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Long-range Josephson effect in mesoscopic T-shaped superconductor–normal-metal–superconductor junctions

P. Samuelsson
Department of Microelectronics and Nanoscience, Chalmers University of Technology and Göteborg University, S-41296 Göteborg, Sweden

V. S. Shumeiko
Department of Microelectronics and Nanoscience, Chalmers University of Technology and Göteborg University, S-41296 Göteborg, Sweden
and B. Verkin Institute for Low Temperature Physics and Engineering, 47 Lenin Avenue, 310164 Kharkov, Ukraine

G. Wendin
Department of Microelectronics and Nanoscience, Chalmers University of Technology and Göteborg University, S-41296 Göteborg, Sweden

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It is found that the dc Josephson current in a ballistic superconductor–normal-metal–superconductor junction connected to an electron reservoir does not decay with the length of the junction if voltage is applied between the reservoir and the junction. At finite temperature, this nonequilibrium Josephson current is proportional to the applied voltage and saturates at $eV>\Delta$ at a level typical for the critical current of short junctions. In asymmetric junctions the current-phase dependence is π periodic. [S0163-1829(97)50934-9]

Josephson coupling in superconducting junctions is determined by the properties of electron states in the normal region of the junction. Disturbance of these states due to tunnel injection or electromagnetic irradiation significantly affects the Josephson current.[1,2] In this paper we address the possibility of nonequilibrium long-range Josephson coupling in long superconductor–normal-metal–superconductor (SNS) junctions.

It is well known that the Josephson current in long SNS junctions is strongly suppressed, being exponentially small if the length of the junction exceeds the coherence length ξ_T.[3–5] In mesoscopic junctions with large normal electron phase breaking lengths, a paradoxical situation may occur with superconducting correlations decaying more rapidly than the coherence of normal electrons. Such behavior of the Josephson current has indeed been observed in a number of recent experiments.[6,7] At the same time, the presence of coherent Andreev reflections on a scale greater than the coherence length has been found in experiments on nonequilibrium transport in mesoscopic SNS junctions[7,8] and discussed theoretically.[9] One may conclude from these studies that long-range Josephson coupling is fundamentally allowed. In this paper we show that it can be realized in ballistic three-terminal SNS junctions similar to the one proposed by van Wees *et al.*[1] and Nakano and Takayanagi.[10]

It has been realized already in early works on the Josephson effect in SNS junctions that the Josephson current in ballistic junctions is suppressed by cancellation mechanisms,[11] and that the exponentially small, for $L\gg\xi_T$, critical current $I_c\sim(eT/\hbar)e^{-2\pi L/\xi_T}$ results from compensation of the Andreev bound state currents by the continuum current[5] ($\xi_T=\hbar v_F/T$, v_F is the Fermi velocity in normal metal, L is the length of the junction). The main cancellation effect results, however, from an interplay among the Andreev bound state currents. In long SNS junctions, $L\gg\xi_0=\hbar v_F/\Delta$, a current $I_n\sim(e\Delta/\hbar)(\xi_0/L)$ flows through the nth Andreev level.[12] This estimate can be deduced from the exact quantum mechanical relation $I_n=(2e/\hbar)dE_n/d\phi$ between the Andreev level current and the Andreev level spectrum $E_n(\phi)$,[13] and is obtained by dividing the available energy interval Δ by the number of levels $N\sim L/\xi_0$ per normal electron mode. If all the currents had the same sign, the total current of Andreev states would be of the same order of magnitude as the critical current in a short junction $I_c=e\Delta/\hbar$. In reality, this does not happen because the functions $E_n(\phi)$ have alternating positive and negative slopes[4] and the Andreev level currents cancel each other, yielding in the best case an uncompensated current of one level, $I_c\sim(e\Delta/\hbar)(\xi_0/L)$.

