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Direct observation of two-dimensional self-focusing of spin waves in magnetic films

M. Bauer, C. Mathieu, S. O. Demokritov, and B. Hillebrands*
Fachbereich Physik, Universität Kaiserslautern, Erwin-Schrödinger-Strasse 46, D-67663 Kaiserslautern, Germany

P. A. Kolodin
St. Petersburg Electrotechnical University, 197376 St. Petersburg, Russia

S. Sure and H. Dötsch
Fachbereich Physik, Universität Osnabrück, D-49069 Osnabrück, Germany

V. Grimalsky and Yu. Rapoport
Kiev State University, 252601 Kiev, Ukraine

A. N. Slavin
Department of Physics, Oakland University, Rochester, Michigan 48309-4401

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An observation of self-focusing of dipolar spin waves in garnet film media is reported. In particular, we show that the quasistationary diffraction of a finite-aperture spin-wave beam in a focusing medium leads to the concentration of the wave power in one focal point rather than along a certain line (channel). The obtained results demonstrate the wide applicability of nonlinear spin-wave media to study nonlinear wave phenomena using an advanced combined microwave–Brillouin-light-scattering technique for a two-dimensional mapping of the spin-wave amplitudes. [S0163-1829(97)51638-9]

Spin waves propagating in thin magnetic films are a superb object for experimental investigations of linear and nonlinear wave dynamics in anisotropic and dispersive media. The threshold of nonlinearity for spin waves is relatively low, and many important nonlinear effects like formation of spin-wave solitons, parametric and kinetic instabilities, as well as transitions to chaos have been observed in magnetic films at moderate input power levels of $P_{in}<1W$.[1,2] An additional important advantage of film media is the accessibility of the wave process from the film surface. The combination of microwave spectrometry with Brillouin light scattering (BLS) from coherent magnetic excitations provides a unique possibility to observe directly spin-wave processes in magnetic films with high spatial resolution. First attempts have been performed for uniform modes in the sixties and seventies.[3] Recently this technique has been used in experiments by Boyle *et al.*[4] where the observation of self-channeling and beam shaping for magnetostatic waves in a yttrium iron garnet (YIG) film has been reported.

In this paper we report direct experimental evidence of two-dimensional self-focusing for dipole-dominated spin waves propagating in a tangentially magnetized magnetic film along the direction of magnetization [backward volume magnetostatic waves (BVMSW) in the classification of Ref. 5]. We show that the quasistationary diffraction of a nonlinear BVMSW beam of finite transverse width at sufficiently large wave amplitudes leads to the concentration of the wave power at one focal point (self-focusing) rather than along a certain line, or channel (self-channeling) as it was proposed in Ref. 4. The experimental results are qualitatively reproduced by numerical simulations of the mode propagation. Our experimental observations have been made possible by advancing the stability and the data acquisition procedure of the BLS spectrometer enabling us to obtain high-resolution, two-dimensional maps of the spin-wave amplitude distribution across the sample.

The theory of nonlinear wave diffraction of finite wave beams was developed in Refs. 6 and 7, and it was applied to the case of magnetostatic wave beams in ferrite films in Ref. 8. The evolution of a spectrally narrow magnetostatic wave beam with an almost plane front in a ferrite film in the (y,z) plane propagating in the z direction can be described in terms of the two-dimensional parabolic equation:[7,8]

$$i\left(\frac{\partial\varphi}{\partial t}+V_g\frac{\partial\varphi}{\partial z}\right)+\frac{1}{2}D\frac{\partial^2\varphi}{\partial z^2}+S\frac{\partial^2\varphi}{\partial y^2}-N|\varphi|^2\varphi=-i\omega_r\varphi, \tag{1}$$

where φ is the dimensionless amplitude of the beam envelope ($|\varphi|^2=m^2/2M_0^2$, where m is the amplitude of the variable magnetization and M_0 is the saturation magnetization), $V_g=\partial\omega/\partial k_z|_{k_0}$ is the group velocity, $D=\partial^2\omega/\partial k_z^2|_{k_0}$ is the dispersion coefficient, $S=\partial\omega/\partial k_y^2|_{k_0}$ is the diffraction coefficient, $N=\partial\omega/\partial|\varphi|^2$ is the nonlinear coefficient, and $\omega_r=\gamma\Delta H_k$ is a dissipation parameter, with γ the gyromagnetic ratio and ΔH_k the half-linewidth of the ferromagnetic resonance in the film. It is well known that the dispersion (D) and the diffraction (S) coefficients for the BVMSW are positive, while the nonlinear coefficient N is negative (see Ref. 8 and chapter 9 in Ref. 1), so that the Lighthill criterion[9] for modulational instability is fulfilled for both the longitudinal (along z) and the transverse (along y) perturbations of a spectrally narrow input wave packet.

