtain after integration over z

$$\mathcal{F}=\frac{H_c^2}{8\pi}\int dxdy\left\{\left[\pi^2\xi_c^2+\frac{\pi^2+3}{3}\xi_i^2 f_i^2-4d^2\right]\frac{\Phi^2}{4d}+d\xi_i^2\Phi_i^2 + \xi_i^2 f_i\Phi\Phi_i+\frac{3d}{8}\Phi^4\right\}, \qquad (9)$$

where $i=a,b$, $f_i=\partial f/\partial x_i$, and $\Phi_i=\partial\Phi/\partial x_i$. Minimization of the free energy (9) results in the following equation for $\Phi(x,y)$:

$$\xi_i^2\Phi_{ii}+\frac{\xi_i^2 f_i}{2d}\Phi_i+\Phi\left[1-\frac{\pi^2\xi_c^2}{4d^2}-\frac{(3+\pi^2)\xi_i^2 f_i^2}{12d^2}+\frac{\xi_i^2 f_{ii}}{2d}\right] -\frac{3}{4}\Phi^3=0. \qquad (10)$$

We are interested in a solution of Eq. (10) imposed by the roughness of the SN interface. This solution has a length scale of the order of the length scale l of variations of the film thickness $f(x,y)$. We take $\Phi(x,y)$ in the form

$$\Phi=\Phi_0+\Phi_1(x,y), \qquad (11)$$

where $\Phi_0=\text{const}$, $|\Phi_1|\ll\Phi_0$, and the length scale of $\Phi_1(x,y)$ is of the order of l. It follows then from Eq. (10) that with the accuracy of $|f(x,y)|/d\ll 1$ we have

$$\Phi_1=-\Phi_0\frac{f}{2d}. \qquad (12)$$

Finally, using Eqs. (9), (11), and (12) we obtain for the Ginzburg-Landau free energy

$$\mathcal{F}=\mathcal{F}_0\left\{\left[\frac{6+\pi^2}{12}\frac{\xi_i^2\langle f_i^2\rangle}{d^2}+\frac{\pi^2}{4}\frac{\xi_c^2}{d^2}-1\right]\Phi_0^2+\frac{3}{8}\Phi_0^4\right\}, \qquad (13)$$

where $\mathcal{F}_0=AdH_c^2/128\pi$. The temperature of the superconducting-to-normal transition is determined by the change of sign of the coefficient at Φ_0^2 in Eq. (13) for the free energy. Taking the temperature dependence of the coherence lengths near T_c as

$$\xi_\mu=\gamma\xi_\mu(0)(1-T/T_c)^{-1/2}, \qquad (14)$$

we obtain the reduction of the transition temperature ΔT_c in the following final form:

$$\frac{\Delta T_c}{T_c}=-\gamma^2\frac{\pi^2}{4}\left[\frac{\xi_c^2(0)}{d^2}+\frac{6+\pi^2}{3\pi^2}\frac{\xi_i^2(0)\langle f_i^2\rangle}{d^2}\right]. \qquad (15)$$

In the case of a plain SN interface the slopes $f_i=0$ and Eq. (15) coincides with Eq. (1). For a rough SN interface there is an increase of ΔT_c.

To estimate the effect of the roughness of the SN interface on ΔT_c assume, for simplicity, that $\xi_a=\xi_b=\xi_{\mathrm{ab}}$ and $\langle f_a^2\rangle=\langle f_b^2\rangle=\langle f'^2\rangle$. In this case Eq. (15) transforms to a simple formula

$$\frac{\Delta T_c}{T_c}=-1.35\left[\frac{\xi_c^2(0)}{d^2}+1.07\langle f'^2\rangle\frac{\xi_{\mathrm{ab}}^2(0)}{d^2}\right]. \qquad (16)$$

It follows from Eq. (16) that even if the roughness is moderate ($\langle f_i^2\rangle\sim 1$) a considerable increase of the reduction of the critical temperature appears for an extremely anisotropic superconductor with $\xi_{a,b}\gg\xi_c$.

The formula for ΔT_c, Eq. (15), is valid for a three-dimensional (3D) anisotropic superconductor. The Lawrence-Doniach model[7] allows one to calculate the reduction of the critical temperature for a layered superconductor with weakly coupled 2D superconducting layers. This extension of the problem is under consideration.

In conclusion, we have studied a thin rough film of a superconductor covered by a thick layer of a normal metal. We have found that in the case of an extremely anisotropic superconductor the roughness of the SN interface can significantly increase the reduction of the critical temperature compared with the case of a plain film.

We acknowledge the support of the German-Israeli Foundation for Research and Development, Grant No. 1-300-101.07/93. R.G.M. is grateful to G. Deutscher, E. H. Brandt, V. G. Kogan, and E. Polturak for stimulating and informative discussions.

[1] G. Deutscher and P. G. de Gennes, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, Inc., New York, 1969) Vol. 2, p. 1005.

[2] P. G. de Gennes, *Superconductivity of Metals and Alloys* (W. A. Benjamin, New York, 1966), pp. 227–234.

[3] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988), pp. 483–486.

[4] R. O. Zaitsev, Zh. Éksp. Teor. Fiz. **48**, 644 (1965) [Sov. Phys. JETP **21**, 426 (1965)]; **48**, 1759 (1965) [**21**, 1178 (1965)].

[5] K. A. Delin and A. W. Kleinsasser, Supercond. Sci. Technol. **9**, 227 (1996).

[6] A detailed discussion of the boundary condition at the SN interface is given in Ref. 3.

[7] W. E. Lawrence and S. Doniach, in *Proceedings of 12th International Conference on Low Temperature Physics*, Kyoto, 1970, edited by E. Kanda (Keigaku, Tokyo, 1971), p. 361.