The outlined mechanism works on a mesoscopic level and leads to current suppression even in one-mode junctions. The crucial point is the rigorous relation between the current and the Andreev spectrum, $I_n=(2e/\hbar)dE_n/d\phi$, which is a specific feature of the Andreev bound states. The situation is quite different in T-shaped SNS junctions with the normal region connected to an electron reservoir, as shown in the inset in Fig. 1. In such junctions each Andreev bound state is split into two degenerate quasibound states related to the scattering of electrons and holes incoming from the reservoir. The current between the NS interfaces carried by the pair of quasibound states is equal to the current of the corresponding Andreev bound level.[1] However, as we will show, the contributions of the electron-like and the hole-like quasibound states are not equal. Moreover, the difference current of the quasibound states has the same sign for all of the Andreev resonances [see Fig. 1 and Eq. (3) below]. The difference current is not observable in equilibrium when fill-

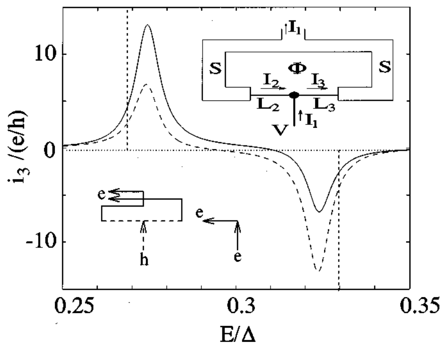

FIG. 1. The charge current density in the SNS junction for electronlike (solid) and holelike (dashed) quasibound states. The positions of the bound Andreev states are indicated by dashed vertical lines. Insets show the scheme of the junction, I_j being the total current in the jth lead, and the paths of injected electrons and holes.

ing factors for electrons and holes in the reservoir are equal. However, it can be revealed by applying a voltage between the SNS junction and the reservoir.

Qualitatively, the difference between the currents of the quasibound states can be explained by comparing the coupling of the Andreev states to incoming electrons and holes. Let us consider, for example, transition into an electron state in the horizontal lead (inset in Fig. 1). In the case of an incoming electron, the transition is obviously direct, and the corresponding factor in the transition amplitude is equal to unity. In the case of an incoming hole, the transition is indirect since the hole must be converted to an electron. This conversion occurs along two equivalent paths: hole injection into either the left or the right side of the junction, then subsequent Andreev reflection at the respective SN interface, and finally normal scattering at the injection point. This yields a factor $re^{-i\phi/2}+de^{i\phi/2}$ in the transition amplitude, where d and r are amplitudes of forward and backward normal electron scattering at the injection point, and ϕ is the superconducting phase difference between the SN interfaces. As a result, the currents created by injected electrons (~ 1) and holes ($\sim |re^{-i\phi/2}+de^{i\phi/2}|^2$) differ by the interference term 2 Re$(rd^*e^{-i\phi})$.

To evaluate the quasibound state currents we consider a one-mode SNS junction (Fig. 1), and model the connection to the normal electron reservoir by a symmetric S matrix[14,10]

$$S=\begin{pmatrix} \sqrt{1-2\epsilon} & \sqrt{\epsilon} & \sqrt{\epsilon} \\ \sqrt{\epsilon} & r & d \\ \sqrt{\epsilon} & d & r \end{pmatrix}, \qquad (1)$$

where ϵ describes the coupling of the SNS junction to the vertical normal lead ($0\leqslant\epsilon\leqslant 0.5$), and the scattering amplitudes d and r obey the relations Re$(rd^*)=-\epsilon/2$ and $D+R=1-\epsilon$ ($D=|d|^2$, $R=|r|^2$). Normal electron reflection at the NS interfaces is neglected in order to avoid the appearance of Breit-Wigner resonances. We consider a loop geometry of the superconducting electrodes which allows us to control the superconducting phase difference by magnetic flux, and which also guarantees equal chemical potentials of the electrodes.[15] The voltage applied between the junction and the reservoir drops at the connection point between the horizontal and vertical normal leads.

In mesoscopic junctions with lengths smaller than the phase breaking length, the currents in each of the normal leads $j=1,2,3$ can be calculated using Bogoliubov-de Gennes scattering states. The scattering state wave functions consist of superpositions of electron and hole waves $\exp(\pm ik^{e,h}x)$, where $k^{e,h}=\sqrt{(2m/\hbar^2)(E_F\pm E)}$, and the coefficients of the superpositions $c_j^{\pm,e,h}$ determine the current at given energy, $i_j(E)=(e/h)(|c_j^{+,e}|^2-|c_j^{-,e}|^2-|c_j^{+,h}|^2+|c_j^{-,h}|^2)$. The conservation of current implies $i_1=i_3-i_2$. In the subgap region, $|E|<\Delta$, the currents $i_2^{e,h}$ of the electron-like and the hole-like quasibound states in symmetric junctions ($L_2=L_3$) have the form

$$i_2^{e,h}=\frac{2e}{h}\frac{\epsilon}{Z}\{-D\sin 2\theta\sin\phi \pm\sin^2\theta[2\,\mathrm{Re}(rd^*e^{-i\phi})-\epsilon]\}, \qquad (2)$$

where $Z=[(1-\epsilon)\cos 2\theta-R-D\cos\phi]^2+\epsilon^2\sin^2 2\theta$, and $\theta=\arccos(E/\Delta)-(k^e-k^h)L/2$. The currents in lead 3 can be obtained from Eq. (2) by using the symmetry relation $i_3^{e,h}(\phi)=-i_2^{e,h}(-\phi)$.