In the stationary case $\partial\omega/\partial t=0$, the input wave is strictly monochromatic and the third term in Eq. (1) containing the dispersion coefficient D is not important. The resultant equation, describing the stationary nonlinear diffraction of a monochromatic two-dimensional wave beam, is a standard nonlinear Schrödinger equation (NSE) with renormalized ''time'' $\tau=z/V_g$.[6] This equation in a lossless medium ($\omega_r=0$) has a single soliton solution describing a wave channel localized along the transverse coordinate y and having constant amplitude along the longitudinal coordinate z. The existence of such a solution leads to the assumption that the formation of a narrow wave channel (i.e., the effect of self-channeling) can be observed as a result of a stationary two-dimensional diffraction of a nonlinear wave.[4]

We note, however, that the single soliton solution of the NSE (describing a constant-amplitude channel) exists only in a lossless medium, and in a very limited interval of input wave beam amplitudes. When the amplitude of the input wave beam is increased beyond this interval, the nonlinear diffraction of the beam leads to the two-dimensional intensity distributions described by multisoliton solutions of the NSE.[6,10] These solutions can be interpreted as bound states of several single solitons, and the amplitude of these solutions changes periodically along the direction of wave propagation (longitudinal coordinate z). Therefore, the two-dimensional wave intensity distributions in the (y,z) plane, described by these solutions, have pronounced intensity maxima (or foci) separated by a certain distance (soliton period) along the z axis. Thus, for sufficiently high input beam amplitudes, the transverse modulational instability alone (without any contribution from the longitudinal modulational instability) could lead to the concentration of the wave beam intensity in several well-localized points (or foci) on the (y,z) plane, and therefore to the effect of nonlinear self-focusing of a wave beam. In a dissipative medium the periodically oscillating amplitude of a multisoliton solution of the NSE will also exponentially decay along the longitudinal coordinate z, leading to a situation when only one (first) wave intensity maximum (or focus) will be large enough to be clearly seen in the experiment.

The aim of this paper is to study experimentally the wave intensity distributions in a quasistationary finite-aperture nonlinear beam of a BVMSW mode propagating in a tangentially magnetized ferrite garnet film, to obtain direct experimental evidence of two-dimensional nonlinear diffraction effects in this beam leading to self-focusing, and to clarify the mechanism of this self-focusing effect.

The microwave equipment consists of a standard delay-line structure (see chapter 9 in Ref. 1 and Refs. 3 and 5) of input and output antennas (width $r=35$ μm, length $W=2$ mm separated by a distance of 8 mm), connected to a microwave network analyzer. Precautions have been taken to control overheating of the samples by the microwave power. An intermittent source with a modulation frequency of 4 kHz and with variable duty cycles and peak powers was used to keep the average power less than 100 mW. This periodic modulation of the input signal made our input beam only quasimonochromatic and, therefore, the two-dimensional beam diffraction patterns described below were only quasistationary. Comparative experiments have shown that all observed diffraction effects (associated with the transverse modulational instability of the beam) depend on the peak power, but not on the average one.

The film of the composition $Lu_{2.04}Bi_{0.96}Fe_5O_{12}$ with in-plane dimensions of 2×10 mm^2 epitaxially grown onto a single crystalline (111)-oriented gallium gadolinium garnet (GGG) substrate was mounted onto the delay line structure. The ferrite film sample has the following parameters: thickness $L=1.5$ μm, saturation magnetization $4\pi M_o=1750$ G, full line width of the ferromagnetic resonance $2\Delta H_k=0.9$ Oe. The in-plane bias magnetic field $H=2130$ Oe was applied parallel to the z direction so that a BVMSW beam was excited in the film. The beam divergence due to the finite length W of the input antenna was thoroughly minimized. For this reason the working frequency was chosen as $\nu=8.10$ GHz, which corresponds to a carrier wave number of $k_0=k_{z0}\cong370$ cm^{-1}. The initial angular beam divergence is thus not larger than $\Theta_{max}=(\pi/W)/k_0=0.042$ rad.

The spin-wave amplitude was obtained from spatially resolved (resolution: 50 μm) forward BLS from the spin waves, using a laser beam (514.5 nm Ar^+ line) directed along the x direction perpendicular to the sample plane. In this case the intensity of the inelastic scattering process is proportional to m^2 or $|\varphi|^2$ and it is sensitive to the k interval $0-10^4$ cm^{-1}.[11] The scattered light was analyzed by a Sandercock-type (3+3)-pass tandem Fabry-Pérot interferometer. Special care has been taken to yield long-time stability of the instrumental transmission. A full computer control of the stabilization, the data acquisition and the sample positioning allows for an automated, high contrast (>60 dB), two-dimensional mapping of the spin-wave intensity across the film. The mesh used was 0.1 mm along y and 0.5 mm along z. Since the accumulation time was 45 s per position, all the nonstationary effects were beyond our BLS measurements. The upper panels in Fig. 1 show two-dimensional maps of the inelastically scattered light, proportional to $|\varphi|^2$ in a *logarithmic* color code. The data in each panel is normalized to the respective maximum intensity at the minimum measured distance to the antenna of 0.5 mm. From left to right the intensity of the microwave input powers measured at the antenna input are 30, 100, 800, and 1000 mW, respectively. Note here once more, that these values correspond to the microwave peak power, the average power being always lower than 100 mW. Even in the logarithmic scale the measured data show clearly a narrowing of the mode profile at a distance to the antenna of 0.8 to 2.2 mm at a microwave power of 800 mW. A comparison of the absolute values of the light scattering intensities shows that $|\varphi|^2$ is not proportional to the input power even near the input antenna. This intensity at 1000 mW input power, for example, is about 8 times higher than the corresponding value at 800 mW. A reason for that is not clear at present.

Despite the complicated mode intensity profile in the yz plane and its dependence on the input power the average spin-wave attenuation along the propagation z direction obeys a simple law, as demonstrated in Fig. 2. Shown is the measured BLS intensity (proportional to $|\varphi|^2$) integrated across a line perpendicular to the mode propagation direction (along the y axis) as a function of z, the distance to the antenna. It is clearly seen that for all values of the microwave input power used in the experiment the mode attenuation along z is strictly exponential, $I\propto|\varphi|^2\propto\exp(-\kappa z)$ and the

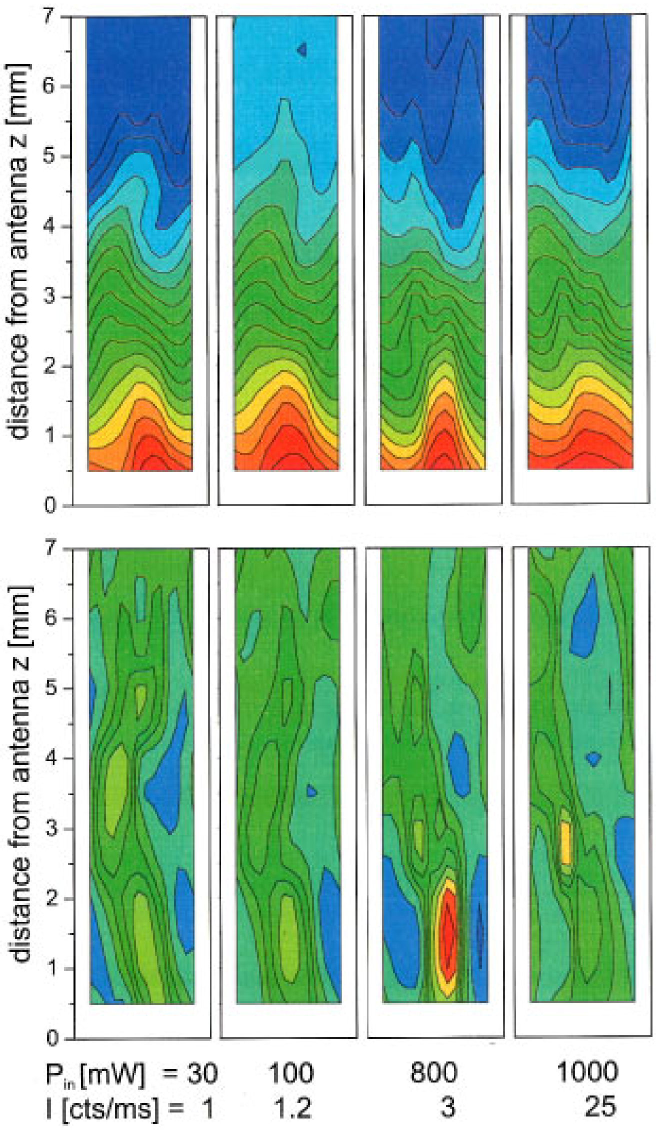

FIG. 1. (Color) Distribution of the spin-wave intensity $|\varphi|^2$ normalized to the maximum intensity at minimum measured distance to the antenna of 0.5 mm. The microwave input power and the absolute values of the light scattering intensity, measured near the input antenna, are indicated in the figure. The mode propagation is from bottom to top on each panel. The upper panels show the raw data in a logarithmic scale. In the lower panels $|\varphi|^2\exp(\kappa z)$ is shown in a linear color scale.