The sum current of the quasibound states, $i_2^+=i_2^e+i_2^h=i_3^+$, is given by the first term in Eq. (2) multiplied by a factor of 2. In the weak coupling limit $\epsilon\ll 1$ it takes the form $i^+=(2e/\hbar)(dE_n/d\phi)\delta(E-E_n)$, which is the conventional form of the Andreev bound state currents [E_n is the Andreev level spectrum given by zeros of the function $Z(E)$]. The difference current of the quasibound states, $i_{2,3}^-=(i_{2,3}^e-i_{2,3}^h)$ tends to a finite value:

$$i_a=-\frac{e}{2\hbar}\frac{\mathrm{Im}(rd^*)\sin\phi}{\sqrt{D}a(\phi)|\cos(\phi/2)|}\left|\frac{dE_n}{d\phi}\right|\delta(E-E_n) \qquad (3)$$

in both arms of the SNS junction when $\epsilon\to 0$ [$a(\phi)=[1-D\sin^2(\phi/2)]^{1/2}$ and $\mathrm{Im}(rd^*)=2\sigma\sqrt{RD-\epsilon^2/4}$, $\sigma=\pm 1$]. This motivates us to define the current in Eq. (3) as the *anomalous Josephson current*. The remaining part of the difference current is of first order in ϵ and is equal to half of the current i_1 injected into lead 1. This injection current has been calculated in Ref. 10. The injection and anomalous Josephson currents are related as

$$i_1=-\frac{\sigma\epsilon}{\sqrt{RD-\epsilon^2/4}}\frac{1+\cos\phi}{\sin\phi}i_a. \qquad (4)$$

The striking feature of the anomalous Josephson current in Eq. (3) is that the *modulus* of the Andreev bound state currents $|dE_n/d\phi|$, rather than the currents themselves, enter the equation. This implies that the anomalous current flows through all of the Andreev resonances *in the same direction*. The same is true for the injection current which gives a natural explanation for the long-range effect of conductance oscillations with the superconducting phase.[8,9] There are other unusual properties of the anomalous Josephson current caused by its interference origin: (i) the current vanishes in completely transparent junctions ($R=0$); (ii) the direction of the current flow depends on the normal electron scattering phases through the quantity σ, the sign of which uniquely

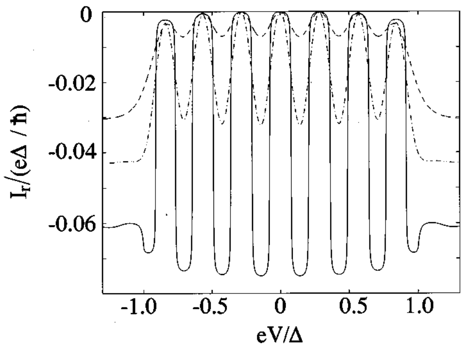

FIG. 2. The regular current I_r vs voltage. $L_2=L_3=5\xi_0$, $\phi=\pi/2$, $D=0.8$, $\epsilon=0.05$. Solid line, $T=0$; dashed-dotted, $T=0.04\Delta$; dashed, $T=0.07\Delta$.