corresponding value of $\kappa = 77.4 \pm 1.5$ dB/cm does not depend on the input power. This fact can be used in a straightforward procedure to improve the contrast of the raw data. By plotting $|\varphi|^2\exp(\kappa z)$ instead of $|\varphi|^2$ one can avoid the large changes in the plotted values and therefore a logarithmic scale in the figure. The lower panels of Fig. 1 show $|\varphi|^2\exp(\kappa z)$, normalized in the same way as the upper panels, in a *linear* color code. Two pronounced features are seen: first, at 800 mW a sharp, high-intensity spot of 0.25 mm width (FWHM) and 1.5 mm length is observed, which we interpret as evidence for the self-focusing effect, as discussed below. Second, a snakelike intensity distribution is observed for both the linear and nonlinear regime. This structure is caused by the fact that the input antenna excites not only the zero order, but also the first-order lateral width

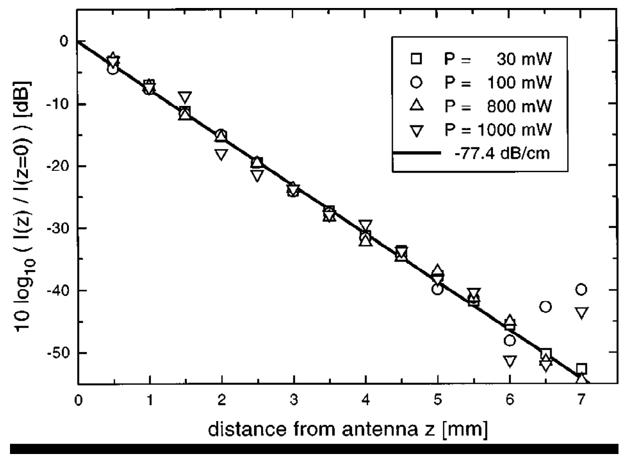

FIG. 2. Integrated intensity of the spin-wave modes $I(z)$ as a function of the distance to the antenna normalized to its extrapolated value at $z=0$. The attenuation parameter obtained from a fit (full line) is $\kappa = 77.4 \pm 1.5$ dB/cm.

mode of the film waveguide due to the asymmetric connection of the antenna. These two modes have slightly different group velocities, and interference of these modes creates this structure. Experiments on the films of pure YIG yielded very close results, which are not shown due to space limitations, and which fully corroborate our findings on the self-focusing effect.[12]

Calculations of the coefficients of the two-dimensional parabolic equation (1) yield the following values of the parameters (see chapter 9 in Ref. 1): $V_g = -8.6 \times 10^5$ cm/s; $D = 71.1$ cm^2/s; $S = 3.06 \times 10^3$ cm^2/s; $N = -1.14 \times 10^{10}$ s^{-1}; $\omega_r = 7.92 \times 10^6$ s^{-1}. A two-dimensional nonlinear dif-

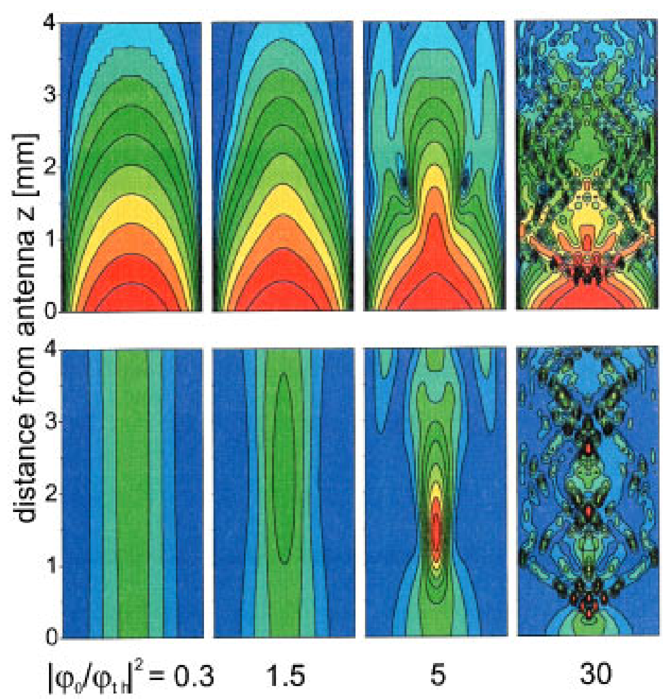

FIG. 3. (Color) Numerically calculated stationary distribution of the spin-wave intensity. Upper panels show $|\varphi(y,z)|^2$ normalized to the maximum intensity $|\varphi_0|^2$ of the initial ($z=0$) beam intensity distribution in a logarithmic scale. Lower panels show $|\varphi/\varphi_0|^2\exp(\kappa z)$ in a linear scale.