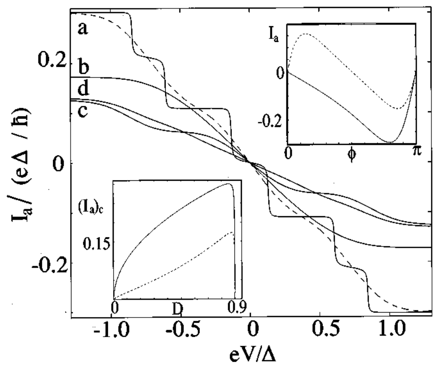

FIG. 3. The anomalous current I_a vs voltage. $D=0.8$, $\epsilon=0.05$. (a) $L_2=L_3=5\xi_0/\pi$, $\phi=3\pi/4$. Solid line, $T=0$; dashed, $T=0.1\Delta$. The dashed line also shows the current in the longer junction, $L=60\xi_0/\pi$, (b)–(d) show the effect of asymmetry for $L=60\xi_0/\pi$, $l=4,10,40\xi_0/\pi$. Insets: I_a vs ϕ (upper) and $(I_a)_c$ vs D (lower) for $eV=\Delta$ and $T=0.1\Delta$. Solid line $l=0$ and dashed line $l=40$.

determined by the position of the Fermi level with respect to the transmission resonances at the injection point $[r(E)=0]$. The discovery of the anomalous Josephson current is the central result of this paper.

Proceeding to the calculation of the total nonequilibrium Josephson current, we subtract the equilibrium Josephson current from the total current in horizontal lead and divide the nonequilibrium Josephson current into regular I_r and anomalous I_a parts associated with current densities i^+ and i_a respectively:

$$I_r+I_a=\int dE\left[\frac{i^+}{2}(n^e+n^h-2n)+\frac{i_a}{2}(n^e-n^h)\right]. \quad (5)$$

In Eq. (5) $n=n_F(E)$, while $n^{e,h}=n_F(E\mp eV)$ are filling factors for electrons and holes in the reservoir. The latter implies that the weak coupling of the SNS junction to the reservoir nevertheless is assumed to dominate over intrinsic inelastic relaxation in the junction, i.e., the width of the Andreev resonances $\Gamma\sim\epsilon\Delta\xi_0/L$ is greater than the inelastic relaxation frequency. This yields a window for the coupling constant $L/l_{\rm in}\ll\epsilon\ll 1$ ($l_{\rm in}$ is inelastic mean free path). The regular Josephson current manifests the pure effect of nonequilibrium population of the Andreev states, while the anomalous current manifests the effect of transformation of the Andreev bound states themselves, due to coupling to the normal reservoir. Switching on of the voltage gives rise to successive population or depopulation of the quasibound levels. Thus, the current-voltage characteristics (IVC) of the nonequilibrium Josephson current possess sharp structures at low temperature at voltages equal to the level energies.

The dependence of the regular current on the applied voltage is shown in Fig. 2. In long junctions, $L\gg\xi_0$, the current rapidly oscillates, where the amplitude of oscillation is equal to the current of individual bound states, $\delta I_r=-(e\Delta/\hbar)\sqrt{D}\xi_0\cos(\phi/2)/La(\phi)$; see Ref. 16. The anomalous current has a staircase dependence on the applied voltage shown in Fig. 3, with step heights $\delta I_a=-(e\Delta/\hbar)[\sigma\sin\phi\xi_0\sqrt{DR}/2La^2(\phi)]{\rm sgn}V$. The currents saturate at voltages $|eV|>\Delta$ because of the absence of resonances outside the energy gap $|E|>\Delta$.

At finite temperatures exceeding the interlevel distance, $T\gg\Delta\xi_0/L$, the oscillations of the regular current are completely washed out, while the anomalous current exhibits a long-range behavior. Explicit equations for the IVC can be derived by introducing the density of levels $dn/dE_n\approx L/\pi\xi_0\Delta$ and calculating the sum over the Andreev levels in the continuum limit. The result for the regular current is $I_r=\delta I_r\arcsin[\sqrt{D}\sin(\phi/2)]\pi^{-1}n_F(\Delta-|eV|)$ at $T\ll eV,\ \Delta$, i.e., the current is exponentially small in the subgap voltage regime $|eV|<\Delta$. The anomalous current, in contrast, is large and independent of the length of the junction,

$$I_a(V,\phi)=-\frac{e}{\hbar}\frac{\sigma\sqrt{DR}\sin\phi}{\pi a^2(\phi)}f(V,T), \quad (6)$$

$$f(V,T)=T\ln\frac{\cosh[(eV+\Delta)/2T]}{\cosh[(eV-\Delta)/2T]}=\min(eV,\Delta), \quad T\ll\Delta.$$

The current in Eq. (6) resembles the equilibrium Josephson current with the major difference that the critical current here depends on the applied voltage, $(I_a)_c=(e/\pi\hbar)\sqrt{D}f(V,T)$, $1-D\gg\epsilon$. The critical current is proportional to first power of Δ at $T\approx T_c$.