fraction of the propagating wave beam manifests itself only if the initial amplitude of the beam exceeds φ_{th} — the minimum threshold of modulational instability in the medium.[13] Using Eq. (1) in the stationary case ($\partial\varphi/\partial t=0$) with the above calculated values of coefficients, we performed a numerical modeling of the observed process of two-dimensional quasistationary diffraction of the BVMSW beam. We assumed that the initial (at $z=0$) distribution of the beam amplitude along the y axis was cosinusoidal $\varphi(y,z=0)=\varphi_0\cos[\pi(y-y_0)/W)]$, where $W=2$ mm is the film width, and $y_0=1$ mm. Therefore, the calculated distributions do not reproduce the snakelike structure, which is not of importance for the self-focusing effect. We also took into account the finite width ($W=2$ mm) of our film sample by imposing the ''magnetic wall'' boundary conditions at the film edges. We present in Fig. 3 the obtained normalized wave intensity as the distribution of $|\varphi/\varphi_0|^2$ in a logarithmic scale (upper panels) and of $|\varphi/\varphi_0|^2\exp(\kappa z)$ in a linear scale (lower panels). The main feature observed in our experiment — a clear intensity maximum (Fig. 1, low part, $P_{in}=800$ mW) — is also clearly seen in Fig. 3 for $|\varphi_0/\varphi_{th}|^2=5$. It is important to note, that the numerical calculations describe the position, size, and the contrast of the observed focus very well. Numerical calculations at higher values of the input amplitude φ_0 demonstrate the destruction of the focus and the formation of several foci with lower contrast and different spatial positions compatible with the evolution observed in the experiments with $P_{in}=1000$ mW (right panel in Fig. 1). It is difficult to correlate directly the results of our calculations with the experimental data, because the experimental value of the threshold could not be determined exactly. Our estimations show that the crossover from the linear to the nonlinear regime corresponds to an input power between 30 and 70 mW.

In summary the two-dimensional nonlinear diffraction of quasistationary finite-width beams of the BVMSW mode caused by the transverse modulational instability has been observed. At sufficiently high values of the input power this diffraction leads to self-focusing (i.e., concentration of wave power in one focal point) rather than to self-channeling (i.e., concentration of wave power along a certain line or channel).

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*Author to whom correspondence should be addressed.

[1] *Nonlinear Phenomena and Chaos in Magnetic Materials*, edited by P. E. Wigen (World Scientific, Singapore, 1994).

[2] M. Chen, M. A. Tsankov, J. M. Nash, and C. E. Patton, Phys. Rev. B **49**, 12 773 (1994).

[3] V. G. Zhotikov and N. M. Kreines, Pis'ma Zh. Eksp. Teor. Fiz. **26**, 360 (1977) [JETP Lett. **26**, 496 (1977)]; J. T. Hanlon and J. F. Dillon, J. Appl. Phys. **36**, 1269 (1965).

[4] J. W. Boyle, S. A. Nikitov, A. D. Boardman, J. G. Booth, and K. Booth, Phys. Rev. B **53**, 12 173 (1996).

[5] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids **19**, 308 (1961).

[6] V. E. Zakharov and A. B. Shabat, Zh. Eksp. Teor. Fiz. **64**, 1627 (1973) [Sov. Phys. JETP **37**, 823 (1973)].

[7] V. P. Lukomskii, Zh. Eksp. Teor. Fiz. **84**, 513 (1983) [Sov. Phys. JETP **57**, 297 (1983)].

[8] A. K. Zvezdin and A. F. Popkov, Zh. Eksp. Teor. Fiz. **80**, 704 (1981) [Sov. Phys. JETP **53**, 355 (1981)].

[9] M. J. Lighthill, J. Inst. Math. Appl. **1**, 269 (1965).

[10] G. Agrawal, *Nonlinear Fiber Optics*, 2nd ed. (Academic, San Diego, 1995).

[11] W. Wettling, M. G. Cottam, and J. R. Sandercock, J. Phys. C **9**, 211 (1975).

[12] M. Bauer, C. Mathieu, S. O. Demokritov, B. Hillebrands, P. A. Kolodin, V. Grimalsky, Yu. Rapoport, and A. N. Slavin (unpublished).

[13] V. S. L'vov, *Wave Turbulence under Parametric Excitation* (Springer-Verlag, New York, 1994).