The effect of asymmetry of the junction, $L_2\neq L_3$, is interesting. The smeared IVCs of the anomalous Josephson current at $L\gg\xi_T$ are shown in Fig. 3. If the asymmetry is small, $l=L_2-L_3<\xi_T$, a slow periodic modulation of the IVC develops on the scale of $eV\sim\Delta\xi_0/l$ caused by the additional dephasing factor $e^{i(k^e-k^h)l}$ in the quasibound level currents. This structure is smeared out at larger l, and at $l\gg\xi_T$ the current obtains a form similar to the one in Eq. (6), but with a different phase dependence (see insets in Fig. 3):

$$I_a(V,\phi)=-\frac{e}{\hbar}\frac{\sigma D\sin\phi}{\pi\sqrt{R}}\left(\frac{|\sin\phi/2|}{b(\phi)}-\frac{|\cos\phi/2|}{a(\phi)}\right)f(V,T),$$

where $b(\phi)=[1-D\cos^2(\phi/2)]^{1/2}$. This current is a π periodic with respect to the superconducting phase difference. Similarly, a π-periodic component also appears in the injection current.

The anomalous Josephson current can be directly detected by measuring the magnetic flux through the superconducting loop as a function of applied voltage by a SQUID magnetometer. Another possibility is to use the injection current. The

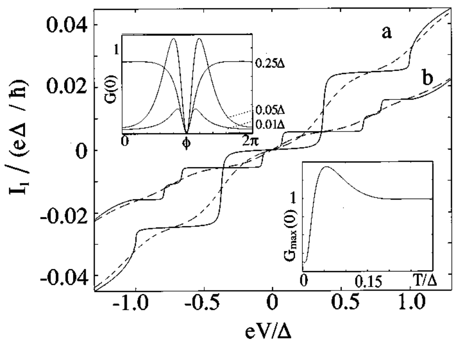

FIG. 4. Injection current I_1 vs voltage. $L_2=L_3=5\xi_0/\pi$, $D=0.9$, $\epsilon=0.05$. Solid line, $T=0$; dashed line, $T=0.1\Delta$. (a) $\phi=0$, (b) $\phi=7\pi/8$. Insets: $G(0)$ vs ϕ (upper) for different T and $G_{\max}(0)$ vs T/Δ (lower) in units of G_N.

behavior of a small ($\sim\epsilon$) injection current is very similar to the behavior of the anomalous Josephson current. At zero temperature, the injection current has a steplike IVC in the subgap region (Fig. 4),[17] which is smeared out at $L\gg\xi_T$ yielding in symmetric junctions $I_1=G_N[(1+\cos\phi)/2a^2(\phi)]V$ at $T\ll eV<\Delta$, where $G_N=4e^2\epsilon/h$ is the normal junction conductance. At the Andreev resonances, the maximum differential conductance achieves a magnitude $G_{\max}=4e^2/h$, while at the plateaus of the IVC it is of the order of G_N or smaller. The conductance is always small at zero voltage because the Andreev level spectrum has a gap near the Fermi level (unless the junction is completely transparent), at $D\leqslant R$ the conductance has a universal magnitude $G_{\max}(0)=\epsilon G_N$. The conductance rapidly increases with voltage (at $eV\sim\Delta\xi_0/L$) and with temperature (at $T\sim\Delta\xi_0/L$, see the inset of Fig. 4). Such suppression of the conductance at low voltage (reentrance effect) has been observed in diffusive junctions.[18] The period of the oscillation of the injection current with respect to applied magnetic flux depends on the magnitude of the Josephson current:[19] large current in the superconducting loop violates monotonic phase-flux dependence and causes phase slips. This will show up in the decrease of the oscillation period with the applied voltage.

In conclusion, we have considered the Josephson current in mesoscopic ballistic SNS junctions with the normal region coupled to an electron reservoir. When voltage is applied between the junction and the reservoir, an anomalous Josephson current emerges with a critical magnitude independent of the length of the junction and proportional to the applied voltage. This anomalous current results from the interference of quasiparticle wave functions injected into the left and right sides of the junction. The anomalous Josephson current can be uniquely defined only at a weak coupling to the reservoir; however, the phenomenon exists in the general case of arbitrary coupling in the form of a phase-dependent nonsymmetric distribution of the injection current between the arms of the SNS junction.

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